Homework 3 Solution

Question 1

Let the relation A(MNOPQRST) satisfy the following functional dependencies: N → P, MO → Q, RS → T, Q → S, OP → M, PT → R. Which of the following FD's is also guaranteed to be satisfied by A? Recall that an FD of the form X → BC, where X is a set of attributes and where each of B and C is an attribute, is actually two FDs X → B and X → C. We say that an FD X → BC is guaranteed to be satisfied by a relation schema if and only if each of X → B and X → C is guaranteed to be satisfied by this relation schema.

Question Explanation

The secret is to compute the closure the left side of each FD that you get as a choice. Recall that the closure of a set of attributes is computed by starting with that set, and repeatedly looking for left-hand sides of given FD's that are wholly contained within your current set of attributes. If you find such an FD, you add its right-hand side to your set, until you can add no more. If the right-hand side of the candidate FD is contained in your set, then the candidate does follow from the given FD's; otherwise not.

Correct Answer

MNO → PS
NOT → MS
PQR → ST
OSP → MQ
NRS → PT

Incorrect Answer

RST → MP
ORS → TQ
MRT → NO
OST → PQ
NQS → PT
QRS → MT

Question 2

Determine the keys and superkeys of the relation R(MNOPST) with FD's:

NS → T, MNO → P, NO → T, MPST → N

Then, demonstrate your knowledge by selecting the true statement from the list below. Each statement must include all the possible values.

Question explanation

First, notice that M, O, and S do not appear in any right-hand side, so they must be in any key. Thus, we might as well assume that M, O, and S are in any closure we care about, and remove them from left-hand sides. That leaves us with N → T, N → P, and PT → N. To get N, P, T in a closure, we can start with either N alone, or P and T; it is easy to check that these are the only two sets that we need to explore. Thus, the only keys are MNOS and MOPST. There are three superkeys other than the two keys above. These are the proper supersets of MOPST or MNOS, namely MNOPST, MNOPS, and MNOST.

Thus, there are two keys, five superkeys, and three superkeys that are not keys.
Correct Answers

Keys: MNOS, MOPST
Superkeys: MOPST, MNOS, MNOPST, MNOPS, MNOST
Superkeys that are not keys: MNOPST, MNOPS, MNOST
Incorrect Answers

Keys: MNOPST, MNOPS, MNOST
Superkeys: MOPS, MNPST
Keys: MNOPT, ST
Superkeys: NOPS, NPST, OST, MNOP
Keys: MNOPS
Superkeys: MNOP, NST, MOPT
Keys: MNOT, MNOPS, MNOST
Superkeys that are not keys: MOPST, MNOS

Question 3

Which of the following relations is in Boyce-Codd Normal Form (BCNF)?

Question Explanation

It is easy to check whether a given relation is in BCNF --- just check that the left-hand side of each given FD is a superkey. We test it is a superkey by computing its closure and verifying that the closure is all the attributes.

Correct Answers

R(LMNOP) FD's: M → P; P → LO; O → MN
R(LMNO) FD's: LN → M; MO → L; MO → N; LM → O
R(LMNOP) FD's: M → NO; NO → L; L → MP
R(LMNO) FD's: LMN → O; LNO → M; MNO → L; MN → O

Incorrect Answers

R(LMNOP) FD's: MNO → P; LMN → O
R(LMNO) FD's: MNO → L; LNO → M; M → L
R(LMNOP) FD's: L → P; M → OP; OP → N; P → O
R(LMNO) FD's: M → L; LO → N; L → O; L → N
R(LMNOP) FD's: LM → P; N → O; MP → N
R(LMNO) FD's: LMN → O; LNO → M; MNO → L; L → O

Question 4

Which of the following relations is in Third Normal Form (3NF)?

Question Explanation

If the relation is in BCNF, then it surely is in 3NF. Since there is an easy test for BCNF --- check that every left-hand side is a superkey, i.e., its closure is all attributes --- that is an easy way to show some relations in 3NF. But what if some left-hand sides have a closure that is less than all attributes. Then the relation violates BCNF, but might not violate 3NF if the right-hand side is prime --- that is, a member of some key. To check, we have to discover a key such that the given right-hand side (of an FD) is a member of that key.

For example, assume a relation R(WXYZ) with FD's WYZ → X, WY → Z, WY → X, and Z → Y. WXYZ is the closure of each of WYZ and WY, so those FD's do not violate BCNF or 3NF. But Z+ = YZ, so Z → Y is a BCNF violation. To check that it is not a 3NF violation, we need to verify that Y is in some key. We already know that WY is a superkey. But it is easy to check that neither W nor Y is superkeys, and thus WY is a key. We conclude this relation is in 3NF.

Correct Answers

R(VWXYZ) FD's: V → WX; Z → VY; W → Z
R(VWXY) FD's: XY → W; XY → V; V → X
R(VWXYZ) FD's: WX → V; Y → WX; V → YZ; Z → X
R(VWXY) FD's: VXY → W; W → X

Incorrect Answers

R(VWXYZ) FD's: V → WZ; X → V; W → XY; Z → Y
R(VWXY) FD's: V → W; V → X; WX → Y
R(VWXYZ) FD's: XY → Z; W → XY; Z → VW; X → V
R(VWXY) FD's: X → W; V → Y
R(VWXY) FD's: WXY → V; VW → X; Y → V; W → X
R(VWXY) FD's: V → Y; Y → X; Y → W

Question 5

Which of the following relations is correctly decomposed into the minimal number of relations that are collectively in BCNF (Boyce-Codd Normal Form)?

Question Explanation

To satisfy the BCNF criterion, any relation that is not in BCNF can be decomposed into several relations that are in BCNF by repeatedly choosing suitable decompositions. For example, given R(ABCD) with FD's A → B and C → D, the relation is not in BCNF since the left-hand sides of the FD's are not superkeys. Therefore, we need to decompose the relation R into R1(AB), R2(CD), and R3(AC).

Correct Answers

R(ABCDE) FD's: AB → C; C → D; D → E into R1(ABC), R2(CD), R3(DE)
R(ABCD) FD's: C → D; C → A; B → C into R1(BC), R2(ACD)
R(ABCDE) FD's: BDE → A; A → C into R1(ABDE), R2(AC)
R(ABCD) FD's: A → BD; D → C into R1(ABD), R2(CD) (Why?)

Incorrect Answers

R(ABCD) FD's: A → B; A → C; D → A into R1(AB), R2(AC), R3(DA)
R(ABCDE) FD's: E → A; D → E; BC → D into R1(BCD), R2(DEA)
R(ABCD) FD's: C → B; C → D; B → A into R1(CD), R2(AB)
R(ABCDE) FD's: A → B; B → D; D → E; C → A into R1(ABE), R2(CD)
R(ABCD) FD's: AB → D; D → C into R1(CD), R2(ABC)
R(ABCDE) FD's: B → CD; A → E into R1(ABCDE), R2(AE)

Question 6

Suppose relation R(A, B, C, D) has the tuples:

A	B	C	D
a	1	4	e

b	2	10	e
c	7	6	f
a	3	19	e

And the relation S(F, G, H) has tuples:

F	G	H
b	15	21
b	4	5
c	7	2
b	5	4
a	20	11
d	6	3
b	17	12

Which of the following tuples is in the theta-join of R and S with the condition A = F AND C < G AND (D = 'e' OR D = 'f') AND (A = 'a' OR A = 'b') AND G > H?

Question Explanation

For the condition (D = 'e' OR D = 'f'), (A = 'a' OR A = 'b') and the condition G >H, each field is related to only one relation. So, it is possible to apply those conditions before executing the join in order to reduce the number of tuples to be joined.

Correct Answers

(a, 1, 4, e, a, 20, 11)
(b, 2, 10, e, b, 17, 12)
(a, 3, 19, e, a, 20, 11)

Incorrect Answers

(c, 7, 6, f, c, 7, 2)
(b, 2, 10, e, b, 5, 4)
(b, 2, 10, e, b, 4, 5)
(a, 1, 4, e, c, 7,2)
(a, 1, 4, e, b,17,12)
(c, 7, 6, f, b,5, 4)
(c, 7, 6, f, b, 15, 21)
(a, 3, 19, e,d, 6, 3)
(a, 3, 19, e, b, 4, 5)

<u>Question 7</u>

A basis for a set of FD's F is any set G of FD's whose closure is the same as the closure of F. That is, exactly the same FD's follow from F as from G. In addition, a basis must consist of a minimal set of nontrivial FD's. Suppose we have a relation R(W, M,X, Y, Z) with FD's W → M, M → X, X → Y, Y → Z, Z → W. Suppose we project R onto attributes WMXY. Describe all the bases for the set of FD's that hold in WMXY. Given a set of FD's, select statements that correctly explain if the set is a basis or not.

Correct Answers

W → M, M→ X, X → W, X → Y: NOT a basis

W → M, M→ X, X → W, X → Y, Y → X: a basis

W → M, M→ Y, X → Y, Y → W, Y → X: a basis

Y → W, Y → X, M→ W, W → X, X → W: NOT a basis

Incorrect Answers

W → M, M→ Y, Y → X, X → W: NOT a basis

X → W, X → Y, W → Y, Y → W, M→ X: a basis

Y → W, W → X: a basis

W → M, M→ X, X → Y, Y → W: NOT a basis

W → X, W → Y, X → Y, Y → X, M→ X, M→ Y: a basis

W → M, M→ Y, X → Y, Y → W, Y → X: NOT a basis

W → M, M→ X ,X → W, X → Y, Y → X: NOT a basis