

Mathematical Foundations

Instructions

Please share your answers wherever applicable in-line with the word document.
Submit code separately wherever applicable. Mathematical calculations which are manually performed should be uploaded with a picture along with the explanation in a word document.

Please ensure you update all the details:

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Topic: Mathematical Foundations

Note: Submit pictures of mathematical calculations

Guidelines:

1. An assignment submission is considered complete only when correct and executable code(s) are submitted along with the documentation explaining the method and results. Failing to submit either of those will be considered an invalid submission and will not be considered as correct submission.
2. Ensure that you submit your assignments correctly and in full. Resubmission is not allowed.
3. Post the submission you can evaluate your work by referring to keys provided. (will be available only post the submission).

Problem Statements

Q1) Find the maximum and minimum values of the function: $x^3 - 3x^2 - 9x + 12$

Q2) Calculate the slope and the equation of a line which passes through the points $(-1, -1)$, $(3, 8)$

Q3) Solve for $w'(z)$ when

$$w(z) = \frac{4z-5}{2-z}$$

Q4) Consider $Y = 2x^3 + 6x^2 + 3x$. Identify the critical values and verify if it is the maxima or minima.

Q5) Determine the critical points and obtain relative minima or maxima of a function defined by

$$y = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 6x_1$$

Assignment : Mathematical Foundations :

Q1) Given $y = x^3 - 3x^2 - 9x + 12$ $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

$$y' = \frac{d}{dx}(x^3 - 3x^2 - 9x + 12)$$

$$y' = 3x^2 - 6x - 9$$

$$= ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-9)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 + 108}}{6}$$

$$x = \frac{6 \pm \sqrt{144}}{6} = \frac{6 \pm 12}{6}$$

$$x = \frac{6+12}{6} = 3, \quad x = \frac{6-12}{6} = -1 \quad (\text{critical values})$$

$$y'' = 6x - 6$$

$$y''(3) = 6(3) - 6 = 12 > 0 \rightarrow \text{Minima}$$

$$y''(-1) = 6(-1) - 6 = -12 < 0 \rightarrow \text{Maxima}$$

Q2)

$$\begin{pmatrix} -1, -1 \\ x_1, y_1 \end{pmatrix}$$

$$\begin{pmatrix} 3, 8 \\ x_2, y_2 \end{pmatrix}$$

slope =

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8 - (-1)}{3 - (-1)} = \frac{9}{4} = 2.25$$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{9}{4}(x - (-1))$$

$$4y + 4 = 9x + 9$$

$$9x - 4y + 5 = 0$$

Equation of line = $9x - 4y + 5 = 0$

Q3)

$$w(z) = \frac{4z-5}{2-z}$$

$$\frac{d}{dz} \left(\frac{u}{v} \right) = \frac{u'v - uv'}{v^2}$$

$$w'(z) = \frac{(2-z)(4) - (4z-5)(-1)}{(2-z)^2}$$

$$(2-z)^2$$

$$= \frac{8 - 4z + 4z - 5}{(2-z)^2}$$

$$= \frac{3}{(2-z)^2}$$

$$Q1) \quad y = 2x^3 + 6x^2 + 3x$$

$$y' = 6x^2 + 12x + 3 = 0$$

$$3(2x^2 + 4x + 1) = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(2)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{8}}{4}$$

$$= \frac{-4 \pm 2\sqrt{2}}{4} = -1 \pm 0.7071$$

$$\text{Critical values} = -0.2928, -1.7071$$

$$y'' = 4x + 4$$

$$y''(-0.2928) = 4(-0.2928) + 4 = 2.8288 > 0 \rightarrow \text{minima}$$

$$y''(-1.7071) = -2.8284 < 0 \rightarrow \text{Maxima}$$

$$Q9) \quad y = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 6x_1 \quad (1)$$

Differentiating w.r.t x_1 (x_2 is constant)

$$\frac{dy}{dx_1} = 4x_1 + 2x_2 + 6$$

$$\frac{dy}{dx_1} = 0 = 4x_1 + 2x_2 + 6$$

$$2x_1 + x_2 + 3 = 0 \rightarrow (1)$$

$$\frac{d}{dx_1} \left(\frac{dy}{dx_1} \right) = 4$$

$$\frac{d}{dx_2} \left(\frac{dy}{dx_1} \right) = 2$$

$$\frac{dy}{dx_2} = 4x_2 + 2x_1 = 0$$

$$\frac{dy}{dx_2} = 4$$

$$4x_2 + 2x_1 = 0$$

$$x_1 = -2x_2 \rightarrow (2)$$

put equation (2) in equation (1)

$$2(-2x_2) + x_2 + 3 = 0$$

$$x_2 = 1$$

$$x_1 = -2$$

$$\text{Critical point} = (-2, 1)$$

For relative maxima and minima

$$\begin{aligned} y &= 2(-2)^2 + 2(1)(-2) + 2(1)^2 + 6(-2) \\ &= 8 - 4 + 2 - 12 = 10 - 16 = -6 < 0 \end{aligned}$$

Relative maximum at $(-2, 1)$