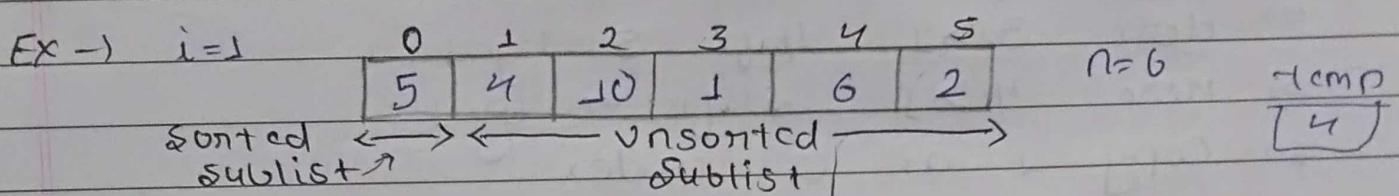


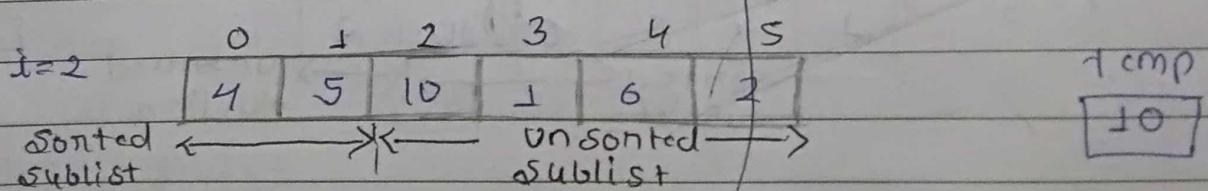
## # Insertion Sort Algorithm ->

In this technique the given array is divided into two parts sorted sublist and unsorted sublist from left to right and take one item from unsorted sublist and going insert at correct location in sorted sublist.



Here by default we are going to take first item in sorted sublist. Because in this way we have only one item this means that it is always sorted.

- Round 1 -> Step 1) copy 4 in temp  
 2) compare 4 with 5 (elements in sorted sublist)  
 3) If  $5 > 4$ ,  
 Then shift 5 by 1 and copy  $\text{temp} = 4$  at 0 index



- Round 2 -> Step 1) copy 10 in temp.

- 2) Compare 5 with 10  
 3) Here 5 is not greater than 10 so 5 will not shift by 1  
 4) 10 will be at index 2.

$i=3$	0 1 2 3 4 5	temp
	4 5 10 1 6 2	1

Sorted sublist      Unsorted sublist

- Round 3 → 1) Copy 1 in temp  
 2) compare 1 with 10.  
 3) Here  $10 > 1$ , shift 10 by 1.  
 Now I will compare with each and every element in sorted sublist.  
 4) Here  $5 > 1$ , shift 5 by 1.  
 5) Here  $4 > 1$ , shift 4 by 1.  
 6) Copy 1 at 0 index

$i=4$	0 1 2 3 4 5	temp
	1 4 5 10 6 2	6

Sorted sublist      Unsorted sublist

- Round 4 → 1) copy 6 in temp  
 2) Compare 6 with 10, Here  $6 < 10$ , shift 10 by 1.  
 3) Compare 6 with 5, Here  $6 > 5$  so,  
 4) copy 6 at index 3.

$i=5$	0 1 2 3 4 5	temp
	1 4 5 6 10 2	2

Sorted sublist      Unsorted sublist

- Round 5 → 1) copy 2 in temp  
 2) compare 2 with 10  
 3)  $10 > 2 \rightarrow$  shift 10 by 1  
 4)  $6 > 2 \rightarrow$  shift 6 by 1  
 5)  $5 > 2 \rightarrow$  shift 5 by 1  
 6)  $4 > 2 \rightarrow$  shift 4 by 1

7) 1 is not greater than 2, copy 2 at index 1

Sorted array	0 1 2 3 4 5
	1 2 4 5 6 10

Algorithm →

```
for (int i=1; i<n; i++) → loop for n unsorted
{
    int temp = arr[i];
    int j = i-1;
    while (j ≥ 0 && arr[j] > temp) → loop for sorted
    {
        arr[j+1] = arr[j];
        j--;
    }
    arr[j+1] = temp; → copy temp at
} appropriate place
```

# Bubble sort Algorithm → In this sorting technique we compare two adjacent elements. If these elements are in correct order then its fine and if these elements are not in correct order then swapping take place.

We are going to arrange elements in ascending order.

Pass	0	1	2	3	4	
	15	16	6	8	5	$i=0$

$15 > 16$  (No swapping)

$15 > 6$  (swap)

$16 > 8$  (swap)

$16 > 5$  (swap)

$15 > 6$  (swap)

$15 > 8$  (swap)

$15 > 5$  (swap)

$15 > 16$  (No swap)

$n=5$

$i=0$

6	5	8	15	16
6	5	8	15	16
5	6	8	15	16
5	6	8	15	16
5	6	8	15	16

pass 4

→ All elements are at correct position  
No swap

Here total 5 elements are there if 4 elements are at correct position then 5<sup>th</sup> one will be automatically at correct position.  
No. of passes = (No. of elements) - 1

In above example in each pass no. of comparison is 4  $\Rightarrow n-1$

Algorithm  $\rightarrow$

```

Fun (int i=0 ; i<n-1 ; i++) ← loop for passes
{
    Fun (int j=0 ; j<n-1 ; j++) ← loop for comparison
    {
        if (arr[j] > arr[j+1])
        {
            int temp = arr[j];
            arr[j] = arr[j+1];
            arr[j+1] = temp;
        }
    }
}

```

Swapping

}

Pass	0	1	2	3	4	
2	15	6	8	5	16	$i=1$

$15 > 6$  (swap)

$15 > 8$  (swap)

$15 > 5$  (swap)

$15 > 16$  (swap)

$15 > 6$  (No swap)

Pass	0	1	2	3	4	
3	6	8	5	15	16	$i=2$

$6 > 8$  (No swap)

$6 > 5$  (swap)

$6 > 15$  (swap)

$6 > 16$  (swap)

$6 > 8$  (No swap)

$6 > 5$  (No swap)

$6 > 15$  (No swap)

$6 > 16$  (No swap)

$6 > 8$  (No swap)

$6 > 5$  (No swap)

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$6 > 16$  (No swap)

$6 > 8$  (No swap)

$6 > 5$  (No swap)

$$

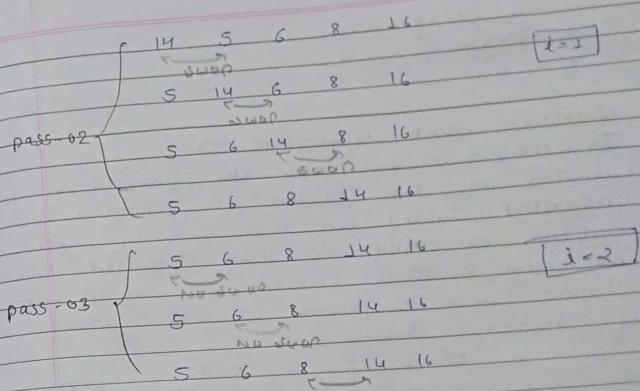
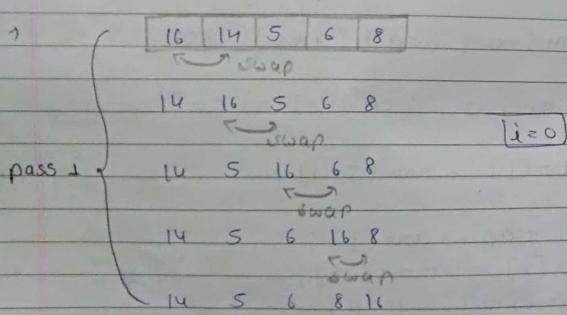
Improved algorithm →

```

for (int j=0; i<n-1; i++)
{
    for (int j=0; j<n-i-1; j++)
        if (arr[j] > arr[j+1])
            int temp = arr[j];
            arr[j] = arr[j+1];
            arr[j+1] = temp;
}

```

\* Suppose we have an array of size 10 and it is sorted after 3 passes. But according to algorithm No. of passes = 9. How we use flag variable.



If we get any pass in which no swapping is there then we can break our passing loop and array is sorted.

Optimized algorithm →

```

for (int i=0; i<n-1; i++)
{
    int flag = 0;
    for (int j=0; j<n-1-i; j++)
        if (arr[j] > arr[j+1])
            int temp = arr[j];
            arr[j] = arr[j+1];
            arr[j+1] = temp;
            flag++;
}

```

if (flag == 0)  
break;

### # Selection Sort Algorithm →

In this technique also the given array is divided into two sublist, i.e. sorted and unsorted sublist, but here initially sorted sublist is empty and from unsorted sublist we found minimum element and swap with element which is present at the starting of unsorted sublist.

Ex -	0 1 2 3 4 5
	7 4 10 8 3 1      (n=6)

Sorted      Unsorted sublist (min)

Pass-1	1 4 10 8 3 7
	Sorted      Unsorted sublist

Pass-2	1 3 10 8 4 7
	Sorted      Unsorted sublist

Pass-3	1 3 4 8 10 7
	Sorted      Unsorted sublist

Pass-4	1 3 4 7 10 8
	Sorted      Unsorted sublist

Pass-5	1 3 4 7 8 10
	Sorted      Unsorted sublist

Algorithm →

For (int i=0; i<n-1; i++) → for passes

{

int min=i;

For (j=i+1; j<n; j++) → for finding

minimum element

{

if (arr[j] < arr[min])

{

min=j;

}

if (min==i)

{

Swap (arr[i], arr[min])

}

}

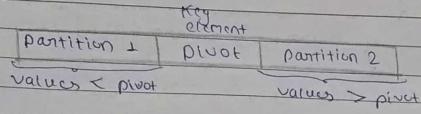
### # Quick Sort Algorithm →

In this type of sorting technique a key (pivot) element is chosen, pivot element may be starting element, last element or any random element.

It is based on divide and conquer technique. In this technique complete array is divided into subarrays and dividing of arrays is called partitioning.

Ex -	0 1 2 3 4 5 6
	10 15 + 2 9 16 11 ↑ Pivot element

partition take place in such a way that all elements less than pivot move to left side and all elements greater than pivot move to right side and elements equal to pivot element move either left or right.



0	1	2	3	4	5	6
2	1	9	10	15	11	16

may be sorted  
or may not be  
on appropriate  
place

same algorithm  
also apply for  
sub partition also

2, 1, 9

1, 2, 9

9, 2, 1

16, 15, 11

11, 15, 11

16, 11, 15

0	1	2	3	4	5	6	7	8
7	6	10	5	9	2	1	15	7

Ex)

lower bound

upper bound

Pivot = 7

a[0]

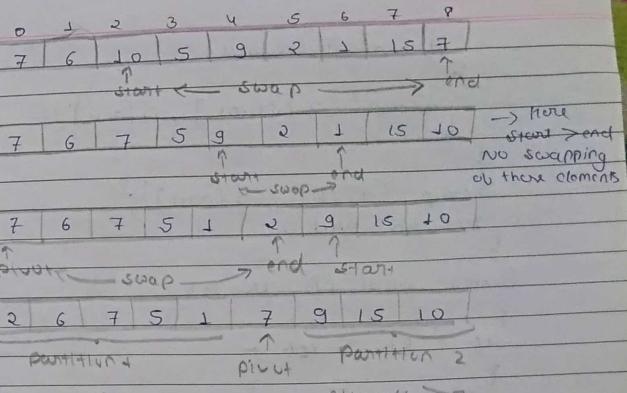
,

start →

if item ≤ point start++;

end

ii (item > pivot) end--



- same algo for these partition also

# Algorithm →

partition (A, lb, ub)

{

    pivot = A[lb];

    start = lb;

    end = ub;

    while (start < end)

    {

        while (a[start] ≤ pivot)

        {

            start ++;

        }

        while (a[end] > pivot)

        {

            end --;

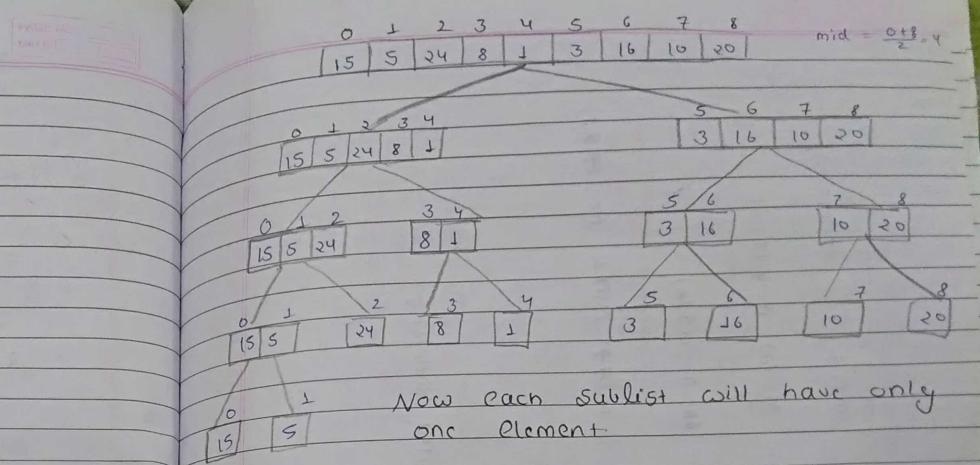
        }

```

if (start < end)
{
    swap (a[start], a[end]);
}
swap (a[16], a[end]);
return end;
}
QuickSort (A, lb, ub)
if (lb < ub)
{
    loc = partition (A, lb, ub);
    QuickSort (A, lb, loc-1);
    QuickSort (A, loc+1, ub);
}

```

**Merge Sort Algorithm →**  
This technique also work on the principle of divide and conquer rule in this case the complete list is divided into n sublist and each sublist contain 1 element. Now we merge the sublist and keep on merging each sublist to get a sorted list.

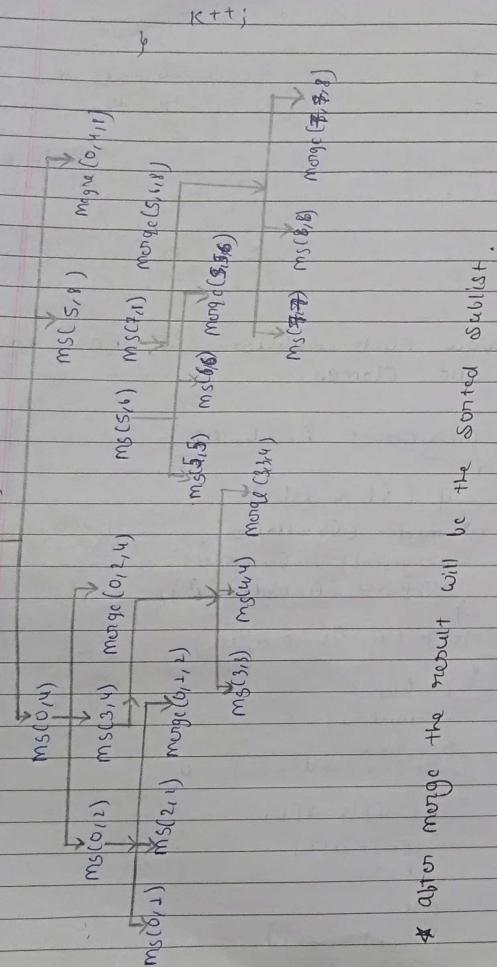


Now each sublist will have only one element

Algorithm → MergeSort (A, lb, ub)

```

if (lb < ub)
{
    mid = (lb+ub)/2; } recursive case
    mergeSort (A, lb, mid);
    mergeSort (A, mid+1, ub); } function
}
merge (A, lb, mid, ub)
{
    i = lb;
    j = mid+1;
    k = lb;
    while (i <= mid & & j <= ub)
    {
        if (a[i] <= a[j])
        {
            b[k] = a[i];
            i++;
        }
        else
        {
            b[k] = a[j];
            j++;
        }
    }
}
```



```

if (i > mid)
{
    while (j <= ub)
        b[k] = a[j];
        j++;
        k++;
}
else
{
    while (i <= mid)
        b[k] = a[i];
        i++;
        k++;
}
for (k = lb; k <= ub; k++)
    a[k] = b[k];
}

```

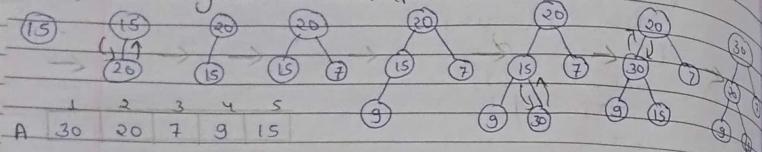
# Heap sorting → Heap is a tree based data structure. In Heap sorting we have to follow two steps -

- 1) To create a heap or build a heap
- 2) deletion of elements from heap

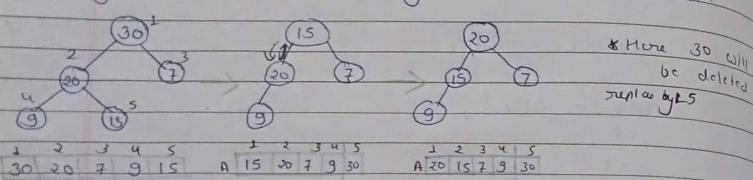
After deletion of all data we find that data is sorted in increasing order

Ex-1 A 15 20 7 9 30

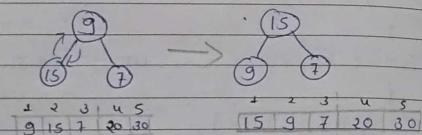
→ Building a max heap



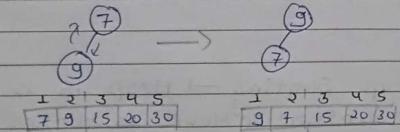
→ Deletion of element → root node element is deleted firstly and is replaced by most last element.



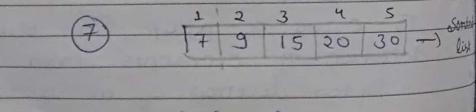
→ Now 20 will be deleted and replace by 9



→ Now 15 will be deleted and replace by 7



→ 9 will be deleted and replace by 7



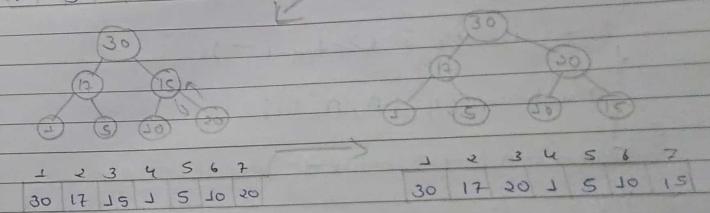
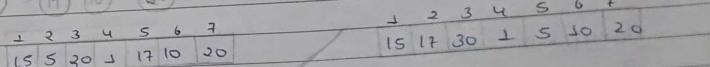
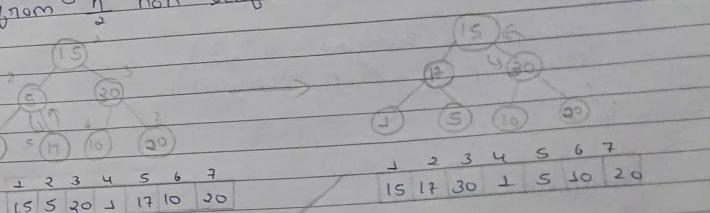
→ TC of heap sort =  $O(n \log n)$

\* Heapsify method → Heapsify is the process of creating a heap data structure from a binary tree. It is used to create a min or max heap.

Ex-2 A 15 5 20 17 10 30 Index start from 1

\* Leaf node  $\rightarrow \lceil \frac{N+1}{2} \rceil$  to N nodes.

\* Heapsify method will start from  $\lceil \frac{N}{2} \rceil$  non leaf nodes.



→ Algorithm →

max\_heapify(A, n, i)

y

int i = largest;

int l = (2 \* i);

```

int i = (x * i) + 1;
while (l <= n && a[r] > a[largest])
{
    largest = l;
}
while (r <= n && a[r] > a[largest])
{
    largest = r;
}
if (largest != i)
{
    swap (a[largest], a[i]);
    maxHeapify (A, n, largest);
}

```

HeapSort (A, n)

```

for (i = n / 2; i >= 1; i--)
{
    maxHeapify (A, n, i);
}
for (i = n; i >= 1; i--)
{
    swap (arr[1], arr[i]);
    maxHeapify (A, n, 1);
}

```

# Radix Sort → The idea of Radix sort is to do digit sort starting from least significant digit to most significant digit. Radix sort is non comparison based algorithm

Ex) 904      \* firstly we have to place 3 no's in order to maintain three digits  
 46  
 5  
 74  
 62  
 ↓                 arranged  
 904    001    001    001  
 046    062    904    005  
 005    904    005    046    sorted  
 074    074    046    062  
 062    005    062    074  
 601    046    074    904

\* If two numbers having same least significant digit that arrange them in their significant order

# Hashing → It is basically a searching technique with a constant time complexity of  $O(1)$  whereas in linear or binary search time complexity is  $O(n)$  and  $O(\log n)$ . It is basically a concept of storing and retrieving the data. Here we use Hash table, hashfunction and key.

$M = 10 \rightarrow$  hashtable size

Key	Location	Probe
3	$\lceil (2 \times 3) + 3 \rceil \% 10 = 9$	1
2	$\lceil (2 \times 2) + 3 \rceil \% 10 = 7$	1
9	$\lceil (2 \times 9) + 3 \rceil \% 10 = 1$	1
6	$\lceil (2 \times 6) + 3 \rceil \% 10 = 5$	1
11	$\lceil (2 \times 11) + 3 \rceil \% 10 = 5$	1
13	$\lceil (2 \times 13) + 3 \rceil \% 10 = 9$	1
7	$\lceil (2 \times 7) + 3 \rceil \% 10 = 7$	1
12	$\lceil (2 \times 12) + 3 \rceil \% 10 = 7$	1

Fun 26,  $26 \% 10 = 6 \rightarrow$  also stored at index 6,  
 $\therefore$  Collision occurs.

\* To resolve these collisions we have some techniques  
 Types of hashing →

- open hashing (closed addressing) → chaining
  - Closed hashing (open addressing)
- ↓      ↓      ↓  
 linear    quadratic    double  
 probing    probing    masking

1) open hashing - closed addressing [chaining]  
 linked list used to store data.

E.g.,  
 Array[] → 3, 2, 9, 6, 11, 13, 7, 12  
 Used division method and closed addressing  
 to store the values

$$h(k) = 2k + 3 \quad [m=10]$$

Division method →  $h(k) = k \% m$

Key	Location	Probe
3	$\lceil (2 \times 3) + 3 \rceil \% 10 = 9$	1
2	$\lceil (2 \times 2) + 3 \rceil \% 10 = 7$	1
9	$\lceil (2 \times 9) + 3 \rceil \% 10 = 1$	1
6	$\lceil (2 \times 6) + 3 \rceil \% 10 = 5$	1
11	$\lceil (2 \times 11) + 3 \rceil \% 10 = 5$	1
13	$\lceil (2 \times 13) + 3 \rceil \% 10 = 9$	1
7	$\lceil (2 \times 7) + 3 \rceil \% 10 = 7$	1
12	$\lceil (2 \times 12) + 3 \rceil \% 10 = 7$	1

\* Linear probing → whenever open addressing is asked  
 we always use linear probing by default.

→ same example for open addressing.

Key	Location	Probe
3	$\lceil (2 \times 3) + 3 \rceil \% 10 = 9$	1
2	$\lceil (2 \times 2) + 3 \rceil \% 10 = 7$	1
9	$\lceil (2 \times 9) + 3 \rceil \% 10 = 1$	1
6	$\lceil (2 \times 6) + 3 \rceil \% 10 = 5$	1
11	$\lceil (2 \times 11) + 3 \rceil \% 10 = 5$	2
13	$\lceil (2 \times 13) + 3 \rceil \% 10 = 9$	2
7	$\lceil (2 \times 7) + 3 \rceil \% 10 = 7$	2
12	$\lceil (2 \times 12) + 3 \rceil \% 10 = 7$	6

Total probes = 16

Order of elements in hash table → 13, 9, 12, -, -, 6, 11, 2, 7, 3

Linear probing → In case of collision insert  $K_i$  at first free location from  $(u+1) \% m$  where  $i = 0 \dots m-1$ .

