Algorithms and Data Structures

COMP261
Tutorial Week 6

Yi Mei

yi.mei@ecs.vuw.ac.nz

3D Rendering

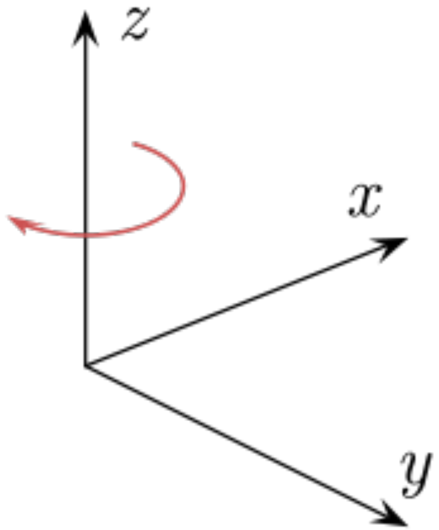
- Given a 3D object
  - A set of triangle polygons, each polygon has three vertices (3D points)

- **Step 1: calculate the coordinates based on any transformation**
- **Step 2: identify which polygons are visible**
  - Calculate normal: visible if the z-values of the normal is negative
- **Step 3: calculate shading color of the visible polygons**
  - Ambient light + incident light
- **Step 4: render**
  - Line-by-line, calculate edge list, and z-buffer

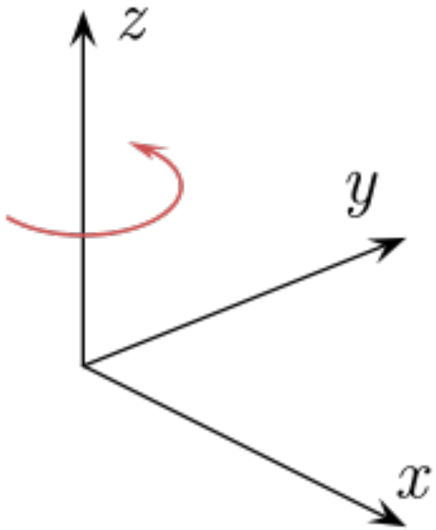
- Whenever transformed (rotate -> scale -> translate)
  - Calculate new coordinates of the vertices using the unified transformation operator (4x4 matrix)

3D Coordinate System

- A standard 3D coordinate system based on **right-hand rule**
  - Order x, y and z
  - Use right hand, thumb point to the **z** axis
  - Curl of other fingers from x to y (anti-clockwise viewing from z axis)



Left-hand rule



Right-hand rule

Right-hand rule

Unified Transformation Operator

Input: original point = {x, y, z}, 4x4 transformation matrix T
Output: new point {x’, y’, z’} after transformation

```
Initialise newpoint[4] = {0, 0, 0, 1};
//multiply (x, y, z, 1) with the 4x4 transformation matrix T on the left
for (row = 0 to 3) {
   for (col = 0 to 3) {
      newpoint[row] += T[row][col]*point[col];
   }
}
// only keep the first 3 elements
newpoint = newpoint[0:2];
return newpoint;
```

Transformation Matrices

Translation:

$$\begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{bmatrix}$$

Scaling:

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x \cdot s_x \\ y \cdot s_y \\ z \cdot s_z \\ 1 \end{bmatrix}$$

Rotation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ \cos\theta \cdot y - \sin\theta \cdot z \\ \sin\theta \cdot y + \cos\theta \cdot z \\ 1 \end{bmatrix}$$

Composite Transformation

- Apply the transformation matrix from right to left
  - First transformation to the right most, last to the left most

```
// Calculate the composite transformation matrix
Input: a sequence of 4x4 transformation matrices (M_1, …, M_k)
Output: the 4x4 composite transformation matrix CM
CM = M_2 * M_1;
for (i = 3:k) {
   CM = M_i * CM;
}
```

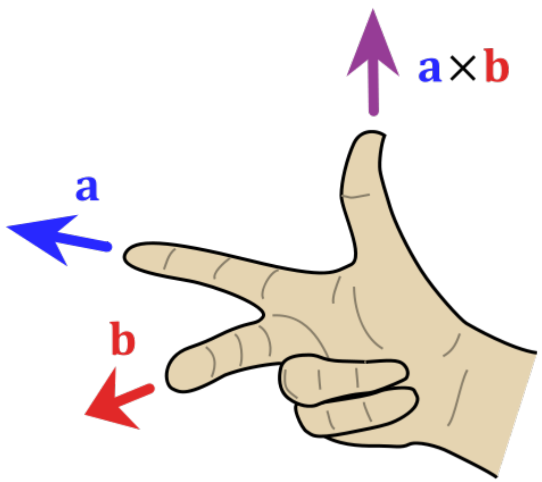
- Order of transformations
  - Rotation -> Scaling -> Translation

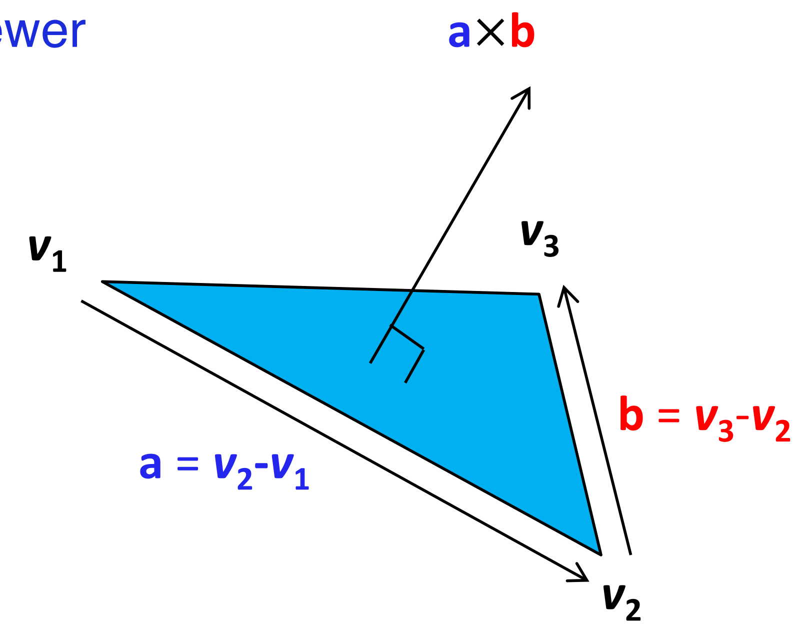
Cross Product

- **Cross product** is an operation between vectors. It returns another vector that is perpendicular with the input vectors.

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ z_1 x_2 - x_1 z_2 \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

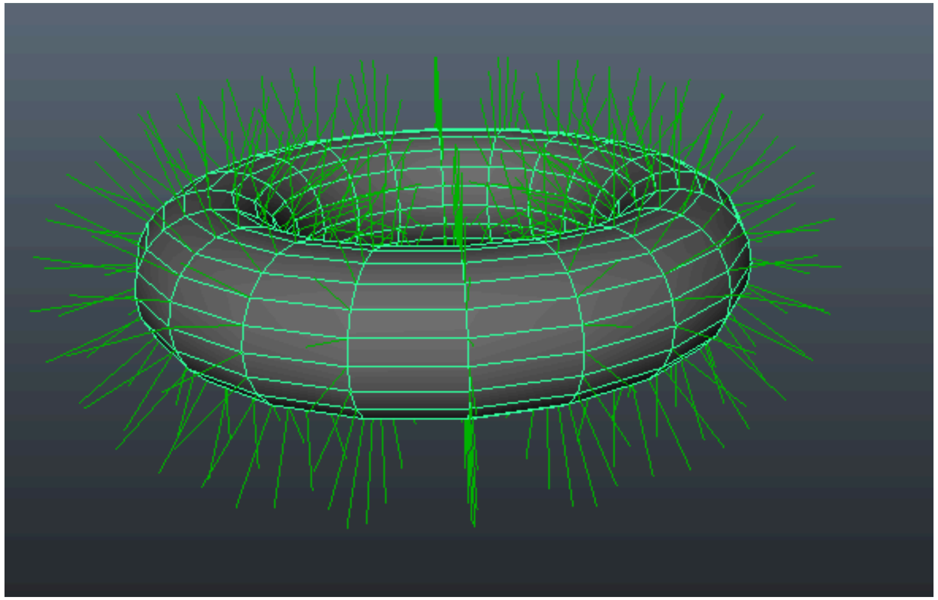
- The three vectors follow the right-hand rule.
- Use cross product to get which direction the polygon is facing
  - **v_1 -> v_2 -> v_3** is anti-clockwise
  - **$(v_2 - v_1)$ x $(v_3 - v_2)$** is facing the viewer

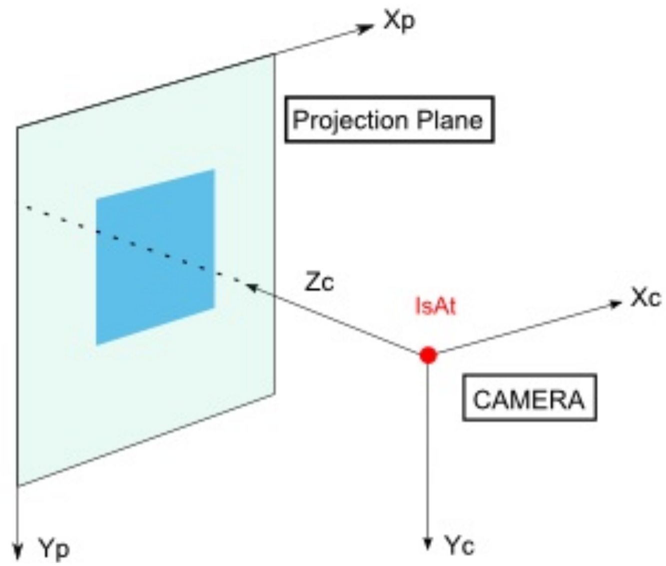




Visible/Invisible Polygons

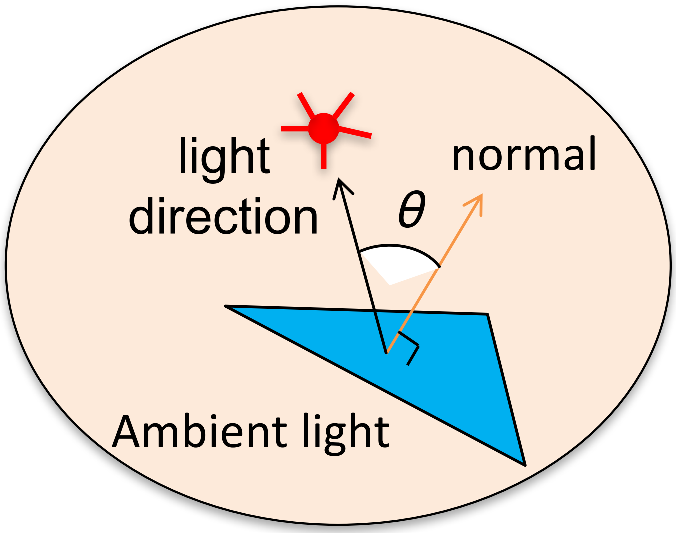
- The cross product **$(v_2 - v_1) \times (v_3 - v_2)$** is called the normal of the polygon
- A polygon is visible to viewer, if its normal has negative z value





Shading

- A simple method:
  - Uniform reflectance for red, green, blue
  - Ambient light: intensity (0, 1]
    - ambientlight = ambient light intensity * reflectance
  - Indicdent light source: intensity (0, 1], and its direction
    - incidentlight = incident light intensity * reflectance * cos(θ)
  - Light (shading color = ambientlight + incidentlight



$$\cos(\theta) = \frac{lightDirection \cdot normal}{|lightDirection| \times |normal|}$$

Shading Computation

Input:

- three vertices ordered anti-clockwise when facing the viewer: $\boldsymbol{v}_i, i = 1,2,3$
- Ambient light intensity $\boldsymbol{AL} = (AL.r, AL.g, AL.b)$, each color is within the range (0, 1]
- Incident light intensity $\boldsymbol{IL} = (IL.r, IL.g, IL.b)$, each color is within the range (0, 1]
- Incident light direction $\boldsymbol{D} = (D.x, D.y, D.z)$
- Reflectance $\boldsymbol{R} = (R.r, R.g, R.b)$, each color within the range [0, 255]

Output: the shading color $(S.r, S.g, S.b)$

// calculate normal

$\boldsymbol{a} = \boldsymbol{v}_2 - \boldsymbol{v}_1, \boldsymbol{b} = \boldsymbol{v}_3 - \boldsymbol{v}_2$

$\boldsymbol{n} = \boxed{\boldsymbol{a} \times \boldsymbol{b}}$ ← Cross product

// calculate $\cos(\theta)$

$\cos(\theta) = \frac{\boxed{\boldsymbol{n} \cdot \boldsymbol{D}}}{|\boldsymbol{n}| \times |\boldsymbol{D}|}$ ← Dot product

// calculate the shading

for (c in {r, g, b}) {

$\quad S.c = AL.c \times R.c + IL.c \times R.c \times \cos(\theta);$

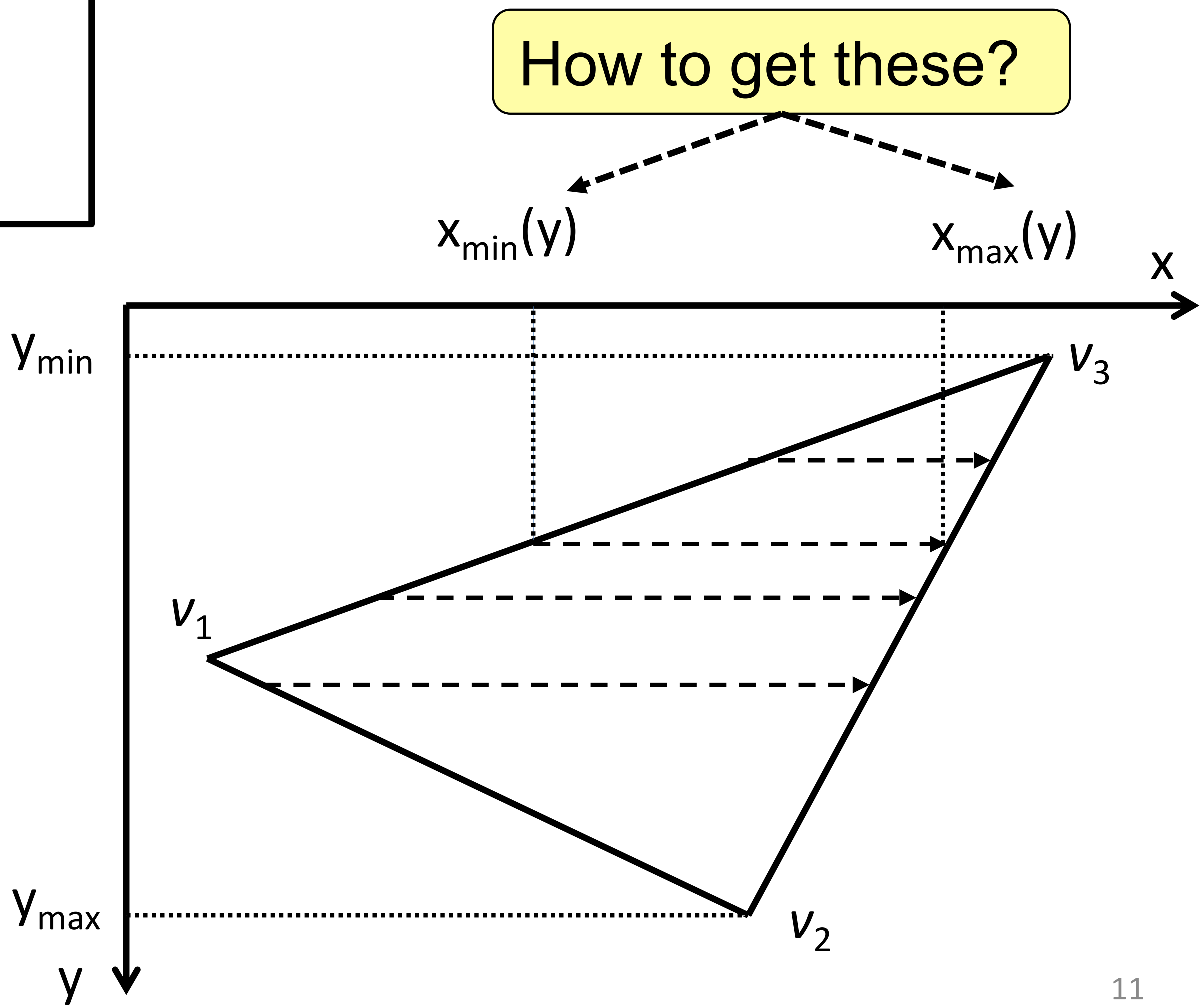
}

Polygon Rendering

- z-axis is the viewing direction, so the screen is x-y plane
  - Render pixels line by line

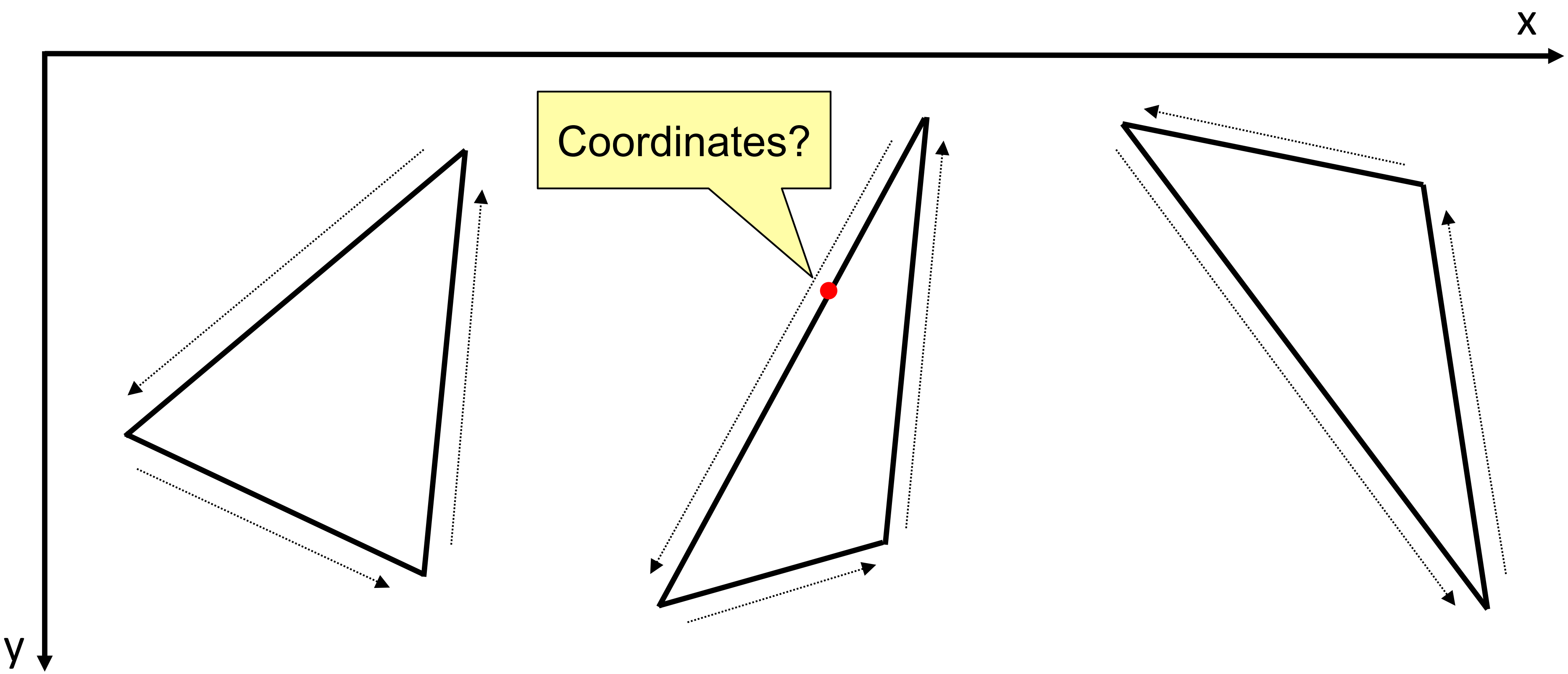
```
for (y = y_min to y_max) {
   for (x = x_min(y) to x_max(y))
      pixel(x,y) = shading color;
}
```

$y_{min} = v_3.y;$
$y_{max} = v_2.y;$



Polygon Rendering

- For any y value, get $x_{min}(y)$ and $x_{max}(y)$
  - All the $x_{min}(y)$ and $x_{max}(y)$ are on the edges of the polygon
  - If scanning the edges anti-clockwise, then
    - When the scan is going down, then visit $x_{min}(y)$
    - When the scan is going up, then visit $x_{max}(y)$



Edge List

- Scan each of the three edges, and update 2-column **EdgeList**
  - If scanning **up**, then update $\mathbf{x_{max}(y)}$ column
  - If scanning **down**, then update $\mathbf{x_{min}(y)}$ column
  - Use liner interpolation for each edge

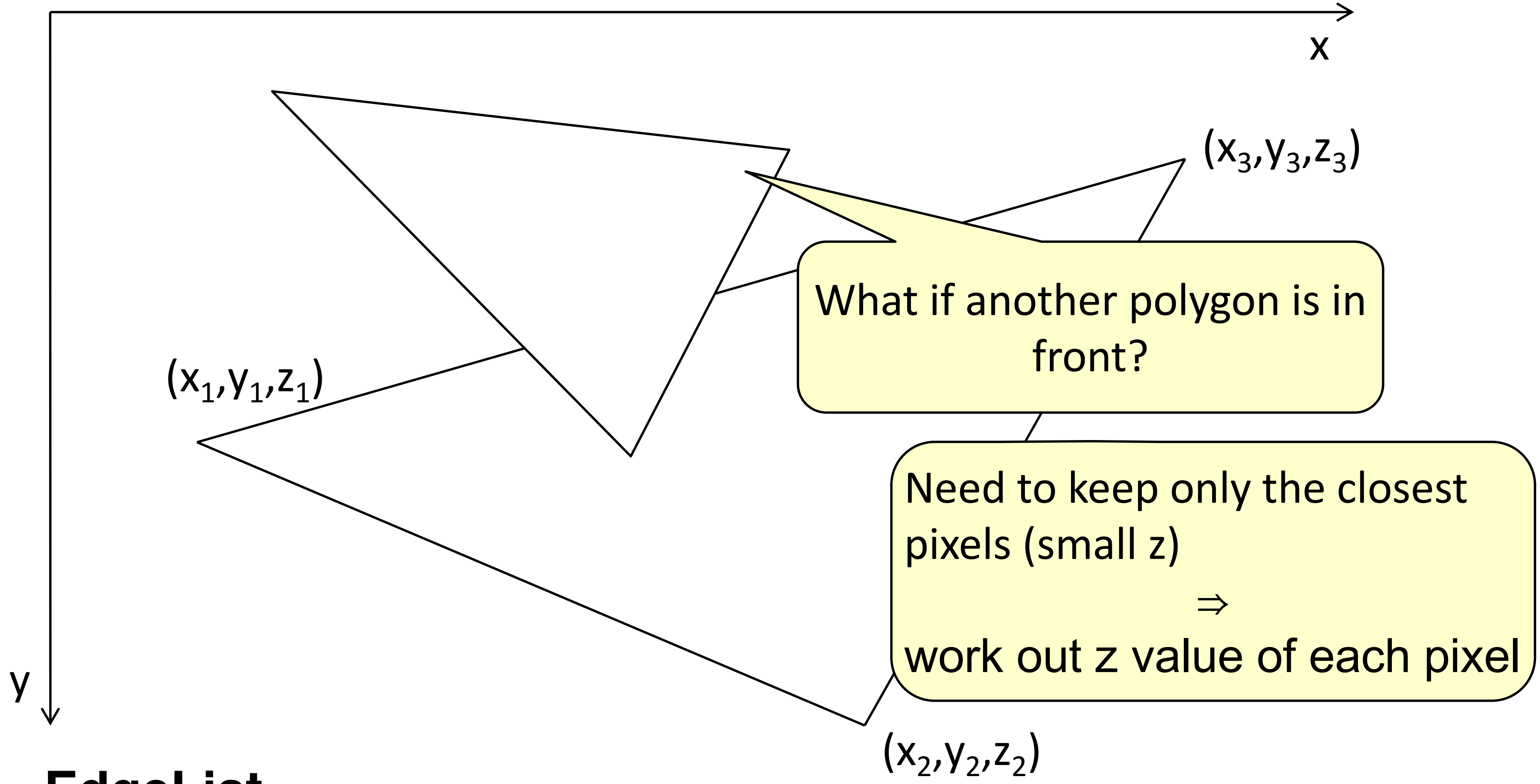
```
for (edge (a, b) in {(v_1,v_2), (v_2, v_3), (v_3, v_1)}) {
    slope = (b.x – a.x) / (b.y – a.y);
    x = a.x, y = round(a.y);
    if (a.y < b.y) {// going down, update x_min(y)
        while (y <= round(b.y))
            x_min(y) = x, x = x + slope, y++;
    else // going up, update x_max(y)
        while (y >= round(b.y))
            x_max(y) ← x, x ← x – slope, y--
```

Anti-clockwise ordered

EdgeList

	$x_{min}(y)$	$x_{max}(y)$
y_{min}		
$y_{min}+1$		
⋮		
y_{max}		

Multiple Polygons



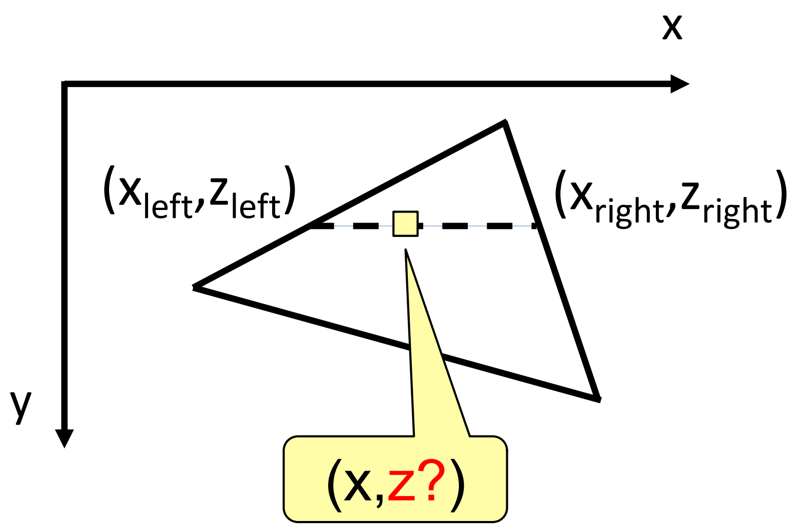
- **EdgeList**
  - **x_{min}(y)** and **x_{max}(y)** for the edges
  - **z(x, y)** for each pixel
    - If a pixel is on multiple polygons, render the polygon where it has the smallest z value

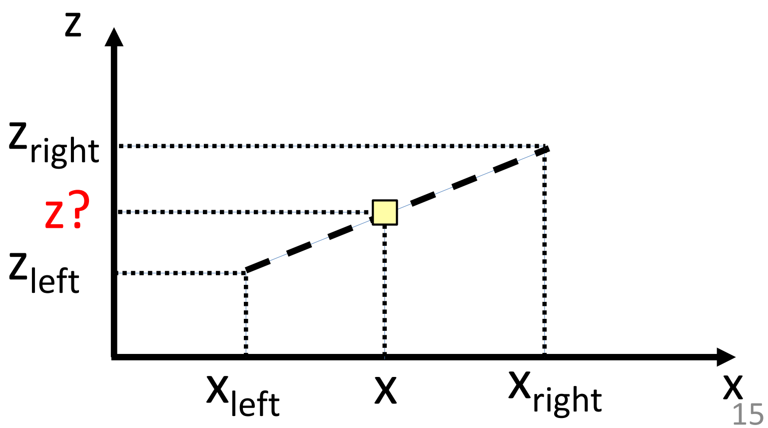
Render with EdgeList and z-buffer

- Compute the EdgeList for both x and z for the vertices on the edges
  - Compute the z value for each pixel inside the polygon using another linear interpolation

EdgeList

	Left		Right	
	x	z	x	z
y_{min}				
$y_{min}+1$				
⋮				
y_{max}				





Render with EdgeList and z-buffer

```
renderedImg = new Color[imageWidth][imageHeight];
zdepth = new double[imageWidth][imageHeight], initialise all entries to ∞;
for (each polygon) {
    calculate the x and z EdgeList (EL) of this polygon;
    for (y from EL.y_min to EL.y_max) {
        slope = (EL.z_right(y) – EL.z_left(y)) / (EL.x_right(y) – EL.x_left(y));
        x = round(EL.x_left(y)), z = EL.z_left(y) + slope * (x – EL.x_left(y));
        while (x <= round(EL.x_right(y))) {
            if (z < zdepth(x,y)) {
                renderedImg(x,y) = shading color of this polygon, zdepth(x,y) = z;
                z ← z + slope, x++;
}}}}
return renderedImg;
```