COMP261 Lecture 18

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String Searching 2 of 2

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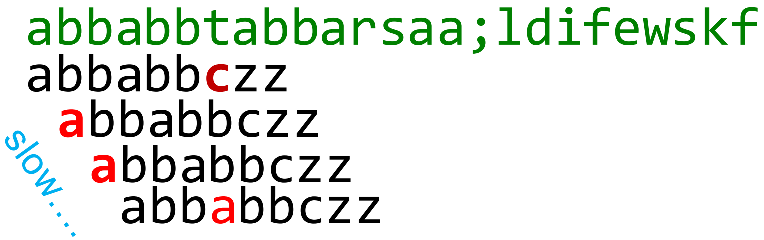
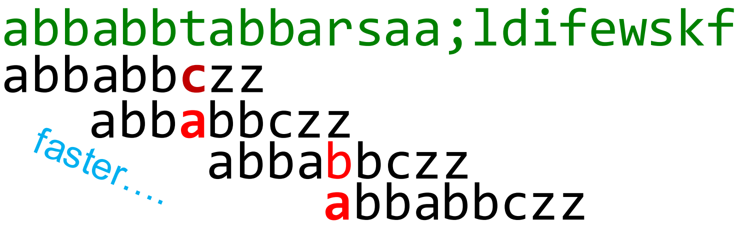
String search - recap

- Simple search
  - Slide the window by 1
    - `t = t+1;`



```
abbabbtabbarsaa;ldifewskf
abbabbczz
 abbabbczz
  abbabbczz
   abbabbczz
```

- Knuth-Morris-Pratt (KMP)
  - Slide the window faster
    - `t = t + s – M[s]`



```
abbabbtabbarsaa;ldifewskf
abbabbczz
   abbabbczz
      abbabbczz
          abbabbczz
```

- is there a "suffix ==prefix"?
  - If No, skip these characters altogether (big jump ahead for S)
    - » `M[s] = 0`
  - If Yes, reuse: no need to recheck those characters!

    (smaller jump for S, but start further along it)
    - » `M[s]` is the length of the "reusable" suffix

Knuth-Morris-Pratt (KMP) algorithm

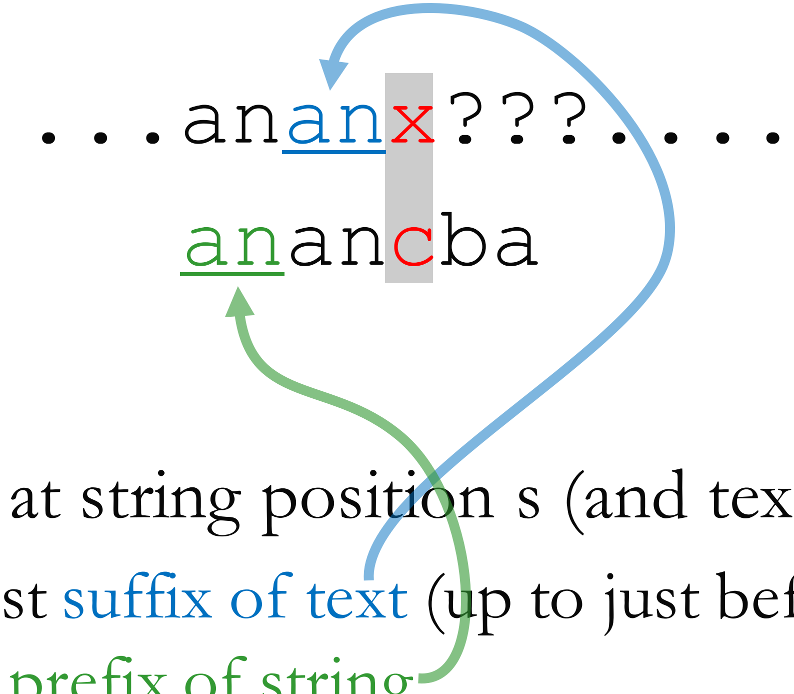
After a mismatch, advance to the earliest place where search string could possibly match.

- avoids the re-checking of characters that brute-force does

How far can we advance safely? (as much as possible, no further)

- Use a table based on the search string.
- Let M[0..m-1] be a table showing how far to back up the search if a prefix of S has been matched.

KMP - how far to move along? (in general)



- long text: `...ananx???....`
- string: `anancba`

- If mismatch at string position s (and text position t+s)
  - find longest suffix of text (up to just before the fail point) that matches a prefix of string
  - move k forward by (i – length of substring)
  - keep matching from i ← length of substring
- special case:
  - if i = 0, then move k to k + 1 and match from i ← 0

KMP

fail: not ‘g’

- anzn#??????????????????????????
  anzngfg
  - having got to a fail point, where should we check next?
  - jump ahead, and re-start at… where?

the fail point

fail: not ‘g’

but it could be ‘a’!

- anan#??????????????????????????
  anangfg
  - what about this one?
  - unsafe to jump straight to the fail point!

move S by 2, but restart from the fail point (#)

- anan#??????????????????????????
  ananafg
  - what about this one?
  - (nb: in theory, could jump further in such cases, for a small extra saving)

simplest: treat same as above

KMP

MOVING FROM THE LEFT of the search string S, on mismatch with T we check for a suffix == prefix, skip ahead that many, and continue checking matches from the fail point.

```
T:   abbabbtabbabbczzrsaldifewsk
S:   abbabbczz
```

suffix of 3 **in the matched part**:
skip ahead 3, and restart from "**t**"

```
T:   abbabbtabbabbczzrsaldifewsk
S:      abbabbczz
```

no suffix: move to "**t**", and restart

```
T:   abbabbtabbabbczzrsaldifewsk
S:         abbabbczz
```

no suffix: move to "**t**", and restart

```
T:   abbabbtabbabbczzrsaldifewsk
S:          abbabbczz
```

and we could precompute
all these jumps, just from S

Knuth Morris Pratt, the algorithm

input: string S[0 .. m-1] , text T[0 .. n-1], jump table M[0 .. m-1]

output: the position in T at which S is found, or -1 if not present

variables: k ← 0 *start of current match in T*

i ← 0 *position of current character in S*

```
while  k + i  <  n
    if  S[ i ] = T[ k + i ]   then          //  match at i
        i ← i + 1
        if  i = m   then return  k          // found S
    else if M[ i ]  = -1   then             // mismatch, no self overlap
        k ← k + i + 1, i ← 0
    else                                    // mismatch, with self overlap
        k ← k + i - M[ i ]                  // match position jumps forward
        i ← M[ i ]

return  -1      // failed to find S
```

How do we build the “jump” table? Example.

- Consider the search string `abcdabd`.
- Look for a proper suffix of failed match, which is a prefix of S, starting at each position in S
  – so suffix ends at previous position.
- `0:` `abcdabd`
  We can’t have a failed match at position 0.
  Special case, set M[0] to -1.
- `1:` a**b**cdabd
  a not a proper suffix.
  Special case, set M[1] to 0.
- `2:` ab**c**dabd
  b not a prefix, set M[2] to 0.

How do we build the "jump" table? Example.

- `3:` `abc`**`d`**`abd`
  abc has no suffix which is a prefix, set M[3] to 0.
- `4:` `abcd`**`a`**`bd`
  abcd has no suffix which is a prefix, set M[4] to 0.
- `5:` `abcda`**`b`**`d`
  a is longest suffix which is a prefix, set M[5] to 1.
- `6:` `abcdab`**`d`**
  ab is longest suffix which is a prefix, set M[6] to 2.

- Knowing what we matched before allows us to determine length of next match.

How do we precompute the “jump” table, M?

Look for *suffix of a failed match* which is *prefix of the search string*. eg:

- `abcmndsjhhhsjgrjgslagfiigirnvkfir`
  `abcefg`
  - No suffix. Resume checking at ‘`m`’:

  `abcefg`

- `ananfdfjoijtoiinkjjkjgfjgkjkkhgklhg`
  `ananaba`
  - Yes (‘an’). Resume checking at the second ‘`a`’:

  `ananaba`

- NB: suffix of a partial match is also part of the search string…
  We can find partial matches just by analysing the search string!

KMP – Partial Match Table

Index	0	1	2	3	4	5	6
S	a	b	c	d	a	b	d
M	-1						

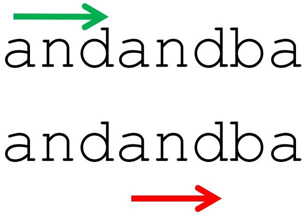
KMP – Partial Match Table

Index	0	1	2	3	4	5	6
S	a	n	a	n	a	b	a
M	-1						

Building the table.

M:	0	1	2	3	4	5	6	7

```
input:    S[0 .. m-1]    // the string
output:   M[0 .. m-1]    // match table

initialise:  M[0] ← -1
             M[1] ← 0
             j ← 0              // position in prefix
             pos ← 2            // position in table

while pos < m
    if S[pos - 1] = S[ j ]          // substrings ...pos-1 and 0..j match
        M[pos] ← j+1,
        pos++,  j++
    else if  j > 0                  // mismatch, restart the prefix
        j ← M[ j ]
    else   // j = 0                 // we have run out of candidate prefixes
        M[pos] ← 0,
        pos++
```



String search: Knuth Morris Pratt

- Cost?

- What happens for the worst case for brute force search?

```
S = aaaaab
T = aaaaaaaaaaaaaaaaaaaaaa
```

String search: can we do even better?!

- The previous lecture said: "ideally, we'd have an algorithm that never needs to re-trace its steps in the long string. Can we check each letter just once?" (Answer: yes, it's KMP).

fail

- aabah???????????????????????????
  aabaacb

  - but notice h is *nowhere* in the key string, so we can jump past…
  - Boyer-Moore exploits this notion to the absolute max, so much so that it does *better* than our "aim" of only checking everything once!

String search: Boyer-Moore (details not examinable)

- KMP searches forwards, and gets worse as the search sequence gets longer.
- It seems implausible that one could do better than looking at each T element only once, and yet…

- Boyer-Moore algorithm searches backward, gets *better* as search sequence gets longer!

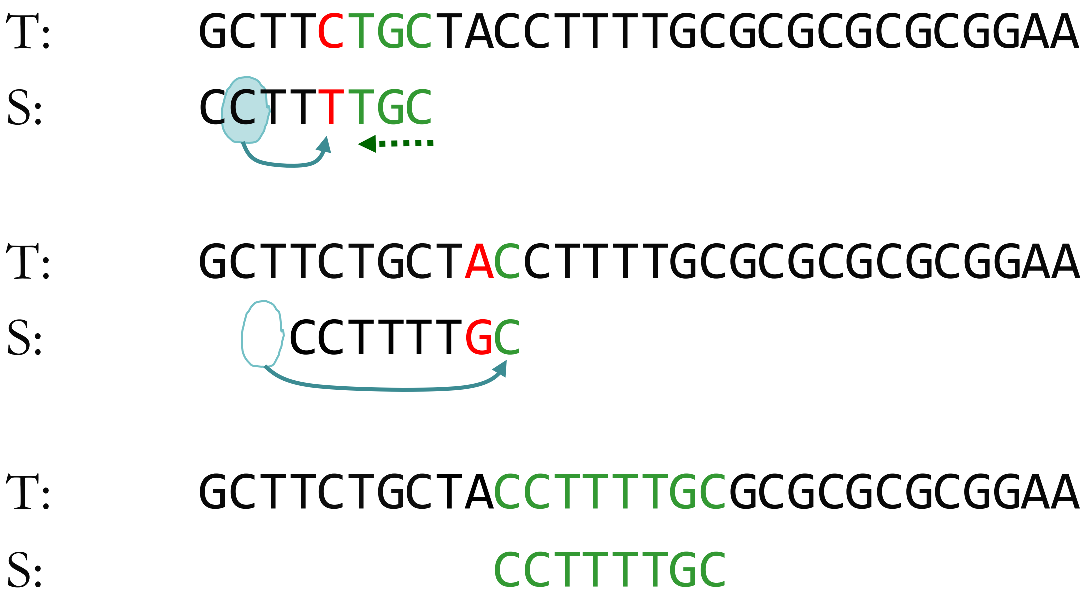
1. Bad character rule – tries to turn mis-match into match
2. Good suffix rule – tries to keep existing matches okay

Boyer Moore’s “Bad Character rule” (details not examinable)

Go *FROM THE RIGHT* within the search string S, upon a mis-match, we skip until either

CCTTTTGC

- mismatch becomes a match, or
- S moves past the mis-match character



T: GCTTCTGCTACCTTTTGCGCGCGCGGAA
S: CCTTTTGC

T: GCTTCTGCTACCTTTTGCGCGCGCGGAA
S: CCTTTTGC

T: GCTTCTGCTACCTTTTGCGCGCGCGGAA
S: CCTTTTGC

Boyer Moore’s “Good Suffix rule” (details not examinable)

Let ***t*** be the substring matched by the inner loop.

On mismatch we skip until either

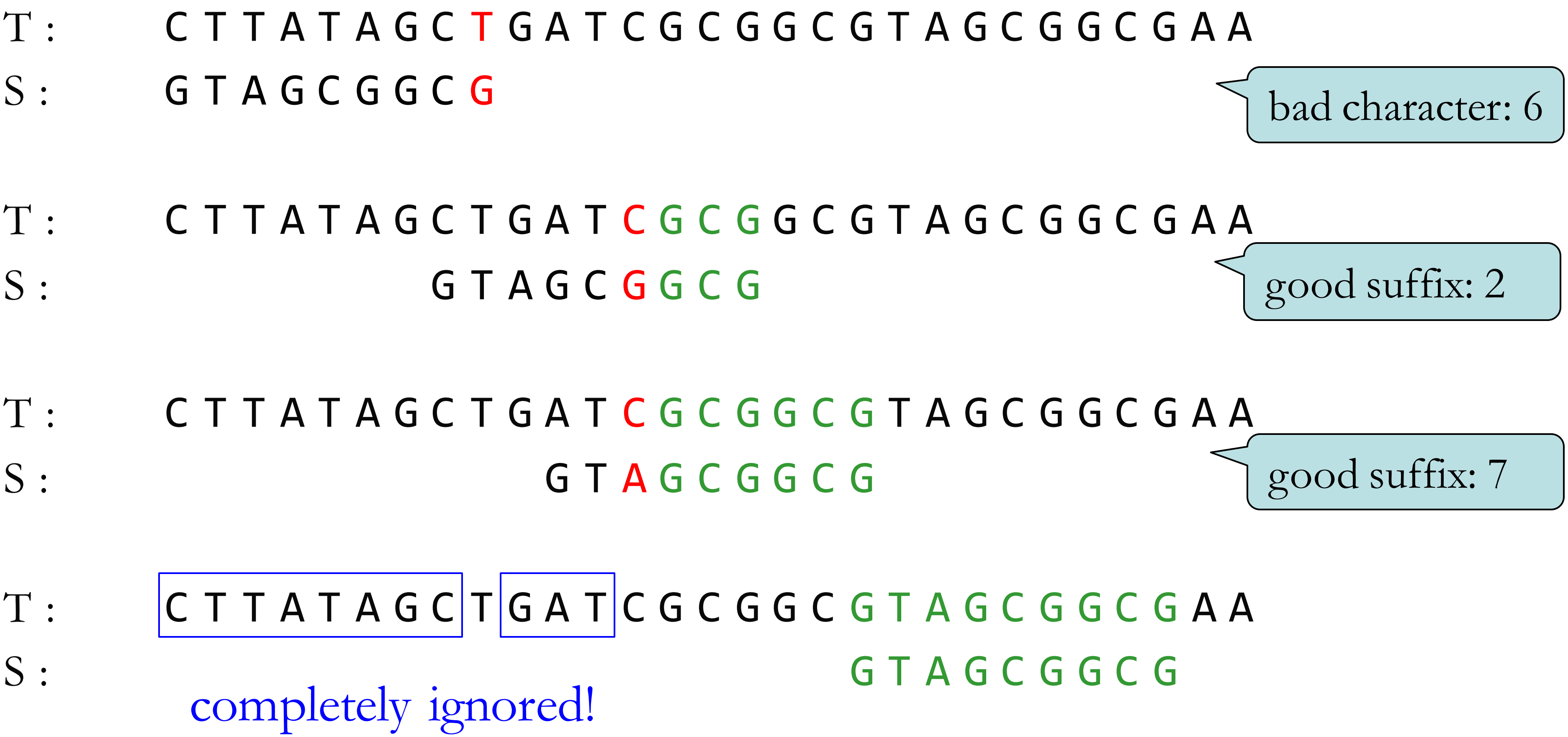
- no mismatch between S and ***t***, or
- S moves past ***t***

```
               t
T:      CGTGCCTACTTACTTACTTACTTACTTACGCGAA
S:      CTTACTTAC

                 t
T:      CGTGCCTACTTACTTACTTACTTACTTACGCGAA
S:          CTTACTTAC

T:      CGTGCCTACTTACTTACTTACTTACTTACGCGAA
S:              CTTACTTAC
```

Boyer-Moore algorithm
(details not examinable)

This is the go-to algorithm for fast string search in most practical cases.

- At each step, look up *both* jumps, and take max!



```
T : C T T A T A G C T G A T C G C G G C G T A G C G G C G A A
S : G T A G C G G C G
```

bad character: 6

```
T : C T T A T A G C T G A T C G C G G C G T A G C G G C G A A
S :               G T A G C G G C G
```

good suffix: 2

```
T : C T T A T A G C T G A T C G C G G C G T A G C G G C G A A
S :                     G T A G C G G C G
```

good suffix: 7

```
T : C T T A T A G C T G A T C G C G G C G T A G C G G C G A A
S :                                     G T A G C G G C G
```

completely ignored!