Questions on random variables, p.m.f., p.d.f., c.d.f, mean, variance, standard deviation

Q-1 A random variable X has the following probability function :

Values of X, x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k.

(ii) Evaluate P(X < 6), P(X ≥ 6), and P(0 < X < 5).

(iii) If P(X≤ a) > $\frac{1}{2}$, find the minimum value of a.

(iv) Determine the distribution function of X.

(v) Find mean, variance and standard deviation.

Q-2

If $p(x) = \begin{cases} \frac{x}{15}; x = 1,2,3,4,5 \\ 0, elsewhere \end{cases}$

(i) Find P{X = 1 or 2}

(ii) $P\left\{\frac{1}{2} < X < \frac{5}{2} | X > 1|\right\}$

Q-3 Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces. Construct a table giving the non-zero values of the probability mass function and draw the probability chart. Also find the distribution function, mean, variance and standard deviation of X .

Q-4 A variable X is distributed at random between the values 0 and 4 and its probability density function is given by: f(x) = $f(x) = kx^3(4-x)^2$. Find the value of k and the standard deviation of the distribution.

Q-5 The diameter, say X, of an electric cable, is assumed to be a continuous random variable with p.d.f.: $f(x) = 6x\,(1 - x),\ 0\ 0 \leq x \leq 1$

(i) Check that f (x) is a p.d.f.

(ii) Obtain an expression for the c.d.f. of X. Find mean, variance and standard deviation.

(iii) Compute $\boldsymbol{P\left(X \leq \frac{1}{2} \mid \frac{1}{3} \leq X \leq \frac{2}{3}\right)}$.

(iv) Determine the number k such that P(X < k) = P(X > k).

Q-6 Let X be a continuous random variable with p.d.f. given by

$$f(x) = \begin{cases} kx, & 0 \leq x < 1 \\ k, & 1 \leq x < 2 \\ -kx + 3k, & 2 \leq x < 3 \\ 0, & elsewhere \end{cases}$$

Determine k, c.d.f., mean, variance, standard deviation.