Comparison of Ancient Greek and Indian Epicyclic Geocentric Models

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Abstract

The study of the night sky has been of interest to societies over the world for thousands of years. The advancement of technology has given modern societies valuable insight on planetary orbits in our solar system and celestial bodies well beyond. However, the accurate predictions of ancient astronomers with ancient techniques are even more astonishing. The ancient Indian astronomers, in particular, developed geocentric models to understand the orbit of planets using only human-eye measurements from sun dials and celestial positions in the night sky. At the intersection of modern scientific techniques and ancient measurements lie some useful insight on the power of the former in light of the former and vice versa. In our analysis, we conclude that modern machine learning techniques have the accurate ability of modelling the geocentric orbit of planets given ancient measurements, illustrating how advanced both modern techniques and ancient data can be used in tandem.
https://github.com/NaveenGop/astro-data

1 Background

Understanding the movement of celestial bodies has long been a source of interest for civilizations and astronomers in times past. As astronomy was the first science, understanding the thought process and derivation of the mathematical formulas used to map the movements of planets and location of stars offers introspection towards these ancient civilizations. As a basis, the use of astronomy for astrology has long been documented and the influence of the works of past astronomers has shown cultural significance in concepts such as calendars and religion.

One of the earliest forms of astronomy owes thanks to Indian scientists and was motivated by the Vedas and the interest in cosmological movement prevalent throughout Hindu religion as early as 1500 BCE. With Greek influences introduced in 4th century BCE, the Indian astronomical field started becoming its own with the *Aryabhata* in the 6th century. The keystone text of ancient Indian astronomy was the *Surya Siddhanta*, which described a geoentric model of the visible planets for astrological purposes. This classical era of Indian astronomy was the forefront of the field at the time, with far reaching influence across the globe on Chinese, European, Muslim, and various other civilizations' study of space [2].

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2 Introduction

To ancient philosophers, a perfect celestial model of the solar system would consist of all the visible planets following circular orbits around the Earth. Such a model, however, fails to account for the true orbit being elliptical around the Sun, so, as the planets follow their orbits, they would sometimes appear to speed up, slow down, or go entirely in reverse. In order to create a more accurate model, the ancient Greeks, as we know through remarks by Ptolemy in *Almagest*, used an epicycle model, wherein each planet rotates around a smaller epicycle circle whose center rotates around a larger circle (deferent) centered on Earth, thus yielding a geocentric perspective of the solar system. Ancient Indian astronomers took this further by having the radius of the epicycle circle vary as a function of time, as opposed to a constant radius.

3 Methods

Two anomalies arise when the ancient astronomers attempted to model the motion of heavenly bodies with regular circular motion: the *sighra*, or solar anomaly, in which the speed of the heavenly body varies along its body, and the *manda*, or zodiacal anomaly, in which the planets appear to sometimes move in reverse due to their relationship with the Sun. Like the ancient Greeks, the ancient Indians sought to correct these anomalies with the introduction of epicyclic orbits. But unlike the ancient Greeks, the ancient Indian astronomers introduced the additional innovation of having the epicycle radius be a function of time, leading to a more accurate geocentric model of the heavens.

3.1 Ancient Greek Astronomy

Assuming all orbits were to be perfectly circular, the main orbit of the epicycle's center around the Earth could be simply modeled as a pair of parametric equations:

$$x = R_0 \cos(\omega_0 t)$$
$$y = R_0 \sin(\omega_0 t)$$

where x and y are the coordinates of the body's location on a 2D grid, R_0 is the radius of the main orbit, ω_0 is the angular frequency of the planet's orbit, and t is our current time. However, as previously mentioned, the orbits of the planets as viewed from Earth are not circular, so the ancient Greek astronomers added an epicycle to this main orbit. In this case, the parametric equations of the planetary positions would be as follows:

$$x = R_0 \cos(\omega_0 t) + R_1 \cos(\omega_1 t)$$
$$y = R_0 \sin(\omega_0 t) + R_1 \sin(\omega_1 t)$$

where R_1 and ω_1 are the radius and the angular frequency respectively of this epicycle orbit.

3.2 Ancient Indian Astronomy

To adjust for retrograde motion, ancient Indian astronomers also choose to add another epicycle to the main circular orbit, but, instead of R_1 being a constant, $R_1(t)$ is a function of time. Specifically, $R_1(t)$ is a periodic waveform, which would make the radius of the epicycle appear to pulsate. Furthermore, by constraining the angular velocities of the main orbit (deferent) and epicycle itself to be equal, the ancient astronomers were able to operate under the assumption that the line between the celestial and the center of its epicycle was horizontal and parallel to the x-axis. This implies that at any point in time, only the planet's horizontal position is affected by epicycle radius:

$$x = R \cos(t) + R_t$$
$$y = R \sin(t)$$

where R_t is the radius of the epicycle [1].

This more complex pulsating epicycle model was used to explain the orbits of the five visible planets, as both the *sighra* and *manda* corrections needed to be applied. The ancient Indian astronomers calculated the contraction and expansion of these epicycles through very intricate computations, and we aim to examine these further in our machine learning model. For our purposes, we loosen the constraint that the angular velocity of the main orbit (deferent) and epicycle itself to be equal ($\omega_0 = \omega_r$), which allows the model to learn a richer function [3].

3.3 Modelling using Fourier Features

It naturally follows for us to model the radius of the epicycle as a time-varying sinusoid as follows:

$$R_1(t) = \frac{R_1^{(b)} - R_1^{(a)}}{2} \sin(\omega_r t + \beta) + \frac{R_1^{(b)} + R_1^{(a)}}{2}$$

where $R_1^{(b)}$ is the maximal possible epicycle radius, $R_1^{(a)}$ is the minimal possible epicycle radius, β is the phase shift, and ω_r is the radial angular velocity for the epicycle. The central orbit, or deferent orbit (as mentioned earlier), will be defined as the 0-th epicycle with a non-varying radial function as such: $R_0(t) = R_0$.

To learn these orbits, we convert the position data from $\mathbb{R}^2$ to $\mathbb{C}$ by letting each position be $x + jy$ where j is the imaginary unit. This conversion allows us to learn using Fourier features since the parametric equations would reduce to one equation for the true orbit:

$$\phi(t) = R_0 e^{j\omega_0 t} + R_1(t) e^{j\omega_1 t}$$

where $x = Re\{\phi(t)\}$ and $y = Im\{\phi(t)\}$.

Furthermore, as $R_1(t)$ is a sinusoid, it can be expressed as a difference of exponentials, namely

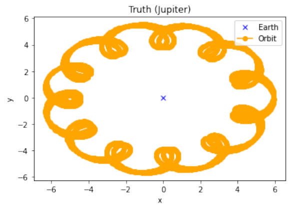
$$\phi(t) = R_0 e^{j\omega_0 t} + \frac{R_1^{(b)} + R_1^{(a)}}{2} e^{j\omega_1 t} + \frac{R_1^{(b)} - R_1^{(a)}}{4i} [e^{j(\omega_1 + \omega_r + \beta)t} - e^{j(\omega_1 - \omega_r - \beta)t}].$$

To learn this orbit function, we use ridge regression to learn the weights of the three terms in the sum ($e^{j\omega_0 t}, e^{j\omega_1 t}, e^{j(\omega_1 + \omega_r + \beta)t} - e^{j(\omega_1 - \omega_r - \beta)t}$), which, by the formulation of the problem, encapsulates learning the min/max radii, radial angular velocity, phase shift, and orbital angular velocity of the epicycle and deferent. The angular frequencies are learned simply by doing a grid search over the parameter space.

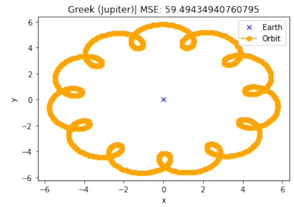
We used the data from Project SKEP for our training and test data, though we adjusted the observer location to be in Ujjain, India [4].

4 Results

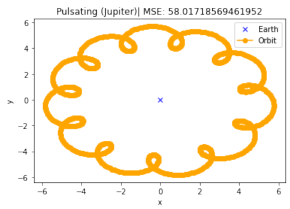
We ran our learning model on all five of the visible planets as well as the Sun and Moon. The following figures showcase the various models we have been discussing so far, with Greek referring to the Ancient Greek model, Static Pulsating referring to the Ancient Indian model with $\omega_r = \omega_0$, and Pulsating referring to the Ancient Indian model with the ω_r constraint loosened as previously mentioned. The orbits of all the planets as well as the Moon rotate slightly year over year, meaning that, as a predictor, the models lose accuracy over time. We were unable to learn this orbit shifting along with the orbits because shifting of the orbit does not appear to be periodic, and the data required to learn and show some almost periodic nature would be truly massive.



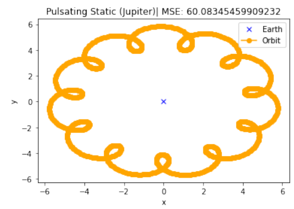
(a) True Orbit



(b) Greek



(c) Pulsating

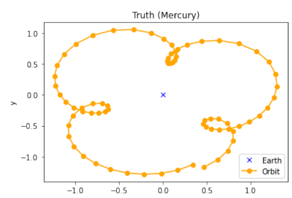


(d) Static Pulsating

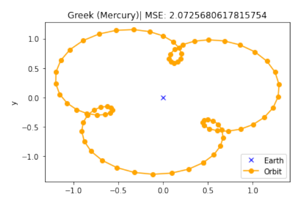
Figure 1: Jupiter

The figures to the left are the true orbit of Jupiter along with all the learned orbits of Jupiter. The models were trained from two full orbits of Jupiter and tested on two separate orbits. Note that the Pulsating yields the orbit with the lowest mean-squared error (MSE), Static Pulsating is slightly worse, and Greek is the worst of the three. Intuitively, since the Pulsating models have an extra feature, there are more parameters to absorb the error of the data. As Indian Pulsating model has the most leeway in regards to this third feature, it should have the most expressiveness to predict the orbit data. However, the highly periodic nature of Jupiter's orbit along with its long period actually render it much easier to learn for all models.

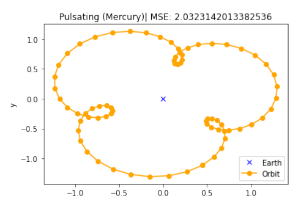
By contrast, the orbit of Mercury (shown on the right) is much more irregular in that it does not follow the same overall circular shape as Jupiter. Interestingly, the Greek model now has the lowest MSE, the Static Pulsating is slightly worse, and Pulsating the worst of the three. Qualitatively, the Pulsating model appears more similar to the true orbit even as the other two look like rougher versions of the true orbit, which indicates that perhaps the Pulsating model did learn the training orbit better than the other models. That being said, the Pulsating model did not overfit the data, but rather the shifting of the orbit over time renders the better shape of the model over the training data worse in regards to MSE than the more generic shapes shown in the other two.



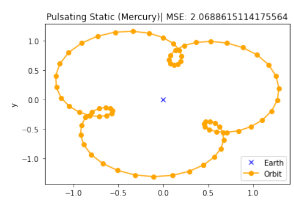
(a) True Orbit



(b) Greek



(c) Pulsating



(d) Static Pulsating

Figure 2: Mercury

The plots for the other visible planets, Moon, and Sun are shown in the Jupyter notebook. The orbit of the sun is rather trivial as the true orbit is, for the purposes of ancient astronomy, a circle. For all the models, learning a circle was not too challenging a task. For the other planets, as a heuristic, the Pulsating model did better with the orbits that appeared more periodic, even being more robust to orbit shifting over time, whereas highly irregular orbits such as Mercury or Venus displaying variable behavior. The Moon data was sampled too infrequently, and, as a result, the learned models were poor. Indeed, for planets with a shorter orbital period, the models benefit from data sampled more frequently than the five day period currently used.

5 Conclusion

The biggest bottleneck to the predictive nature of the models is the slight shifting of the orbits over time; this continual shifting renders the learned models useless after a sufficiently long period of time. We attempted to learn the orbital shift of each planet using gradient descent to test the models over long timeframes, but the results were inconclusive. As a whole, the qualitative effects of the models were discussed above: highly irregular orbits are ill-suited to be modelled in this epicyclic way, a high degree of orbital shift causes the better learning of the Pulsating Model to be worse in terms of MSE for other orbits, and the Pulsating model does learn the approximate shape much better than the other models. Our work sets basic frameworks for analyzing a pulsating epicycle model in relation to the standard epicycle model. In terms of expanding on our work, we only modelled the orbit as the deferent and one epicycle, but generalizing our work to an arbitrary number of epicycles is straightforward and could yield interesting results in terms of the optimal number of pulsating epicycles relative to constant radii epicycles. Similarly, we did grid search to find optimal angular frequencies, but a better heuristic to approach this problem might exist.

With regards to problems related to this work, doing an analysis of the frequency response of the planetary orbits could yield relationships between the frequencies present in the data and the optimal frequencies learned by the epicyclic models. We did do some basic data exploration on the Fourier Transfrom of the Right Ascension and Declination data, experimenting with the largest magnitude frequencies. Our results were inconclusive as we were unable to find any elementary relationship between the Fourier Transfrom frequencies and the epicycle frequencies, though more work can definitely be done.

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