

# **Thermal Protection Solver Final Report**

Date: 05/01/2025

**Group Members:** Nishi Mishra, Layan Samandar, Parina Patel, Hassan Niaz, Bihao Zhang, Naveen Jagadeesan

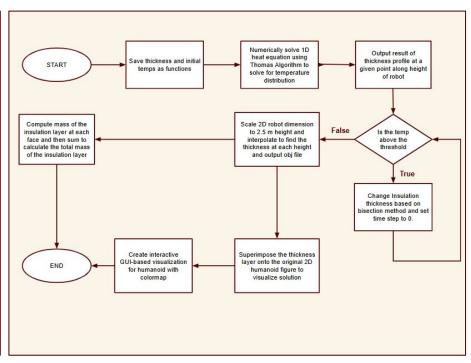
# Flow Diagrams



#### **Initial:**

#### Assign faces a Is the Set initial Parse mesh file material / thickness material an conditions insulator False True For each surface point. For each surface point, assign depth d and lower assign depth d and higher number of "depth points" number of "depth points" Time step False Solve 1d heat problem Save solution at time (w/ matrix inversion) reached? True Is the temp Plot temperature Mesh temperature above the animation with time threshold True Add layers and set Time step to 0

#### Final:





$$egin{aligned} rac{\partial u}{\partial t} &= lpha rac{\partial^2 u}{\partial x^2} \ rac{u_{i,n+1} - u_{i,n}}{\Delta t} - lpha^2 rac{u_{i+1,n+1} - 2u_{i,n+1} + u_{i-1,n+1}}{(\Delta x)^2} &pprox 0 \ u_{i,n} &pprox u_{i,n+1} - lpha^2 \Delta t \cdot rac{u_{i+1,n+1} - 2u_{i,n+1} + u_{i-1,n+1}}{(\Delta x)^2} \ u_{i,n} &pprox u_{i,n+1} \left(1 + rac{2lpha^2 \Delta t}{(\Delta x)^2}
ight) - rac{lpha^2 \Delta t}{(\Delta x)^2} (u_{i+1,n+1} + u_{i-1,n+1}) \end{aligned}$$

$$\lambda = \frac{\alpha^2 \Delta t}{(\Delta x)^2}$$

$$\lambda = rac{lpha^2 \Delta t}{(\Delta x)^2} \quad \quad u_{i,n} pprox u_{i,n+1} \left(1 + 2\lambda
ight) - \lambda \left(u_{i+1,n+1} + u_{i-1,n+1}
ight)$$

#### **Matrix Inversion - Continued**



In compact form:

$$\mathbf{u}_n \approx A\mathbf{u}_{n+1}$$

where:

$$A = egin{bmatrix} 1+2\lambda & -\lambda & 0 & \cdots & 0 \ -\lambda & 1+2\lambda & -\lambda & \ddots & dots \ 0 & \ddots & \ddots & \ddots & 0 \ dots & \ddots & -\lambda & 1+2\lambda & -\lambda \ 0 & \cdots & 0 & -\lambda & 1+2\lambda \end{bmatrix}$$

$$\mathbf{u}_{n+1} = A^{-1}\mathbf{u}_n$$

So, The heat equation solution boils down to solving Ax=b. Since A has a special structure, i.e., A is tridiagonal, we'll be using **Thomas Algorithm** for the solution

# **Tridiagonal Matrices**



For Tridiagonal systems, an observation is that their inverse has a special

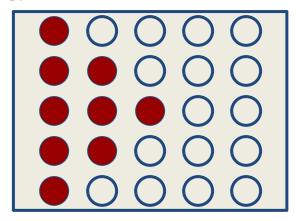
structure:

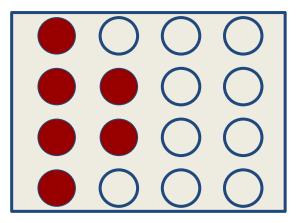
```
A =
     >> inv(A)
ans =
   0.0455
          -0.0021
                 0.0001
                           -0.0000
  -0.0021 0.0456 -0.0021
                          0.0001
   0.0001
          -0.0021
                 0.0456
                           -0.0021
  -0.0000
          0.0001
                  -0.0021
                           0.0455
```

# Tridiagonal Matrices



For Tridiagonal systems, an observation is that their inverse has a special structure:





For even number of columns: n(n+2)/4

For odd number of columns:  $(n+1)^2/4$ 

# **Why Thomas Algorithm**



- Specialized form of Gaussian elimination tailored for tridiagonal matrices
- Solves the inverse of an nxn matrix in O(n) time as opposed to standard inverse algorithms which take O(n<sup>3</sup>) time
- Has low memory head where instead of storing the whole matrix (n<sup>2</sup> terms), we only need to store three columns (3n-2 terms)

# **Why Thomas Algorithm**



87 times faster in this case

```
Command Window
```

Thomas Algo Finished in 0.0042196000 s
MATLAB's Inverse Finished in 0.3688809000 s

difference =

1.0e-13 \*

0.4529

-0.0297

-0.5420

0.0715

0.6464

Generally, 50-100x faster



# **Thermal Simulation Assumptions**



## External boundary (robot surface):

- Constant temperature from initialTemp(z) function
  - 900 K at toes

## Innermost boundary:

- Assumed to be in contact with still air modeled as an
  - $\circ$  insulator  $\rightarrow$  zero heat flux

# Interface Handling:

Assume heat flux to be continuous across material boundaries

## **Interface Handling**



We assume heat flux is continuous between materials:

$$k_1 \frac{\partial T}{\partial x} = k_2 \frac{\partial T}{\partial x}$$

We used thermal resistance to enforce this:

$$R = rac{\Delta x}{k}$$
  $T_{
m interface} = rac{T_L R_2 + T_R R_1}{R_1 + R_2}$ 

 This guarantees the flux entering neighboring elements equals flux leaving

# Thermal Resistance & Harmonic Mean Equivalence



Thermal resistances add in series:

$$R_{ ext{total}} = rac{\Delta x_1}{k_1} + rac{\Delta x_2}{k_2}$$

So for equal spacing ( $\Delta x_1 = \Delta x_2 = \Delta x/2$ ):

$$R_{ ext{total}} = rac{\Delta x}{2k_1} + rac{\Delta x}{2k_2} = rac{\Delta x}{2} \left(rac{1}{k_1} + rac{1}{k_2}
ight)$$

The effective conductivity is defined by:

$$R_{
m total} = rac{\Delta x}{k_{
m eff}} \Rightarrow rac{\Delta x}{k_{
m eff}} = rac{\Delta x}{2} \left(rac{1}{k_1} + rac{1}{k_2}
ight)$$

Cancel  $\Delta x$ :

$$oxed{rac{1}{k_{\mathrm{eff}}} = rac{1}{2} \left(rac{1}{k_1} + rac{1}{k_2}
ight)} \Rightarrow k_{\mathrm{eff}} = rac{2k_1k_2}{k_1 + k_2}}$$

#### **Evolution of the Heat Solver**



#### **Previous Approaches:**

- Insulation Only
  - Simulated just thermal protection layer
  - Only interface temp stayed above carbon fiber's glass transition temp (350K)
- 2 Thermal Resistance + Inertia

$$rac{\partial T}{\partial t} = rac{1}{
ho c_p} rac{\partial}{\partial x} \left( k rac{\partial T}{\partial x} 
ight)$$

- Added transient term using ρ·cp (thermal inertia)
- Used half-cell  $R = \Delta x / k$  and BTCS with varying properties
- Sequential Layer Solver
  - Solved each layer one after another for part of dt
  - Assumed constant outer temp + insulated inner face
  - Layers temp calcs evolved sequentially over time

#### **Final Method:**

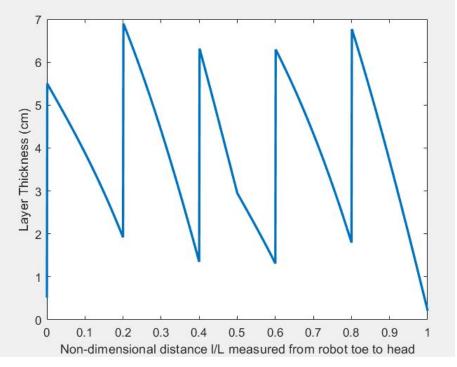
#### Harmonic Mean Solver

- Uses harmonic mean of k at interfaces
  - equivalent to thermal resistance
- Based on steady-state conduction flux, applied over time
- Solves full stack with BTCS, synchronized in time

### **Validation**



```
z=0.00 units -> T CF=339.43K, T GLUE=339.43K, T STL=339.43K
z=0.04 units -> T CF=317.55K, T GLUE=317.53K, T STL=317.05K
z=0.08 units -> T CF=318.31K, T GLUE=318.30K, T STL=318.01K
z=0.12 units -> T CF=319.53K, T GLUE=319.52K, T STL=319.38K
z=0.16 units -> T CF=321.39K, T GLUE=321.38K, T STL=321.34K
z=0.20 units -> T CF=324.30K, T GLUE=324.30K, T STL=324.30K
z=0.24 units -> T CF=313.26K, T GLUE=313.25K, T STL=312.88K
z=0.28 units -> T CF=314.58K, T GLUE=314.57K, T STL=314.34K
z=0.36 units -> T CF=319.65K, T GLUE=319.65K, T STL=319.61K
z=0.40 units -> T CF=324.71K, T GLUE=324.71K, T STL=324.71K
z=0.44 units -> T CF=313.55K, T GLUE=313.54K, T STL=313.16K
z=0.48 units -> T CF=315.98K, T GLUE=315.97K, T STL=315.72K
z=0.52 units -> T CF=318.44K, T GLUE=318.44K, T STL=318.31K
z=0.56 units -> T CF=320.40K, T GLUE=320.40K, T STL=320.36K
z=0.60 units -> T CF=323.11K, T GLUE=323.11K, T STL=323.11K
z=0.64 units -> T CF=311.81K, T GLUE=311.80K, T STL=311.47K
z=0.68 units -> T CF=312.81K, T GLUE=312.80K, T STL=312.60K
z=0.72 units -> T CF=314.24K, T GLUE=314.23K, T STL=314.13K
z=0.76 units -> T CF=316.37K, T GLUE=316.37K, T STL=316.34K
z=0.80 units -> T CF=309.93K, T GLUE=309.92K, T STL=309.50K
z=0.84 units -> T CF=311.09K, T GLUE=311.08K, T STL=310.77K
z=0.88 units -> T CF=312.79K, T GLUE=312.79K, T STL=312.59K
z=0.92 units -> T CF=315.37K, T GLUE=315.36K, T STL=315.25K
z=0.96 units -> T CF=319.52K, T GLUE=319.52K, T STL=319.48K
z=1.00 units -> T CF=327.10K, T GLUE=327.10K, T STL=-0.00K
```



# **Our Solving Functions**



```
ectorcvectorcdouble>> solveMultiLayer(
 const vectoredoubles& layerThickness,
  double missionDuration.
  bool dummy
  double totalThickness - 8:
  for (double t_cm : layerThickness) totalThickness +- t_cm/188.8;
  int N - static_castcint>(totalThickness / dx) + 1;
  int timePoints - static castcint>(missionDuration / dt) + 1:
  vector(double> T(N, 300.8); // Initialize temperatures
  vector<vector<double>> T out(timePoints, vector<double>(N, 300.0));
  vector(double) alpha(N); // thermal diffusivity at each node
  int idx - 8;
  for (size t L = 0; L < materials.size(); ++L) {
         alpha[idx] = materials[L].thermalConductivity / (materials[L].density * materials[L].specificHeatCapacity);
     alpha[idx++] - materials.back().thermalConductivity / (materials.back().density * materials.back().specificHeatCapacity
  vector<double> a(N-1, 0.8), b(N, 0.8), c(N-1, 0.0);
         double alphat - alpha[i-1];
         double alphaR - alpha[i]:
         double alpha_eff = 2.8 * alphat * alphaR / (alphat + alphaR);
         double lambda - alpha eff * dt / (dx * dx):
         a[i-1] - -lambda;
         b[i] - 1.0 + lambda:
         double alphat - alpha[i-1];
         double alphaC - alpha[i];
         double alphaR - alpha[i+1];
         double alpha eff L = 2.0 * alphaL * alphaC / (alphaL + alphaC);
         double alpha_eff_R = 2.0 * alphaC * alphaR / (alphaC + alphaR);
         double lambda L = alpha eff L * dt / (dx * dx);
         double lambda_R = alpha_eff_R * dt / (dx * dx);
         a[i-1] - -lambda L;
         b[i] - 1.8 + lambda_L + lambda_R;
         c[i] - -lambda R;
     T = thomas_algorithm(a, b, c, d);
```

# calculateRequiredInsulation ThicknessMultiLayer()

MakeTimeSolution()

```
ector doubles calculateRequiredInsulationThicknessMultiLayer(double missionDuration, double dz, double dt, bool normalized)
  int Nz = static_castcint>(totalHeight / dz) + 1;
  vectorcdouble> insThick(Nz), zVals(Nz);
  vector (Material Properties) mats - { THERMAL PROTECTION, CARBON FIBER, GLUE, STEEL };
  vector<double> zValues((int)(totalHeight/dz)+1, 0.0);
      double gl = glueThickness(z);
      double st = steelThickness(z):
           vector (double > layers - { mid, cf, gl, st };
          double T ext = initialTemp(z):
          auto Tdist - solveMultiLayer(
              layers, mats,
              dt dx,
          double T if - Tdist.back()[idx if];
          if (T if <= CARBON FIBER.glassTransitionTomp-3) {
             hi - mid;
          cout << "z-" << normalizePosition(z) << " units, ins thick-" << insThick[i]
      auto Tfin - solveMultiLayer({insThick[i], cf, gl, st}, mats,
                                 missionDuration, dt, dx, initialTemp(z), normalized);
      int idx = static castcint>((insThick(i)/100.0) / dx + 0.5):
     court or Tfin back(\Tidy) or " K\n":
      zValues[i] - z;
 // outputting thickness values in a csv
string filename = "Thickness" + to string((int)/missionDuration/3608))*" hr.csv":
  writeVectorsToCSV(filename, zValues, insThick);
```

## **OBJ Scaling and Thickness Parsing**

TEXAS A&M

- Scale the 2D robot dimensions to the 2.5 m height
- Interpolate to find the thickness at each height
  - Uses csv with height vs thickness
- Optimization: Error handling for input obj

4	Α	В			
1	z value	thickness (cm)	24	22	7.10805
2	0	21.7333	25		
3	0.1	8.18621	25	2.3	9.16082
4	0.2	9.30092	26	2.4	12.5963
5	0.3	10.9029	27	2.5	20.5089

```
1 # Blender v2.71 (sub 0) OBJ File: 'Man.blend'
2 # www.blender.org
3 mtllib Man.mtl
4 o Human_Cylinder
5 v 0.271502 3.129741 0.0
6 v 0.341832 4.004711 0.0
7 v 0.417949 3.129741 0.0
8 v 0.488278 4.004711 0.0
9 v 0.271502 3.129741 0.0

1 v 0.128951 1.48648 0.171449
2 v 0.128951 1.48648 0.171449
4 v 0.23191 1.90205 0.114159
5 v 0.128951 1.48648 0.171449
Updated OBJ file
```

```
// Generate scaled/thickened vertices
for (const auto& v : original_vertices) {
    // Normalize y to [0, 2.5] meters
    double normalized_y_m = ((v.y - y_min) / (y_max - y_min)) * 2.5;
    double t_cm = InterpolateThickness(normalized_y_m, thickness_data);
    double t_m = t_cm / 100.0; // Convert to meters

    // Scale x and y; apply thickness to z
    Vertex front = {v.x * scale, v.y * scale, v.z + t_m};
    Vertex back = {v.x * scale, v.y * scale, v.z};

    front_vertices.push_back(front);
    back_vertices.push_back(back);
}
```

```
// ------ Interpolate Thickness ------
// Given a y-position, return the interpolated thickness from the CSV data
double InterpolateThickness(double y, const std::vector<ThicknessEntry>& data) {
    if (data.empty()) return 0.0;

    // Clamp to the bounds
    if (y <= data.front().y_m) return data.front().thickness_cm;
    if (y >= data.back().y_m) return data.back().thickness_cm;

    // Linear interpolation
    for (size_t i = 0; i < data.size() - 1; ++i) {
        if (y >= data[i].y_m && y <= data[i + 1].y_m) {
            double y0 = data[i].y_m;
            double t0 = data[i].thickness_cm;
            double y1 = data[i + 1].y_m;
            double t1 = data[i + 1].thickness_cm;
            return t0 + (t1 - t0) * (y - y0) / (y1 - y0);
        }
    }
    return data.back().thickness_cm;
}</pre>
```

# Thermal Insulation Layer Mass



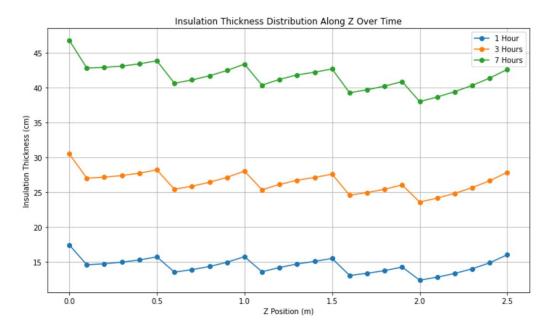
- Uses scaled dimensions obj file
- Calculates area at each face
  - Splits quadrilaterals into 2 triangles
- Computes mass at each face
  - m = Area (m^2) x thickness (m) x density (kg/m^3)
- Add up total mass across all faces
  - 1 hr: 49.3702 kg
     3 hrs: 90.796kg
  - o 7 hrs: 143.202 kg

```
// Handle triangle face.
if (face.indices.size() == 3) {
  const Vertex& v1 = vertices[face.indices[0]];
 const Vertex& v2 = vertices[face.indices[1]];
 const Vertex& v3 = vertices[face.indices[2]];
  face area = TriangleArea2D(v1, v2, v3);
 avg thickness = (v1.z + v2.z + v3.z) / 3.0f;
// Handle quadrilateral face (split into 2 triangles)
else if (face.indices.size() == 4) {
  const Vertex& v1 = vertices[face.indices[0]];
  const Vertex& v2 = vertices[face.indices[1]];
  const Vertex& v3 = vertices[face.indices[2]];
  const Vertex& v4 = vertices[face.indices[3]];
  float area1 = TriangleArea2D(v1, v2, v3);
  float area2 = TriangleArea2D(v1, v3, v4);
  face area = area1 + area2;
 avg thickness = (v1.z + v2.z + v3.z + v4.z) / 4.0f;
// Calculate volume and mass.
float volume = face area * avg thickness;
float mass = volume * density;
total mass += mass;
```

## **Insulation Thickness Profile Plot**



- Thickening varies along z-position, following a periodic pattern.
- Longer submersion time results in greater thickness.
- Average thicknesses for each time interval:
  - o 1 hr: 14.43 cm
  - o 3 hr: 26.43 cm
  - o 7 hr: 41.59 cm

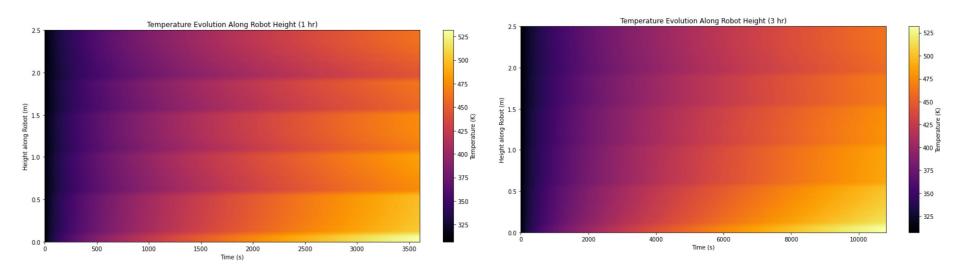


# Temperature Evolution Along Robot Height



1 hr:

3 hr:

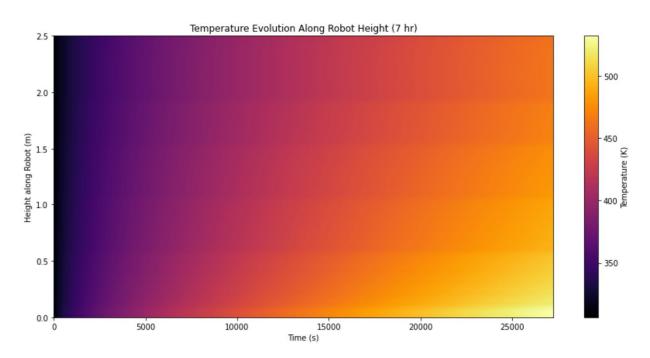


# Temperature Evolution Along Robot Height

0 0 0



# 7 hr:



#### **Visualization-Thickness**







- View modes: Face, Wireframe, Material.
- Material mode visualizes the total thickness with colormap.

```
Colorbar
Max:
9.24
Min:
8.16
```

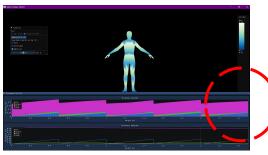
```
for (int i = 0; i < V.size(); i++) {
    const auto& v = V[i];
    float normalized_y_at_v = (v.y - minY) / (maxY - minY);
    float tps_at_v = interp1D(tps_metric_stacked, normalized_y_at_v);

    float normalized_tps_at_v = Normalize(tps_at_v, tps_metric_stacked);
    vertex_colors[i] = InterpolateColormapToVec3(cmap, normalized_tps_at_v);
}

glBindBuffer(GL_ARRAY_BUFFER, CBO);
glBufferSubData(GL_ARRAY_BUFFER, 0, vertex_colors.size() * sizeof(glm::vec3),</pre>
```

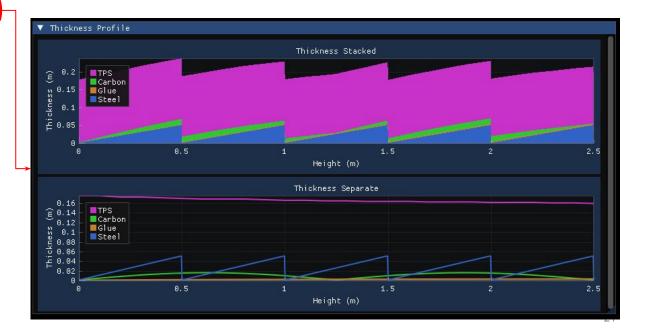
#### **Visualization-Thickness**





```
ImGui::Begin("Thickness Profile");
if (ImPlot::BeginPlot("Thickness Stacked", ImVec2(-1, plc
    ImPlot::SetupAxisLimits(ImAxis Y1, 0.0f, *std::max el
    ImPlot::SetupAxisLimits(ImAxis_X1, robotLength, 0.0f,
    ImPlot::SetupAxis(ImAxis_Y1, "Thickness (m)");
    // ImPlot::SetupAxis(ImAxis_X1, "Height (m)", ImPlotA
    ImPlot::SetupAxis(ImAxis_X1, "Height (m)");
    ImPlot::PushStyleColor(ImPlotCol_Fill, magenta);
    ImPlot::PlotShaded("TPS", heights.data(), tps_metric_
    ImPlot::PopStvleColor();
    ImPlot::PushStvleColor(ImPlotCol Fill, green):
    ImPlot::PlotShaded("Carbon", heights.data(), carbon_m
    ImPlot::PopStyleColor();
    ImPlot::PushStyleColor(ImPlotCol_Fill, orange);
    ImPlot::PlotShaded("Glue", heights.data(), glue_metri
    ImPlot::PopStyleColor();
    ImPlot::PushStvleColor(ImPlotCol Fill, blue):
    ImPlot::PlotShaded("Steel", heights.data(), steel_met
    ImPlot::PopStvleColor():
```

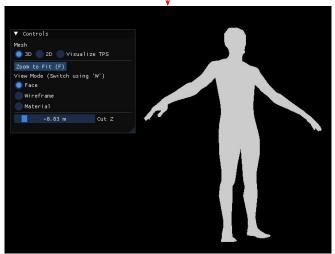
 Material thickness is plotted both separately and stacked.



#### Visualization-GUI







**Menu** for interaction with GUI.

**F** key zooms the model to fit.

**W** key toggles between view modes.

**Mouse** pan, rotate, zoom supported. **Slider** for the cutting plane.

### **Visualization-Error Handling**



```
std::vector<float> loadProfile1(const std::string& fname) {
      std::vector<float> vals:
      std::ifstream in(fname);
      if (!in) {
           std::cerr << "[CSV] ERROR: Cannot open file " << fname << "\n":
           return vals;
      std::string line;
      if (!std::getline(in, line)) {
           std::cerr << "[CSV] ERROR: File " << fname << " is empty or inva
           return vals;
// Check material vectors
if (carbon.empty() || glue.empty() || steel.empty()) {
   std::cerr << "[CSV] ERROR: One or more thickness CSV files are empty.\n":
   return 1; // Exit or handle the error
if (carbon.size() != glue.size() || glue.size() != steel.size()) {
   std::cerr << "[CSV] ERROR: Thickness CSV files have inconsistent sizes.\n";
   return 1: // Exit or handle the error
   if (!(ss >> z >> comma >> t) || comma != ',') {
      throw std::runtime_error("Invalid CSV format (expected 'z,t').");
  thermZ.push_back(z);
  thermT.push_back(t);
catch (const std::exception& e) {
  std::cerr << "[CSV] WARNING: Skipping invalid line: " << line << " (" << e.wha
  continue;
```

**Hard stop** if the file cannot be opened.

**Soft skip** for individual malformed lines viewer continues with the rest.

Mesh upload path filters invalid face indices.

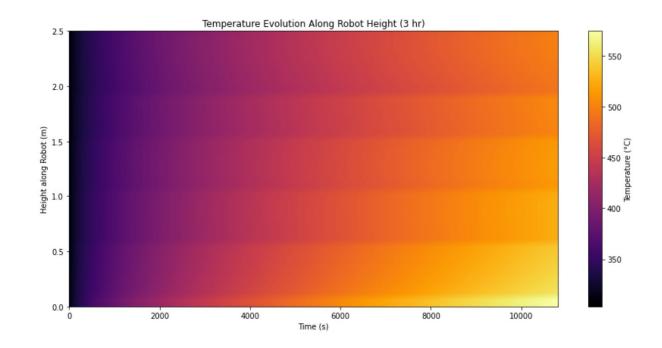
Similar pattern is used for CSV and OBJ loading and empty-vector size checks.

#### **Final Simulation Exercise - Results**



**Total mass of insulation for 3 hours:** 72.8 kg

Average thickness of insulation for 3 hours: 21.3015 cm



# **Final Simulation Exercise - Results**



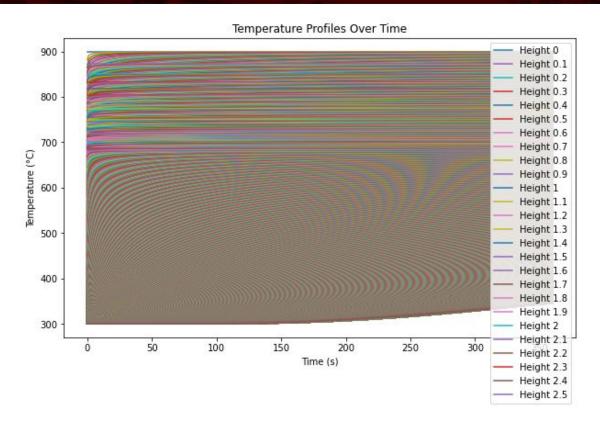


# Temperature Profile Plot for 1 Hr



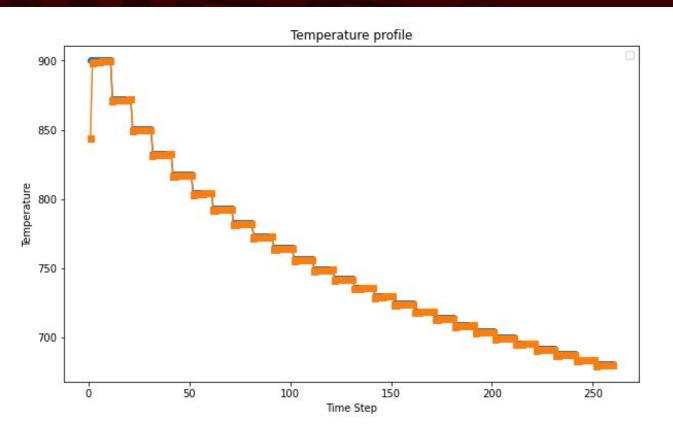
 Each z-value block reaches steady state at different times.

0 0 0



# Temperature profile at 0 m z value







Member	Contributions		
Nishi Mishra	Matrix math for solving heat equation		
Layan Samandar	OBJ scaling, thickness parsing, and thermal insulation mass calculation, insulation thickness plot		
Parina Patel	OBJ scaling, thickness parsing, and thermal insulation mass calculation, insulation thickness plot		
Hassan Niaz	1D heat equation solution - Algorithm selection, implementation and everything in between		
Bihao Zhang	Visualization - GUI, thickness plots, color Map CMakeLists, Error Handling		
Naveen Jagadeesan	Visualization - GUI Interactive Viewer, Rendering the thickness plots in the viewer, Color Map, Error Handling		



# Thank You!

**Any Questions?**