① Problem Statement 1 :

$$\text{Mean} = \frac{\Sigma x_i}{N} = \frac{137}{20} = 6.85$$

Sorted dataset : 4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10

$$\text{Median} = \frac{7+7}{2} = 7$$

Mode = 7

$$\text{Standard deviation} = \sqrt{\frac{\Sigma (x_i - m)^2}{N}} = \sqrt{\frac{(4-6.85)^2 + (4-6.85)^2 + \cdots + (10-6.85)^2}{35}}$$

$= 1.59$ //

② Problem Statement 2 :

$$\text{Mean} = \frac{\Sigma x_i}{N} = \frac{3763}{35} = 107.51 \text{ //}$$

Sorted dataset : 28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90, 97, 97, 100, 100, 100, 104, 104, 109, 113, 120, 120, 120, 122, 123, 123, 130, 140, 145, 170, 174, 194, 217

Median = 100

Mode = 75

$$\text{Standard deviation} = \sqrt{\frac{\Sigma (x_i - m)^2}{N}} = \sqrt{\frac{(28-107.51)^2 + (40-107.51)^2 + \cdots + (217-107.51)^2}{35}}$$

$= 38.77$ //

③ Prob statement 3 :

$$\text{Mean} = E(X) = \Sigma x\, f(X) = 0\times0.09 + 1\times0.15 + 2\times0.4 + 3\times0.25 + 4\times0.10 + 5\times0.01$$

$= 2.15$

$$E(X^2) = \Sigma x^2 f(X) = 0^2\times0.09 + 1^2\times0.15 + 2^2\times0.4 + 3^2\times0.25 + 4^2\times0.10 + 5^2\times0.01$$

$= 5.85$

$$\text{Variance} = E(X^2) - (E(X))^2$$

$= 5.85 - 2.15^2 = 1.2275$

(5) Problem Statement 5:

$$p = \frac{30}{100} = 0.3$$

$$n = 6 \qquad P(x) = \frac{n!}{(n-x)!\,x!}(1-p)^{n-x}p^{x}$$

Prob of having 2 faulty LEDs in my sample =

$$P(x=2) = \frac{6!}{4!\,2!}(0.7)^4 \times (0.3)^2 = 0.324$$

$$P(x=1) = \frac{6!}{5!\,1!}(0.7)^5 \times (0.3)^1 = 0.302$$

$$P(x=0) = \frac{6!}{6!\,0!}(0.7)^6 \times (0.3)^0 = 0.117$$

$$P(x=3) = \frac{6!}{3!\,3!}(0.7)^3 \times (0.3)^3 = 0.185$$

$$P(x=4) = \frac{6!}{2!\,4!}(0.7)^2 \times (0.3)^4 = 0.059$$

$$P(x=5) = \frac{6!}{1!\,5!}(0.7)^1 \times (0.3)^5 = 0.010$$

$$P(x=6) = \frac{6!}{0!\,6!}(0.7)^0 \times (0.3)^6 = 0$$

$$\text{Mean} = E(x) = \sum x P(x)$$

$$= 0\times 0.117 + 1\times 0.302 + 2\times 0.324 + 3\times 0.185 + 4\times 0.059 + 5\times 0.010 + 6\times 0 = 1.791$$

$$E(x^2) = \sum x^2 P(x) = 4.457$$

$$\text{Variance} = E(x^2) - (E(x))^2 = 1.249$$

$$\text{standard deviation} = \sqrt{\text{Variance}} = 1.117$$

(6) Problem Statement 6:

$$n_1 = 8 \qquad n_2 = 12$$

$$p_1 = \frac{75}{100} = 3/4 \qquad p_2 = \frac{45}{100} = 9/20$$

$$P(x_1 = 5) = \frac{8!}{3!\,5!}(1-3/4)^3\,(3/4)^5 = 0.207$$

$$P(x_2 = 5) = \frac{12!}{7!\,5!}(1-9/20)^7\,(9/20)^5 = 0.222$$

$$P(x_1=4) = \frac{8!}{4!\,4!}(1-3/4)^4 (3/4)^4 = 0.086$$

$$P(x_2=4) = \frac{12!}{8!\,4!}(1-9/20)^8 (9/20)^4 = 0.169$$

$$P(x_1=6) = \frac{8!}{2!\,6!}(1-3/4)^2 (3/4)^6 = 0.311$$

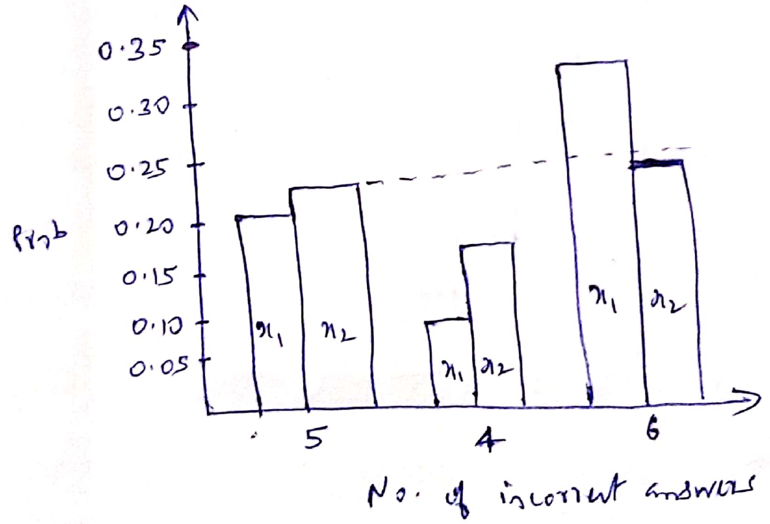
$$P(x_2=6) = \frac{12!}{6!\,6!}(1-9/20)^6 (9/20)^6 = 0.212$$

Inference

$$P(x_1=4) < P(x_1=5) < P(x_1=6)$$

$$P(x_2=4) < P(x_2=6) < P(x_2=5)$$

Correction rate and Number of questions they attempts per day are factors affecting their ability to solve questions correctly.



(7) Problem Statement 7 :

Prob of k customers arriving in 4 mins = P(k)

$$M = \frac{72}{60} \times 4 = 4.8$$

$$P(k) = \frac{e^{-4.8}(4.8)^k}{k!}$$

a) $P(k=5) = \frac{e^{-4.8}(4.8)^5}{5!} = 0.174$

b) $P(k \leq 3) = P(k=0) + P(k=1) + P(k=2) + P(k=3)$
$= 0.008 + 0.039 + 0.094 + 0.151 = 0.292$

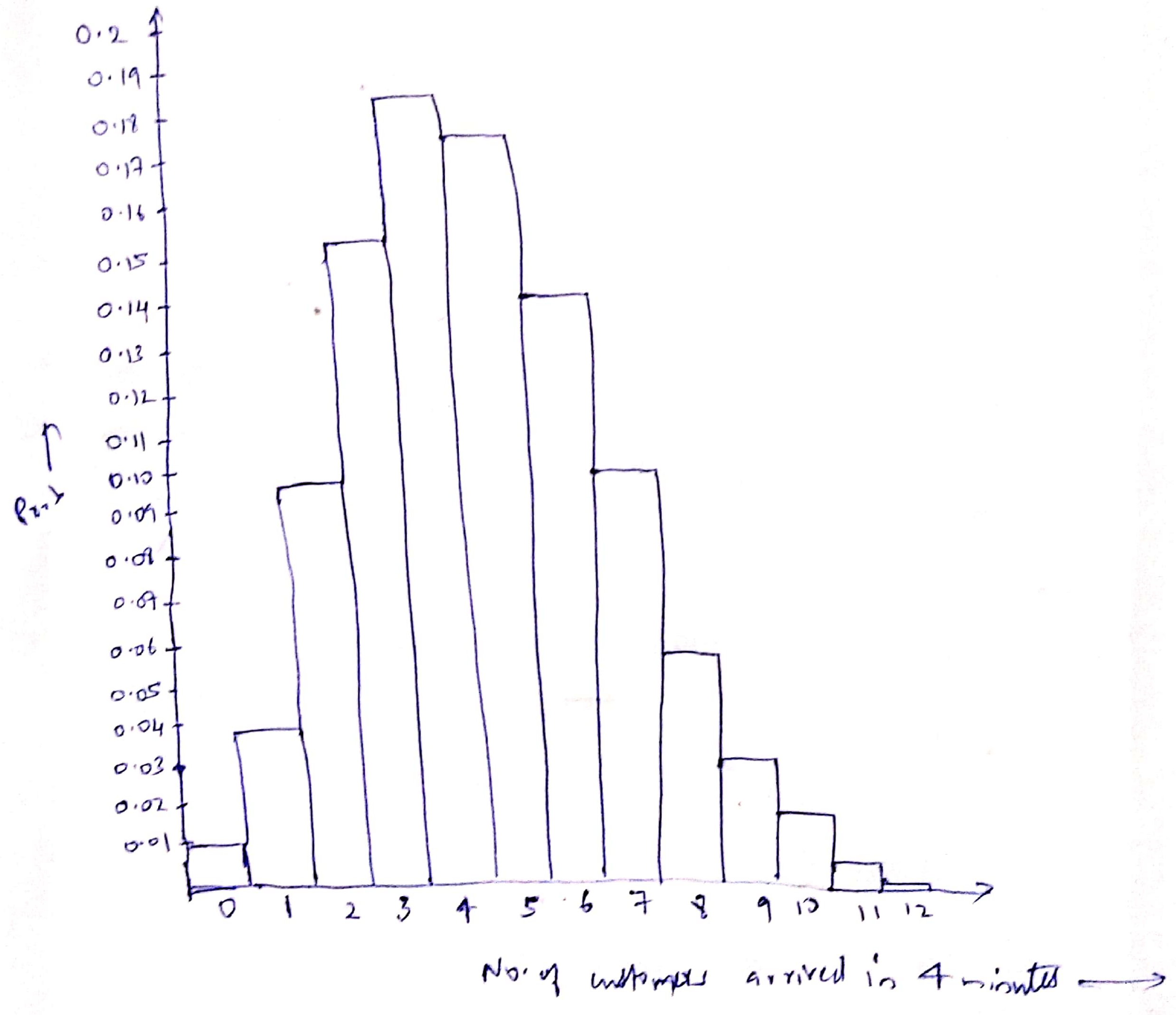
c) $P(k>3) = 1 - P(k \le 3)$

$= 1 - 0.292 = 0.708$ //

$P(k=4) = 0.182$ $P(k=5) = 0.174$ $P(k=6) = 0.139$

$P(k=7) = 0.095$ $P(k=8) = 0.057$ $P(k=9) = 0.030$

$P(k=10) = 0.014$ $P(k=11) = 0.006$ $P(k=12) = 0.002$ $P(k=13) = 0$



(8) Problem Statement 9:

455 words report

Time taken to type 455 words $= \frac{455}{77} = \frac{65}{11}$ minutes

$\lambda = \frac{6}{60} \times \frac{65}{11} = 0.59$

$P(x=2) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.59}(0.59)^2}{2!} = 0.096$ //

<u>1000 words report</u>

Time taken to type 1000 words $= \frac{1000}{77}$ minutes

$\lambda = \frac{6}{60} \times \frac{1000}{77} = 1.298$

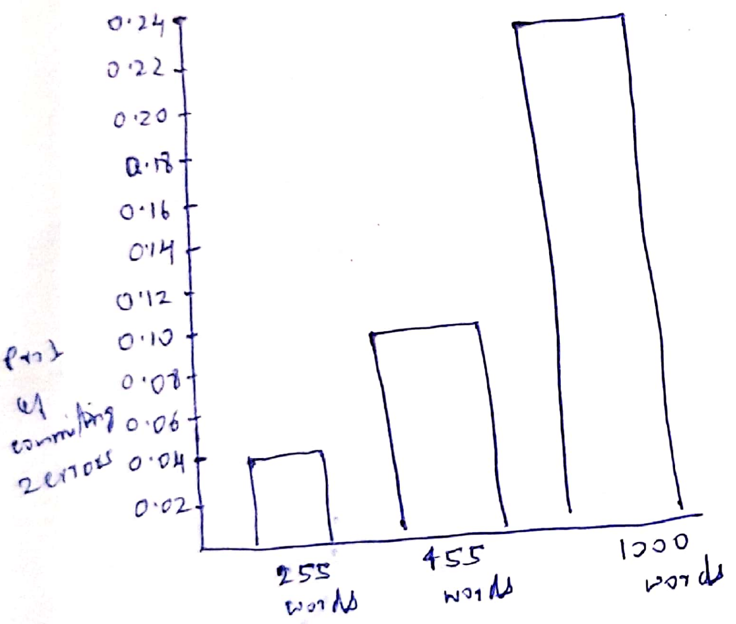
$P(x=2) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-1.298}(1.298)^2}{2!} = 0.23$ //

<u>255 words report</u>

Time taken to type 255 words $= \frac{255}{77}$ minutes

$\lambda = \frac{6}{60} \times \frac{255}{77} = 0.331$

$P(x=2) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.331} \times (0.331)^2}{2!} = 0.039$ //



(10) Problem Statement 10 :

a) $P(Z > 1.26) = 1 - P(Z \le 1.26) = 1 - 0.8962 = 0.1038$

$P(Z < 0.86) = 0.1949$

$P(Z > -1.37) = 1 - P(Z \le -1.37) = 1 - 0.0853 = 0.9147$

$P(-1.25 < Z < 0.37) = P(Z < 0.37) - P(Z \le -1.25)$

$= 0.6443 - 0.1056 = 0.5387$

$P(Z \le -4.6) = 0$

b) $P(Z > z) = 0.05$

$\Rightarrow 1 - P(Z \le z) = 0.05 \Rightarrow P(Z \le z) = 1 - 0.05 = 0.95$

$z = 1.65$ //

c) $P(-z < Z < z) = 0.99$

$1 - 2P(Z \leq -z) = 0.99$

$P(Z \leq -z) = 0.005$

$z = 3.3$ //

(11)

Problem Statement - 11

$I \rightarrow$ current in mA

$\mu = 10mA \quad \sigma = 2mA$

$P(I > 13) = P(Z > z)$

$z = \frac{x-\mu}{\sigma} = \frac{13-10}{2} = 3/2 = 1.5$

$P(z > 1.5) = 1 - P(Z \leq 1.5) = 1 - 0.9332 = 0.0668$

$P(9 < I < 11) = P(z_1 < z < z_2)$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{9-10}{2} = -1/2 = -0.5$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{11-10}{2} = 1/2 = 0.5$

$P(-0.5 < Z < 0.5) = P(z < 0.5) - P(z \leq -0.5)$

$= 0.6915 - 0.3085$

$= 0.3830$

$\times$ Next question is "determine the current measurements which has a probability of 0.98"

Which means value of current for which its ~~propt~~ probability of occurring is 0.98

Since current follows normal distribution

Pdf of normal distribution, $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$

$P(x) = 0.98 \qquad$ Let $\frac{x-\mu}{\sigma} = z$

$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2 z^2} = 0.98$

$e^{-1/2 z^2} = 0.98 \times \sqrt{2\pi \times 4}$

$e^{-z^2/2} = 4.913$

$\frac{-z^2}{2} = \ln(4.913)$

$z^2 = -3.18$

∴ There is no real number of current measurement for which its probability value is 0.98

∴ Suppose if the question meant " Determine the value of current for which the probability of current measurement below this value is 0.98

Then $P(I \leq x) = P(Z \leq z) = 0.98$

$z = 2.06$

$$\frac{x - \mu}{\sigma} = 2.06$$

$$\frac{x - 10}{2} = 2.06 \Rightarrow x = 14.12 \text{ mA}$$

(12) Problem Statement 12 :

$\mu = 0.2508$ inch

$\sigma = 0.0005$ inch

Specification of shaft

lower limit $= 0.2500 - 0.0015$
$= 0.2485$

upper limit $= 0.2500 + 0.0015$
$= 0.2515$

Proportion of shaft in sync with the specifications =

$P(0.2485 < X < 0.2515) = P(z_1 < z < z_2)$

$$z_1 = \frac{0.2485 - 0.2508}{0.0005} = -4.6$$

$$z_2 = \frac{0.2515 - 0.2508}{0.0005} = 1.4$$

$P(-4.6 < z < 1.4) = P(z < 1.4) - P(z \leq -4.6)$
$= 0.9192 - 0 = 0.9192$ //

If the process is centered

→ $\mu_{new} = 0.2500$ inch $\sigma = 0.0005$ inch

Proportion of shaft that conform to specification =

$P(0.2485 < X < 0.2515) = P(z_1 < z < z_2)$

$$z_1 = \frac{0.2485 - 0.2500}{0.0005} = -3 \qquad z_2 = \frac{0.2515 - 0.2500}{0.0005} = 3$$

$P(-3 < z < 3) = P(z < 3) - P(z \leq -3) = 0.9987 - 0.0013$
$= 0.9974$

Proportion of shaft is (7) increased from 91.92% to 99.74%

④ + ⑨

Problem Statement 4 & Problem Statement 9

$P(X) = 20 e^{-20(X-12.5)}, X \geq 12.5$

$P(X \leq 12.6) = \int_{12.5}^{12.6} 20 e^{-20(x-12.5)} dx$

$= \left. \frac{20 e^{-20(x-12.5)}}{-20} \right|_{12.5}^{12.6} = 1 - e^{-20\times0.1} = 0.864$

~~P(X ≥ 12.6) = 1 - P~~

$P(12.5 \leq X \leq \infty) = \int_{12.5}^{\infty} 20 e^{-20(x-12.5)} dX$

$= \left. \frac{20 e^{-20(X-12.5)}}{-20} \right|_{12.5}^{\infty} = -e^{-\infty} + e^{-0}$

$= 0 + 1 = 1$ //

$P(X > 12.6) = P(12.5 \leq X < \infty) - P(X \leq 12.6)$

$= 1 - 0.864 = 0.136$ //

Proportion of those scrap parts = 0.136 //

~~CDF when d = 11 mm~~ =

$CDF_{d=11} + CDF_{11<d<12.5} + CDF_{12.5 \leq d < \infty} = 1$

$CDF_{d=11} + CDF_{11<d<12.5} + 1 = 1$

$CDF_{d=11} + CDF_{11<d<12.5} = 0$

$CDF_{d=11} = -CDF_{11<d<12.5}$

Since $CDF_{d=11}$ is negative of $CDF_{11<d<12.5}$

and CDF can't be negative

$\Rightarrow CDF_{d=11} = 0$ & $CDF_{11<d<12.5} = 0$

∴ CDF when diameter is 11 mm = 0

Conclusion: Minimum diameter of the hole that can be achieved in this experiment is 12.5mm and maximum can be any value.