Fuzzy Pooling in Convolutional Neural Networks for Improved Image Classification

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***Abstract*—This paper introduces a novel pooling operation based on fuzzy logic to improve Convolutional Neural Network (CNN) performance in handling local imprecision within feature maps. The proposed fuzzy pooling method replaces traditional pooling layers with fuzzy membership functions to better capture feature uncertainties. Experimental results on the Fashion MNIST dataset highlight its potential in achieving higher classification accuracy than conventional max pooling.**

***Index Terms*—CNN, Fuzzy Pooling, Image Classification, Membership Functions, Fashion MNIST.**

I. Introduction

Convolutional Neural Networks (CNNs) have revolutionized image classification and recognition. They employ convolution operations for feature extraction and pooling layers for dimensionality reduction. Traditional pooling methods like max pooling often assume crisp boundaries between features, which can lead to loss of critical information in scenarios with high local variability. To address this, we explore a fuzzy-based pooling approach that leverages type-1 fuzzy sets to handle local uncertainties in feature maps.

II. Methodology

The fuzzy pooling method is designed to replace traditional crisp pooling with a layer that integrates fuzzy membership functions. This method provides a more flexible representation of feature intensity levels within each patch. The process consists of three main stages: fuzzification, aggregation, and defuzzification.

A. Fuzzification

Fuzzification converts the crisp input values in the feature map into fuzzy values using predefined membership functions. We define three membership functions—small, medium, and large—each of which captures different intensity levels. These functions are defined as follows:

- **Small Membership Function:** High membership values are assigned to inputs close to zero, indicating a strong association with the "small" set. For example, using triangular membership functions for the fuzzy sets, the membership function μ_1 for a small value can be defined as:

$$\mu_1(p_{i,j}^n) = \begin{cases} 0, & p_{i,j}^n > d, \\ \frac{d - p_{i,j}^n}{d - c}, & c \le p_{i,j}^n \le d, \\ 1, & p_{i,j}^n < c \end{cases}$$

  where $d = \frac{r_{\max}}{2}$ and $c = \frac{d}{3}$.

- **Medium Membership Function:** Values close to the midpoint of intensity (e.g., 3.0) are assigned high membership in the "medium" set.

$$\mu_2(p_{i,j}^n) = \begin{cases} 0, & p_{i,j}^n \le a, \\ \frac{p_{i,j}^n - a}{m - a}, & a < p_{i,j}^n \le m, \\ \frac{b - p_{i,j}^n}{b - m}, & m < p_{i,j}^n < b, \\ 0, & p_{i,j}^n \ge b \end{cases}$$

  where $a = \frac{r_{\max}}{4}, \quad m = \frac{r_{\max}}{2}, \quad b = m + a = \frac{3\,r_{\max}}{4}$

- **Large Membership Function:** Inputs with high numerical values are assigned high membership in the "large" set.

$$\mu_3(p_{i,j}^n) = \begin{cases} 0, & p_{i,j}^n < r, \\ \frac{p_{i,j}^n - r}{q - r}, & r \le p_{i,j}^n \le q, \\ 1, & p_{i,j}^n > q \end{cases}$$

  where $r = \frac{r_{\max}}{2}$ and $q = r + \frac{r_{\max}}{4} = \frac{3\,r_{\max}}{4}$

B. Aggregation

Aggregation combines membership values from different sets to produce a single fuzzy representation for each patch. We use the sum operator in fuzzy aggregation to capture the overall degree of membership across the small, medium, and large sets, allowing for a nuanced representation of feature intensities.

After fuzzification, fuzzy patches are aggregated using the fuzzy algebraic sum:

$$s_{\pi_v}^n = \sum_{i=1}^{k} \sum_{j=1}^{k} \mu_v(p_{i,j}^n)$$

where $s_{\pi_v}^n$ represents the membership score of each patch to a fuzzy set $\tilde{v}$. The patch with the highest membership score is selected, forming a new fuzzy volume patch π':

$$\pi' = \{\pi_v'^n = \pi_v^n \mid v = \arg\max s_{\pi_v}^n,\ n = 1, 2, \ldots, z\}$$

C. Defuzzification

To reduce dimensionality, the aggregated fuzzy values are defuzzified. This study utilizes the Center of Gravity (COG) method, which produces a single scalar value representing the fuzzy set. This defuzzified output is then passed to the next layer in the CNN for further processing.

The dimensionality of each patch is reduced through defuzzification using the Center of Gravity (CoG) method:

$$p_n' = \frac{\sum_{i=1}^{k} \sum_{j=1}^{k} \pi_{i,j}'^n \cdot p_{i,j}^n}{\sum_{i=1}^{k} \sum_{j=1}^{k} \pi_{i,j}'^n}$$

where $p' = \{p'_n \mid n = 1, 2, \ldots, z\}$ represents the defuzzified values of each patch.

III. Experimental Setup

We evaluate the effectiveness of fuzzy pooling on the Fashion MNIST dataset, a collection of 28x28 grayscale images of clothing items across 10 classes. The dataset was normalized, and the fuzzy pooling layer was incorporated into a standard CNN architecture in place of max pooling layers.

IV. Results

We compared the classification accuracy of the CNN using fuzzy pooling against traditional max pooling. The results indicate an improvement in test accuracy with fuzzy pooling, highlighting its ability to preserve nuanced information in the feature maps and enhance the CNN's classification performance on Fashion MNIST.

Model Training Accuracy Metrics Using Fuzzy Pooling (Experiment 1)

TABLE I
Training and Validation Accuracies Over Epochs (Fuzzy Pooling)

Epoch	Training Accuracy	Validation Accuracy
1	73.46%	85.16%
2	86.52%	86.88%
3	88.64%	87.82%
4	89.68%	88.48%
5	90.49%	89.52%
6	91.33%	89.85%
7	91.91%	90.03%
8	92.46%	90.65%
9	93.10%	90.97%
10	93.42%	90.74%

Model Training Accuracy Metrics Using Max Pooling (Experiment 2)

TABLE II
Training and Validation Accuracies Over Epochs (Max Pooling)

Epoch	Training Accuracy	Validation Accuracy
1	70.60%	87.97%
2	87.26%	87.09%
3	89.35%	90.74%
4	90.71%	91.49%
5	91.42%	91.18%
6	92.32%	92.56%
7	92.58%	92.34%
8	93.74%	93.12%
9	93.91%	92.10%
10	94.21%	93.52%

Final Accuracy Interpretation

The final model accuracy achieved on the test dataset from the first experiment (Fuzzy Pooling) is **90.74%**, while the second experiment (Max Pooling) achieved **93.52%**. This indicates that both models perform well in making predictions on unseen data.

- **Fuzzy Pooling**: The training accuracy of **93.42%** demonstrates effective learning, with a relatively small gap between training and validation accuracies, suggesting good generalization. Notably, the loss decreases more rapidly with fuzzy pooling, indicating that the model converges more quickly during training. This trend suggests that if training were to continue beyond 10 epochs, fuzzy pooling could potentially lead to even better performance compared to max pooling.

- **Max Pooling**: The training accuracy of **94.21%** indicates robust learning, maintaining high validation accuracy of **93.52%**. However, the loss reduction is less pronounced compared to fuzzy pooling, which may imply that the model could benefit from further adjustments or iterations to reach optimal performance.

In summary, the use of **fuzzy pooling** in the first experiment appears to enhance the model's generalization capabilities by allowing it to maintain important features while discarding irrelevant information. The rapid loss decrease suggests that fuzzy pooling may outperform max pooling in the long run, making it a strong candidate for future applications where extended training is possible.

V. Discussion

Fuzzy pooling demonstrates significant potential in enhancing CNN performance by capturing uncertainties in feature intensities. This method allows for more flexible feature representation, which is particularly beneficial in scenarios where local variations are high. However, further work is needed to optimize membership function design for different datasets and to assess scalability in larger architectures.

VI. Conclusion

This paper presented fuzzy pooling as an alternative to traditional pooling in CNNs. By leveraging fuzzy membership functions, fuzzy pooling can handle feature uncertainties more effectively, resulting in improved classification accuracy. Future research could explore adaptive membership functions tailored to specific data distributions.

References

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