

# Logistic Regression (classification problem)

## IIT JEE Example

Study	Play hrs	o/p (Pass/Fail)
1	8	Fail
2	7	Fail
3	7	Fail
6	3	Pass
7	2	Pass
6	4	Pass
5	3	Pass

□ → outliers.

## Data Set

Study hours

o/p (Pass/Fail)

UPSC

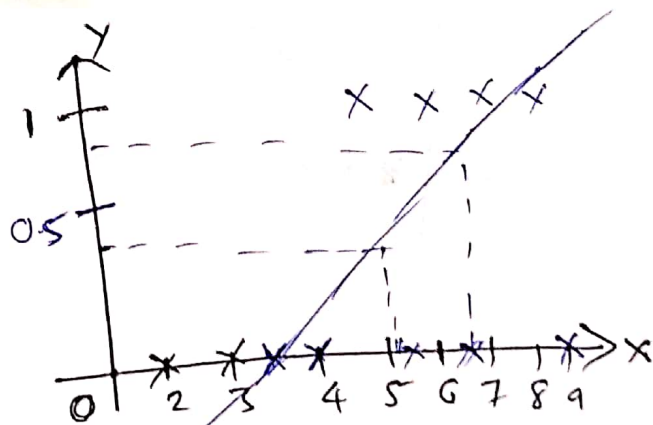
1 → Pass

0 → Fail

① Can we solve this program

Using regression.

2	Fail
3	Fail
4	Fail
5	Pass
6	Pass
7	Pass
8	Pass
9	Pass



## Regression

0.5  $\Rightarrow$  Threshold

$$y \leq 0.5 = 0$$

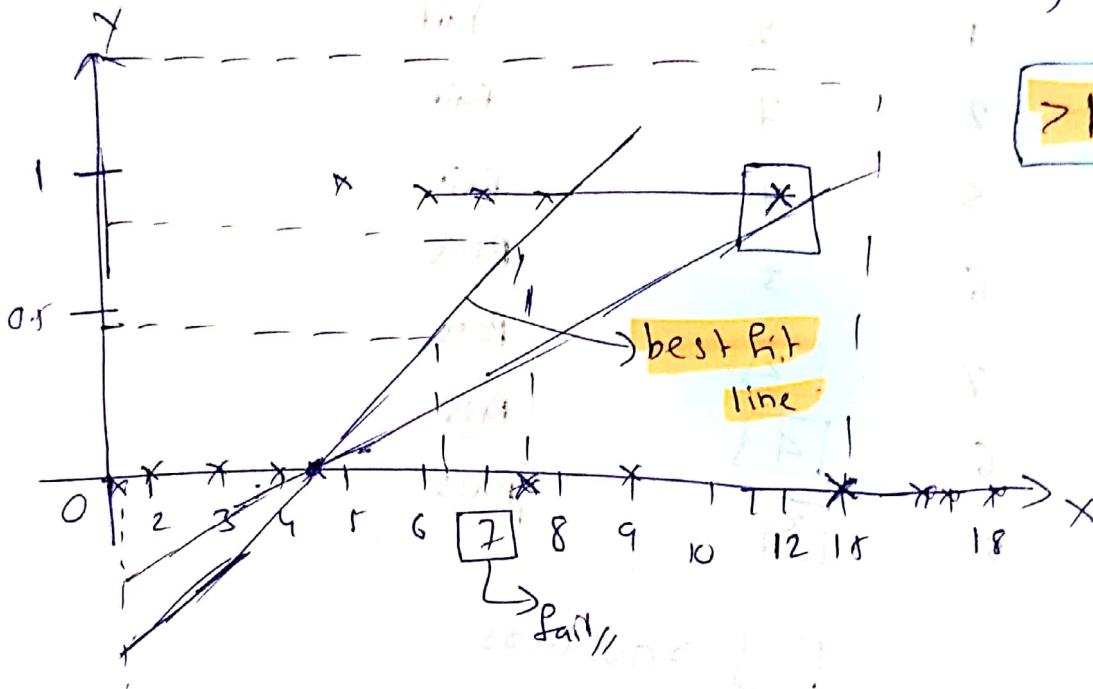
$$y > 0.5 = 1$$

## Regression

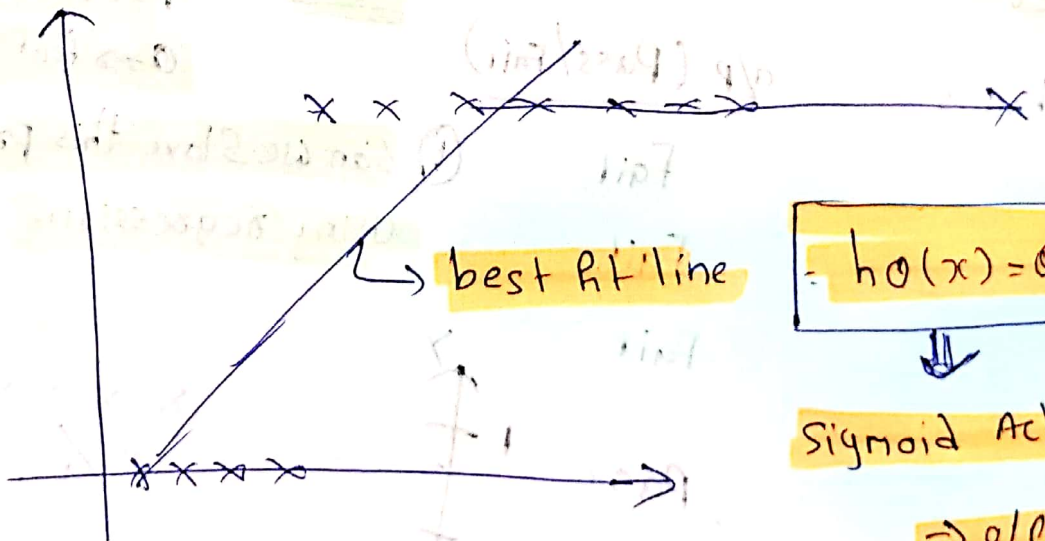
$$y \leq 0.5 = 0$$

$$y > 0.5 = 1$$

$$y \neq 0$$



## Sigmoid Activation



$$h_0(x) = \theta_0 + \theta_1 x$$

Sigmoid Activation

$$\Rightarrow 0/p = 0$$

$$① Z = h_0(x) = \theta_0 + \theta_1 x$$

Sigmoid  $\sigma$

$$= \frac{1}{1 + e^{-z}} \Rightarrow \text{0 to 1}$$

$$Z = \theta_0 + \theta_1 x$$

## Linear Regression Cost Function

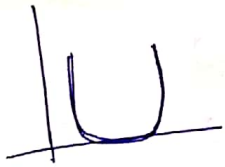
$$J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_0(x^{(i)}) - y^{(i)})^2$$

$$h_0(x) = \theta_0 + \theta_1 x$$

MSE

Convex Function

1 global minima



① Create a best fit line

② Squashing  $\rightarrow$  Sigmoid function

$$\sigma = \frac{1}{1 + e^{-z}}$$

$$Z = \theta_0 + \theta_1 x$$

## Logistic Regression Cost Function

$$J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_0(x) - y^{(i)})^2$$

$$h_0(x) = \sigma(\theta_0 + \theta_1 x) \rightarrow \text{best fit line.}$$

Sigmoid Activation

$$= \sigma(z)$$



$$= \sigma(z)$$

$$= \frac{1}{1 + e^{-z}}$$

$$h_0(x) = \frac{1}{1 + e^{-z}}$$

$$h_0(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}$$

$\Rightarrow 0 \text{ to } 1$   $\Rightarrow$  out will be b/w (0 to 1)

$\leq 0.5 \Rightarrow 0 \Rightarrow$  fail

$> 0.5 \Rightarrow 1 \Rightarrow$  Pass

Threshold = 0.5

0  $\Rightarrow$  fail

1  $\Rightarrow$  Pass

let  $\Rightarrow 0.35 \Rightarrow 0$

$0.25 \Rightarrow 0$

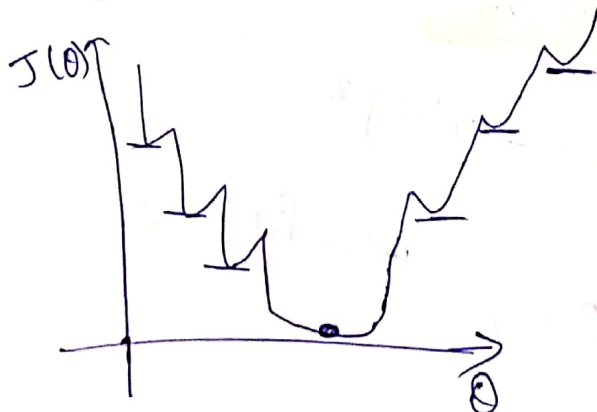
$0.95 \Rightarrow 1$

$0.5 \Rightarrow 0$

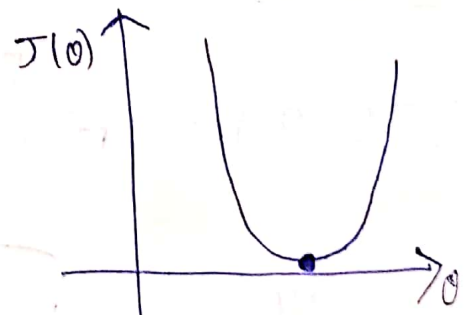
$0.54 \Rightarrow 1$

1  $\Rightarrow 0.7$

Non-convex function



Convex function



\* log loss cost function

$$Cost(h_0(x^{(i)}, y^{(i)})) = \begin{cases} -\log(h_0(x)) & \text{if } y = 1 \\ -\log(1-h_0(x)) & \text{if } y = 0 \end{cases}$$

$$h_0(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}$$

$$cost(h_0(x^{(i)}, y^{(i)})) = -y \log(h_0(x)) - (1-y) \log(1-h_0(x))$$

→ convex function

Never get local minima

minimize cost function  $J(\theta_0, \theta_1)$  by changing  $\theta_0, \theta_1$

converging algorithm

Repeat converging

$j = 0$  and  $1$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

Threshold = 0.5

# Performance metrics



① Confusion matrix

② Accuracy

③ Precision

④ Recall

⑤ F-beta Score

Data Set		y
$f_1$	$f_2$	o/p
-	-	0
-	-	1
-	-	0
-	-	1
-	-	0
-	-	1
-	-	0
-	-	0

Model output  
 $\hat{y} = y_{car}$

## Confusion matrix:

	1	0	$\Rightarrow$ y
1	2	3	Actual Value
0	1	1	
$\Downarrow$			Predicted

Confusion matrix

	1	0	$\Rightarrow$ Actual
1	TP	FP	
0	FP	TN	
$\Downarrow$			Predict

TP  $\Rightarrow$  True prediction (Positive)

TN  $\Rightarrow$  True negative

FP  $\Rightarrow$  False prediction (Positive)

FN  $\Rightarrow$  False Negative

Accuracy =

$$\text{Accuracy} = \frac{TP + TN}{TP + FP + FN + TN} = \frac{2 + 1}{2 + 3 + 1 + 1} = \frac{3}{7}$$



# Dataset: Binary Classification

1000 data points  $\left\{ \begin{array}{l} \rightarrow 900 \rightarrow 1 \\ \rightarrow 100 \rightarrow 0 \end{array} \right\}$  Imbalanced Dataset

Dum model  $\rightarrow 1 \Rightarrow 90\%$  Accuracy  $\Rightarrow \rightarrow \times$  sufficient

④ Precision =

$$\frac{TP}{TP + FP}$$

	1	0	Actual
1	TP	FP	
0	FN	TN	
	1	0	Predicted

Out of all the actual values how many are correctly predicted.

## Problem statement:

Mail  $\rightarrow$  Spam or Ham

i am getting a mail which is spam (1) and model predicted as spam (1) then it is TP (True Positive)

while  $\Rightarrow$  i am getting an important (1) and model is predicting as spam (1) then it is FP (False Positive)

So to reduce the False Positive

So we have to focus on False Positive

Model  $\rightarrow$  diabetes

or not diabetes

Recall :-

$$\frac{TP}{TP + FN}$$

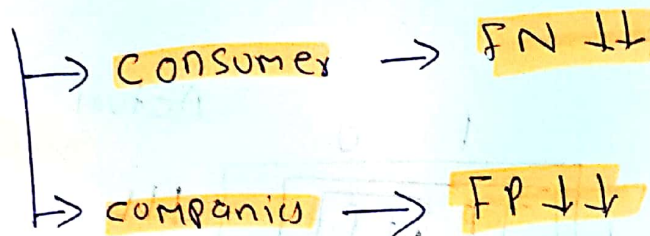
⇒

out of all the predicted values how many are correctly predicted.

Ex:-

Tomorrow the stock market is going to crash

⇓



	1	0	⇒ Actual
1	TP	FP	
0	FN	TN	

⇓  
predicted

then we use

⊕ F - Beta Score

$$\frac{(1 + \beta^2) \text{ Precision} * \text{Recall}}{(\beta^2) \text{ precision} + \text{Recall}}$$

① If FP and FN are both important

$$\beta = 1$$

$$F1 \text{ score} = 2 \frac{P * R}{P + R}$$

$$P + R$$



② if FP is more important than FN

$$\beta = 0.5$$

$$F_{0.5} \text{ score} = \frac{(1 + 0.25) P * R}{(0.25)(P + R)}$$

③ IF  $FN \gg FP$

$$F_2 \text{ score} = \frac{(1 + 4) P * R}{(4 * P + R)}$$