Sensitivity of Au + Au collisions to the symmetric nuclear matter equation of state at 2 – 5 nuclear saturation densities

Nuclear Journal Club Winter 2026, Wayne State University

Syed Afrid Jahan

WAYNE STATE UNIVERSITY

Outline

- Introduction: Equation of State (EoS) of nuclear matter
- Motivation
- Methodology: Simulation framework
- Results
- Conclusions and what do we learn from them
- How will my future work tie into this

Equation of State (EoS) of Nuclear Matter

EoS of Ideal Gas of Particles → $P = P(T, V)$

EoS of Nuclear Matter → $P = P(T, \mu_B, \mu_S, \mu_Q) = P(\epsilon, n_B, n_S, n_Q)$

Nuclear EoS

"stiff"
(Pressure changes a lot under compression)

"soft"
(Pressure changes mildly under compression)

Depending on the type of EoS, different observables may be affected. This can constrain the EoS.

Bayesian Analysis: Constraints on EoS

Assume a parameterization for the EoS → Comparison with data

Prior distribution of parameters

Observables to Constrain the EoS

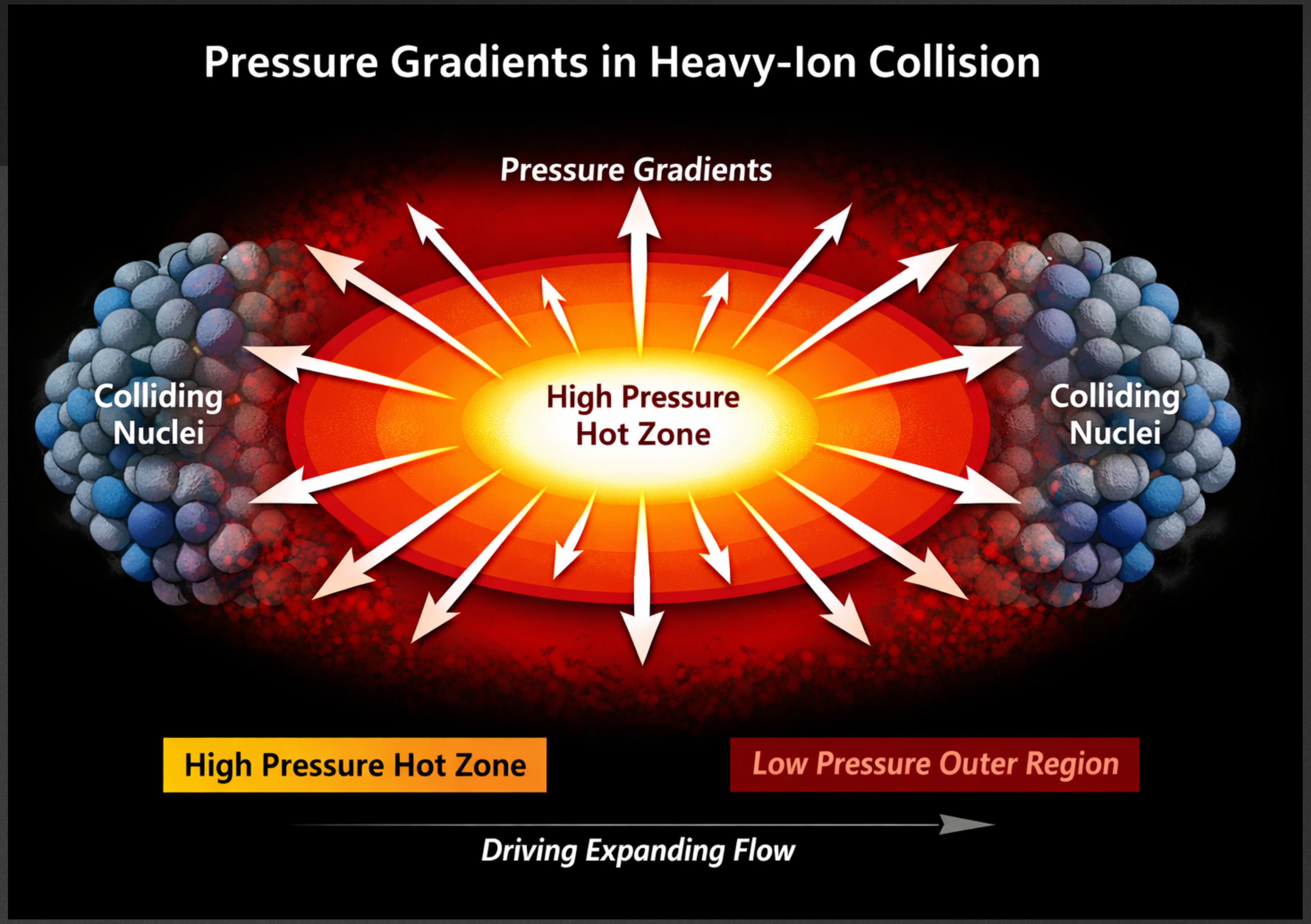
EoS quantifies how much pressure increases if the matter is compressed.

"Stiff" $\Rightarrow$ Like steel, it is hard to compress

"Soft" $\Rightarrow$ Like foam, it is easier to compress

The expansion in heavy ion collisions right after the impact is driven by the pressure gradient:

Particle emission in different directions will depend on the compressibility of the nuclear matter i.e the EoS.



An AI generated image of Heavy Ion Collisions

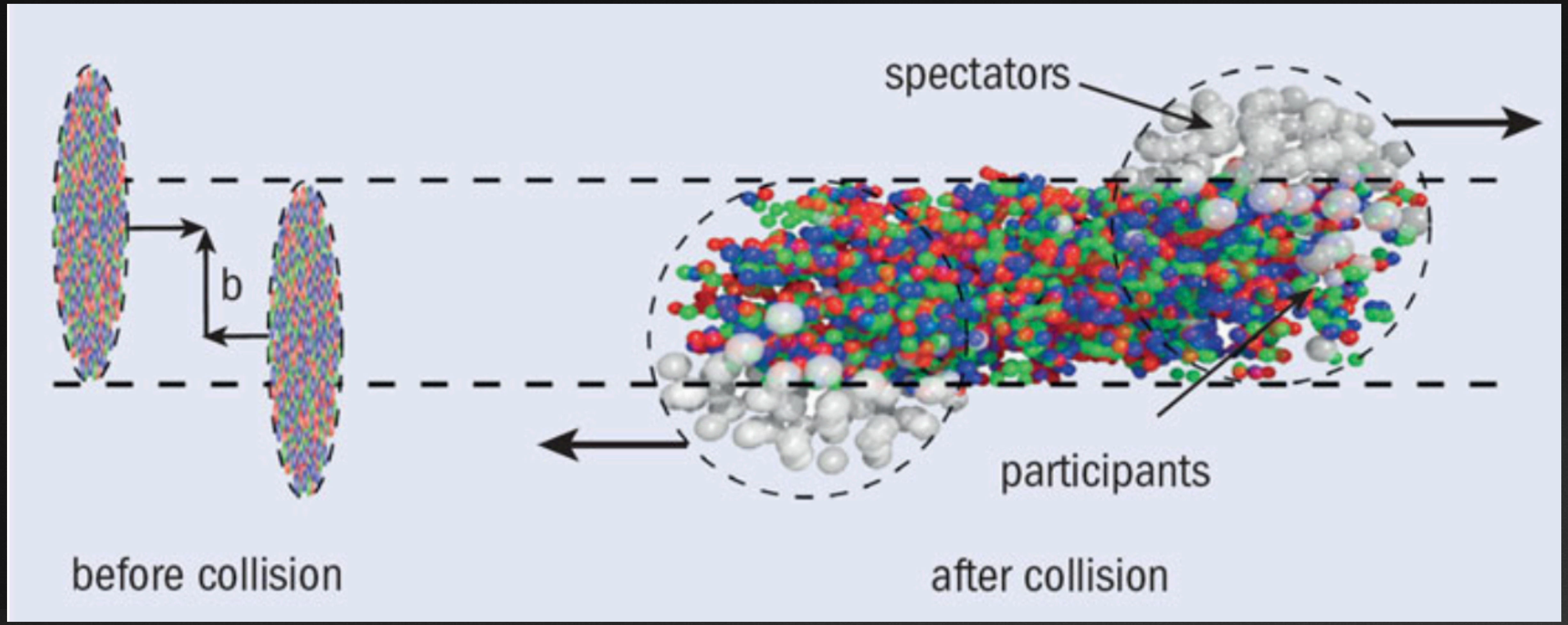
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_{n=0}^{\infty} v_n \cos(n\phi)\right)$$

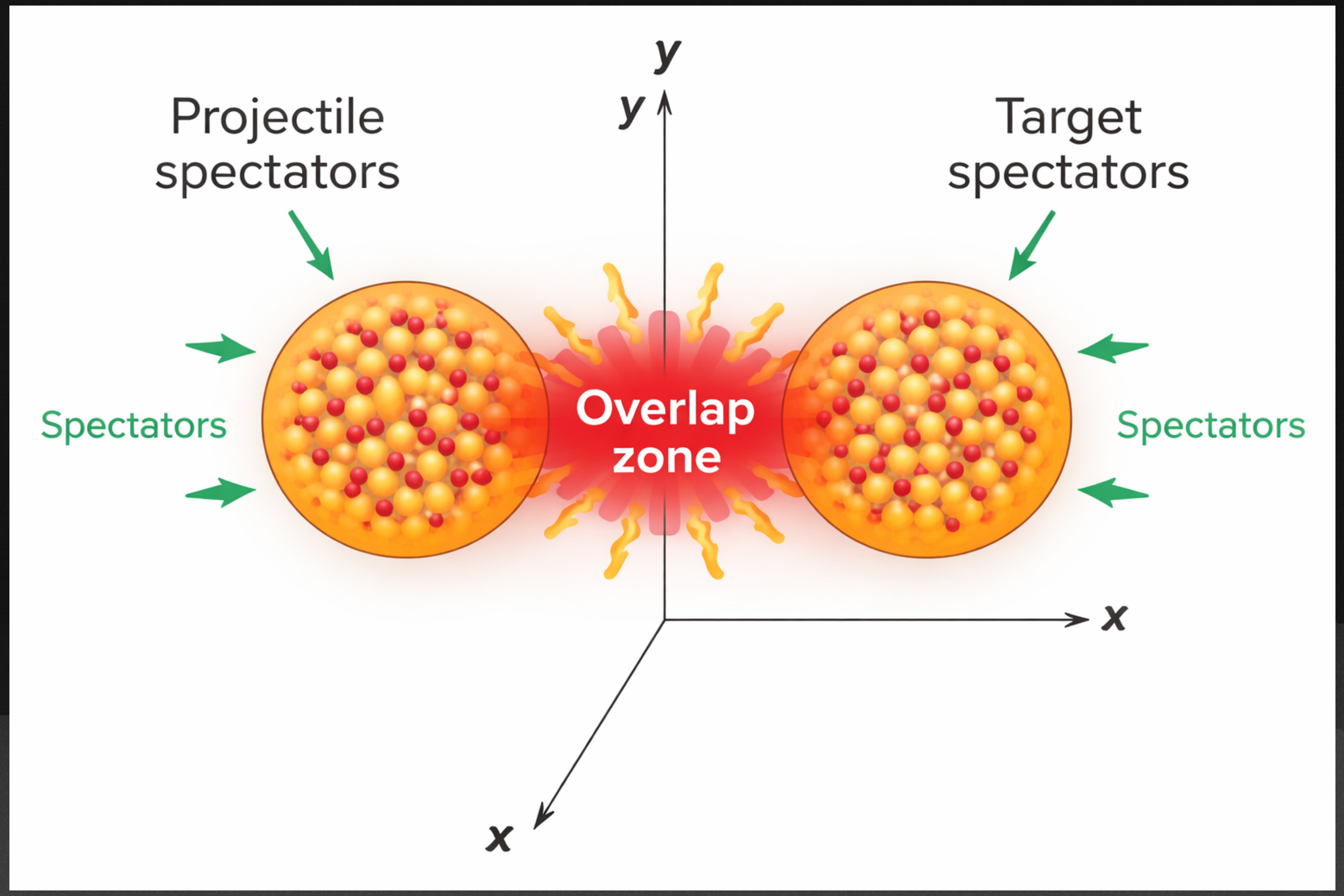
v_1: the directed flow $\Downarrow$ $\langle \cos(\phi) \rangle$

v_2: the elliptic flow $\Downarrow$ $\langle \cos(2\phi) \rangle$

Flow harmonics will depend strongly on EoS

Dependence of v_2 and $\frac{dv_1}{dy'}$ on Compressibility





- For E_{lab} = 1 - 10 GeV/nucleon the spectators block the emitted particles in the x direction. Instead they are emitted preferentially in the y-direction i.e $\phi = \pi/2$ or $3\pi/2$. At these energies negative v_2 is expected.
- $\frac{dv_1}{dy'}$ quantifies how v_1 changes when you move away from mid rapidity $y' = 0$ i.e the spectator deflection. This will depend on the stiffness of the EoS.

Our model must be able describe both of these observations well…which is an ongoing challenge

Motivation for the Presented Work

- A previous study have obtained constraints on the EoS using same flow data. But…

Many limitations:

- EoS only depended on incompressibility *K* at nuclear saturation density n_0 , not speed of sound c_s.
- The models could not describe v_2 and $\dfrac{dv_1}{dy'}$ at the same time.

In the present work:

- A more flexible parameterization for EoS with both *K* and c_s^2.
- K is allowed to vary at different nuclear densities, not a constant parameter.

Simulation Framework

E_{lab} = 2 - 20 GeV/nucleon → Deviations from the assumption of local thermal equilibrium

A hadronic transport approach SMASH is used: this solves the Boltzman-Ueling-Uhlenbeck equation in its kinetic form by using hadron-hadron interactions

$$\Pi_\mu\, \partial^x_\mu f_i(x,p) + \Pi_\nu\, \partial^x_\mu A^\nu\, \partial^p_\mu f_i(x,p) = I^{(i)}_{\mathrm{coll}}$$

where, $\Pi_\mu = \pi_\mu - A_\mu$

$$A^\mu = \alpha(n_B) j^\mu_B$$

The Energy-Momentum Tensor

$$T^{\mu\nu} = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} \tilde{f}_i + A^\mu j^\nu_B + g^{\mu\nu}(n_B U(n_B) - \int_0^{n_B} dn' U(n')$$

$U(n_B) = A^0(n_B)$ at rest frame $= \alpha(n_B) n_B$

We incorporate EoS dependence through α, it is a function of n_B

Vector Potential $U(n_B)$

$$U(n_B) = \mu_B n_B^{(0)} \exp\left[\frac{n_B}{n_B^{(0)}}\int dn'_B \frac{c_s^2(n')}{n_B^{(0)}} n' - m_N^2 + \left(\frac{6\pi^2 n_B}{g}\right)^{2/3}\right]^{1/2}$$

$U(n_B)$ As a function of speed of sound squared

$$U(n_B) = \begin{cases} U_{\mathrm{Sk}}(n_B), & n_B < n_1 = 2n_0, \\ [U_{\mathrm{Sk}}(n_1) + \mu^*(\rho_1)]\left(\frac{n_B}{n_1}\right)^{c_1^2} - \mu^*(n_B), & n_1 < n_B < n_2, \\ [U_{\mathrm{Sk}}(n_1) + \mu^*(n_1)]\left(\frac{n_B}{n_k}\right)^{c_k^2} \prod_{i=2}^{k}\left(\frac{n_i}{n_{i-1}}\right)^{c_{i-1}^2} - \mu^*(n_B), & n_k < n_B < n_{k+1}, \end{cases}$$

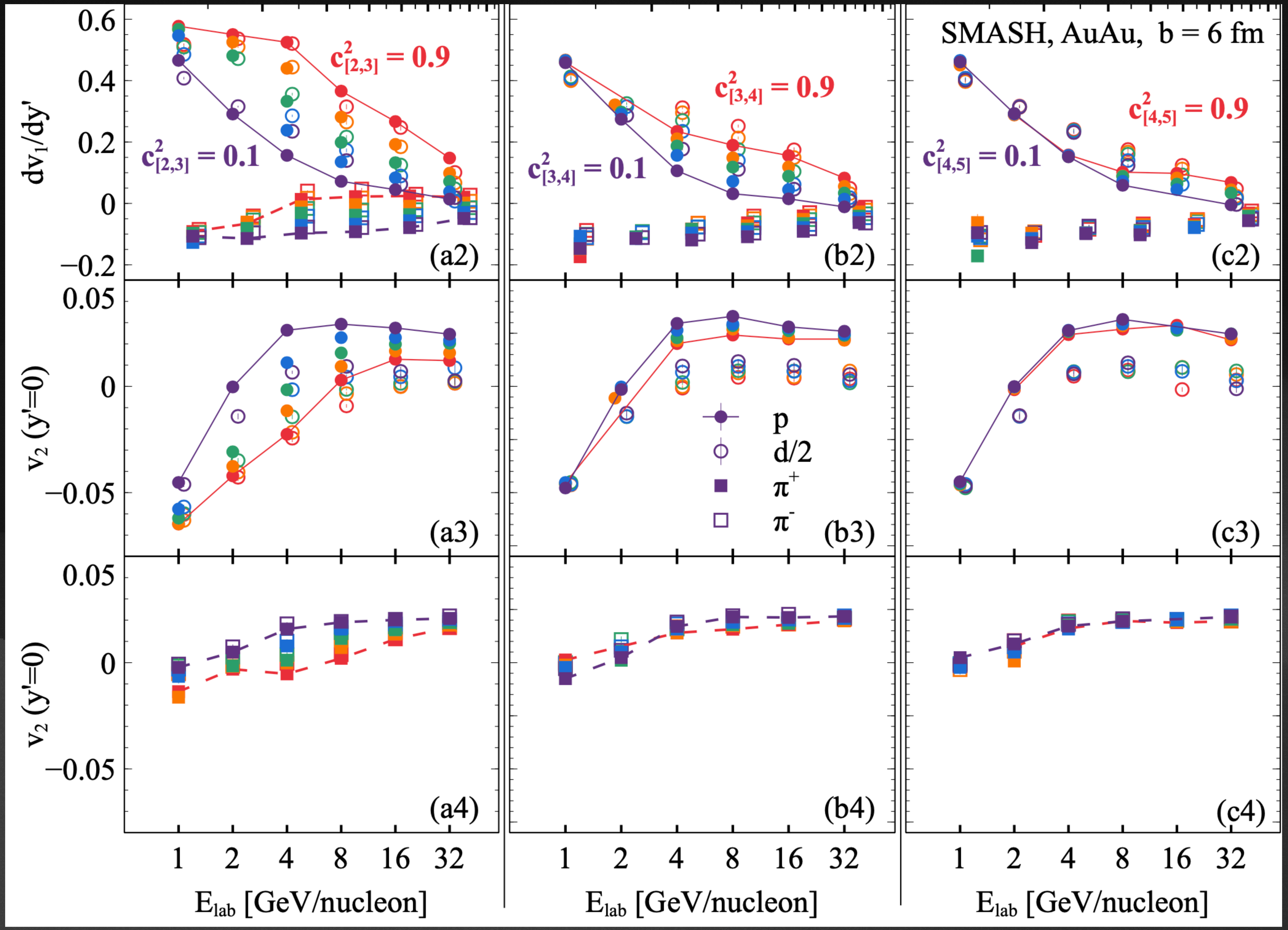
$U(n_B)$
as a function of c_s^2 at different densities

U_{Sk} is the Skyrme potential because we know the potential between n_0 and $2n_0$

Parameters we vary in our Bayesian analysis: $c_1^2 \equiv c_s^2[2,3]n_0,\ \ c_2^2 \equiv c_s^2[3,4]n_0,\ K_0$

All the c_s^2 parameters have a uniform distribution between -1 and 1.

I. Results: Sensitivity of Flow Measurements to the EoS

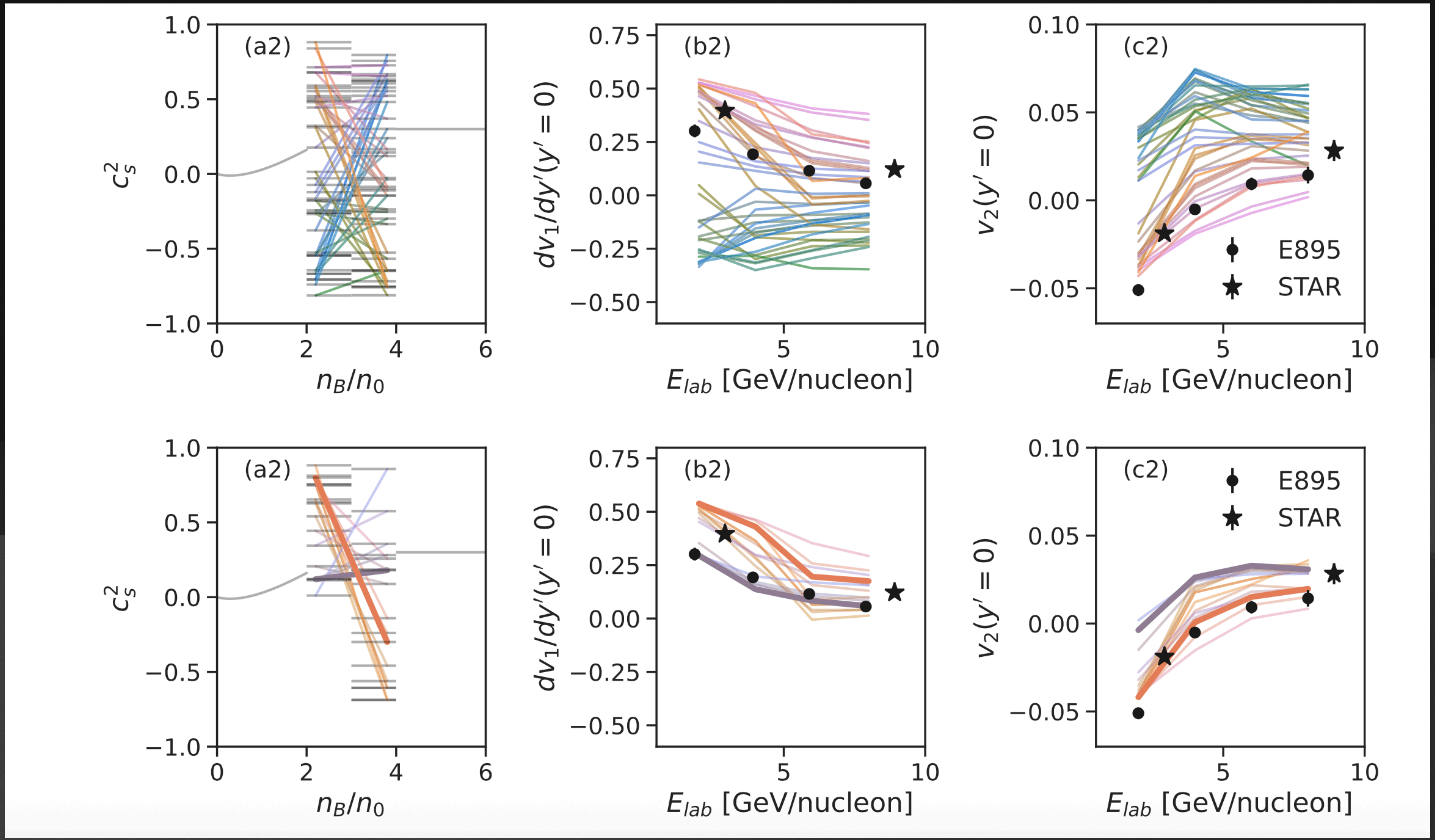


$$\begin{pmatrix} \frac{dv_1}{dy'}\big|_{2\text{ GeV}} \\ \frac{dv_1}{dy'}\big|_{4\text{ GeV}} \\ \frac{dv_1}{dy'}\big|_{8\text{ GeV}} \\ v_2|_{2\text{ GeV}} \\ v_2|_{4\text{ GeV}} \\ v_2|_{8\text{ GeV}} \end{pmatrix} \approx \begin{pmatrix} 0.40 \\ 0.04 \\ -0.04 \\ 0.00 \\ 0.04 \\ 0.04 \end{pmatrix} + \begin{pmatrix} 0.35 & 0.00 \\ 0.50 & 0.14 \\ 0.36 & 0.24 \\ -0.06 & 0.00 \\ -0.06 & -0.03 \\ -0.03 & -0.02 \end{pmatrix} \begin{pmatrix} c^2_{[2,3]n_0} \\ c^2_{[3,4]n_0} \end{pmatrix}.$$

- Solid circle: Protons, Open circle: Deuterons, Solid square: π^+, Open square: π^-

- Proton and Deuteron data is most sensitive to flow measurements.
- $c^2_{[2,3]n_0}$ is constrained more by 4 GeV system and $c^2_{[3,4]n_0}$ by 8 GeV system.

II. Results: Bayesian Analysis with E895 and STAR data



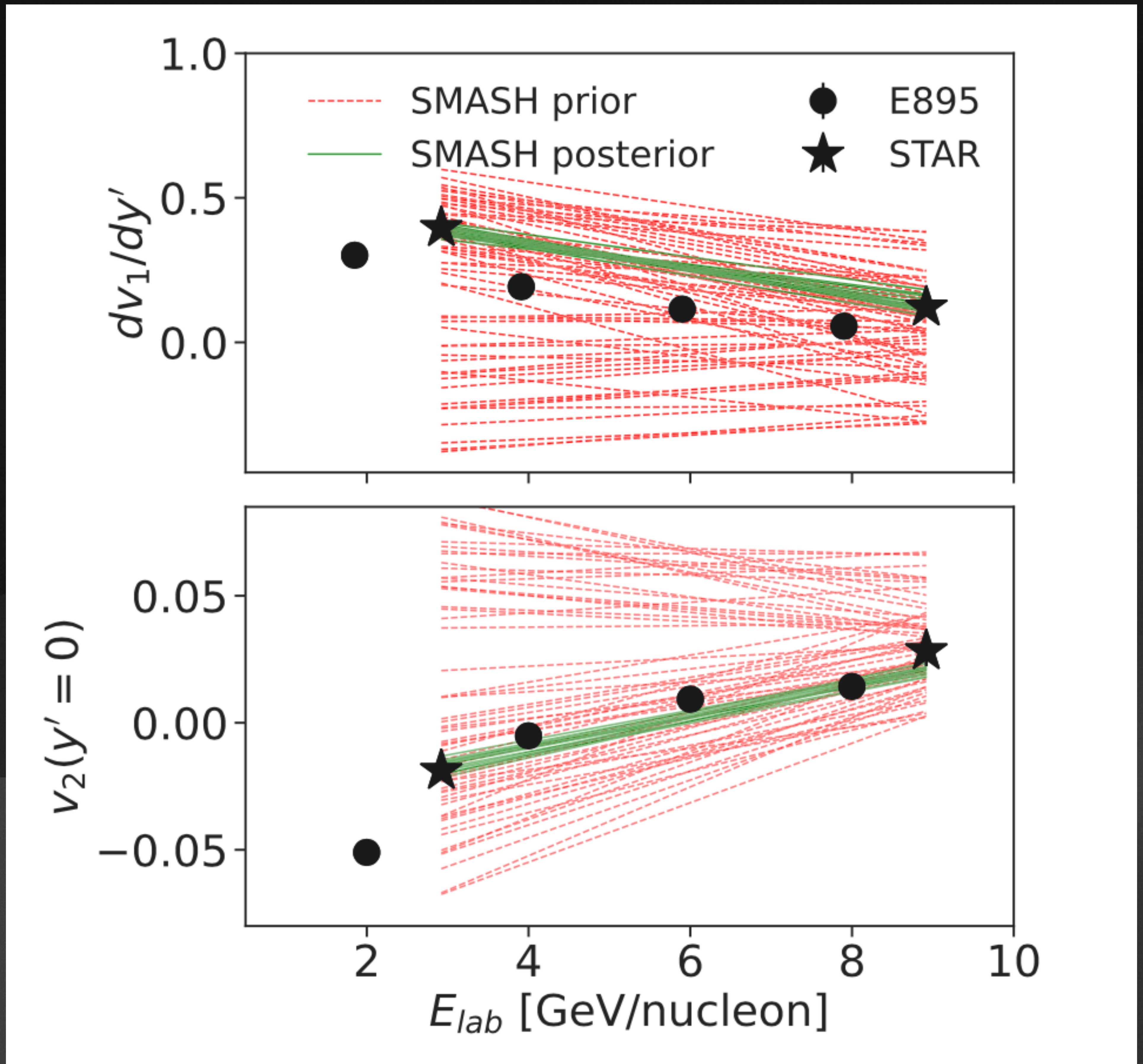
Prior

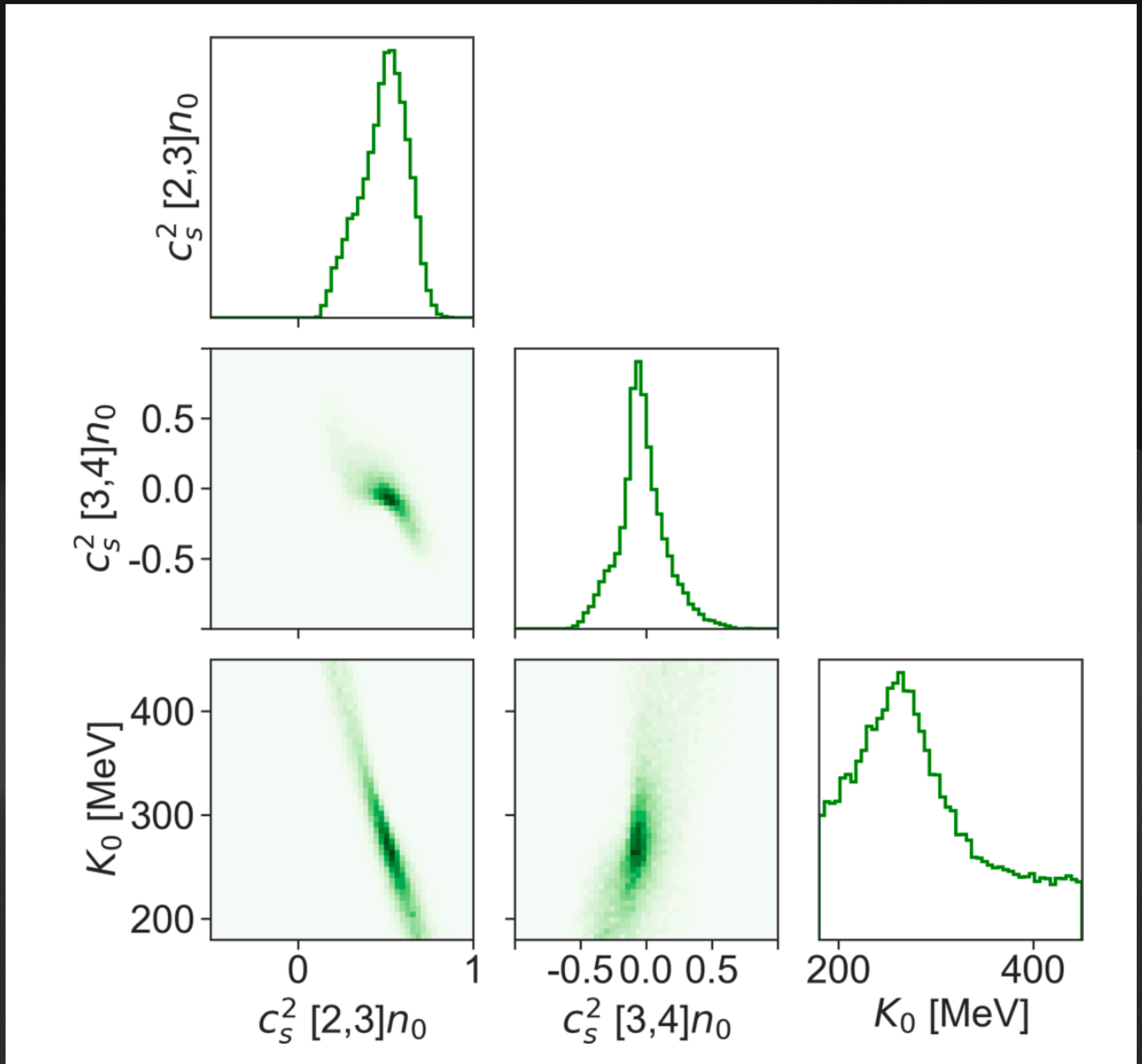
Posterior

- dv_1/dy at $y'=0$ and $v_2(y'=0)$ requires a soft and stiff EoS respectively for a better agreement

This shows a conflict between E895 and STAR data.

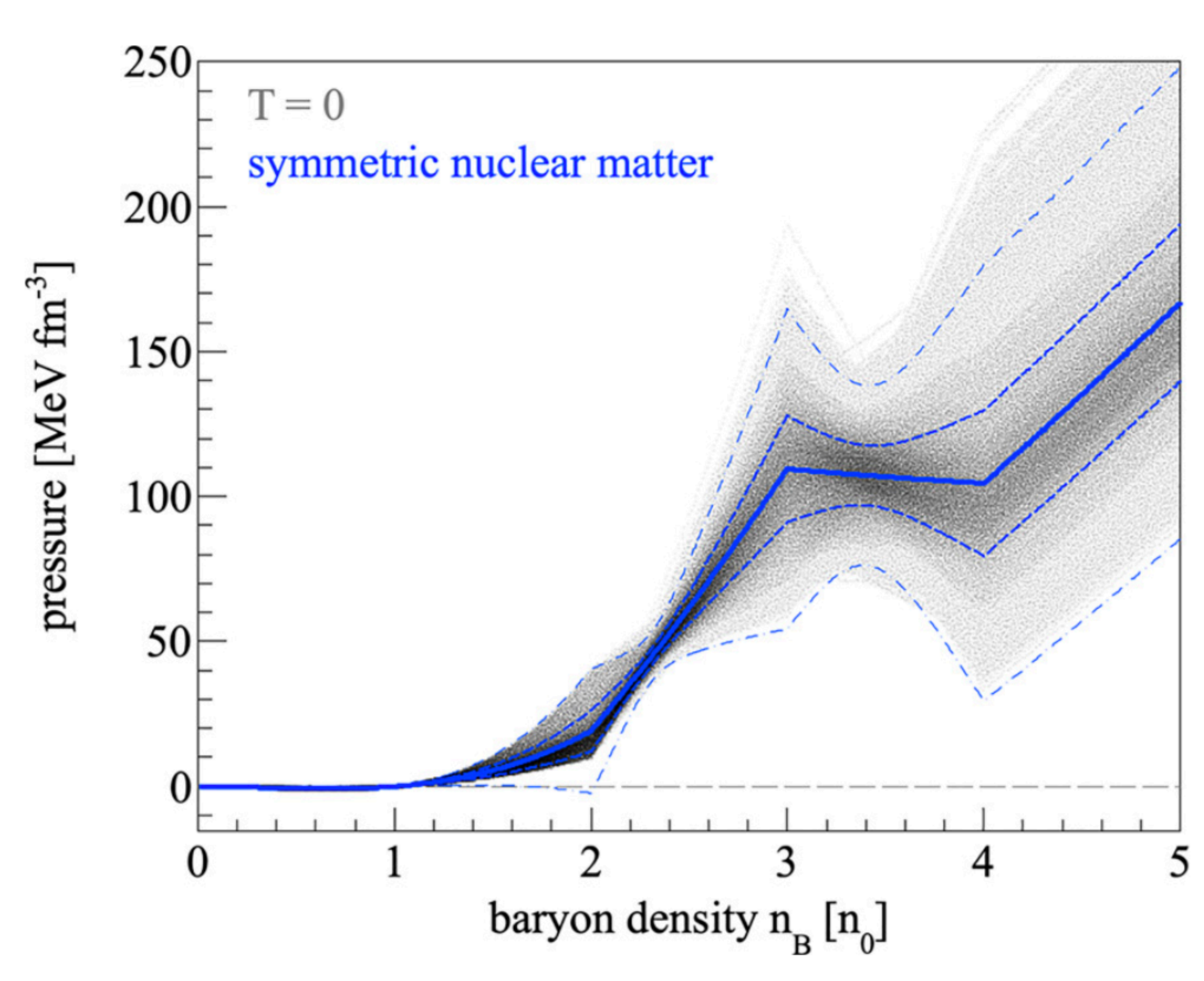
II. Results: Bayesian Analysis with STAR data





- Bayesian analysis with only STAR data can describe $v_2(y'=0)$ and $\frac{dv_1}{dy'}$ at $y'=0$ simultaneously.

II. Results: Equation of State (EoS) with STAR data

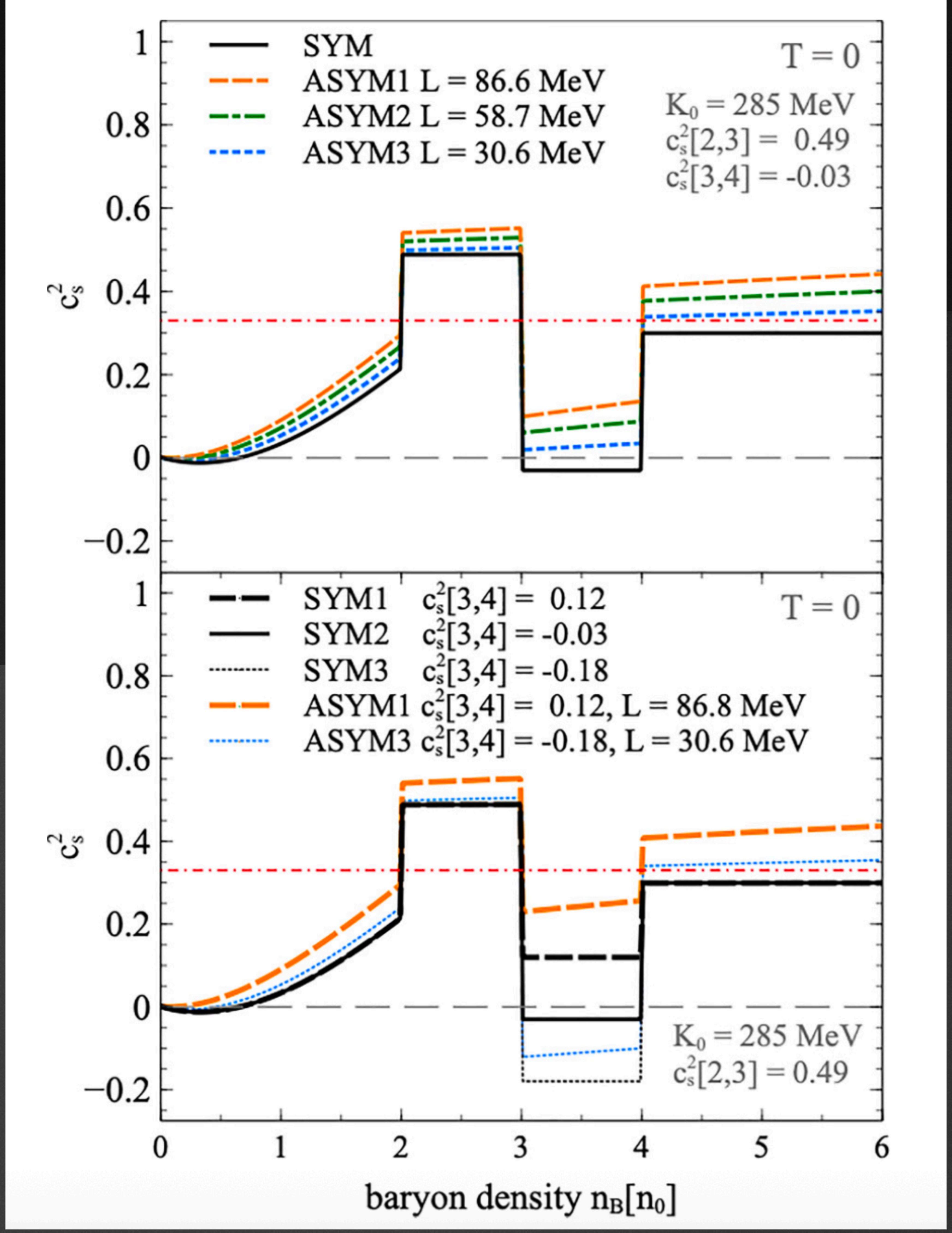


50000 EoS are sampled from the posterior distributions from the EoS parameters

The solid blue line is the mean pressure and the dark gray and light gray bands correspond to 1σ and 3σ respectively.

- Significant possibility: Slope is less between $n_B = 3n_0$ and $n_B = 4n_0$. Negative c_s^2. Sign of a possible phase transition between this baryon density.

III. Results: Comparison with Neutron Star



Unlike symmetric nuclear matter, neutron matter has mostly neutrons.

We can take a Taylor expansion of symmetry energy and use that to find c_s^2. The parameter *L* comes out of this.

Looking at different values of L, we see that the negative value of c_s^2 goes away at any value of L.

A first order phase transition for symmetric nuclear matter can still be consistent with neutron matter EoS.

Conclusion

- A Bayesian analysis was performed using hadronic transport framework SMASH and constraints were obtained on EoS parameters, speed of sound c_s^2 and compressibility K_0. We included data for E_{lab} = 2 - 8 GeV/nucleon for Au+Au collisions.

- This framework were able to describe both $v_2(y' = 0)$ and $\frac{dv_1}{dy'}$ at $y' = 0$

- Our results show that we have a possible softening of the EoS at n_B from $3n_0$ to $4n_0$. The MAP parameters obtained from the STAR data are $c^2[2,3]n_0 = 0.49 \pm 0.13$, $c^2[3,4]n_0 = -0.03 \pm 0.15$, and $K_0 = 285 \pm 67 MeV$.

Relation to my work

- A new energy profile in 3D MC Glauber model to better describe the $v_1(y)$ data.

- An even more flexible EoS - randomly generated from Gaussian processes with constraints from Hadron Resonance Gas and Lattice QCD.

- Bulk viscosity and Shear viscosity similarly generated.

- RHIC BES energies 200 GeV, 19.6 GeV, 7.7 GeV with more observables.