P1:

A variate X_k has the distribution

$$P(X_k = 0) = 1 - \left(\frac{2}{3^{2k+2}}\right), \; P\left(X_k = \pm 3^k\right) = 3^{-(2k+2)}$$

If $\{X_k\}$ is a sequence of independent r.vs, then show that $\{X_k\}$ obeys $WLLN$

Solution:

Here $E(X_k) = 3^k.3^{-(2k+2)} - 3^k.3^{-(2k+2)} = 0$ and

$$V(X_k) = E(X_k^2) = 3^{2k}3^{-(2k+2)} + 3^{2k}\;3^{-(2k+2)}$$

$$= 2.3^{2k}3^{-(2k+1)} = \frac{2}{9}$$

Let $S_n = \sum_{k=1}^{n} X_k$. Then

$$B_n = V(S_n) = \sum_{k=1}^{n} V(X_k) \qquad (\because\; X_k\; s \text{ are independent})$$

$$= \sum_{k=1}^{n} \frac{2}{9} = \frac{2n}{9}$$

and hence $\frac{B_n}{n^2} = \frac{2n}{9n^2} = \frac{2}{9n} \longrightarrow 0$ as $n \longrightarrow 0$

Since $\frac{B_n}{n^2} \longrightarrow 0$ as $n \longrightarrow \infty$, $\{X_k\}$ obeys the $WLLN$.