2.6. Functions of Random Variables:

Exercise

1) The p.m.f. of X is given by

x	-2	-1	1	2
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Find the probability distribution of $Y = 4X + 3$.

2) A r.v. X is exponentially distributed with parameter1. Find the p.d.f. of $y = \sqrt{x}$

3) Let X be a c.r.v with p.d.f.

$$f(x) = \begin{cases} \frac{x}{12} & , \quad 1 < x < 5 \\ 0 & , \quad otherwise \end{cases}$$

find the p.d.f. of $y = 2X + 4$

4) If the c.r.v X has the p.d.f.

$$f(x) = \begin{cases} \frac{2}{9}(x+1) & , \quad -1 < x < 2 \\ 0 & , \quad otherwise \end{cases}$$

find the p.d.f. of $y = X^2$

5) Find the $p.d.f$ of $W = X + Y$ where X and Y are independent r.v s with the following p.d.f s:

$$f_X(x) = \lambda e^{-\lambda x}, x \geq 0$$

and $\quad f_Y(y) = \mu e^{-\mu y}, \ \ y \geq 0,$ where $\lambda \neq \mu$.

6) The j.p.d.f of two random variables X and Y is given by $f_{XY}(xy)$. Find the p.d.f of $V = \frac{X}{Y}$

ANSWERS

1.

Y	-5	-1	7	11
$p(y)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

2. $f(y) = \begin{cases} 2y\, e^{-y^2}, & y \geq 0 \\ 0 \quad , & otherwise \end{cases}$

3. $f(y) = \begin{cases} \frac{y-4}{48}\,, & 6 < y < 14 \\ 0\,, & otherwise \end{cases}$

4. $f(y) = \begin{cases} \frac{2}{9\sqrt{y}} & ,0 < y < 1 \\ \frac{1}{9}\left(\frac{\sqrt{y}+1}{\sqrt{y}}\right) & ,1 < y < 4 \\ 0 & , \ otherwise \end{cases}$

5. $f_W(w) = \begin{cases} \frac{\lambda\mu}{\lambda-\mu}\left(e^{-\mu w} - e^{-\lambda w}\right), & w \geq 0 \\ 0, & otherwise \end{cases}$

6. $f_V(v) = \int_{-\infty}^{\infty} |w|\, f_{XY}(vw, w) dw$