3.2. Moment Generating Function

Exercise:

1. Find the m.g.f of a.r.v. whose moments are given by $\mu_r' = (r+1)!\,2^r$

2. If $M_X(t) = \frac{3}{3-t}$, find standard deviation of X

3. Find the m.g.f. of a.r.v X whose p.d.f is given by

$$f(x) = \begin{cases} \frac{x}{2} \ , & 0 \le x \le 2 \\ 0 \ , & otherwise \end{cases}$$

4. Find the m.g.f and hence find the mean and variance of a.r.v. X whose p.d.f. is given by

i. $$f(x) = \begin{cases} 2e^{-2x} \ , & x \ge 0 \\ 0 \ , & otherwise \end{cases}$$

ii. $$f(x) = \begin{cases} \frac{1}{3} \ , & -1 < x < 2 \\ 0 \ , & otherwise \end{cases}$$

iii. $$f(x) \begin{cases} \frac{1}{3}e^{-\frac{x}{3}} \ , & x \ge 0 \\ 0 \ , & otherwise \end{cases}$$

iv. $$f(x) \begin{cases} \frac{1}{k} \ , & 0 < x < k \\ 0 \ , & otherwise \end{cases}$$

v. $$f(x) = \lambda e^{-\lambda(x-a)} \ , x \ge a$$

Answers:

1. $M_X(t) = \frac{1}{(1-2t)^r}$

2. $\frac{1}{3}$

3. $\frac{1}{2t^2}(1 + 2t\, e^{2t} - e^{2t})$

4.

   i. $M_X(t) = \frac{2}{2-t}$, $\mu = \frac{1}{2}$, $\sigma^2 = \frac{1}{4}$

   ii. $M_X(t) = \begin{cases} \frac{e^{2t} - e^{-t}}{3t} & , \quad t \neq 0 \\ 1 & , \quad t = 0 \end{cases}$

   iii. $M_X(t) = (1 - 3t)^{-1}$, $\mu = 3$, $\sigma^2 = 9$

   iv. $M_X(t) = \frac{e^{tk} - 1}{kt}$, $\mu = \frac{k}{2}$, $\sigma^2 = \frac{k^2}{\sqrt{2}}$

   v. $M_X(t) = \frac{\lambda e^{at}}{\lambda - t}$, $\mu = \frac{9\lambda + 1}{\lambda}$, $\sigma^2 = \frac{1}{\lambda^2}$