P1:

Find the j.p.d.f of extremes ($i.e.,\ X_{(1)}$ and $X_{(n)}$).

Solution:

The j.p.d.f. of $X_{(i)}$ and $X_{(j)}$ is given by

$$f_{X_{(i)},\ X_{(j)}}(u,v)$$

$$= \frac{n!}{(i-1)!(j-i-1)!(n-j)!}[F(u)]^{i-1}[F(v)-F(u)]^{j-i-1}[1-F(v)]^{n-j}f(u)f(v)$$

for $1 \leq i < j \leq n$ and $-\infty < u < v < \infty$.

Let $i = 1$ and $j = n$. Then the j.p.d.f. of $X_{(1)}$ and $X_{(n)}$ is given by

$$f_{X_{(1)},X_{(n)}}(u,v) = n(n-1)[F(v)-F(u)]^{n-2}f(u)f(v), \quad -\infty < u < v < \infty$$