

## 2.3. Mathematical expectation

### Exercise:

1. Find the mean and variance of the following probability distribution:

$x$	8	12	16	20	24
$p(x)$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

2. The probability distribution of a random variable is given below:

$x$	-2	3	1
$p(x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Find (a)  $E(X)$       (b)  $E(2X + 5)$       (c)  $E(X^2)$

3. A fair coin is tossed three times. Let  $X$  denote the number of tail appearing.  
Find the mean and variance of  $X$ .

4. The j.p.m.f of  $(X, Y)$  is given in the following table:

$X \backslash Y$	1	2	3
1	$\frac{5}{27}$	$\frac{4}{27}$	$\frac{2}{27}$
2	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{3}{27}$
3	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{2}{27}$

Find (a)  $E(X)$       (b)  $E(Y)$       (c)  $E(X + Y)$       (d)  $V(X)$       (e)  $V(Y)$

5. A discrete random variable can take all possible integer values from 1 to  $k$ , each with a probability  $\frac{1}{k}$ . Find its mean and variance.

6. The j.p.d.f of  $(X, Y)$  is given by:

$$f(x, y) = \begin{cases} \frac{e\left(-\frac{x}{y}\right) e^{-y}}{y}, & 0 < x < \infty, y > 0 \\ 0 & , otherwise \end{cases}$$

Find  $E(X|Y = y)$

**(Hint:** Find the c.p.d.f. of  $X$  given  $Y$  and hence find its mean.

7. The j.p.m.f. of  $(X, Y)$  is given by

$$f(x, y) = \begin{cases} 25e^{-5y}, & 0 < x < 0.2, y > 0 \\ 0 & , otherwise \end{cases}$$

a. Find the m.p.d.fs of  $X$  and  $Y$ .

b.  $\text{Cov}(X, Y)$

8. The j.p.m.f. of  $(X, Y)$  is given by

$$p(x, y) = \begin{cases} \frac{1}{18}(2x + y), & x = 1, 2 \text{ and } y = 1, 2 \\ 0 & , otherwise \end{cases}$$

Find the conditional p.m.f. of

a.  $X$  given  $Y$

b.  $Y$  given  $X$

## Answers:

1.  $E(X) = 16$  and  $V(X) = 20$

2. a. 1                      b. 7                      c. 6

3. Mean =  $\frac{3}{2}$  and variance =  $\frac{3}{4}$

4. a. 1.93                      b. 1.93                      c. 3.86                      d. 0.58                      e. 0.72

5. mean =  $\frac{k+1}{2}$  and variance =  $\frac{k^2-1}{12}$

6.  $f_{1|2}(x|y) = \frac{1}{y} e^{-\frac{x}{y}}$  and  $E(X|Y = y) = y$

7. a.  $f_1(x) = \begin{cases} 5, & 0 < x < 0.2 \\ 0, & \text{otherwise} \end{cases}$  ,  $f_2(y) = \begin{cases} 5e^{-5y}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$   
b. 0

8. a.  $p_{1|2}(x|y) = \frac{2x+y}{2y+6}$  for  $x = 1, 2$  and  $y = 1, 2$

b.  $p_{2|1}(y|x) = \frac{2x+y}{4x+3}$  for  $x = 1, 2$  and  $y = 1, 2$