4.5. Central Limit Theorem

Exercise:

1. The lifetime of a certain brand of an electric bulb may be considered as a r.v. with mean $1200\ hrs$ and standard deviation $250\ hrs$. Find the probability, using CLT that the average lifetime of 60 bulbs exceeds 1250 hrs.

2. A distribution has unknown mean μ and variance σ^2 equal to 1.5. Use CLT to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean.

3. A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Use CLT, with what probability can we assert that the mean of the sample will not differ from $\mu = 60$ by more than 47.

4. The guaranteed average life of a certain type of electric light bulb is $1000\ hrs$ with a standard deviation of $125hrs$. It is decided to sample the output so as to ensure that 90% of the bulbs do not fall short of the guaranteed average by more than 2.5%. Use CLT to find the minimum sample size.

5. If X_i , $i = 1,2, \dots ,50$ are independent r.vs, each having a Poisson distribution with parameter $\lambda = 0.03$ and $S_n = X_1 + \cdots + X_n$, evaluate $P(S_n \geq 3)$, using CLT.

Answers:

1. 0.0606
2. at least 24
3. 0.9544
4. 41
5. 0.1112