P3:

Comparison of the Interconnection Models:

So far we have considered three models for interconnecting two components, namely the **stand by model**, **serial model**, and **parallel model**, the following figure is a review of the three models. Let X be the random variable that denotes the **lifetime of component $\boldsymbol{A}$**, and Y be the random variable that denotes the **lifetime of component $\boldsymbol{B}$**.

We define the following random variables:

$$S = X + Y$$
$$U = \min(X, Y)$$
$$W = \max(X, Y)$$

Assume that the p.d.f. of X and Y are defined respectively as follows:

$$f_X(x) = \lambda e^{-\lambda x} \qquad x \geq 0\,,\ \lambda > 0$$
$$f_Y(y) = \mu e^{-\mu y} \qquad y \geq 0\,,\ \mu > 0$$

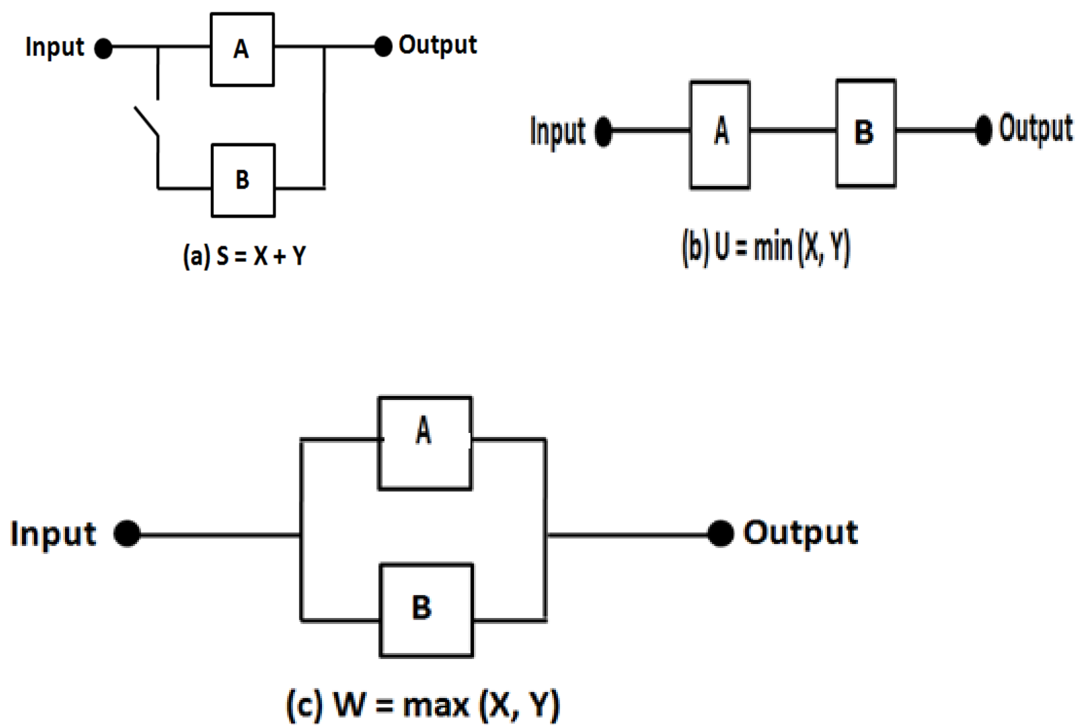

Figure: The three Interconnection Models

The following is a summary of the results obtained earlier for the p.d.fs of S, U and W in terms of those of X and Y:

$$f_S(s) = \frac{\lambda\mu}{\lambda - \mu}\{e^{-\mu s} - e^{-\lambda s}\} \qquad s \geq 0$$

$$f_U(u) = (\lambda + \mu)e^{-(\lambda+\mu)u} \qquad u \geq 0$$

$$f_W(w) = \lambda e^{-\lambda w} + \mu e^{-\mu w} - (\lambda + \mu)e^{-(\lambda+\mu)w} \quad w \geq 0$$

Similarly, the mean values of the random variables are given by:

$$E(S) = \frac{1}{\lambda} + \frac{1}{\mu}$$

$$E(U) = \frac{1}{\lambda+\mu}$$

$$E(W) = \frac{1}{\lambda} + \frac{1}{\mu} - \frac{1}{\lambda+\mu}$$

From the above result, we find that $E(S) > E(W)$. We then compare $E(U)$ and we have that

$$E(W) - E(U) = \frac{1}{\lambda} + \frac{1}{\mu} - \frac{1}{\lambda+\mu} - \frac{1}{\lambda+\mu} = \frac{1}{\lambda} + \frac{1}{\mu} - \frac{2}{\lambda+\mu}$$
$$= \frac{\mu(\lambda+\mu)+\lambda(\lambda+\mu)-2\lambda\mu}{\lambda\mu(\lambda+\mu)} = \frac{(\lambda+\mu)^2-2\lambda\mu}{\lambda\mu(\lambda+\mu)} = \frac{\lambda^2+\mu^2}{\lambda\mu(\lambda+\mu)} > 0$$

Thus, we have that $E(S) > E(W) > E(U)$. That is, the standby connection has the greatest mean lifetime, followed by the parallel connection, and the serial connection has the smallest mean lifetime. This result is not surprising because the failure rate of the serial connection is the sum of the failure rates of the two components, which means that the mean lifetime of the serial connection is smaller than the mean lifetime of either component. Similarly, the mean lifetime of the standby connection is the sum of the mean lifetime of the individual components. Finally, the mean lifetime of the parallel connection is equal to the mean lifetime of the component that lasts the longer time, which means that it lies somewhere between those of the other two models.