3.1 .Probability inequalities

Exercise:

1. The Chebychev's inequality for random variable X is $(-2 < X < \infty) \geq \frac{21}{25}$, find $E(X)$ and $V(X)$.

2. Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P(|X - 7| \geq 3) \leq \frac{35}{54}$. Compare this with the actual probability.

3. If X is the number scored in a throw of a fair die, find the upper bound for $P(|X - \mu| \geq 2.5)$ where $\mu = E(X)$. Also find the actual probability.

4. If X is a r.v. such that $E(X) = 3$ and $E(X^2) = 13$, find the lower bound of $P(-2 \leq X \leq 8)$.

5. A discrete random variable X is specified by $p(-a) = p(a) = \frac{1}{8}$ and $p(0) = \frac{3}{4}$. Compute
   (i) $P(|X| \geq 2\sigma)$ and
   (ii) Chebychev's inequality bound.

Answers:

1. $E(X) = 3$ and $V(X) = 4$
2. Actual probability $= \frac{1}{3}$
3. Upper bound $= 0.47$ and actual probability $= 0$
4. $\frac{21}{25}$
5. (i) $\frac{1}{4}$ (ii) $\frac{1}{4}$