**P4:**

**(a) For a p.m.f**

$$\boldsymbol{p(x) = \begin{cases} \frac{1}{8} & , \quad x = -1 \\ \frac{6}{8} & , \quad x = 0 \\ \frac{1}{8} & , \quad x = 1 \end{cases}}$$

**Find $\boldsymbol{P(|X - \mu| \geq 2\sigma)}$.**

**(b) Compare this result with that obtained by using Chebychev's inequality.**

*Solution:*

(a)

| $x$: | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $p(x)$: | $\frac{1}{8}$ | $\frac{6}{8}$ | $\frac{1}{8}$ |

$\therefore \mu = E(X) = \sum x\, p(x) = -1 \times \frac{1}{8} + 1 \times \frac{1}{8} = 0$ and

$$E(X^2) = \sum x^2 p(x) = 1 \times \frac{1}{8} + 1 \times \frac{1}{8} = \frac{1}{4}$$

$$\therefore Var(X) = E(X^2) - \{E(X)\}^2 = \frac{1}{4} \Longrightarrow \sigma = \frac{1}{2}$$

$$P\{|X - \mu| \geq 2\sigma\} = P\{|X| \geq 1\} = 1 - P(|X| < 1)$$

$$= 1 - P(-1 < X < 1) = 1 - P(X = 0) = \frac{1}{4}$$

(b) $P\{|X - \mu| \geq 2\sigma\} \leq \frac{1}{2^2}$ (By Chebychev's inequality: $P\{|X - \mu| \geq k\sigma\} \leq \frac{1}{k^2}$)

In this case, both results are same.

**Note:** This example shows that, in general, Chebychev's inequality cannot be improved.