P1:

The c.d.f. of bivariate discrete random variable (x, y) is given by

$$F(x,y)=\begin{cases}\frac{1}{8} & for\ \ x=1, y=1\\ \frac{5}{8} & for\ \ x=1, y=2\\ \frac{1}{4} & for\ \ x=2, y=1\\ 1 & for\ \ x=2, y=2\end{cases}$$

Find a) j.p.m.f. of (x, y) b) m.p.m.f. of x c) m.p.m.f. of y

Solution:

The j.p.m.f. is obtained from the relationship $F(x,y)=\sum_{t\le x}\sum_{s\le y} p(t,s)$. Thus

$$F(1,1) = \frac{1}{8} = p(1,1)$$

$$F(1,2) = p(1,1) + p(1,2) = \frac{5}{8} \Longrightarrow p(1,2) = \frac{5}{8} - \frac{1}{8} = \frac{1}{2}$$

$$F(2,1) = p(1,1) + p(2,1) = \frac{1}{4} \Longrightarrow p(2,1) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

$$F(2,2) = p(1,1) + p(1,2) + p(2,1) + p(2,2) = 1 \Longrightarrow p(2,2) = \frac{1}{4}$$

Thus, the j.p.m.f is given by

$$p(x,y)=\begin{cases}\frac{1}{8} & for\ x=1, y=1\\ \frac{1}{2} & for\ x=1, y=2\\ \frac{1}{8} & for\ x=2, y=1\\ \frac{1}{4} & for\ x=2, y=2\end{cases}$$

The m.p.m.f of X is given by

$$p_1(x)=\begin{cases} p(1,1)+p(1,2)=\dfrac{5}{8} & for\ \ x=1 \\ p(2,1)+p(2,2)=\dfrac{3}{8} & for\ \ x=2 \end{cases}$$

The m.p.m.f of Y is given by

$$p_2(y)=\begin{cases} p(1,1)+p(2,1)=\dfrac{1}{4} & for\ \ y=1 \\ p(1,2)+p(2,2)=\dfrac{3}{4} & for\ \ y=2 \end{cases}$$