

2.7 .Correlation coefficient and Bivariate Normal Distribution

Exercise:

1. Find the correlation coefficient between X and Y for each of the j.p.d.f. $f(x, y)$ of (X, Y) given below:

$$\begin{aligned} \text{(i)} \quad f(x, y) &= \begin{cases} \frac{3}{2}(x^2 + y^2) & , \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1 \\ 0 & , \quad \text{otherwise} \end{cases} \\ \text{(ii)} \quad f(x, y) &= \begin{cases} (x + y) & , \quad 0 \leq x, y \leq 1 \\ 0 & , \quad \text{otherwise} \end{cases} \\ \text{(iii)} \quad f(x, y) &= \begin{cases} 2xy & , \quad 0 < x < 1, \quad 0 < y < 1 \\ 0 & , \quad \text{otherwise} \end{cases} \end{aligned}$$

2. If X, Y and Z are uncorrelated r.vs with 0 mean and standard deviations 5, 12 and 9 respectively and $U = X + Y$ and $V = Y + Z$, then find the correlation coefficient between U and V .
3. If X, Y, Z are uncorrelated r.vs having same variance, find the correlation coefficient between $(X + Y)$ and $(Y + Z)$.
4. If the independent r.vs X and Y have variance 36 and 16 respectively, find the correlation coefficient between $(X + Y)$ and $(X - Y)$.

ANSWERS

1. (i) -0.2055 (ii) $-\frac{1}{11}$ (iii) 0.8

2. $\frac{48}{65}$

3. $\frac{1}{2}$

4. $\frac{5}{13}$