2.5. Continuous Probability Distributions

Exercise:

1) X is a normal variate with mean 30 and standard deviation 5. Find the probabilities that
   (i) $26 \leq X \leq 40$ (ii) $X \geq 45$ (iii) $|X - 30| > 5$

2) There are 600 engineering students in the B.Tech. classes of a university and the probability for any student to need a copy of a particular book from the university library on any day is 0.05. How many copies of the book should be kept in the university library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed? (use normal approximation to the binomial distribution)

3) In a distribution exactly normal, 10.03% of the items are under $25kg$ weight and 89.97% of the items are under $70kg$ weight. What are the mean and standard deviatioin of the distribution?

4) In an examination it is laid down that a student passes if he secures 30 percent or more marks. He is placed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. He gets distinction in case he secures 80% or more marks. It is noticed from the result that 10% of the students failed in the examination, where as 5% of them obtained distinction. Calculate the percentage of students placed in the examination. (Assume normal distribution to marks)

5) A sample of 100 items is taken at random from a batch known to contain 40% defectives. What is the probability that the sample contains (i) at least 44 defectives and (ii) exactly 44 defectives? (use normal approximation to the binomial distribution)

6) If X is uniformaly distributed with mean 1 and variance $\frac{4}{3}$, find $P(X < 0)$.

7) Subway trains on a certain line run every half hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes?

8) Suppose the life time of an electric component has exponential distribution with a mean life of 500 hrs.
   (i)   Find the probability that it will give additional 600 hrs life given that the component has been working for the last 300hrs.
   (ii) Find the probability that it will work for more than 600hrs.

9) The life(in hours) of electronic tubes manufactured by a certain process is known to have p.d.f.
$$f(x) = \begin{cases} \dfrac{1}{400} e^{-\frac{1}{400}(x-400)} & , & x \geq 400 \\ 0 & , & otherwise \end{cases}$$
Find the probability of one such tube lasting for
   (i)   at least 800hrs.
   (ii) at most 1200hrs.
   (iii) anywhere between 500 and 800hrs.

Answers:

1) (i) 0.7653 (ii) 0.00135 (iii) 0.3174

2) 37

3) $\mu = 47.5$ and $\sigma = 17.578$

4) 34%

5) (i) 0.2376 (ii) 0.0584

6) $\dfrac{1}{4}$

7) $\dfrac{1}{3}$

8) (i) $e^{-\frac{6}{5}}$ (ii) $e^{-\frac{6}{5}}$

9) (i) e^{-1} (ii) e^{-2} (iii) $e^{-\frac{1}{4}} - e^{-1}$