P2:

A variate $\boldsymbol{X_k}$ has the distribution

$$\boldsymbol{P(X_k = 0) = 1 - \left(\frac{2}{3^{2k+2}}\right), P\left(X_k = \pm 3^k\right) = 3^{-(2k+2)}}$$

If $\{X_k\}$ is a sequence of independent r.vs, then show that $\{X_k\}$ obeys $SLLN$.

Solution:

Here $E(X_k) = 3^k\ 3^{-(2k+2)} - 3^k\ 3^{-(2k+2)} = 0$ and

$\sigma_k^2 = V(X_K) = E(X_n^2) = 3^{2k} 3^{-(2k+2)} + 3^{2k}\ 3^{-(2k+2)} = \frac{2}{9}$

Then we have

$\sum_{k=1}^{\infty} \frac{\sigma_k^2}{k^2} \frac{1}{k^2} = \frac{2}{9} \sum_{k=1}^{\infty} \frac{1}{k^2}$ Converges.

Thus $\{X_n\}$ obeys the $SLLN$.