A.Y. 2017-18

$E_2S_2$ - CSE - EST - KEY

PS

1. a) (i) 3 students out of 6 can be chosen in $6_{C_3}$ ways then have to give the seating in $1^{st}$ hall

$\therefore$ $6_{C_3} \cdot 1$ ways we can arrange

Remaining halls $\rightarrow$ 2

Remaining students $\rightarrow$ 3

each student can be seated in 2 ways (because 2 halls).

$\therefore$ Total $6_{C_3} \cdot 1 \cdot 2 \cdot 2 \cdot 2 = \frac{6 \times 5 \times 4}{3 \times 2} \times 2^3$

$= 5 \times 2^5$

$= 160$

(ii) In the same way

3 particular students can be alloted in $1^{st}$ hall only in 1 way

$\therefore$ Total $1 \cdot 2 \cdot 2 \cdot 2 = 8$

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b) X : Number of selected students from CSE

$\therefore$ X can assumes only 0, 1, 2, 3

$x_1 = 0, \quad x_2 = 1, \quad x_3 = 2, \quad x_4 = 3$

&

$P(X = x_1) = P(X = 0) = \frac{4_{C_0} \cdot 7_{C_3}}{11_{C_3}} = \frac{7}{33}$

$$P(X=x_2) = P(X=1) = \frac{4C_1\, 7C_2}{11C_3}$$

$$P(X=x_3) = P(X=2) = \frac{4C_2\, 7C_1}{11C_3}$$

$$P(X=x_4) = P(X=3) = \frac{4C_3\, 7C_0}{11C_3}$$

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c) $P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

A, B & C are mutually exclusive

$\therefore P(A\cap B) = P(B\cap C) = P(A\cap C) = 0$ & $P(A\cap B\cap C) = 0$

& A, B & C are exhaustive events

$\therefore P(A\cup B\cup C) = P(S) = 1$

$1 = P(A)+P(B)+P(C) - 0 - 0 - 0 + 0$

$\therefore P(A)+P(B)+P(C) = 1 \rightarrow (1)$

but $P(A) = \frac{1}{3}P(B)$ & $P(B) = \frac{3}{4}P(C)$

$\Rightarrow P(C) = \frac{4}{3}P(B)$

$(1) \Rightarrow \frac{1}{3}P(B) + P(B) + \frac{4}{3}P(B) = 1$

$P(B)\left[\frac{1}{3}+1+\frac{4}{3}\right] = 1$

$P(B)\left[\frac{8}{3}\right] = 1$

$\boxed{P(B) = \frac{3}{8}}$

$\therefore \boxed{P(A) = \frac{1}{3}\left[\frac{3}{8}\right] = \frac{1}{8}}$

$P(C) = \frac{4}{3}P(B)$

$= \frac{4}{3}\left(\frac{3}{8}\right)$

$\boxed{P(C) = \frac{1}{2}}$

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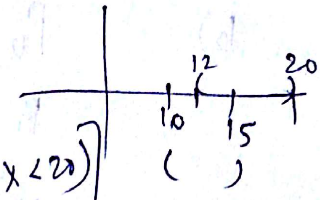
Q. a) $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$




$= \dfrac{P[(10<x<15) \cap (12<x<20)]}{P(10<x<15)}$

$= \dfrac{P[12<x<15]}{P[10<x<15]}$

$P[12<x<15] = \int_{12}^{15} f(x)\,dx$

$= \int_{12}^{15} x e^{-x}\,dx$

$= \left( x \int e^{-x} dx - \int 1 \int e^{-x} dx \right)_{12}^{15}$

$= \left[ \frac{x e^{-x}}{-1} - e^{-x} \right]_{12}^{15}$

$= -\left[ (15 e^{-15} + e^{-15}) - (12 e^{-12} + e^{-12}) \right]$

$= -\left[ 16 e^{-15} - 13 e^{-12} \right]$

$= 13 e^{-12} - 16 e^{-15} = 7.498 \times 10^{-5}$

$\therefore P[10<x<15] = -\left[ (15 e^{-15} + e^{-15}) - (10 e^{-10} + e^{-10}) \right]$

$= -\left[ 16 e^{-15} - 11 e^{-10} \right]$

$= 11 e^{-10} - 16 e^{-15} = 4.945 \times 10^{-4}$

$\therefore P[B/A] = 0.1516$

b) $P_0$ = Politician
$B$ = businessman
$E$ = educationist

$P[P_0]$ = Probability of appointment of Politician as a VC

$= \frac{50}{100}$

$P[B] = \frac{30}{100}$, $P[E] = \frac{20}{100}$

$P[R/P_0]$ = Prob. of Research activity promoted by the politician

$= \frac{30}{100}$

$P[R/B] = \frac{70}{100}$, $P[R/E] = \frac{80}{100}$

$P[R] = ?$

$\therefore$ by Total probability

$P[R] = P[P_0] . P[R/P_0] + P[B] . P[R/B] + P[E] P(R/E)$

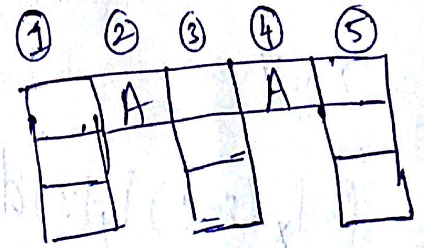
$= \frac{50}{100} \cdot \frac{30}{100} + \frac{30}{100} \cdot \frac{70}{100} + \frac{20}{100} \cdot \frac{80}{100}$

$= \frac{15}{100} + \frac{21}{100} + \frac{16}{100}$

$= \frac{52}{100}$

$= 0.52$

Q. c)




Total Columns = 5
2nd & 4th Columns having only one box each ∴ one A in 2nd column can be put in 1 way & one A in 4th column can be put in only 1 way

Remaining A's = 3
Remaining columns = 3
∴ one A should be in each column (1st, 3rd & 5th)
∴ one 'A' can be put in 1st column in ${}^3C_1$ ways

similly in 3rd Column ${}^3C_1$ ways
in 5th Column ${}^3C_1$ ways

∴ Total ${}^3C_1 \times 1 \times {}^3C_1 \times 1 \times {}^3C_1 = 3\times3\times3$
$= 27$

by Fundamental principle of Multiplication

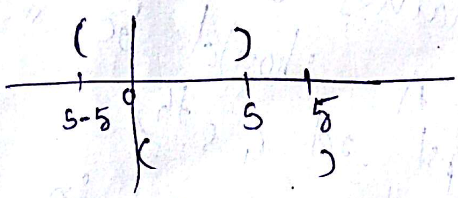
3 a) let $U = X+Y$

$$\therefore f_U(u) = \int_{-\infty}^{\infty} f_X(s-y)\, f_Y(y)\, dy$$

$$f_X(u) = \begin{cases} \frac{1}{5} ; & 0<u<5 \\ 0 ; & o/w \end{cases} \qquad \therefore f_X(s-y) = \begin{cases} \frac{1}{5} ; & 0<s-y<5 \text{ (or) } s-5<y<s \\ 0 ; & o/w \end{cases}$$

$$f_Y(u) = \begin{cases} \frac{1}{5} ; & 0<u<5 \\ 0 ; & o/w \end{cases} \qquad \therefore f_Y(y) = \begin{cases} \frac{1}{5} ; & 0<y<5 \\ 0 ; & o/w \end{cases}$$

Case (i)



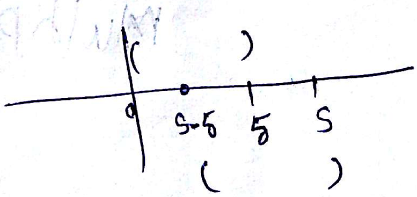

$$\therefore f_X(s-y) \cdot f_Y(y) = \frac{1}{5} \cdot \frac{1}{5} ; \quad 0<y<s$$

$$\therefore f_U(u) = \int_{-\infty}^{\infty} f_X(s-y)\, f_Y(y)\, dy$$

$$= \int_0^s \frac{1}{5} \cdot \frac{1}{5}\, dy = \frac{1}{25}(s-0)$$

$$f_U(u) = \frac{s}{25} ; \quad 0<s<5$$

Case (ii) :-




$$f_X(s-y) \cdot f_Y(y) = \frac{1}{5} \cdot \frac{1}{5} ; \quad s-5<y<5$$

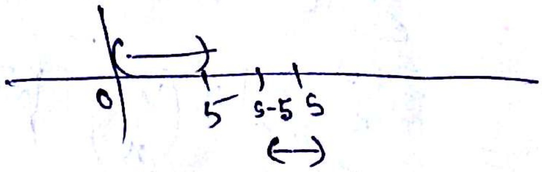
(9)

(7)

$$\therefore f_U(u) = \int_{s-5}^{5} \frac{1}{5}\frac{1}{5}\,dy = \frac{1}{25}\left[5-(s-5)\right]$$

$$= \frac{10-s}{25} \; ; \; 5<s<10$$

Case (iii):



$$f_X(s-y)\cdot f_Y(y) = 0$$

$$\therefore f_U(u) = \int_{-\infty}^{\infty} 0\,dy = 0$$

$$\therefore f_U(u) = \begin{cases} \frac{s}{25} & ; 0<s<5 \\ \frac{10-s}{25} & ; 5<s<10 \\ 0 & ; o/w \end{cases}$$

3 (b) $p(X=x) = \binom{n}{x} p^x q^{n-x}$ ; $x = 0, 1, 2, \ldots n$ (8)

$M_X(t) = E[e^{tX}]$

$= \sum e^{tx} p(X=x)$

$= \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{n-x}$

$= \sum_{x=0}^{n} \binom{n}{x} (pe^t)^x q^{n-x}$

$= [q + pe^t]^n$

$E(X^2) = \mu_2' = \left[\frac{d^2}{dt^2}(M_X(t))\right]_{t=0}$

$\frac{d}{dt}(M_X(t)) = \frac{d}{dt}\left[(q+pe^t)^n\right]$

$= n(q+pe^t)^{n-1} \cdot pe^t$

$\frac{d^2}{dt^2}(M_X(t)) = \frac{d}{dt}\left[\frac{d}{dt} M_X(t)\right] = \frac{d}{dt}\left[n(q+pe^t)^{n-1} \cdot pe^t\right]$

$= n(n-1)(q+pe^t)^{n-2} \cdot pe^t \cdot pe^t$

$+ n(q+pe^t)^{n-1} \cdot p \cdot e^t$

$\therefore E(X^2) = \mu_2' = \left[\frac{d^2}{dt} M_X(t)\right]_{t=0}$

$= n(n-1)(q+p)^{n-2} \cdot p \cdot p + n(q+p)^{n-1} \cdot p$

$= n(n-1)p^2 + np$

$= n^2p^2 - np^2 + np$

$= n^2p^2 + np[1-p] = n^2p^2 + npq$

3 c) $f_X(x) = \begin{cases} \frac{1}{3} ; & 0 < x < 3 \\ 0 ; & o/w \end{cases}$

$\because F_X(x) = P(X \le x)$

$F_U(u) = P[U \le u] = P[\ln(X) \le u]$

$= P[X \le e^u]$

$$\boxed{F_U(u) = F_X(e^u)}$$

$\because \frac{d}{dx} F_X(x) = f_X(x)$

$\because \frac{d}{du} F_U(u) = f_U(u) \Rightarrow \frac{d}{du} F_U(u) = \frac{d}{du}\left[F_X(e^u)\right]$

Let $v = e^u$

$\frac{dv}{du} = e^u \cdot 1$

$= \frac{d}{dv}\left[F_X(v)\right]\frac{dv}{du}$

$= \frac{d}{dv}\left[F_X(v)\right] e^u \cdot 1$

$= f_X(v) \cdot e^u$

$= f_X(e^u) \cdot e^u$

$$\boxed{\frac{d}{du} F_U(u) = \frac{1}{3} \cdot e^u}$$

$\therefore \boxed{f_U(u) = \frac{1}{3} e^u}$ ; $\ln(0+\epsilon) < u < \ln(3)$

$\epsilon > 0$

4 a) $V(X+Y) = V(X) + V(Y) + 2\,Cov(X,Y)$

$\left(\because V(a_1X_1+a_2X_2) = a_1^2 V(X_1) + a_2^2 V(X_2) + 2a_1a_2\,Cov(X_1,X_2)\right)$

$f_X(x) = \int_{y=1}^{3} f(x,y)\,dy$

$= \int_1^3 \frac{xy}{8}\,dy$

$= \frac{x}{8}\left(\frac{y^2}{2}\right)_1^3$

$= \frac{x}{16}(9-1)$

$\boxed{f_X(x) = \frac{x}{2}}$ ; $0 < x < 2$

$f_Y(y) = \int_{x=0}^{2} f(x,y)\,dx$

$= \int_0^2 \frac{xy}{8}\,dx$

$= \frac{y}{8}\left(\frac{x^2}{2}\right)_0^2$

$= \frac{y}{16}(4-0)$

$\boxed{f_Y(y) = \frac{y}{4}}$ ; $1 < y < 3$

$E(X) = \int_0^2 x f_X(x)\,dx$

$= \int_0^2 x\frac{x}{2}\,dx$

$= \frac{1}{2}\left(\frac{x^3}{3}\right)_0^2$

$= \frac{1}{2}\,\frac{8}{3}$

$= \frac{8}{6}$

$\boxed{E(X) = \frac{4}{3}}$

$E(X^2) = \int_0^2 x^2 f_X(x)\,dx$

$= \int_0^2 x^2\frac{x}{2}\,dx$

$= \frac{1}{2}\left(\frac{x^4}{4}\right)_0^2$

$\boxed{E(X^2) = 2}$

$\therefore V(X) = E(X^2) - [E(X)]^2$

$= 2 - \left(\frac{4}{3}\right)^2$

$= 2 - \frac{16}{9}$

$\boxed{V(X) = \frac{2}{9}}$

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$$E(y) = \int_1^3 y f_y(y)\,dy$$

$$= \int_1^3 y\left(\frac{y}{4}\right)dy$$

$$= \frac{1}{4}\left(\frac{y^3}{3}\right)_1^3$$

$$= \frac{1}{12}(27-1)$$

$$= \frac{26}{12}$$

$$\boxed{E(y) = \frac{13}{6}}$$

$$E(y^2) = \int_1^3 y^2 f_y(y)\,dy$$ (11)

$$= \int_1^3 y^2\left(\frac{y}{4}\right)dy$$

$$= \frac{1}{4}\left(\frac{y^4}{4}\right)_1^3$$

$$= \frac{1}{16}(81-1) = \frac{80}{16}$$

$$= 5$$

$$V(y) = E(y^2) - [E(y)]^2$$

36×5

$$= 5 - \left(\frac{13}{6}\right)^2$$

$$= \frac{180 - 169}{36} = \frac{11}{36}$$

$$E(xy) = \int_{y=1}^{3}\int_{x=0}^{2} xy\, f(x,y)\,dx\,dy$$

$$= \int_{y=1}^{3}\left(\int_0^2 xy\,\frac{xy}{8}\,dx\right)dy = \int_{y=1}^{3}\frac{y^2}{8}\left(\frac{x^3}{3}\right)_{x=0}^{2} dy$$

$$= \int_{y=1}^{3}\frac{y^2}{24}(8-0)\,dy = \frac{1}{3}\left(\frac{y^3}{3}\right)_1^3$$

$$= \frac{1}{9}(27-1) = \frac{26}{9}$$

$$\therefore\ Cov(x,y) = E(xy) - E(x)\,E(y)$$

$$= \frac{26}{9} - \left(\frac{\cancel{4}^{\,2}}{3}\right)\left(\frac{13}{\cancel{6}_{\,3}}\right) = 0$$

$$V(X+Y) = V(X) + V(Y) + 2\,Cov(X,Y)$$

$$= \frac{2}{9} + \frac{11}{36} + 0$$

$$\boxed{V(X+Y) = \frac{19}{36}}$$

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4 b) X : number of MI mobiles sold weekly

$E(X) = 16$

(i) $P[X \geq 20] = ?$

$P[X \geq 20] \leq \frac{E(X)}{20}$ [by Bienayme Chebychev's $\Rightarrow$ $P[g(X) \geq k] \leq \frac{E(X)}{k}$]

$= \frac{16}{20} = \frac{4}{5}$

$$\boxed{P[X \geq 20] \leq \frac{4}{5}}$$

(ii) $P[10 < X < 22] = ?$

by Chebychev's inequality

$P[|X - \mu| < \epsilon] > 1 - \frac{\sigma^2}{\epsilon^2}$

$P[\mu - \epsilon < X < \mu + \epsilon] > 1 - \frac{\sigma^2}{\epsilon^2}$

$P[16 - \epsilon < X < 16 + \epsilon] > 1 - \frac{9}{\epsilon^2}$

let $\epsilon = 6$

$P[16 - 6 < X < 16 + 6] > 1 - \frac{9}{36}$

$P[10 < X < 22] > \frac{27}{36} = \frac{3}{4}$

4 c) $f_X(x) = \begin{cases} 2e^{-2x} & ; x \geq 0 \\ 0 & ; o/w \end{cases}$

$$f_{X_{(j)}} = \frac{n!}{(j-1)!(n-j)!} [F(x)]^{j-1} [1-F(x)]^{n-j} \cdot f(x) \quad ; \text{ If } 1 \leq j \leq n$$

<u>If $x \geq 0$ :-</u>

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(x)\,dx$$

$$= \int_{-\infty}^{0} 0 + \int_{0}^{x} f_X(x)\,dx$$

$$= \int_{0}^{x} 2e^{-2x}\,dx = \left(\frac{2e^{-2x}}{-2}\right)_0^x$$

$$= -\left(e^{-2x} - 1\right)$$

$$\boxed{F_X(x) = 1 - e^{-2x}}$$

$$\therefore f_{X_{(3)}}(x) = \frac{5!}{(3-1)!(5-3)!} [1-e^{-2x}]^{3-1} [1-(1-e^{-2x})]^{5-3} \cdot 2e^{-2x}$$

$$= 30 [1-e^{-2x}]^2 [e^{-2x}]^2 \cdot 2e^{-2x}$$

$$= 60 [1-e^{-2x}]^2 e^{-6x}$$

5 a) $P[\text{getting good rank}] = \frac{4}{5} = p$

$P[\text{not getting good rank}] = \frac{1}{5} = q$

(i) $n = 4$

$P[X \geq 2] = ?$

$P[X=x] = \binom{n}{x} p^x q^{n-x} = \binom{n}{x}\left(\frac{4}{5}\right)^x\left(\frac{1}{5}\right)^{n-x}$

$P[X \geq 2] = 1 - P[X < 2]$

$= 1 - \{P[X=0] + P[X=1]\}$

$= 1 - \left\{\binom{4}{0}\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^{4-0} + \binom{4}{1}\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^{4-1}\right\}$

$= 1 - \left\{\left(\frac{1}{5}\right)^4 + 4\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^3\right\}$

$= 1 - \frac{17}{5^4} = \frac{608}{625}$

$= 0.9728$

(ii) $P(X \geq 1) > \frac{9}{10}$

$1 - P[X < 1] > \frac{9}{10}$

$1 - P[X=0] > \frac{9}{10}$

$1 - \binom{n}{0}\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^n > \frac{9}{10}$

$1 - \frac{1}{5^n} > \frac{9}{10}$

$$\frac{1}{5^n} - 1 < -\frac{9}{10}$$

$$\frac{1}{5^n} < 1 - \frac{9}{10}$$

$$\frac{1}{5^n} < \frac{1}{10}$$

$$5^n > 10$$

$\therefore$ $n = 2, 3, 4, \ldots$

$\therefore$ He must to attempt 2 times.

(iii) $E(X) = np$
$= 10 \times \frac{4}{5} = \underline{\underline{8}}$

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5 b) $\lambda = 2.5$

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} \; ; \; x = 0, 1, 2, \ldots$$

$$P(X=x) = \frac{e^{-2.5} (2.5)^x}{x!}$$

(i) $P[X \geq 2] = 1 - P[X < 2]$

$= 1 - \{P[X=0] + P[X=1]\}$

$$= 1 - \left\{ \frac{e^{-2.5} (2.5)^0}{0!} + \frac{e^{-2.5} (2.5)^1}{1!} \right\}$$

$= 0.71$

(ii) $P[X \leq 1] = P[X=0] + P[X=1]$

$$= \frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!}$$

$$= 0.285$$

c) by using NBD

got 6 marks at his 10th question

$\therefore$ Number of failures before 6th Success ~~are~~ are 4

$\therefore r = 6$ & $p = \frac{2}{3}$, $q = \frac{1}{3}$

$x = 4$

$$P(X=x) = \binom{x+r-1}{r-1} p^r q^{x}$$

$$P(X=4) = \binom{4+6-1}{6-1} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^4$$

$$= \binom{9}{5} \frac{2^6}{3^{10}}$$

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6) a) Ed: $\mu = 100$

$\sigma = 16$

(i) $P[X \leq 80] = P\left[\frac{X-\mu}{\sigma} \leq \frac{80-\mu}{\sigma}\right]$

$= P\left[Z \le \frac{80-100}{16}\right]$

$= P\left[Z \le \frac{-5}{4}\right]$

$= P[Z \le -1.25]$


$= P[Z \ge 1.25]$

$= 0.5 - P[0 \le Z \le 1.25]$

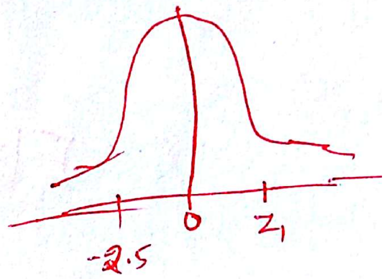
$= 0.5 - 0.3944 = 0.1056$

(iii) $P(60 < X < x_1) = 0.90$

$P\left[\frac{60-100}{16} < Z < \frac{x_1-100}{16}\right) = 0.90$

$P[-2.5 < Z < z_1] = 0.90$

where $z_1 = \frac{x_1 - 100}{16}$




$P[\cancel{0 \le Z \le 2.5] +}$ $P[0 < Z < z_1) = 0.90$

$P[-2.5 < Z < 0) + P[0 < Z < z_1] = 0.90$

$P[0 < Z < 2.5] + P[0 < Z < z_1] = 0.90$

$0.4938 + P[0 < Z < z_1] = 0.90$

$P[0 < Z < z_1] = 0.90 - 0.4938$

$P[0 < Z < z_1] = 0.4062$

$\therefore \cancel{z_1 = 0.4}$ $\therefore z_1 = 1.32$

$\frac{x_1 - 100}{16} = 1.32$

$x_1 - 100 = 21.12$

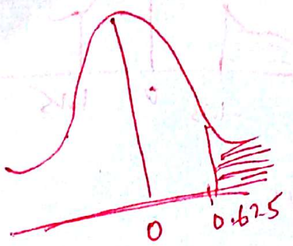
$\boxed{x_1 = 121.12}$

$\cancel{x_1 - 100 = 21.12}$

$\boxed{\cancel{x_1 = 121.12}}$

(iii) $P[X \ge 110] = P\left[\frac{X-\mu}{\sigma} \ge \frac{110-\mu}{\sigma}\right]$

$= P\left[Z \ge \frac{110-100}{16}\right]$




$= P\left[Z \ge \frac{5}{8}\right]$

$= P[Z \ge 0.625]$

$= 0.5 - P[0 \le Z \le 0.625]$

$= 0.5 - 0.2357$

$= 0.2643$

$\therefore E(X) = 100 \times P[X \ge 110]$

$= 100 \times (0.2643)$

$= 26.43$

$\simeq 26$

$\therefore$ Expected number of children having IQ more than 110 is 26

6 b) (i) $\int_{x=0}^{1}\int_{y=0}^{2} k(2x+y)\,dx\,dy = 1$

$k\int_{x=0}^{1}\left(2xy + \frac{y^2}{2}\right)_{y=0}^{2} dx = 1$

(19)

$$k \int_{x=0}^{1} \left[(4x+2) - 0\right] dx = 1$$

$$k \left(4\frac{x^2}{2} + 2x\right)_0^1 = 1$$

$$k \left[2+2-0\right] = 1$$

$$\therefore \boxed{k = \frac{1}{4}}$$

(ii)

$$f_y(y) = \int f(x,y)\,dx$$

$$= \int_0^1 \frac{1}{4}(2x+y)\,dx = \frac{1}{4}\left(2\frac{x^2}{2} + yx\right)_0^1$$

$$= \frac{1}{4}(1+y)$$

$$\therefore f_{x/y} = f_{y/2} = \frac{f(x,y)}{f_y(y)} = \frac{\cancel{\frac{1}{4}}(2x+y)}{\cancel{\frac{1}{4}}(1+y)}$$

$$\boxed{f_{x/y} = \frac{2x+y}{1+y}} \longrightarrow (1)$$

$$E(x/_{y=1}) = \int x \cdot f_{x/_{y=1}}\,dx$$

$$\therefore f_{x/_{y=1}} = \frac{2x+1}{1+1} = \frac{2x+1}{2} \quad \left[\text{substitute } y=1 \text{ in } (1)\right]$$

$$\therefore E\left(x/_{y=1}\right) = \int_0^1 x \cdot \left(\frac{2x+1}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{2}{3} + \frac{1}{2}\right] = \frac{1}{2}\left[\frac{4+3}{6}\right]$$

$\therefore E(X|Y=1) = \frac{7}{12}$

(iii) $f_X(x) = \int_0^2 f(x,y)\,dy$

$= \int_0^2 \frac{1}{4}(2x+y)\,dy$

$= \frac{1}{4}\left(2xy + \frac{y^2}{2}\right)_{y=0}^{2}$

$= \frac{1}{4}\left[(4x+2) - 0\right] = \frac{1}{2}(2x+1)$

$\therefore f_X(x) \cdot f_Y(y) = \frac{1}{4}(1+y)\,\frac{1}{2}(2x+1)$

$\neq f(x,y)$

$\therefore X \,\&\, Y$ are not independent

6 (c) $f_X(x) = \begin{cases} \frac{1}{8} e^{-\frac{1}{8}x} & ; x \geq 0 \\ 0 & ; o/w \end{cases}$

by Memory less ness property

$P\left[X \leq s+t \,/\, X > t\right] = P[X \leq s]$

s = additional time
t = life time

$\therefore \; P\left[ X \leq 3+5 \,/\, X \geq 3 \right] = ?$

If $X$ : Life time of the radio

$\therefore \; P\left[ X \geq 8 \,/\, X \geq 3 \right] = P\left[ X \leq 3 \right]$

---

$\therefore \; P\left[ X \leq 3+5 \,/\, X > 3 \right] = ?$

If $X$ : Life time of the radio

$$\therefore \; P\left[ X \leq 3+5 \,/\, X > 3 \right] = P\left[ X \leq 5 \right]$$

$$= \int_{-\infty}^{5} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0 + \int_{0}^{5}$$

$$= \int_{0}^{5} \frac{1}{8} e^{-\frac{x}{8}}\,dx$$

$$= \frac{1}{8}\left( \frac{e^{-\frac{x}{8}}}{(-1/8)} \right)_{0}^{5}$$

$$= -\left( e^{-\frac{x}{8}} \right)_{0}^{5} = 1 - e^{-\frac{5}{8}}$$

$$= 0.4647$$

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7 a)

$\mu = 8.9$

$n = 50$

$\bar{x} = 9.3$

$s = 1.6$

$\because n = 50$ is large

$\therefore$ we have to use Normal statistic

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{9.3 - 8.9}{1.6/\sqrt{50}}$$

$$= 1.768$$

$H_0: \mu = 8.9$

$H_1: \mu \neq 8.9$

$\therefore$ Two Tailed Test can be applied (should be) at 5% level of significance

$|Z| \leq 1.96$

$|Z| = |1.768| \leq 1.96$ ~~False~~ True

$\therefore H_0$ Can be accepted at 5% level of significance.

$$\bar{x} - (1.96)\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + (1.96)\frac{s}{\sqrt{n}}$$

$$9.3 - 1.96\frac{1.6}{\sqrt{50}} \le \mu \le 9.3 + 1.96\frac{1.6}{\sqrt{50}}$$

$$8.85 \le \mu \le 9.74$$ (95% Confident interval)

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7 b) (i) $x + 2y - 5 = 0 \rightarrow (1)$

$2x + 3y - 8 = 0 \longrightarrow (2)$

$(1) \Rightarrow x = -2y + 5 \rightarrow (3)$

$(2) \Rightarrow y = -\frac{2}{3}x + \frac{8}{3} \rightarrow (4)$

assume (3) is x on y

(4) is y on x

$\therefore b_{xy} = -2$ & $b_{yx} = -\frac{2}{3}$

$r = \sqrt{b_{xy} b_{yx}}$

$= \sqrt{(-2)(-\frac{2}{3})} = \sqrt{\frac{4}{3}} \approx 1.14$

$\therefore \rho(x,y) = 1.14$ which is wrong

$\therefore$ our assumption is wrong

$\therefore$ $x+2y-5=0$ will be the y on x

$2x+3y-8=0$ will be the x on y

(ii)
$$2y = -x + 5$$
$$y = -\frac{1}{2}x + \frac{5}{2} \quad (y \text{ on } x)$$
$$2x = -3y + 8$$
$$x = -\frac{3}{2}y + 4 \quad (x \text{ on } y)$$

$$\therefore b_{yx} = -\frac{1}{2} \quad \& \quad b_{xy} = -\frac{3}{2}$$

$$\therefore r = \sqrt{b_{xy} \cdot b_{yx}}$$
$$= \sqrt{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)} \quad r = \sqrt{\frac{3}{4}}$$
$$= 0.866$$

$$\therefore \boxed{r(x, y) = 0.866}$$

(iii) Both the lines of regression passes through the point $(\bar{x}, \bar{y})$

$$\therefore \bar{x} + 2\bar{y} - 5 = 0 \rightarrow ①$$
$$2\bar{x} + 3\bar{y} - 8 = 0 \rightarrow ②$$

$$①\times 2 \Rightarrow \quad \cancel{2\bar{x}} + 4\bar{y} - 10 = 0$$
$$②\times 1 \Rightarrow \quad \cancel{2\bar{x}} + 3\bar{y} - 8 = 0$$
$$- \quad - \quad +$$
$$\bar{y} - 2 = 0$$
$$\therefore \boxed{\bar{y} = 2}$$

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(1) $\Rightarrow$ $\bar{x}+2\bar{y}-5=0$

$\bar{x}+4-5=0$

$\bar{x}=1$

$\therefore$ Avg marks in Maths $(\bar{x}) = 1$

Avg marks in physics $(\bar{y}) = 2$

c) 5, 6, 2, 3, 1, $x$, 9, 0, 7, $x-1$

Mode = 5

$\therefore$ $x$ must be 5

If $x=6$ then mode becomes 6

If $x=4$ then mode becomes 3

$\therefore$ Mean of 5, 6, 2, 3, 1, 5, 9, 0, 7, 4 is

$$= \frac{5+6+2+3+1+5+9+0+7+4}{10}$$

$$= \frac{42}{10} = 4.2$$

Median :-

0, 1, 2, 3, 4, 5, 5, 6, 7, 9

$$\text{Median} = \frac{4+5}{2} = 4.5$$