P2:

If $\boldsymbol{\lambda = np}$**, then** $\boldsymbol{\lim\limits_{n \to \infty} \lim\limits_{p \to 0} \binom{n}{x} p^x q^{n-x} = \frac{e^{-\lambda}\lambda^x}{x!}}$ **.**

Proof:

$$\binom{n}{x} p^x q^{n-x} = \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \binom{n}{x} \left(\frac{p}{1-p}\right)^x (1-p)^n$$

$$= \frac{n(n-1)(n-2)\ldots(n-x+1)}{x!} \frac{\left(\frac{\lambda}{n}\right)^x}{\left(1-\frac{\lambda}{n}\right)^x} \left(1-\frac{\lambda}{n}\right)^n$$

$$= \frac{n^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{x-1}{n}\right)}{x!\left(1-\frac{\lambda}{n}\right)^x} \frac{\lambda^x}{n^x} \left(1-\frac{\lambda}{n}\right)^n$$

Then $\lim\limits_{n \to \infty} \lim\limits_{p \to 0} \binom{n}{x} p^x q^{n-x} = \frac{e^{-\lambda}\lambda^x}{x!} \quad \left(\because \lim\limits_{n \to \infty} \left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda}\right)$