P1:

A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

Solution:

Let X be the total number of sixes.

Then $X \sim B\left(600, \frac{1}{6}\right)$, $E(X) = np = 600 \times \frac{1}{6} = 100$

and $V(X) = np(1-p) = 600 \times \frac{1}{6} \times \frac{5}{6} = \frac{500}{6}$.

Using Chebychev's inequality, we get

$$P\{|X - E(X)| < k\sigma\} \geq 1 - \frac{1}{k^2} \Rightarrow P\left\{|X - 100| < k\sqrt{\frac{500}{6}}\right\} \geq 1 - \frac{1}{k^2}$$

Therefore, $P\left\{100 - k\sqrt{\frac{500}{6}} < X < 100 + k\sqrt{\frac{500}{6}}\right\} \geq 1 - \frac{1}{k^2}$

Taking $k = \frac{20}{\sqrt{\frac{500}{6}}}$, we get $P(80 < X < 120) \geq 1 - \frac{1}{400 \times \left(\frac{6}{500}\right)} = \frac{19}{24}$

The lower bound for the probability of getting 80 to 120 sixes.