P4:

The j.p.d.f. of (X, Y) is given by

$$f(x,y)=\begin{cases}2\ ,0<x<1,0<y<x\\0,\ otherwise\end{cases}$$

a) Find marginal and conditional p.d.fs of x and y.

b) Check for independence of x and y.

Solution:

a) The m.p.d.f. of X is given by

$$f_1(x)=\int_0^x f(x,y)dy=\int_0^x 2dy=2x$$

$$\Rightarrow f_1(x)=2x,0<x<1$$

The m.p.d.f. of Y is given by

$$f_2(y)=\int_y^1 f(x,y)dx=\int_y^1 2dx=2(1-y)$$

$$\Rightarrow f_2(y)=2(1-y),0<y<1$$

The conditional p.d.f of X given Y is given by

$$f_{1|2}(x|y)=\frac{f(x,y)}{f_2(y)}=\frac{2}{2(1-y)}=\frac{1}{1-y}\ ,y<x<1$$

The conditional p.d.f of Y given X is given by

$$f_{2|1}(y|x)=\frac{f(x,y)}{f_1(x)}=\frac{2}{2x}=\frac{1}{x}\ ,0<x<1$$

b) Since $f_1(x)\cdot f_2(y)=2x\cdot 2(1-y)=4x(1-y)\neq f(x,y)$, X and Y are not independent.