4.2. Convergence of sequences of random variables

Exercise:

1. Let $X_1, X_2, ..$ be a sequence of r.vs with corresponding d.fs given by

$$F_n(x) = \begin{cases} 0 & , \quad x < n \\ \frac{x+n}{2n} & , \quad -n \le x < n \\ 0 & , \quad x \ge n \end{cases}$$

   Does F_n converge to some d.f.

2. Let $X_1, X_2, ..$ be a i.i.d $U(0, \theta)$ r.vs. Let $X_{(1)} = min(X_1, X_2, \dots, \mathrm{X_n})$ and consider the sequence $Y_n = nX_{(1)}$. Does Y_n converge in distribution to some r.v. Y? If so, find the d.f. of r.v. Y.

3. Let $X_1, X_2, ..$ be i.i.d. r. vs with continuous d.f. F .Let $X_{(n)} = max(X_1, X_2, .. \mathrm{X_n})$ and consider the sequence of r.vs $Y_n = n[1 - F(X_{(n)})]$.Find the limiting d.f. of of Y_n.

4. Let $X_1, X_2, ..$ be a sequence of i.i.d r.vs with common p.d.f

$$f(x, \theta) = \begin{cases} e^{-x+\theta}, & if \ x \ge \theta \\ 0 \quad , & if \ x < \theta \end{cases}$$

   write $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$

   (a) Show that $\bar{X}_n \xrightarrow{P} 1 + \theta$

   (b) Show that $\min\{X_1, X_2, .. \mathrm{X_n}\} \xrightarrow{P} \theta$

5. Let $X_1, X_2, ..$ be i.i.d $U[0, \theta]$ r.vs. Show that $\max\{X_1, X_2, .., \mathrm{X_n}\} \xrightarrow{P} \theta$

Answers:

1. No
2. Yes. $Y_n \longrightarrow Y$ where $F(y) = \begin{cases} 0 & if \quad y < 0 \\ 1 - e^{-y/\theta} & if \quad y \geq 0 \end{cases}$
3. $F(y) = \begin{cases} 0 & if \quad y \leq 0 \\ 1 - e^{-y} & if \quad y > 0 \end{cases}$