P1:

If $X \sim B(n, p)$, then show that $V(X) = npq$.

Proof: We already derived that $E(X) = \sum_{x=0}^{n} x\, p(x) = np.$

$$E(X^2) = \sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} \left[x(x-1)+x\right] p(x)$$

$$= \sum_{x=2}^{n} x(x-1)\, p(x) + \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=2}^{n} x(x-1)\binom{n}{x} p^x q^{n-x} + np$$

$$= \sum_{x=2}^{n} x(x-1)\left(\frac{n}{x}\right)\left(\frac{n-1}{x-1}\right)\binom{n-2}{x-2} p^x q^{n-x} + np$$

$$= n(n-1)\, p^2 \sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x} + np$$

$$= n(n-1)\, p^2 (q+p)^{n-2} + np$$

$$\Rightarrow E(X^2) = n(n-1)\, p^2 + np \;\; and \;\; hence$$

$$V(X) = E(X^2) - \left(E(X)\right)^2 = n(n-1)\, p^2 + np - n^2 p^2 = npq$$