P1:

Let $(\boldsymbol{X},\boldsymbol{Y})\sim\boldsymbol{BN}(\boldsymbol{\mu_1},\boldsymbol{\mu_2},\boldsymbol{\sigma_1}^2,\boldsymbol{\sigma_2}^2,\boldsymbol{\rho})$. If $\boldsymbol{\rho}=\mathbf{0}$, then $\boldsymbol{X}$ and $\boldsymbol{Y}$ are independent.

Solution:

Since $(X,Y)\sim BN(\mu_1,\mu_2,\sigma_1{}^2,\sigma_2{}^2)$, the j.p.d.f. of (X,Y) is given by

$$f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}exp\left[-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{x-\mu_1}{\sigma_1}\right)^2-\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right\}\right]$$

If $\rho=0$, then

$$f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2}exp\left[-\frac{1}{2}\left\{\left(\frac{x-\mu_1}{\sigma_1}\right)^2+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right\}\right]$$

$$=\frac{1}{\sqrt{2\pi}\,\sigma_1}exp\left[-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right]\times\frac{1}{\sqrt{2\pi}\,\sigma_2}exp\left[-\frac{1}{2}\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]$$

$\Longrightarrow f(x,y)=f_1(x)f_2(y)$,where $f_1(x)$ and $f_2(y)$ are m.p.d.fs of X and Y respectively. Thus, X and Y are indentent.

Hence, if $\rho=0$, then X and Y are independent.