P3:

A student doing a summer internship in a company was asked to model the life term of certain equipment that the company makes. After a series of tests, the student proposed that the life time of the equipment can be modeled by a random variable X that has p.d.f.

$$f(x) = \begin{cases} \dfrac{xe^{-\frac{x}{10}}}{100}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

a. **Show that $f(x)$ is a valid p.d.f.**
b. **What is the probability that the lifetime of the equipment exceeds 20?**
c. **Find the mean?**

Solution:

a. Consider $\int_0^\infty f(x)dx = \dfrac{1}{100}\int_0^\infty xe^{-\frac{x}{10}}dx$

$$= \frac{1}{100}\left\{\left[-10xe^{-\frac{x}{10}}\right]_0^\infty + 10\int_0^\infty e^{-\frac{x}{10}}dx\right\}$$

$$= \frac{1}{100}\left\{0 - 100\left[e^{-\frac{x}{10}}\right]_0^\infty\right\} = \frac{1}{100}\times 100 = 1$$

Thus, $f(x)$ is a valid p.d.f.

b. $P(X > 20) = \int_{20}^\infty f(x)dx = \dfrac{1}{100}\left\{\left[-10xe^{-\frac{x}{10}}\right]_{20}^\infty + 10\int_{20}^\infty e^{-\frac{x}{10}}dx\right\}$

$$=\frac{1}{100}\left\{200e^{-2}-\left[100e^{-\frac{x}{10}}\right]_{20}^{\infty}\right\}=2e^{-2}+e^{-2}=3e^{-2}=\frac{3}{e^{2}}=0.406$$

c. Mean= $E(X)=\int_0^\infty x\,f(x)dx=\frac{1}{100}\int_0^\infty x^2e^{-\frac{x}{10}}dx$

$$=\frac{1}{100}\left[10x^2e^{-\frac{x}{10}}\right]_0^\infty+\frac{20}{100}\int_0^\infty xe^{-\frac{x}{10}}dx`$$

$$=0+20=20$$