

**P3:**

If  $X$  and  $Y$  are two random variables having j.p.d.f.

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y), & 0 < x < 2, \quad 2 < y < 4 \\ 0, & \text{otherwise} \end{cases}$$

Find (i)  $P(X < 1 \cap Y < 3)$  (ii)  $P(X + Y < 3)$  and (iii)  $P(X < 1 | Y < 3)$ .

**Solution:**

$$(i) P(X < 1 \cap Y < 3) = \int_0^1 \int_0^3 f(x, y) dx dy = \frac{1}{8} \int_0^1 \int_0^3 (6 - x - y) dx dy = \frac{3}{8}$$

$$(ii) P(X + Y < 3) = \frac{1}{8} \int_0^1 \int_2^{3-x} (6 - x - y) dx dy = \frac{5}{24}$$

$$(iii) P(X < 1 | Y < 3) = \frac{P(X < 1 \cap Y < 3)}{P(Y < 3)}$$

$$\text{But } P(Y < 3) = \frac{1}{8} \int_0^2 \int_2^3 (6 - x - y) dx dy = \frac{5}{8}$$

$$\text{Hence } P(X < 1 | Y < 3) = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$