2.1. Random Variables

Exercise

1. State, with reasons, if the following probability distributions are admissible or not.

   (i)

x:	0	1	2
$p(x)$:	0.3	0.2	0.5

   (ii)

x:	– 1	0	2
$p(x)$:	0.4	0.4	0.3

   (iii)

x:	0	1	2	3
$p(x)$:	0.2	0.3	0.3	0.1

   (iv)

x:	–2	–1	0	1	2
$p(x)$:	0.3	0.4	– 0.2	0.2	0.3

2. Two dice are thrown simultaneously and *getting a number less than 3* on a die is termed as a success. Obtain the probability distribution of the number of success.

3. Obtain the probability distribution of the number of sixes in two tosses of a die.
4. Obtain the probability distribution of number of heads of two tosses of a coin.

5. Three cards are drawn at random successively, with replacement, from a well shuffled pack of cards, getting ‘a card of diamonds’ is termed as a success. Obtain probability distribution of the number of successes.

6. Two cards are drawn without replacement, form a well shuffled pack of cards. Obtain the probability distribution of the number of face cards (Jack, Queen, King and Ace).

7. Five defective mangoes are accidently mixed with twenty good ones and by looking at them it is not possible to difference between them. Four mangoes are drawn at random from the lot. Find the probability distribution of x, the number of defective mangoes.

8. Two bad eggs are mixed accidentally with 10 good ones and three are drawn at random from the lot. Obtain the probability distribution of the number of bad eggs drawn.

9. An urn contains 6 red and 4 white balls. Three balls are known at random. Obtain the probability distribution of the number of white balls drawn.

10. Suppose that the life in hours of a certain part of radio tube is a continuous random variable x with p.d. f given by

$$f(x) = \begin{cases} \frac{100}{x^2}, & when\ x \geq 100 \\ 0\ , & otherwise \end{cases}$$

(i) What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?

(ii) What is the probability that none of three of the original tubes will have to be replaced during that first 150 hours of operation?

(iii) What is the probability that a tube will last less than 200 hours if it is known that the tube still functioning after 150 hours of service.

(iv) What is the maximum number of tubes that many be inserted into a set so that there is a probability of 0.5 that after 150 hours of services all of them are still functioning?

Answers:

1.

(i) Yes

(ii) No, since $\sum p(x) > 1$

(iii) No, since $\sum p(x) < 1$

(iv) No, since $p(0) = -0.2$ which is not possible.

2.

x :	0	1	2
$p(x)$:	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

3.

x :	0	1	2
$p(x)$:	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

4.

x :	0	1	2
$p(x)$:	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

5.

x:	0	1	2	3
$p(x)$:	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

6.

x:	0	1	2
$p(x)$:	$\frac{36_{C_2}}{52_{C_2}} = \frac{105}{221}$	$\frac{36_{C_1} \times 16_{C_1}}{52_{C_2}} = \frac{96}{221}$	$\frac{16_{C_2}}{52_{C_2}} = \frac{20}{221}$

7.

x:	0	1	2	3	4
$p(x)$:	$\frac{20_{C_4}}{25_{C_4}} = \frac{969}{2530}$	$\frac{5_{C_1} \times 20_{C_3}}{25_{C_4}} = \frac{1140}{2530}$	$\frac{5_{C_2} \times 20_{C_2}}{25_{C_4}} = \frac{380}{2530}$	$\frac{5_{C_3} \times 20_{C_1}}{25_{C_4}} = \frac{40}{2530}$	$\frac{5_{C_1}}{25_{C_4}} = \frac{1}{2530}$

8.

x:	0	1	2	3
$p(x)$:	$\frac{10_{C_3}}{12_{C_3}} = \frac{12}{22}$	$\frac{2_{C_1} \times 10_{C_2}}{12_{C_3}} = \frac{9}{22}$	$\frac{2_{C_2} \times 10_{C_1}}{12_{C_3}} = \frac{1}{22}$	0

9.

x:	0	1	2	3
$p(x)$:	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

10.

(i) $\frac{1}{3}$

(ii) $\frac{2}{3}$

(iii) $\frac{1}{4}$

(iv) 1.7 (= 2 approximately)