1.5. Bayes' Theorem and Its Applications

Exercise

1. In 2004 there will be three candidates for the position of principal, in the college, Dr. Singhal, Mr. Mehra and Dr. Chatterji, whose chances of getting appointment are in the proportion $4:2:3$ respectvely. The probability that Dr. singhal if selected, will abolish co – education in the college is 0.3. The probability of Mr. Mehra and Dr. Chatterji doing the same are respectively 0.5 and 0.8. What is the probability that co – education will be abolished from the college in 2004?

2. (a) Suppose that one of three men, a politician, a businessman, and an educationist, will be appointed as the vice – chancellor of a university. The respective probabilities of their appointments are $0.50, 0.30, 0.20$. The probability that research activities will be promoted by these people if they are appointed are $0.30, 0.70$ and 0.80 respectively. What is the probability that research will be promoted by the new vice – chancellor?
   (b) A manufacturing firm purchases a certain component for its manufacturing process from three sub – contractors A, B and C. These supply 60 percent, 30 percent and 10 percent of the firm's requirements, respective suppliers are defective items. On a particular day, a normal shipment arrives from each of the three suppliers and the contents get mixed. If a component is chosen at random from the day's shipment, what is the probability that it is defective?

3. Assume that a factory has two machines. Past records show that machine 1 produces 30% of the items of output and machine 2 were produces 70% of the items. Further, 5% of the items produced by machine 1 were defective and only 1% produced by machine 2 defective. If a defective item is drawn at random, what is the probability that it was produced by
   (i) machine 1 , (ii) machine 2 ?

4. In a bolt factory machines A, B and C manufacture respectively $20\%, 30\%$ and 50% of the total of its output. Of them $5, 4$ and 2 percent respectively are defective bolts. A bolt is drawn at random from the product and it found to be defective. What is the probability that it was manufactured by machine B?

5. A factory produces a certain type of outputs by three types of machines. The respective daily production figures are:
   Machine : $3{,}000$ Units ; Machine II : $2{,}500$ Units ; Machine III : $4{,}500$ Units
   Past experience shows that 1 percent of the output produced by Machine I is defective. The corresponding fraction of defectives for the other two machines are 1.2 percent and 2 percent respectively. An item is drawn at random from the day's production run and is found to be defective, what is probability that it comes from the output of
   (a) Machine 1, (ii) Machine II, (iii) Machine III?

6. Suppose that a product is produced in three factories A, B and C. It is known that factory A produces twice as many items as factory B, ad that factories B and C produces the same number of products. Assume that it is known that 2 oercent of the items produced by each of the factories A and B are defective while 4 percent of those manufactured by factory C are defective. All the items produced in three factories are stocked, and an item of product is selected at random. What is the probability that this item is defective?

7. A company has two plants to manufacture scooters. Plant I manufacutes 70% of the scooters and Plant II manufactures 30%. At Plant I, 80% of the scooters produced are of standard quality and at Plant II, 90% of the scooters produced are of standard quality. A scooter is picked at random and found to be of standard quality. What is the chance that it has come from Plant II?

8. Suppose there is a chance for a newly constructed building to collapse, whether the design is faulty or not. The chance that the design in faulty is 10%. The chance that the building collapses in 95%. If the design is faulty and otherwise it is 45%. It is seen that the building collapsed. What is the probability that it is due to faulty design.

Answers:

1. $\frac{23}{45}$
2. $a.\ 0.52$ b. 0.035
3. $(i)\ 0.682\ ,\quad (ii)\ 0.318.$
4. $\left(\frac{3}{8}\right) = 0.375$
5. a. $\frac{1}{5}$ b. $\frac{1}{5}$ c. $\frac{3}{5}$
6. 0.07
7. 0.03253
8. 0.19