

## 1.4. Theorems in Probability

### Exercises:

1. A card is drawn from a well shuffled pack of 52 cards. Find the probability that it is either a diamond or a king.
2. If  $P(A) = 0.4$ ,  $P(B) = 0.7$  and  $P(\text{at least one of } A \text{ and } B) = 0.8$ , find  $P(\text{only one of } A \text{ and } B)$ .
3. Let  $A$  and  $B$  be the two possible outcomes of an experiment and suppose  $P(A) = 0.4$ ,  $P(A \cup B) = 0.7$  and  $P(B) = p$ .
  - (i) For what choice of  $P$  are  $A$  and  $B$  mutually exclusive?
  - (ii) For what choice of  $P$  are  $A$  and  $B$  independent?
4. An urn contains four tickets marked with numbers 112, 121, 211 and 222 and one ticket is drawn at random. Let  $A_i (i = 1, 2, 3)$  be the event that  $i^{\text{th}}$  digit of the number of the ticket drawn is 1. Are  $A_1, A_2, A_3$  (i) pairwise independent (ii) independent?
5. An engineer applies for a job in two firms  $X$  and  $Y$ . He estimates that the probability of his being selected in firm  $X$  is 0.7 and being rejected at  $Y$  is 0.5 and the probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the firms?
6. Probability that a man will be alive 25 years hence is 0.3 and the probability that his wife will be alive 25 years hence is 0.4. Find the probability that 25 years hence
  - (i) both will be alive
  - (ii) one the man will be alive
  - (iii) only the woman will be alive
  - (iv) none will be alive
  - (v) at least one of them will be alive
7. The probability that a contractor will get a plumbing contract is  $\frac{2}{3}$  and the probability that he will not get an electric contract is  $\frac{5}{9}$ . If the probability of getting at least one contract is  $\frac{4}{5}$ , what is the probability that he will get both the contracts?

8. A problem in probability is given to two students  $X$  and  $Y$ . The odds in favour of  $X$  solving the problem are 6 to 9 and against  $Y$  solving the problem are 12 to 10. If both  $X$  and  $Y$  attempt, find the probability of the problem being solved.
9. A piece of equipment will function only when all the three components  $A$ ,  $B$  and  $C$  are working. The probability of  $A$  failing during one year is 0.15, that of  $B$  failing is 0.05 and that of  $C$  failing is 0.10. What is the probability that the equipment will fail before the end of the year?
10. Find the probability of throwing 6 at least once in six throws with a single die.
11. The odds that  $A$  speaks the truth are 3 : 2 and the odds that  $B$  speaks the truth are 5 : 3. In what percentage of cases are they likely to contradict each other on an identical point?
12. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. Find the probability that the selected consist of 1 girl and 2 boys.
13. If  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{2}{5}$  and  $P(A \cup B) = \frac{1}{2}$ , then find
  - (i)  $P(A \cap \bar{B})$
  - (ii)  $P(\bar{A} \cap \bar{B})$
14. If  $A$ ,  $B$  and  $C$  are mutually exclusive and exhaustive event such that  $P(A) = \frac{1}{2}P(B)$  and  $P(B) = \frac{2}{3}P(C)$ , find  $P(A)$ ,  $P(B)$  and  $P(C)$ .
15. If  $P(A) = 0.3$ ,  $P(B) = 0.2$  and  $P(C) = 0.1$  and  $A$ ,  $B$ ,  $C$  are independent events, find the probability of occurrence of atleast one of the three events  $A$ ,  $B$  and  $C$ .

Answers:

1.  $\frac{4}{13}$
2. 0.5
3. (i)  $p = 0.3$  (ii)  $p = 0.5$
4. (i) yes (ii) no
5. 0.8
6. (i) 0.12 (ii) 0.18 (iii) 0.28 (iv) 0.42 (v) 0.58
7.  $\frac{14}{45}$

$$8. \frac{37}{55}$$

$$9. 0.27325$$

$$10. 1 - \left(\frac{5}{6}\right)^6$$

$$11. \frac{19}{40}$$

$$12. \frac{13}{32}$$

$$13. \text{(i) } 0.1 \quad \text{(ii) } 0.85$$

$$14. \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$$

$$15. 0.4\%$$