P3:

The diameter of a telephone cable, say, x is assumed to be continuous random variable with p.d.f. $f(x) = kx(1-x), 0 \le x \le 1$.

i. **Find k for which the above is a p.d.f.**

ii. **Determine b such that $P(x < b) = P(x > b)$.**

Solution:

i. $f(x) = kx(1-x), 0 \le x \le 1$ is the p.d.f. of a continuous random variable x if $\int_0^1 f(x)dx = 1$. That is $k\int_0^1 x(1-x)dx = 1$

$$\Rightarrow k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1 \Rightarrow k = 6$$

(i) Given $P(x < b) = P(x > b)$.That is,

$$\int_0^b f(x)dx = \int_b^1 f(x)dx$$

$$\Rightarrow 6\int_0^b x(1-x)dx = 6\int_b^1 x(1-x)dx$$

$$\Rightarrow \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_b^1$$

$$\Rightarrow \left[3b^2 - 2b^3\right] = \left[1 - 3b^2 + 2b^3\right]$$

$$\Rightarrow 4b^3 - 6b^2 + 1 = 0$$

$$\Rightarrow (2b-1)\left(2b^2 - 2b - 1\right) = 0$$

$$\Rightarrow b = \frac{1}{2}, b = \frac{1 \pm \sqrt{3}}{2}$$

Hence, $b = \frac{1}{2}$ is the only value lying in $[0,1]$ and satisfying $P(x < b) = P(x > b)$