

P1:

Two dice are rolled at random. Obtain the probability distribution of the sum of the numbers on them.

Solution:

When two dice are rolled, the sample space S consists of $6^2 = 36$, sample points as shown.

$$S = \{(1,1), (1,2), \dots, (1,6), (2,1), (2,2), \dots, (2,6), (3,1), (3,2), \dots, (3,6) \\ (4,1), (4,2), \dots, (4,6), (5,1), (5,2), \dots, (5,6), (6,1), (6,2), \dots, (6,6)\}$$

Let X denote the sum of the numbers on the two dice. Then X is a random variable which can take the values 2, 3, 4, ..., 12 with the probability distribution given by:

| Sum of numbers (x) | Favourable sample points | No. of favourable cases | $p(x)$ |
|------------------------|--|-------------------------|----------------|
| 2 | (1,1) | 1 | $\frac{1}{36}$ |
| 3 | (1,2), (2,1) | 2 | $\frac{2}{36}$ |
| 4 | (1,3), (3,1), (2,2) | 3 | $\frac{3}{36}$ |
| 5 | (1,4), (4,1), (2,3), (3,2) | 4 | $\frac{4}{36}$ |
| 6 | (1,5), (5,1), (2,4), (4,2), (3,3) | 5 | $\frac{5}{36}$ |
| 7 | (1,6), (6,1), (2,5), (5,2), (3,4), (4,3) | 6 | $\frac{6}{36}$ |
| 8 | (2,6), (6,2), (3,5), (5,3), (4,4) | 5 | $\frac{5}{36}$ |
| 9 | (3,6), (6,3), (4,5), (5,4) | 4 | $\frac{4}{36}$ |
| 10 | (4,6), (6,4), (5,5) | 3 | $\frac{3}{36}$ |
| 11 | (5,6), (6,5) | 2 | $\frac{2}{36}$ |
| 12 | (6,6) | 1 | $\frac{1}{36}$ |

Hence, the probability distribution of X is given by:

| | | | | | | | | | | | |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| x | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $p(x)$ | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |