

### 4.3. Weak Law of Large Numbers

#### Exercise:

1) For the following sequence of independent r.v.s, does the *WLLN* hold?

a)  $P(X_k = \pm k) = \frac{1}{2\sqrt{k}}, P(X_k = 0) = 1 - \frac{1}{\sqrt{k}}$

b)  $P(X_k = \pm 2^k) = \frac{1}{2^{2k+1}}, P(X_k = 0) = 1 - \frac{1}{2^{2k}}$

c)  $P\left(X_k = \pm \frac{1}{k}\right) = \frac{1}{2}$

d)  $P(X_k = \pm \sqrt{k}) = \frac{1}{2}$

2) Examine if *WLLN* holds for the sequence  $\{X_i\}$  of i.i.d.r.v.s with

$$P[x_i = (-1)^{k-1} \cdot k] = \frac{6}{\pi^2 k^2}, \quad k = 1, 2, 3, \dots, \quad i = 1, 2, \dots$$

Hint:  $\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$

3) Let  $X_1, X_2, \dots, X_n$  be jointly normal with  $E(X_i) = 0, E(X_i)^2 = 1$  for all  $i$  and

$$\text{cov}(X_i, X_j) = \begin{cases} \rho, & |j - i| = 1 \\ 0, & \text{otherwise} \end{cases}$$

Examine if *WLLN* holds for the sequence  $\{X_n\}$ .

#### ANSWERS

1)

a) No

b) Yes

c) Yes

d) No

e) No

2) Yes

3) Yes