P4:

A box contains a white and b black balls. c balls are drawn at random. Find the expected value of the number of white balls drawn.

Solution:

Let a variable X_i associated with i^{th} drawn, be defined as follows:

$$X_i = \begin{cases} 1, if\ the\ i^{th}\ ball\ drawn\ is\ white \\ 0, if\ the\ i^{th}\ ball\ drawn\ is\ black \end{cases}$$

Then the number of S of white balls among c balls drawn is given by

$$S = X_1 + X_2 + \cdots + X_c = \sum_{i=1}^{c} X_i$$

$$\Longrightarrow E(S) = \sum_{i=1}^{c} E(X_i)$$

But $P(X_i = 1) = P(\text{ of drawing a white ball }) = \frac{a}{a+b}$

and $P(X_i = 0) = P(\text{ of drawing a black ball }) = \frac{b}{a+b}$

$\therefore E(X_i) = 1.P(X_i = 1) + 0.P(X_i = 0) = P(X_i = 1) = \frac{a}{a+b}$ and hence

$$E(S) = \sum_{i=1}^{c} E(X_i) = \sum_{i=1}^{c} \frac{a}{a+b} = \frac{ac}{a+b}.$$