P2:

Find the mean and variance of the random variable, whose p.d.f. is given by

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & , x>0 \\ 0 & ,otherwise\end{cases}$$

Solution:

$$E(X)=\int_{-\infty}^{\infty} x f(x)dx=\int_{0}^{\infty} x f(x)dx=\lambda\int_{0}^{\infty} x e^{-\lambda x}dx=\lambda\left[\left(-\frac{xe^{-\lambda x}}{\lambda}\right)_{0}^{\infty}+\frac{1}{\lambda}\int_{0}^{\infty} e^{-\lambda x}dx\right]$$

$$=\lambda\left[0-\left(\frac{e^{-\lambda x}}{\lambda^2}\right)_{0}^{\infty}\right]=\lambda\left(\frac{1}{\lambda^2}\right)=\frac{1}{\lambda}$$

and $E\left(X^2\right)=\int_{-\infty}^{\infty} x^2 f(x)dx=\int_{0}^{\infty} x^2 f(x)dx=\lambda\int_{0}^{\infty} x^2 e^{-\lambda x}dx$

$$=\lambda\left[\left(\frac{x^2.e^{-\lambda x}}{\lambda}\right)_{0}^{\infty}+2\int_{0}^{\infty}\frac{x.e^{-\lambda x}}{\lambda}dx\right]=\lambda\left[0+\frac{2}{\lambda}\int_{0}^{\infty} x.e^{-\lambda x}dx\right]=\lambda\left[\frac{2}{\lambda}.\frac{1}{\lambda^2}\right]=\frac{2}{\lambda^2}$$

Thus $V(X)=E(X^2)-\left(E(X)\right)^2=\frac{2}{\lambda^2}-\frac{1}{\lambda^2}=\frac{1}{\lambda^2}$