4.4. Strong Law of Large Numbers

Exercise:

For the following sequences of independent r.vs, does the SLLN hold or not?

1. $P(X_k = \pm 2^{k+1}) = 2^{-(k+3)}$, $P(X_k = 0) = 1 - 2^{-(k+2)}$

2. $P(X_k = \pm 2^{-k}) = \frac{1}{2^{k+1}}$, $P(X_k = \pm 1) = \frac{1}{2}(1 - 2^{-k})$

3. Examine if SLLN holds for the sequence $\{X_i\}$ of i.i.d.r.vs with

$$P[X_i = (-1)^{k-1}k] = \frac{6}{\pi^2 k^2} \text{ , } k = 1,2,3, \dots \; i = 1,2, \dots$$

4. Let X_1 , $X_2, \dots, X_n$ be jointly normal with $E(X_i) = 0$, $E\left(X_i{}^2\right) = 1$ for all i and

$$cov\left(X_i, X_j\right) = \begin{cases} \rho \text{ , } & |j - i| = 1 \\ 0, & \text{otherwise} \end{cases}$$

Examine if SLLN holds for the sequence $\{X_i\}$.

Answer

1. No
2. Yes
3. Yes
4. Yes