P3:

The time taken by a person while speaking over a telephone is exponentially distributed with mean 4 *minutes.*

i) **Find the probability that he speaks for more than 6 *minutes* but less than 7 *minutes*.**

ii) **Out of 6 calls that he makes, what is the probability that exactly 2 calls take him more than 3 minutes each.**

iii) **How many calls out of 100 are expected to take more than 3 minutes each?**

Solution:

Let X be the time taken (in minutes) per call. We are given that X is exponentially distributed with mean 4 *minutes*.

$$\therefore f(x) = \begin{cases} \frac{1}{4}e^{-\frac{x}{4}} & , \quad x > 0 \\ 0 & , \quad otherwise \end{cases}$$

(i) $$P(6 < X < 7) = \frac{1}{4}\int_6^7 e^{-\frac{x}{4}} = \frac{1}{4}\left[\frac{e^{-\frac{x}{4}}}{-\frac{1}{4}}\right]_6^7 = e^{-\frac{6}{4}} - e^{-\frac{7}{4}} (= 0.04936)$$

(ii) $$P(X > 3) = \frac{1}{4}\int_3^\infty e^{-\frac{x}{4}}dx = \frac{1}{4}\left[\frac{e^{-\frac{x}{4}}}{-\frac{1}{4}}\right]_3^\infty = e^{-\frac{3}{4}} (= 0.4724)$$

Let Y denote the number of calls each with more than 3 minutes out of 6 calls. Then

$$Y \sim B(6, P) \text{ where } P = 0.4724$$

$$\therefore P(Y = 2) = \binom{6}{2}(0.4764)^2(0.5276)^4 = 0.2594$$

(iii) Expected number of calls out of 100 that will be longer than 3 minutes each

$$= 100 \times P(X > 3)$$

$$= 100 \times 0.4724 = 47.24 = \ 47 \text{(approximately)}$$