P4:

The variates $X_1, X_2, ..$ have equal expectations and finite variations. Is $WLLN$ applicable to this sequence if all covariances σ_{ij} are negative?

Solution:

We have

$$V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab\ cov\ (X, Y)$$

$$= a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\sigma_{XY}$$

$$\leq a^2\sigma_X^2 + b^2\sigma_Y^2 \qquad (\because\ \sigma_{XY} < 0)$$

$$\Longrightarrow 0 \leq V(aX + bY) \leq a^2\sigma_X^2 + b^2\sigma_Y^2$$

$$\therefore\ 0 \leq V\left(\frac{X_1 + ... + X_n}{n}\right) \leq \frac{1}{n^2}\sum_{i=1}^{n} V(X_i) < \frac{A}{n} \qquad ...(1)$$

where A is the upper bound of $V(X_i)\ \forall\ i = 1, 2, ... n$

Now, $\frac{B_n}{n^2} = V\left(\frac{X_1 + \cdots + X_n}{n}\right) < \frac{A}{n} \longrightarrow 0$ as $n \longrightarrow \infty$. (from (1))

Thus, $WLLN$ is applicable to this sequence.