2.2. Bivariate Random Variable:

Exercise

1. The joint probability mass function of (X, Y) is given in the following table:

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

   Find (i) $P(X \leq 1, Y = 2)$, (ii) $P(X \leq 1)$, (iii) $P(Y \leq 3)$, iv) $P(X < 3, Y \leq 4)$.

2. The j.p.m.f. of (X, Y) is given by: $P(X = 0, Y = 1) = \frac{1}{3}$, $P(X = 1, Y = -1) = \frac{1}{3}$ and $P(X = 1, Y = 1) = \frac{1}{3}$.
   Find (i) m.p.m.fs of X and Y and (ii) conditional p.m.f. of X given $Y = 1$.

3. The j.p.m.f. of (X, Y) is given by
   $$p(x, y) = \frac{x^2+y}{32} \text{ for } x = 0,1,2,3 \text{ and } y = 0,1$$
   Find the m.p.m.fs of X and Y.

4. The j.p.m.f. of (X, Y) is given by
   $$p(x, y) = \frac{1}{27}(2x + y) \text{ for } x = 0,1,2 \text{ and } y = 0,1,2$$
   Find the conditional p.m.f of Y for given X.

5. The j.p.d.f. of (X, Y) is given by
   $$f(x, y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}, 0 < x < \infty, 0 < y < \infty$$
   Find i) m.p.d.fs of X and Y. ii) Conditional p.d.f. of Y given X.

6. The j.p.d.f. of (X, Y) is given by

$$f(x,y) = \begin{cases} \frac{1}{4}(1+xy) & , \quad |x| < 1, |y| < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

i) Find the m.p.d.fs of X and Y.

ii) Are X and Y independetn?

7. The j.p.d.f. of (X, Y) is given by

$$f(x,y) = \begin{cases} \frac{x^3y^3}{16} & , \quad 0 < x < 2, 0 < y < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find m.p.d.fs of X and Y.

8. The j.p.d.f. of (X, Y) is given by

$$f(x,y) = \begin{cases} \frac{8}{9}xy & , \quad 1 \leq x \leq y \leq 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find i) m.p.d.fs of X and Y. ii) c.p.d.fs of X and Y.

Answers:

1. (i) $\frac{1}{16}$ (ii) $\frac{7}{8}$ (iii) $\frac{23}{64}$ (iv) $\frac{9}{16}$

2. i)

x	0	1
$p_1(x)$	$\frac{1}{3}$	$\frac{2}{3}$

y	-1	1
$p_2(y)$	$\frac{1}{3}$	$\frac{2}{3}$

ii)

x	0	1
$p_{1\|2}(2\|1)$	$\frac{1}{2}$	$\frac{1}{2}$

3.

x	0	1	2	3
$p_1(x)$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{9}{32}$	$\frac{19}{32}$

y	0	1
$p_2(y)$	$\frac{14}{32}$	$\frac{18}{32}$

4.

Y \ X	0	1	2
0	0	$\frac{1}{3}$	$\frac{2}{3}$
1	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$
2	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{6}{15}$

5. (i) $f_1(x) = \frac{3}{4} \cdot \frac{3+2x}{(1+x)^4}$ for $0 < x < \infty$, $f_2(y) = \frac{3}{4} \cdot \frac{3+2y}{(1+y)^4}$ for $0 < y < \infty$.
   (ii) $f_{2|1}(y|x) = \frac{6(1+x+y)}{(1+y)^4(3+2x)}$ for $0 < y < \infty$.

6. (a) $f_1(x) = \frac{1}{2}$, $-1 < x < 1$, $f_2(y) = \frac{1}{2}$, $-1 < y < 1$.
   (b) No

7. $f_1(x) = \frac{x^3}{4}$ for $0 < x < 2$, $f_2(y) = \frac{y^3}{4}$ for $0 < y < 2$.

8. $f_1(x) = \begin{cases} \frac{4}{9}x(4 - x^2) & , \quad 1 \le x \le 2 \\ 0 & , \quad \text{otherwise} \end{cases}$ and
   $f_2(y) = \begin{cases} \frac{4}{9}y(y^2 - 1) & , \quad 1 \le y \le 2 \\ 0 & , \quad \text{otherwise} \end{cases}$.
   $f_{1|2}(x|y) = \frac{2x}{y^2 - 1}$, $1 \le x \le y$ and $f_{2|1}(y|x) = \frac{2y}{4 - x^2}$, $x \le y \le 2$