P4:

Theorem: If A_1 and A_2 are independent events, then $\overline{A_1}$ and $\overline{A_2}$ are also independent.

Proof: Given that A_1 and A_2 are independent events.

Consider $P(\overline{A_1} \cap \overline{A_2}) = P(\overline{A_1 \cup A_2}) = 1 - P(A_1 \cup A_2)$

$$= 1 - \left(P(A_1) + P(A_2) - P(A_1 \cap A_2)\right)$$

$$= 1 - P(A_1) - P(A_2) + P(A_1 \cap A_2)$$

$$= 1 - P(A_1) - P(A_2) + P(A_1) \cdot P(A_2) \text{ } (\because A_1 \& A_2 \text{ are independent})$$

$$= P(\overline{A_1}) - P(A_2)\left(1 - P(A_1)\right)$$

$$= P(\overline{A_1}) - P(A_2) \cdot P(\overline{A_1})$$

$$= P(\overline{A_1})\left(1 - P(A_2)\right)$$

$$= P(\overline{A_1}) \cdot P(\overline{A_2})$$

$\Rightarrow P(\overline{A_1} \cap \overline{A_2}) = P(\overline{A_1}) \cdot P(\overline{A_2})$

Thus, $\overline{A_1}$ and $\overline{A_2}$ are also independent.