P2:

The j.p.m.f. of (X, Y) is given in the following table:

X \ Y	1	2	3	4	Total
1	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{9}{36}$
3	$\frac{5}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{8}{36}$
4	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{9}{36}$
Total	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	1

Find a) The m.p.m.fs of x and y

b) Conditional p.m.f. of x given $y = 1$

c) Conditional p.m.f. of y given $x = 2$

Solution:

a) The marginal p.m.f of x is given by

$$p_1(x) = \sum_y p(x, y)$$

$$\therefore p_1(1) = \sum_{y=1}^{4} p(1, y) = p(1,1) + p(1,2) + p(1,3) + p(1,4)$$

$$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36}$$

Similarly,

$$p_1(2) = \sum_{y=1}^{4} p(2, y) = \frac{9}{36}$$

$$p_1(3) = \sum_{y=1}^{4} p(3, y) = \frac{8}{36} \quad and$$

$$p_1(4) = \sum_{y=1}^{4} p(4, y) = \frac{9}{36}$$

Thus, the m.p.m.f. of x is given in the following table:

x	1	2	3	4
$p_1(x)$	$\frac{10}{36}$	$\frac{9}{36}$	$\frac{8}{36}$	$\frac{9}{36}$

Similarly, we can obtain the m.p.d.f. of y as given in the following table:

y	1	2	3	4
$p_2(y)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{9}{36}$

b) The conditional p.m.f. of x given $y = 1$ is given by

$p_{1|2}(x|1) = \frac{p(x,1)}{p_2(1)}$ for $x = 1,2,3,4.$

$p_{1|2}(1|1) = \frac{p(1,1)}{p_2(1)} = \frac{\frac{4}{36}}{\frac{11}{36}} = \frac{4}{11}.$

Similarly, we can find

$p_{1|2}(2|1) = \frac{1}{11}$, $p_{1|2}(3|1) = \frac{5}{11}$ and $p_{1|3}(4|1) = \frac{1}{11}$

Hence, the conditional p.m.f. of x given $y = 1$ is given in the following table:

x	1	2	3	4
$p_{1|2}(x|1)$	$\frac{4}{11}$	$\frac{1}{11}$	$\frac{5}{11}$	$\frac{1}{11}$

Similarly, we can obtain the conditional p.m.f. of y given $x = 2$ as given in the following table:

y	1	2	3	4
$p_{2|1}(y|2)$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{9}$