Poisson Processes Exercises

#Exercise 1

In the bike exercise above, suppose that 75% of all cyclists stop at the water fountain and stay for a normally distributed amount of time with mean 5 minutes and standard deviation 1 minute. What is the expected number of cyclists at the water fountain at any given time?

```r
pois.sim=function(t=40,lambda=1/2)
{N <- rpois(1, lambda*t)
unifs <- runif(N,0,t)
arrivals <- sort(unifs)
return(arrivals)
}

pois.sim.fountain=function(t=60,lambda=1/3)
{N <- rpois(1, lambda*t)
unifs <- runif(N,0,t)
arrivals <- sort(unifs)
stops=sample(c(0,1),length(arrivals),replace=T,prob=c(0.25,0.75))
fountain.time=rnorm(length(arrivals),5,1)
departure=arrivals+fountain.time*stops
return(data.frame(arrivals,stops,fountain.time,departure))
}

res.vec=0
for(i in 1:1000){
sim=pois.sim.fountain()
check.time=runif(1,0,60)
res.vec[i]=sum(sim$arrivals<=check.time & sim$departure>=check.time & sim$stops==1)
}

mean(res.vec)
```
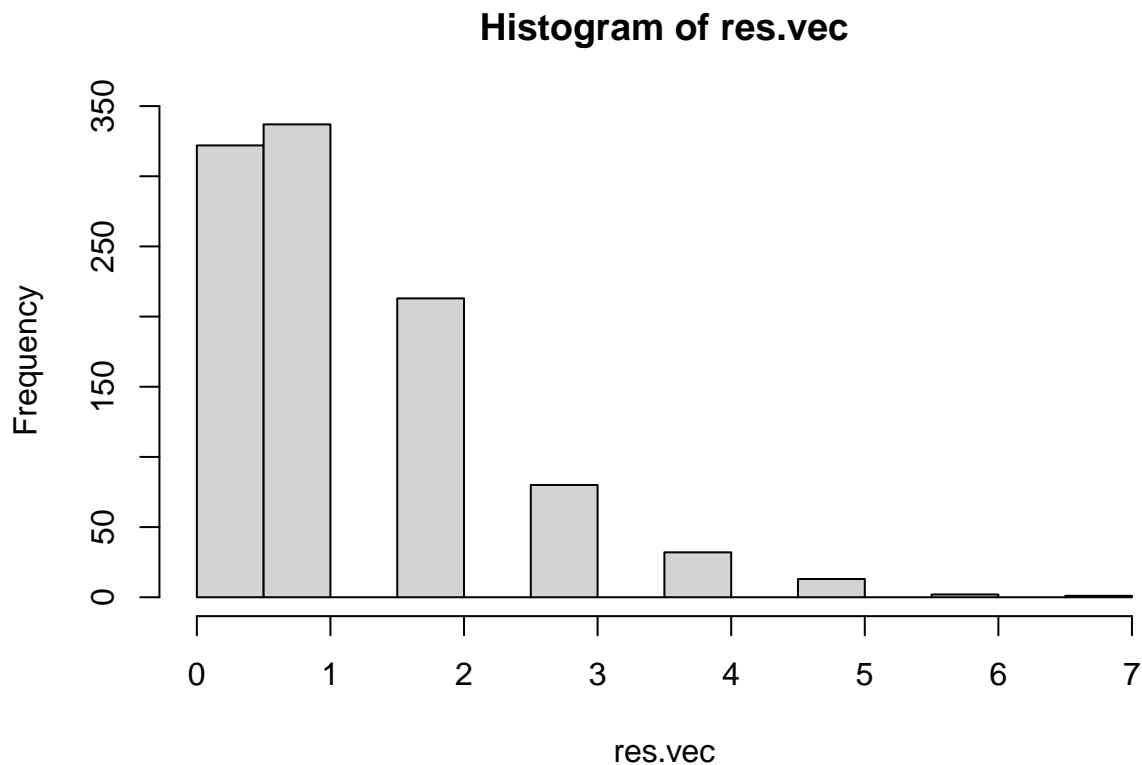
```
## [1] 1.215
```

```r
hist(res.vec)
```

Histogram of res.vec

#Exercise 2

Goals occur in a soccer game according to a Poisson Process. The average total number of goals scored in a 90-minute match is 2.68. Assume that two teams are evenly matched. Use a set of simulations to estimate the probability both teams will score the same number of goals. Does this agree with the theoretical result?

```r
res.vec=0
for(i in 1:10000){
team.a=rpois(1,2.68/2)
team.b=rpois(1,2.68/2)
res.vec[i]=ifelse(team.a==team.b,1,0)
}

mean(res.vec)
```

```
## [1] 0.2615
```

```r
#theoretical
sum(dpois(0:20,2.68/2)^2)
```

```
## [1] 0.259297
```