Dijkstra's Algorithm

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Problem Statement

First Route:

Problem Statement

First Route:

 $\longrightarrow$ 

Problem Statement

First Route:

 $\longrightarrow$  $\longrightarrow$ 

Problem Statement

First Route:

 $\longrightarrow$  $\longrightarrow$ 

$\longrightarrow$ 

Problem Statement

First Route:

Second Route:

Problem Statement

First Route:

 ⟶  ⟶ 

⟶ 

Second Route:

 ⟶ 

Problem Statement

First Route:

 →  → 

→ 

Second Route:

 →  → 

Problem Statement

First Route:

 ⟶  ⟶ 

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Second Route:

 ⟶  ⟶  ⟶ 

Problem Statement

First Route:

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Second Route:

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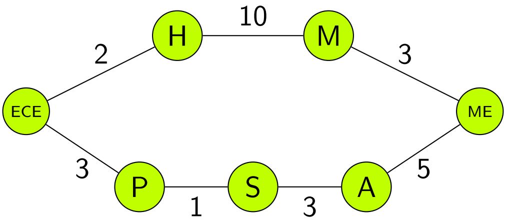
Problem Statement



A Graph Representation of the previous problem

Dijkstra's Algorithm

Definition

Dijkstra's algorithm is an greedy algorithm for finding the shortest paths between nodes in a weighted graph.

Graph

+

Source

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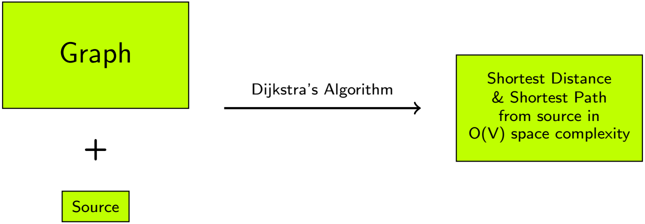

History

O(V^2) Algorithm

What is the shortest way to travel from Rotterdam to Groningen, in general: from given city to given city. It is the algorithm for the shortest path, which I designed in about twenty minutes. One morning I was shopping in Amsterdam with my young fiancée, and tired, we sat down on the café terrace to drink a cup of coffee and I was just thinking about whether I could do this, and I then designed the algorithm for the shortest path. [1]

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O(E + VlogV) Algorithm

In 1984, Fredman & Tarjan proposed use of fibonacci heap to optimize the running time of the algorithm to O($\|E\| + \|V\| logV$)[2]

Pseudocode

```
def Dijkstra(Graph, source):

    Q = PrioriyQueue()
    dist ← ϕ
    prev ← ϕ

    for each vertex v in Graph.Vertices:
        dist[v] ← ∞ if v is not source else 0
        prev[v] ← None
        Q.enqueue(v, dist[v])

    while Q is not empty:
        u ← Q.extract_min()

        for each neighbor v of u:
            alt ← dist[u] + Graph.Edges(u, v)

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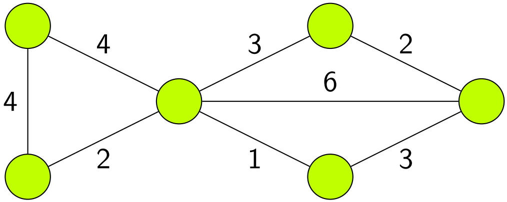
Simulation



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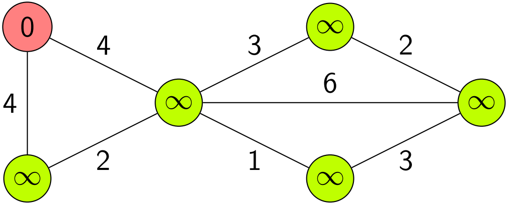

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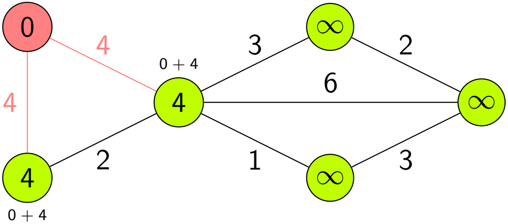

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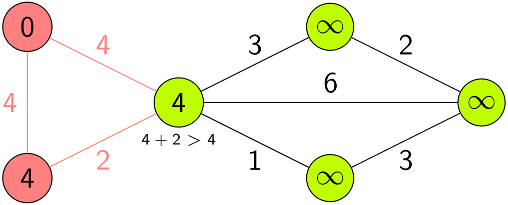

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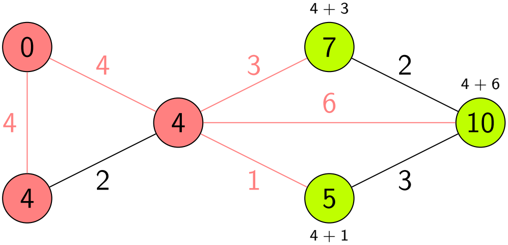

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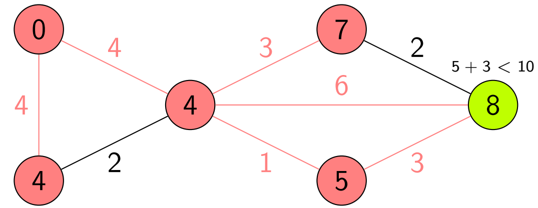

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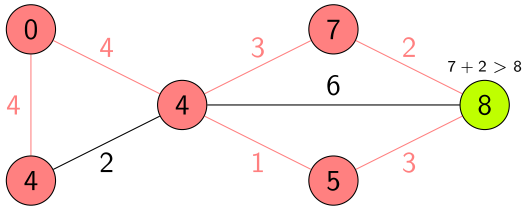

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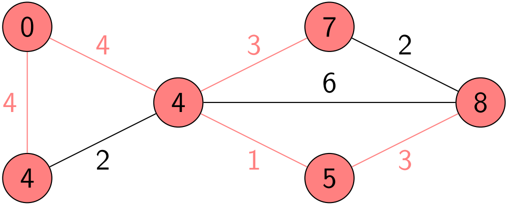

Running time - revisiting the pseudocode

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Running time

Can you identify the costly operations performed here?

Running time

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Running time

Number of calls

Extract min (T_{em}) $\Theta(|V|)$

Decrease priority (T_{dp}) $\Theta(|E|)$

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Total running time

$$T = \Theta(|E|.T_{dp} + |V|.T_{em})$$

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Implementation	$\boldsymbol{T_{dp}}$	$\boldsymbol{T_{em}}$	$\boldsymbol{T}$
Singly Linked List	$\Theta(1)$	$\Theta(\lvert V\rvert)$	$\Theta(V^2)$

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Singly Linked List	$\Theta(1)$	$\Theta(\|V\|)$	$\Theta(V^2)$
Binary Heap	$\Theta(log\|V\|)$	$\Theta(log\|V\|)$	$\Theta\left((E+V)log\|V\|\right)$

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Binary Heap	$\Theta(log\lvert V \rvert)$	$\Theta(log\lvert V \rvert)$	$\Theta\left((E+V)log\lvert V \rvert\right)$
Fibonacci Heap	$\Theta(1)$	$\Theta(log\lvert V \rvert)$	$\Theta(E+Vlog\lvert V \rvert)$

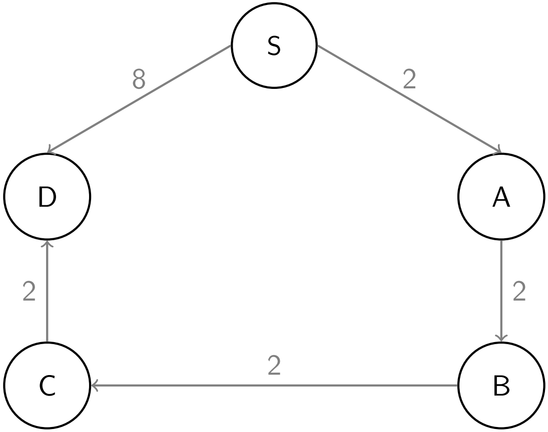
Drawbacks

Traversing unnecessarily

When we are eager to know the distance from the source to a particular node, Dijkstra's algorithm may result in a longer run-time.

Traversing unnecessarily

- Find the shortest distance from S to D



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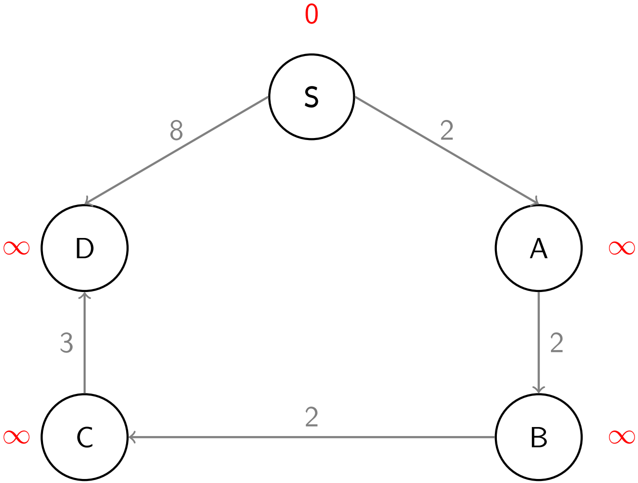

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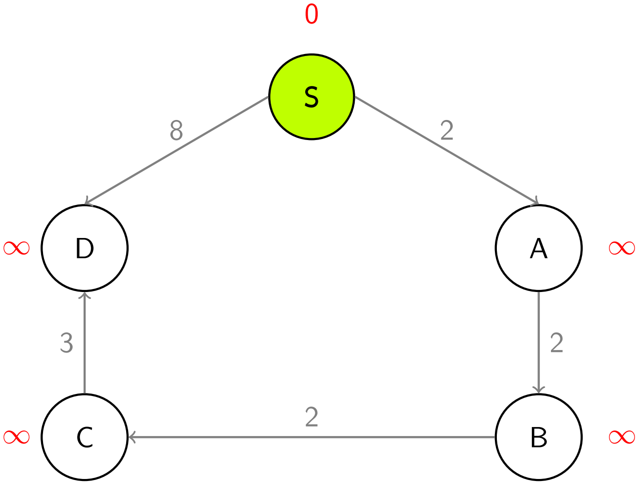

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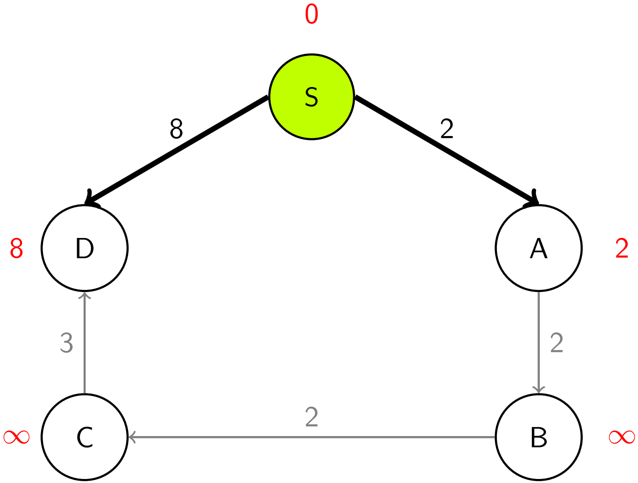

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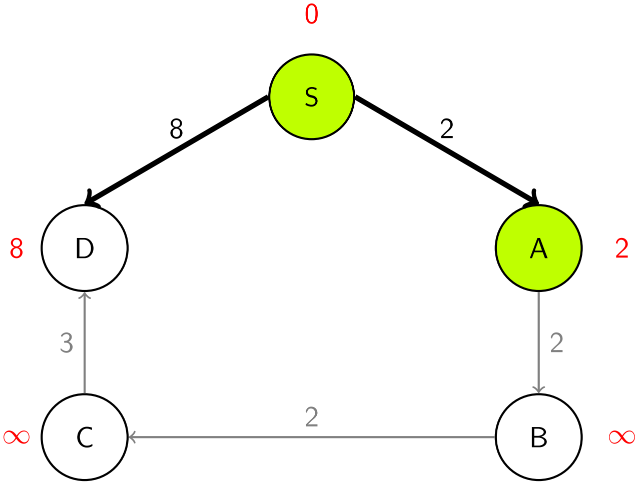

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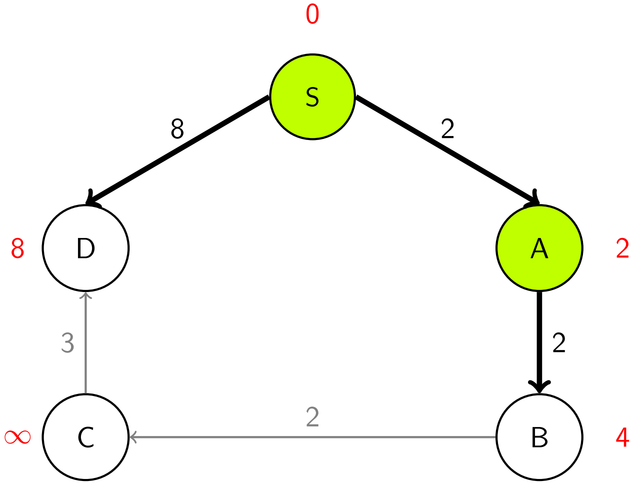

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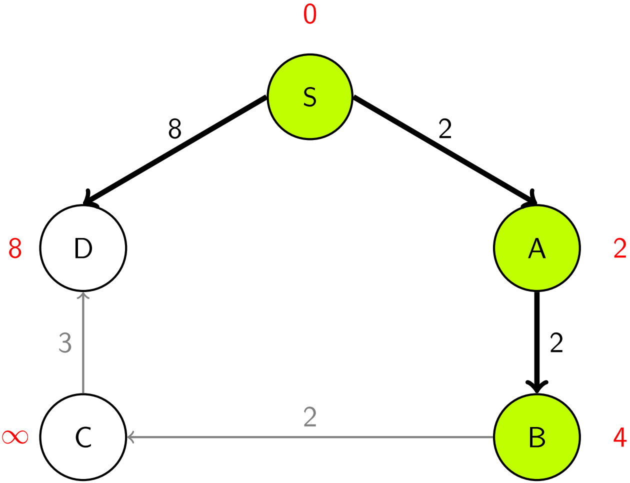

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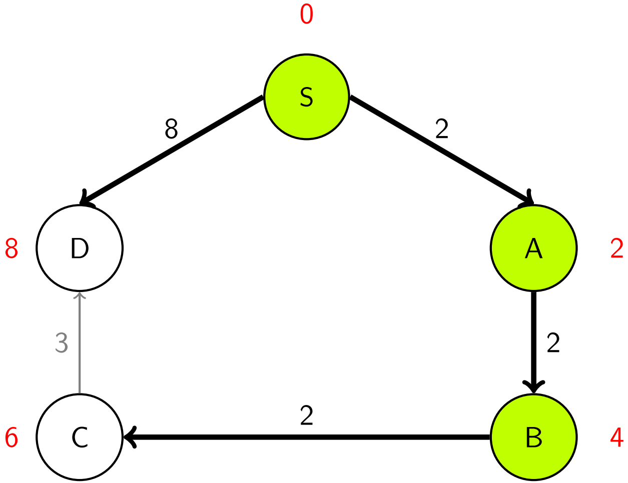

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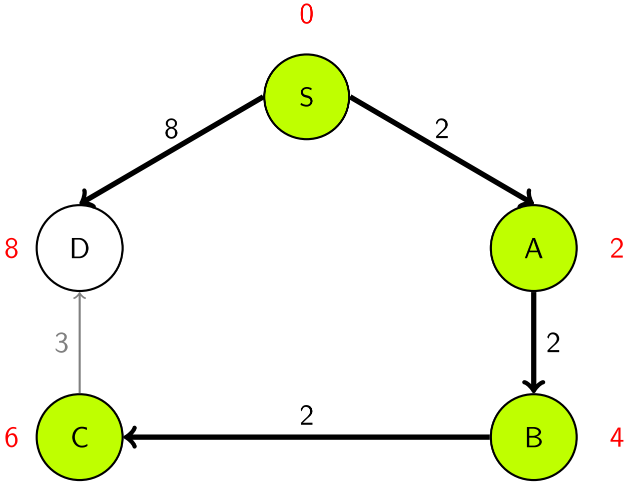

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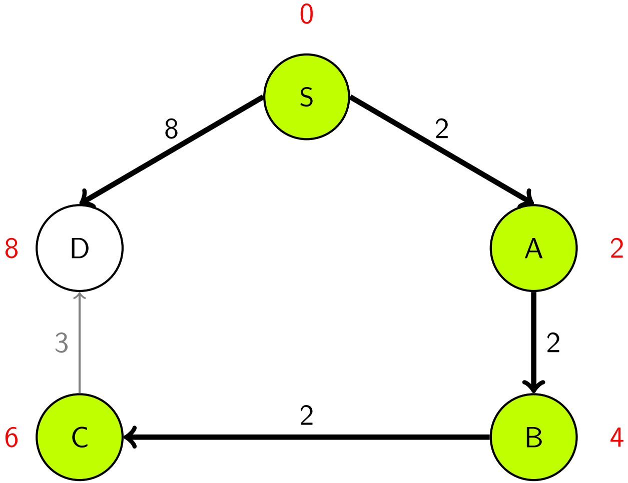

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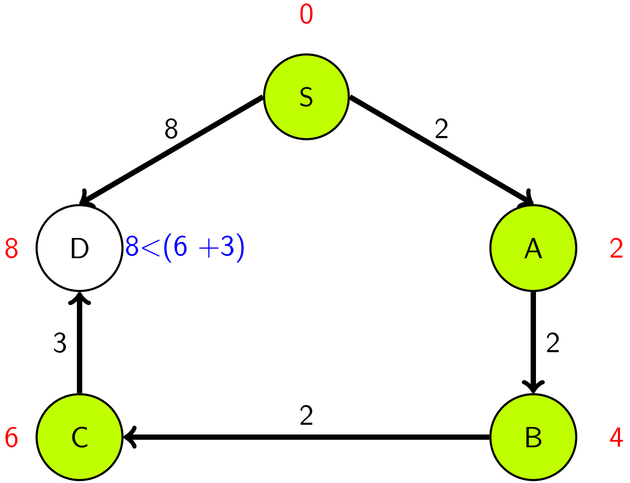

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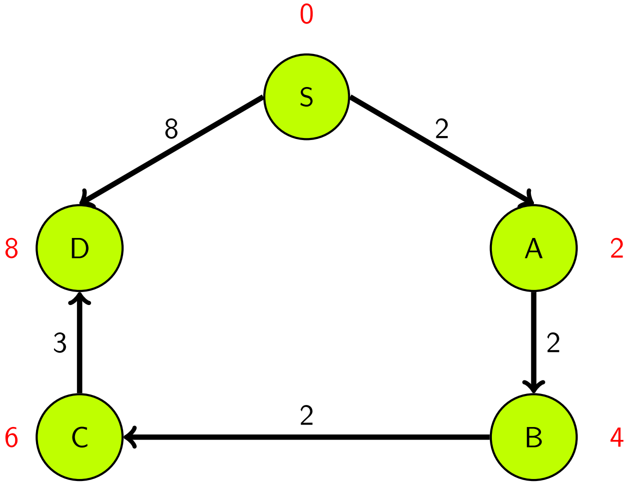


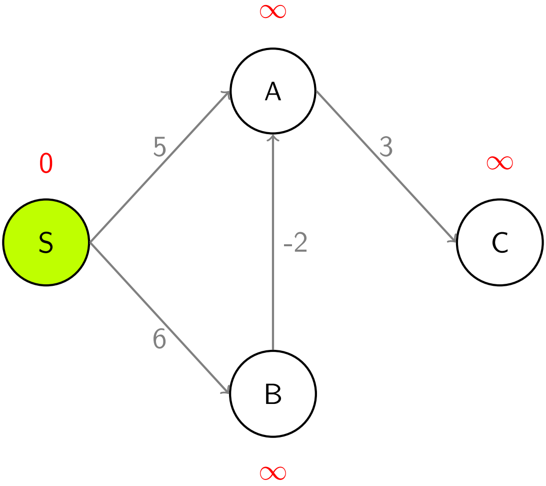
Drawbacks

Fails in case of negative weight edge

If a graph contains negative weight edge(s), the algorithm may often produce wrong results.

Negative Weight Issue

- Find the shortest distance from S to C



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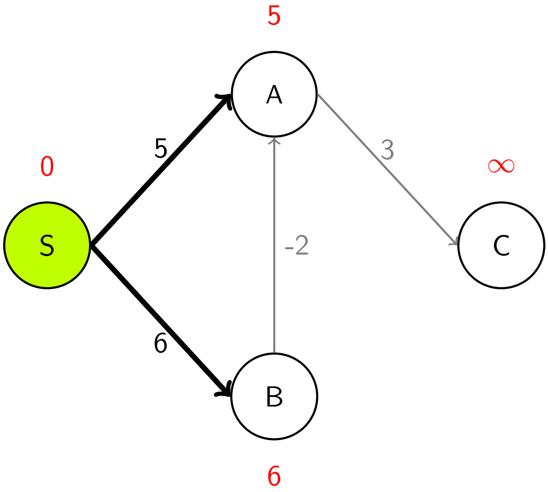

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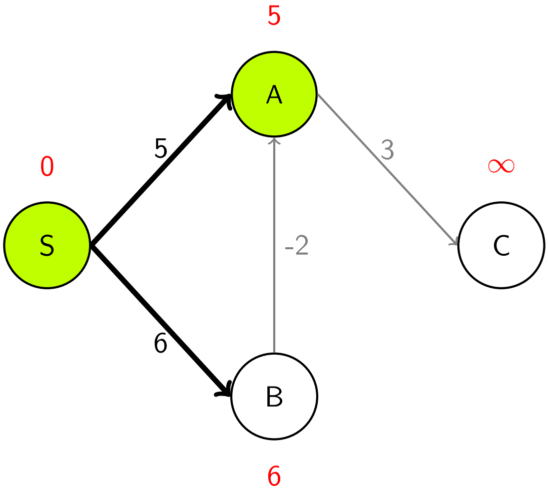

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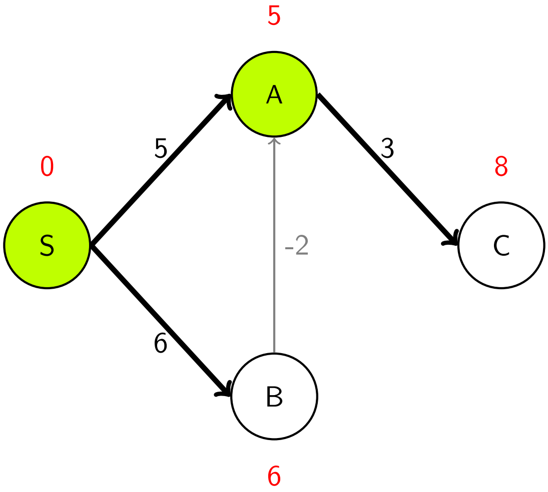

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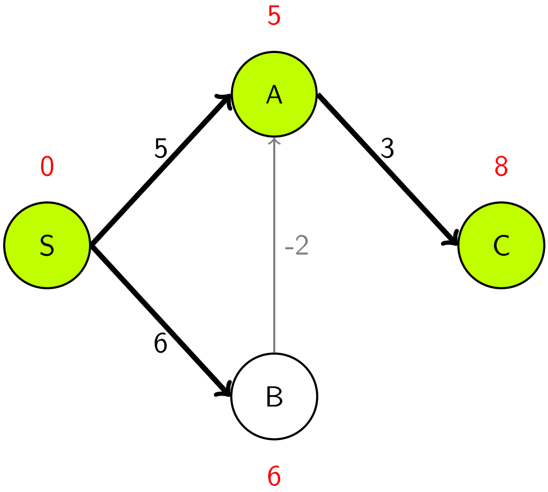

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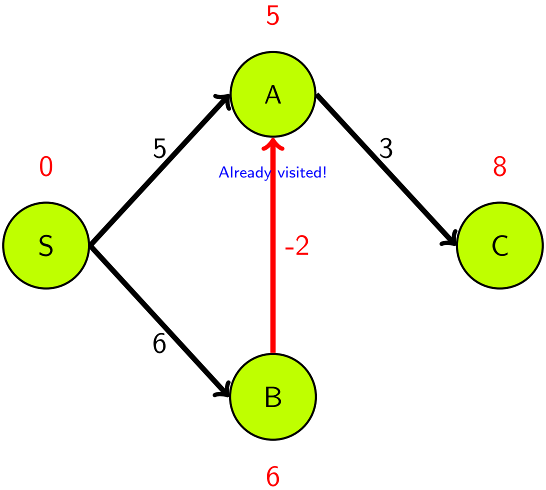

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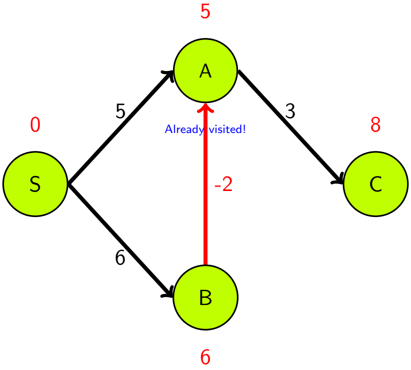

- Thus, the algorithm gives us the shortest distance of 8 by taking the path $S \to A \to C$

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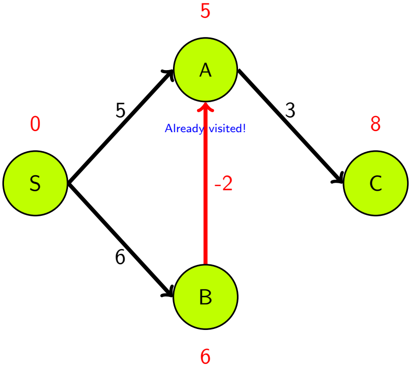

- However, taking the path $S \rightarrow B \rightarrow A$ would result in a shorter distance of $6 - 2 + 3 = 7$

Specialized variants

- When arc weights are small integers (bounded by a parameter C), specialized queues which take advantage of this fact can be used to speed up Dijkstra's algorithm.

Name	Data Structure	Running Time
Dial's algorithm	Bucket queue	$O(\|E\| + \|V\| * C)$

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Ahuja *et al.*	Fibonacci Heap	$O(\|E\| + \|V\|\sqrt{logC})$

Frame Title

- **Routing Protocols in Computer Networks:**
  Dijkstra's algorithm is often used in routing protocols such as OSPF (Open Shortest Path First) and IS-IS (Intermediate System to Intermediate System)

Frame Title

- **GPS Navigation Systems:**
  In GPS navigation, Dijkstra's algorithm is employed to find the shortest route between two locations, considering factors like distance, traffic conditions, and road types.

Frame Title

- **Game Development:**
  In video games, especially those involving maps and navigation, Dijkstra's algorithm can be employed to create intelligent and efficient path-finding for characters or objects.

Frame Title

- **Resource Management in Operating Systems:**
  Dijkstra's algorithm is applied in resource management to allocate resources optimally.

Thank You

References

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