Encrypt "ATTACK" using $K = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}_{2\times 2}$

$$P = \begin{bmatrix} A & T & C \\ T & A & K \end{bmatrix} = \begin{bmatrix} 0 & 19 & 2 \\ 19 & 0 & 10 \end{bmatrix}$$

Encryption:

$$C = KP \bmod 26 = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 19 & 2 \\ 19 & 0 & 10 \end{bmatrix} \bmod 26$$

$$= \begin{bmatrix} 57 & 38 & 34 \\ 114 & 57 & 66 \end{bmatrix} \bmod 26$$

$$= \begin{bmatrix} 5 & 12 & 8 \\ 10 & 5 & 14 \end{bmatrix} \bmod 26$$

Decryption:

$$K = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

1: find det(K)

$$\det(K) = 12 - 9 = 3 \mod 26$$

2 find modular inverse of det(K)

$$\gcd(3, 26) = 1$$

So, modular inverse exists

$$3x + 26y = 1$$

Solve for x: $x = 9$

3: $K^{-1} = [\det(K)]^{-1} * \text{Adj}(K) \mod 26$

$$= 9 * \begin{bmatrix} 6 & -3 \\ -3 & 2 \end{bmatrix} \mod 26$$

$$= \begin{bmatrix} 2 & 25 \\ 25 & 18 \end{bmatrix} \mod 26$$

$$\therefore P = K^{-1} C = \begin{bmatrix} 2 & 25 \\ 25 & 18 \end{bmatrix} \begin{bmatrix} 5 & 12 & 8 \\ 10 & 5 & 14 \end{bmatrix} \mod 26$$

$$= \begin{bmatrix} 0 & 19 & 2 \\ 19 & 0 & 10 \end{bmatrix}$$