03/08/2019 Quarter car model

## Mathematical model for quarter car with passive suspension

The model is derived by applying Newton's Second Law to the two masses. In ordinary differential equations, the model is:

$$f(t) = c v_s(t) - c v_u(t) + k y_s(t) - k y_u(t)$$
 (1)

$$h(t) = f(t) - k_t y_u(t) + k_t y_r(t)$$
 (2)

$$\frac{dy_s(t)}{dt} = v_s(t) \tag{3}$$

$$\frac{dy_s(t)}{dt} = v_s(t)$$

$$\frac{dy_u(t)}{dt} = v_u(t)$$
(3)

$$\frac{dv_s(t)}{dt} = \frac{1}{m_s m_u + (m_s + m_u) b} (-(m_u + b) f(t) + b h(t))$$
 (5)

$$\frac{dv_u(t)}{dt} = \frac{1}{m_s m_u + (m_s + m_u) b} (-b f(t) + (m_s + b) h(t))$$
 (6)

After applying Euler's forward method, the equations become the following.

## Equations (7)-(12) are what you need for simulation.

$$f(t) = c v_s(t) - c v_u(t) + k y_s(t) - k y_u(t)$$
 (7)

$$h(t) = f(t) - k_t y_u(t) + k_t y_r(t)$$
 (8)

$$y_s(t+\Delta) = y_s(t) + v_s(t) \Delta \tag{9}$$

$$y_u(t+\Delta) = y_u(t) + v_u(t) \Delta \tag{10}$$

$$v_s(t+\Delta) = v_s(t) + \left[\frac{(-(m_u+b)f(t)+bh(t))}{m_s m_u + (m_s + m_u)b}\right] \Delta$$
 (11)

$$v_{u}(t+\Delta) = v_{u}(t) + \left[ \frac{(-b f(t) + (m_{s} + b) h(t))}{m_{s} m_{u} + (m_{s} + m_{u}) b} \right] \Delta$$
 (12)