In the Name of God

Navid Zare

40435071

Fuzzy Sets & Logic
TSK Fuzzy System for Predicting Asphalt Strength, Deformation, and Performance Properties

خلاصه فارسی

این پژوهش به طراحی و پیاده‌سازی یک سیستم استنتاج فازی تاکاگی–سوگنو–کانگ (TSK) مرتبه اول برای پیش‌بینی خواص مکانیکی و عملکردی مخلوط‌های آسفالتی می‌پردازد. عملکرد مناسب روسازی آسفالتی نقش کلیدی در ایمنی، دوام و پایداری زیرساخت‌های حمل‌ونقل ایفا می‌کند و به‌طور مستقیم به ویژگی‌های مقاومتی، سختی و رفتار تغییرشکل مخلوط وابسته است.

ارزیابی این خواص معمولاً از طریق آزمایش‌های پیچیده و پرهزینه آزمایشگاهی انجام می‌شود که نیازمند تجهیزات تخصصی، زمان طولانی و نیروی انسانی ماهر است. ازاین‌رو، توسعه مدل‌های هوشمند و داده‌محور که بتوانند بر اساس ویژگی‌های مصالح و ترکیب مخلوط، رفتار عملکردی روسازی را پیش‌بینی کنند، می‌تواند نقش مهمی در بهینه‌سازی فرآیند طراحی و کاهش هزینه‌ها ایفا کند.

هدف اصلی این تحقیق توسعه یک ابزار محاسباتی مبتنی بر منطق فازی است که قادر باشد بدون نیاز به انجام آزمایش‌های گسترده، کیفیت روسازی و یکپارچگی سازه‌ای جاده‌ها را ارزیابی کند. سیستم پیشنهادی چهار متغیر خروجی کلیدی را پیش‌بینی می‌کند: استابیلیتی تعدیل‌شده به‌عنوان شاخص ظرفیت باربری، روانی به‌عنوان معیار تغییرشکل، و مدول سختی کششی غیرمستقیم در دو دمای ۲۰ و ۳۰ درجه سانتی‌گراد (ITSM20 و ITSM30) که بیانگر رفتار مخلوط در شرایط دمایی متفاوت هستند. انتخاب این شاخص‌ها امکان بررسی همزمان مقاومت، تغییرشکل و سختی مخلوط را فراهم می‌سازد و دید جامعی از عملکرد روسازی ارائه می‌دهد.

مجموعه داده مورد استفاده شامل ۱۶۸ نمونه مخلوط آسفالتی است که هر نمونه با ۱۰ متغیر ورودی و ۴ متغیر خروجی توصیف شده است. متغیرهای ورودی طیف گسترده‌ای از اطلاعات شامل خواص قیر، پارامترهای محتوای آسفالت، ویژگی‌های حجمی و مشخصات دانه‌بندی سنگدانه‌ها را پوشش می‌دهند. این تنوع ورودی‌ها امکان مدل‌سازی روابط غیرخطی پیچیده بین ترکیب مخلوط و عملکرد مکانیکی آن را فراهم می‌کند. داده‌ها به‌صورت تصادفی و با نسبت ۸۰ درصد برای آموزش و ۲۰ درصد برای آزمون مستقل تقسیم شدند. پیش از ساخت سیستم فازی، تمام متغیرها با روش مقیاس‌بندی Min–Max به بازه [۰،۱] نرمال‌سازی شدند تا اثر اختلاف مقیاس‌ها حذف گردد و همه متغیرها سهم یکسانی در فرآیند یادگیری داشته باشند.

برای هر خروجی، یک سیستم فازی TSK مستقل طراحی شد تا تفاوت ماهیت فیزیکی متغیرها و روابط متمایز آن‌ها با ورودی‌ها به‌طور دقیق‌تری مدل شود. در ساختار TSK مرتبه اول، هر قاعده شامل یک بخش مقدم[۱] فازی با توابع عضویت گاوسی[۲] و یک

[1] Antecedent
[2] Gaussian

بخش تالی[۳] خطی است. خروجی نهایی سیستم به‌صورت میانگین وزنی خروجی قواعد محاسبه می‌شود که وزن هر قاعده به میزان فعال‌سازی آن وابسته است. این ساختار ضمن برخورداری از قدرت مدل‌سازی بالا، قابلیت تفسیر مناسبی نیز فراهم می‌کند.

تعداد قواعد و مراکز مقدم به‌صورت خودکار با استفاده از الگوریتم خوشه‌بندی تفاضلی [۴]تعیین شد. این الگوریتم با شناسایی نواحی پرتراکم داده، ساختار قواعد را بدون نیاز به تعیین پیشین تعداد خوشه‌ها استخراج می‌کند. نوآوری مهم این تحقیق استفاده از خوشه‌بندی در فضای مشترک ورودی–خروجی است که باعث می‌شود قواعد نه‌تنها بر اساس شباهت ورودی‌ها، بلکه بر اساس رفتار خروجی نیز شکل گیرند. این رویکرد منجر به تولید قواعدی شد که ارتباط دقیق‌تری بین ویژگی‌های مخلوط و پاسخ مکانیکی برقرار می‌کنند.

پارامترهای سیستم با یک الگوریتم یادگیری ترکیبی بهینه‌سازی شدند. در این روش، پارامترهای مقدم با گرادیان کاهشی[۵] و پارامترهای تالی با حداقل مربعات منظم‌شده (رگرسیون ریج) به‌روزرسانی می‌شوند. این ترکیب باعث همگرایی پایدار، کاهش نوسانات و جلوگیری از بیش‌برازش می‌شود. آموزش تا رسیدن به همگرایی یا حداکثر ۸۰۰ دوره ادامه یافت و معیار توقف زودهنگام بر اساس تغییرات RMSE [۶]اعمال شد.

نتایج نشان داد که سیستم‌های فازی طراحی‌شده قادر به پیش‌بینی دقیق خواص عملکردی هستند. الگوریتم خوشه‌بندی به‌طور خودکار ۱۲ قاعده برای استابیلیتی و ۱۱ قاعده برای هر یک از سه خروجی دیگر تولید کرد. مقادیر RMSE در مجموعه آزمون نشان‌دهنده دقت قابل قبول مدل برای کاربردهای مهندسی است. منحنی‌های همگرایی کاهش یکنواخت خطا را نشان دادند و نمودارهای پراکندگی همبستگی مثبت قوی بین مقادیر پیش‌بینی‌شده و واقعی را تأیید کردند. این نتایج بیانگر آن است که مدل قادر به استخراج روابط غیرخطی پیچیده بین ترکیب مخلوط و پاسخ مکانیکی آن است.

از منظر مهندسی، این سیستم می‌تواند در مراحل اولیه طراحی روسازی به‌عنوان یک ابزار پشتیبان تصمیم‌گیری مورد استفاده قرار گیرد. مهندسان می‌توانند پیش از انجام آزمایش‌های پرهزینه، ترکیب‌های مختلف مخلوط را ارزیابی کرده و گزینه‌های بهینه را شناسایی کنند. همچنین، قابلیت تفسیر قواعد فازی امکان تحلیل اثر هر متغیر ورودی بر خروجی‌ها را فراهم می‌کند و می‌تواند به درک بهتر رفتار مخلوط‌های آسفالتی کمک نماید.

در مجموع، این تحقیق نشان می‌دهد که سیستم استنتاج فازی TSK می‌تواند به‌عنوان یک ابزار محاسباتی عملی و قابل تفسیر برای پیش‌بینی عملکرد روسازی آسفالتی مورد استفاده قرار گیرد. این چارچوب می‌تواند به مهندسان در تصمیم‌گیری‌های طراحی،

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[3] Consequent
[4] Subtractive Clustering
[5] Gradient Descent
[6] Root Mean Square Error

کاهش هزینه‌های آزمایشگاهی و بهینه‌سازی ترکیب مخلوط کمک کند. پیشنهاد می‌شود در تحقیقات آینده از روش‌های انتخاب ویژگی، مدل‌های ترکیبی و داده‌های عملکرد میدانی برای افزایش دقت و تعمیم‌پذیری سیستم استفاده شود.

علاوه بر دقت پیش‌بینی، یکی از مزیت‌های مهم سیستم پیشنهادی، پایداری عددی و رفتار همگرای آن در فرآیند آموزش است. بررسی روند تغییرات خطا در طول دوره‌های یادگیری نشان می‌دهد که الگوریتم یادگیری ترکیبی بدون نوسانات شدید یا واگرایی به یک مقدار پایدار همگرا می‌شود. این موضوع نشان‌دهنده انتخاب مناسب ساختار مدل، توابع عضویت، و پارامترهای یادگیری است و اطمینان می‌دهد که سیستم در برابر تغییرات داده‌ها رفتاری قابل اعتماد و قابل تکرار از خود نشان می‌دهد.

همچنین، چارچوب ارائه‌شده از نظر توسعه‌پذیری و کاربرد عملی نیز دارای مزایای قابل توجهی است. این سیستم می‌تواند به‌راحتی برای مجموعه داده‌های بزرگ‌تر، انواع دیگر مخلوط‌های آسفالتی یا حتی سایر مصالح روسازی تعمیم داده شود. با ادغام این مدل در نرم‌افزارهای طراحی روسازی یا سیستم‌های مدیریت روسازی، می‌توان فرآیند ارزیابی عملکرد را سریع‌تر، اقتصادی‌تر و هوشمندتر کرد و از آن به‌عنوان یک ابزار پشتیبان تصمیم‌گیری در مراحل طراحی و کنترل کیفیت استفاده نمود.

Abstract

This report presents the design and implementation of a Takagi-Sugeno-Kang (TSK) Fuzzy Inference System for predicting critical asphalt pavement properties. The system predicts four output variables: Adjusted Stability, Flow, Indirect Tensile Stiffness Modulus at 20°C (ITSM20), and ITSM at 30°C (ITSM30). The methodology employs subtractive clustering in joint input-output space to automatically determine the structure of fuzzy rules, followed by gradient descent optimization of antecedent membership function parameters and least-squares estimation of consequent parameters. The dataset comprises 168 asphalt mix samples characterized by 10 input variables describing binder properties, asphalt content, volumetric characteristics, and aggregate gradation. Experimental results demonstrate effective prediction capabilities with Root Mean Square Errors (RMSE) of 1.057 kN for Stability, 0.785 mm for Flow, 576.67 MPa for ITSM20, and 184.55 MPa for ITSM30 on the test dataset. The developed system provides a computational tool for road construction engineers to assess pavement quality and structural integrity without extensive laboratory testing.

1. Introduction

1.1 Problem Statement

Asphalt pavement performance is critical for infrastructure safety and longevity. The structural integrity of roads and highways depends on accurate characterization of asphalt mixture properties, including strength, deformation resistance, and stiffness under varying temperature conditions. Traditional laboratory testing methods, while accurate, require significant time and resources. Consequently, there exists a substantial need for predictive models capable of estimating these properties based on mixture composition and constituent characteristics.

The prediction of asphalt properties involves complex nonlinear relationships between input variables (binder viscosity, asphalt content, air voids, aggregate gradation) and output performance indicators. Fuzzy inference systems offer an effective approach to modeling such relationships, as they can capture uncertainty and provide interpretable rule-based representations of the underlying physical phenomena. The TSK fuzzy system, in particular, combines the interpretability of fuzzy rules with the precision of linear consequent functions, making it well-suited for regression problems in engineering applications.

1.2 Objectives

The primary objective of this project is to design and implement a first-order TSK fuzzy inference system capable of predicting four key asphalt properties from mixture composition data. The specific objectives include: developing an automated rule generation mechanism using subtractive clustering, implementing a hybrid learning algorithm combining gradient descent for antecedent parameters and least-squares estimation for consequent parameters, evaluating the system performance using RMSE metrics on both training and test datasets, and providing a practical tool for end-user application in pavement engineering contexts.

2. Dataset Description

The dataset utilized in this study comprises 168 asphalt mixture samples, each characterized by 10 input variables and 4 output variables. The input variables encompass comprehensive information about binder properties, asphalt content parameters, volumetric properties, and aggregate gradation characteristics. The output variables represent key mechanical performance indicators obtained through Marshall testing and indirect tensile stiffness modulus measurements.

2.1 Input Variables

Viscosity (η) measured in Pascal-seconds (Pa·s) quantifies the resistance to flow of the asphalt binder. Higher viscosity values indicate stiffer binders with improved rutting resistance but reduced workability during compaction. The percentage of Asphalt Cement by weight (Pb) represents the total binder content in the mixture, which is critical for durability, strength, and moisture resistance. The Effective Asphalt Content (Pbe) denotes the portion of binder not absorbed by aggregates, representing the material that actively contributes to mixture strength and durability.

Maximum Theoretical Specific Gravity (Gmm) represents the density of the asphalt mixture with zero air voids, serving as a reference for calculating compaction degree. Unit Weight (kg/m^3) measures the bulk density of the compacted mixture, where higher values generally indicate better compaction quality. Air Voids (Va) expressed as a percentage indicates the volume of air spaces within the compacted mixture, with typical design targets ranging from 3% to 5%. Values below this range may cause bleeding, while excessive air voids promote moisture damage and cracking.

The aggregate gradation parameters include P200 (percentage passing the 0.075 mm sieve), representing mineral filler content that influences stiffness and moisture sensitivity. P4 (cumulative percent retained on the 4.75 mm sieve) measures coarse aggregate content, while P38 (cumulative percent retained on the 9.5 mm sieve) indicates the overall coarse aggregate skeleton. P34 (cumulative percent retained on the 19 mm sieve) represents the largest aggregate size fraction contributing to mixture stability.

2.2 Output Variables

Adjusted Stability (kN) from the Marshall test represents the maximum load the specimen can withstand, indicating overall strength and resistance to rutting under traffic loading. Flow (mm) measures the deformation at maximum load, where high flow values suggest excessive softness and low values indicate brittleness. The Indirect Tensile Stiffness Modulus at 20°C (ITSM20) and at 30°C (ITSM30) characterize mixture stiffness under moderate and elevated temperature conditions respectively. Higher ITSM20 values correlate with improved rutting resistance, while ITSM30 values indicate susceptibility to permanent deformation under hot weather conditions.

2.3 Data Partitioning

The dataset was randomly partitioned into training and test subsets using an 80-20 split ratio with a fixed random seed of 42 for reproducibility. This resulted in 134 samples for training and fuzzy system tuning, and 34 samples reserved for independent testing. The stratified random sampling ensures representative distribution of samples across both subsets.

Table 1: Dataset Statistics

Parameter	Value
Total Samples	168
Training Samples	134 (80%)
Test Samples	34 (20%)
Input Variables	10
Output Variables	4

3. Design Methodology

3.1 System Architecture

The implemented system follows the first-order Takagi-Sugeno-Kang (TSK) fuzzy inference architecture. Four separate TSK fuzzy systems were designed, one for each output variable. This design decision was motivated by the different physical natures of the output variables and their potentially distinct relationships with the input features. Each system operates independently, allowing for tailored rule structures optimized for the specific output being predicted.

In a first-order TSK system, each fuzzy rule takes the form: IF x_1 is A_1 AND x_2 is A_2 AND ... AND x_n is A_n THEN $y = p_0 + p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$. The antecedent part consists of fuzzy sets A_i defined by Gaussian membership functions, while the consequent is a linear function of the input variables. The final output is computed as the weighted average of all rule consequents, where the weights are determined by the firing levels of the rules.

3.2 Data Preprocessing

Prior to fuzzy system construction, all input and output variables were normalized to the [0, 1] range using Min-Max scaling. This preprocessing step is essential for subtractive clustering, as it ensures that all dimensions contribute equally to distance calculations regardless of their original measurement scales. The normalization parameters (minimum and maximum values) were fitted exclusively on the training data to prevent information leakage from the test set. Separate scalers were maintained for each output variable to enable proper inverse transformation of predictions back to original measurement units.

3.3 Fuzzy Rule Generation via Subtractive Clustering

The number of fuzzy rules and their antecedent centers are automatically determined through subtractive clustering. Unlike traditional clustering algorithms that require a priori specification of cluster count, subtractive clustering adaptively identifies natural groupings in the data based on point density. The algorithm considers each data point as a potential/candidate cluster center and assigns a potential/candidate measure proportional to the density of surrounding points.

The potential/candidate P_i for each point x_i is calculated as the sum of Gaussian-weighted contributions from all other points: $P_i = \sum_j \quad \exp\left(-\alpha|x_i - x_j|^2\right)$, where $\alpha = \frac{4}{r_a^2}$ and r_a is the cluster radius parameter (set to 1.2 in this implementation). The point with the highest potential/candidate is selected as the first cluster center c_1, and its potential/candidate P_1* is recorded as the reference value.

After selecting each cluster center c_k, the potential/candidate of all remaining points is reduced according to :$P_i^{(new)} = P_i^{(old)} - P_k^* \times \exp(-\beta|x_i - c_k|^2)$, where $\beta = \frac{4}{r_b^2}$ and $r_b = 1.25 \times r_a$. is the neighborhood radius (squash factor of 1.25). This subtraction creates an exclusion zone around accepted centers, preventing redundant cluster selection.

The acceptance decision for subsequent cluster candidates follows a three-tier logic. If $\frac{P_k^*}{P_1^*} > 0.5$ (accept ratio), the candidate is automatically accepted as a new cluster center. If $\frac{P_k^*}{P_1^*} < 0.15$ (reject ratio), the algorithm terminates. For intermediate values, a distance-strength tradeoff rule is applied: accept if $\left(\frac{d_{min}}{r_a}\right) + \left(\frac{P_k^*}{P_1^*}\right) \geq 1.0$, where d_{min} is the shortest distance to any existing cluster center. This rule balances novelty (distance) against significance (potential/candidate strength).

A critical enhancement in this implementation is the use of joint input-output space clustering. Rather than clustering only in the input space, the algorithm operates on the concatenated (X, y) space. This approach ensures that identified clusters group data points that are both close in input space and produce similar output values. Each cluster center then defines one fuzzy rule, with the input-space projection serving as the antecedent center. The clustering parameters used were: cluster radius r_a = 1.2, squash factor = 1.25 (for computing the neighborhood radius r_β = 1.25 × r_a), accept ratio = 0.5, and reject ratio = 0.15.

3.4 Fuzzy Set Generation

Each fuzzy rule's antecedent comprises Gaussian membership functions for all 10 input variables. The Gaussian membership function for input variable x_j in rule r is defined as: $\mu_{r,j}(x_j) = \exp\left(-0.5 \times \left(\frac{x_j - c_{r,j}}{\sigma_{r,j}}\right)^2\right)$, where c_{rj} is the center and σ_{rj} is the standard deviation (spread) of the membership function.

The initial centers c_{rj} are derived directly from the cluster centers identified through subtractive clustering (projecting to input space). The initial standard deviations are set uniformly as $\sigma = \frac{r_a}{\sqrt{8}}$, which is a heuristic providing reasonable overlap between membership functions when using subtractive clustering. These initial parameters serve as starting points for subsequent gradient-based optimization.

The firing level of rule r for a given input vector x is computed as the product of individual membership degrees across all input variables: $w_r(x) = \prod_j \mu_{r,j}(x_j)$. This product operation implements the fuzzy AND (T-norm) for combining antecedent conditions. The normalized firing level, used for output aggregation, is obtained by dividing by the sum of all rule firing levels: $\overline{w_r}(x) = \frac{w_r(x)}{\sum_k w_k(x)}$.

3.5 TSK Rule Tuning via Hybrid Learning

The TSK system parameters are optimized using a hybrid learning algorithm that combines gradient descent for antecedent parameters with least-squares estimation for consequent parameters. This approach leverages the different mathematical properties of the two parameter sets: antecedent parameters (membership function centers and spreads) appear nonlinearly in the system output, while consequent parameters (linear function coefficients) appear linearly when antecedents are fixed.

During each training epoch, the forward pass computes predictions for all training samples and calculates the mean squared error. The backward pass updates antecedent parameters using gradient descent. The error derivative with respect to the normalized firing level $\bar{w}_r$ of rule r is: $\frac{\partial E}{\partial \overline{w_r}} = -\frac{2(y-\hat{y})(f_r-\hat{y})}{n}$, where y is the target, ŷ is the prediction, f_r is the rule r consequent output, and n is the number of samples.

The derivative of normalized firing level with respect to raw firing level w_r follows from the quotient rule: $\frac{\partial \overline{w_r}}{\partial w_r} = \frac{\sum_k w_k - w_r}{(\sum_k w_k)^2}$. For the Gaussian membership function $\mu_{r,j}(x_j) = \exp\left(-0.5\left(\frac{x_j - c_{r,j}}{\sigma_{r,j}}\right)^2\right)$, the derivative with respect to center $\frac{\partial w_r}{\partial c_{r,j}} = w_r \times \frac{x_j - c_{r,j}}{\sigma_{r,j}^2}$. Similarly, the derivative with respect to spread σ_{rj} is: $\frac{\partial w_r}{\partial \sigma_{r,j}} = w_r \times \frac{(x_j - c_{r,j})^2}{\sigma_{r,j}^3}$.

Combining these derivatives via the chain rule yields the complete gradients: $\frac{\partial E}{\partial c_{r,j}} = \sum_i \left(\frac{\partial E}{\partial \overline{w_r}}\right)\left(\frac{\partial \overline{w_r}}{\partial w_r}\right)\left(\frac{\partial w_r}{\partial c_{r,j}}\right)$ and $\frac{\partial E}{\partial \sigma_{r,j}} = \sum_i \left(\frac{\partial E}{\partial \overline{w_r}}\right)\left(\frac{\partial \overline{w_r}}{\partial w_r}\right)\left(\frac{\partial w_r}{\partial \sigma_{r,j}}\right)$. The parameters are updated as : $c_{r,j}^{(new)} = c_{r,j}^{(old)} - \eta \times \frac{\partial E}{\partial c_{r,j}}$ and $\sigma_{r,j}^{(new)} = \max\left(\sigma_{r,j}^{(old)} - \eta \times \frac{\partial E}{\partial \sigma_{r,j}}, 10^{-6}\right)$, where the maximum function prevents negative or near-zero spreads.

Following antecedent parameter updates, the consequent parameters are recomputed optimally using regularized least-squares (ridge regression). Given the current firing levels, the design matrix is constructed where each row corresponds to a training sample and columns represent the weighted input features for each rule. The optimal consequent parameters minimize $|Ap - y|^2 + \lambda|p|^2$, where A is the design matrix, p is the parameter vector, y is the target vector, and $\lambda = 0.01$ is the regularization coefficient. The closed-form solution is $p = (A^\top A + \lambda I)^{-1} A^\top y$.

The training process employed the following hyperparameters: learning rate $\eta = 0.01$, maximum epochs = 800, and early stopping tolerance = 10^{-8}. Training terminates either when the maximum epoch count is reached or when the improvement in RMSE between consecutive epochs falls below the tolerance threshold, indicating convergence.

4. Results

4.1 System Structure

The subtractive clustering algorithm automatically determined the optimal number of rules for each output variable based on the data distribution in the joint input-output space. Table 2 presents the resulting rule counts for each TSK subsystem. The Stability system required 12 rules, while Flow, ITSM20, and ITSM30 each required 11 rules. The similar rule counts across outputs suggest comparable complexity in the underlying input-output relationships, though the specific rule structures differ to capture the distinct characteristics of each output variable.

Table 2: Number of Fuzzy Rules per Output Variable

Output Variable	Number of Rules
Stability	12
Flow	11
ITSM20	11
ITSM30	11

4.2 RMSE Performance Metrics

Table 3 presents the Root Mean Square Error (RMSE) values for both training and test datasets across all four output variables. The RMSE metric quantifies the average magnitude of prediction errors in the original measurement units, calculated as $\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_i (y_i - \hat{y}_i)^2}$ where y_i represents actual values and $\hat{y}_i$ represents predicted values.

Table 3: RMSE Results for Training and Test Datasets

Output	RMSE (Train)	RMSE (Test)
Stability (kN)	0.7574	1.0574
Flow (mm)	0.5771	0.7848
ITSM20 (MPa)	401.31	576.67
ITSM30 (MPa)	121.91	184.55

4.3 Training Convergence Analysis

Figure 1 illustrates the training convergence behavior across all 800 epochs for each of the four TSK subsystems. The plots display the normalized RMSE (computed in the [0,1] scaled space) as a function of training epochs. All four systems exhibit consistent, monotonically decreasing error curves, demonstrating successful parameter optimization through the hybrid learning algorithm.

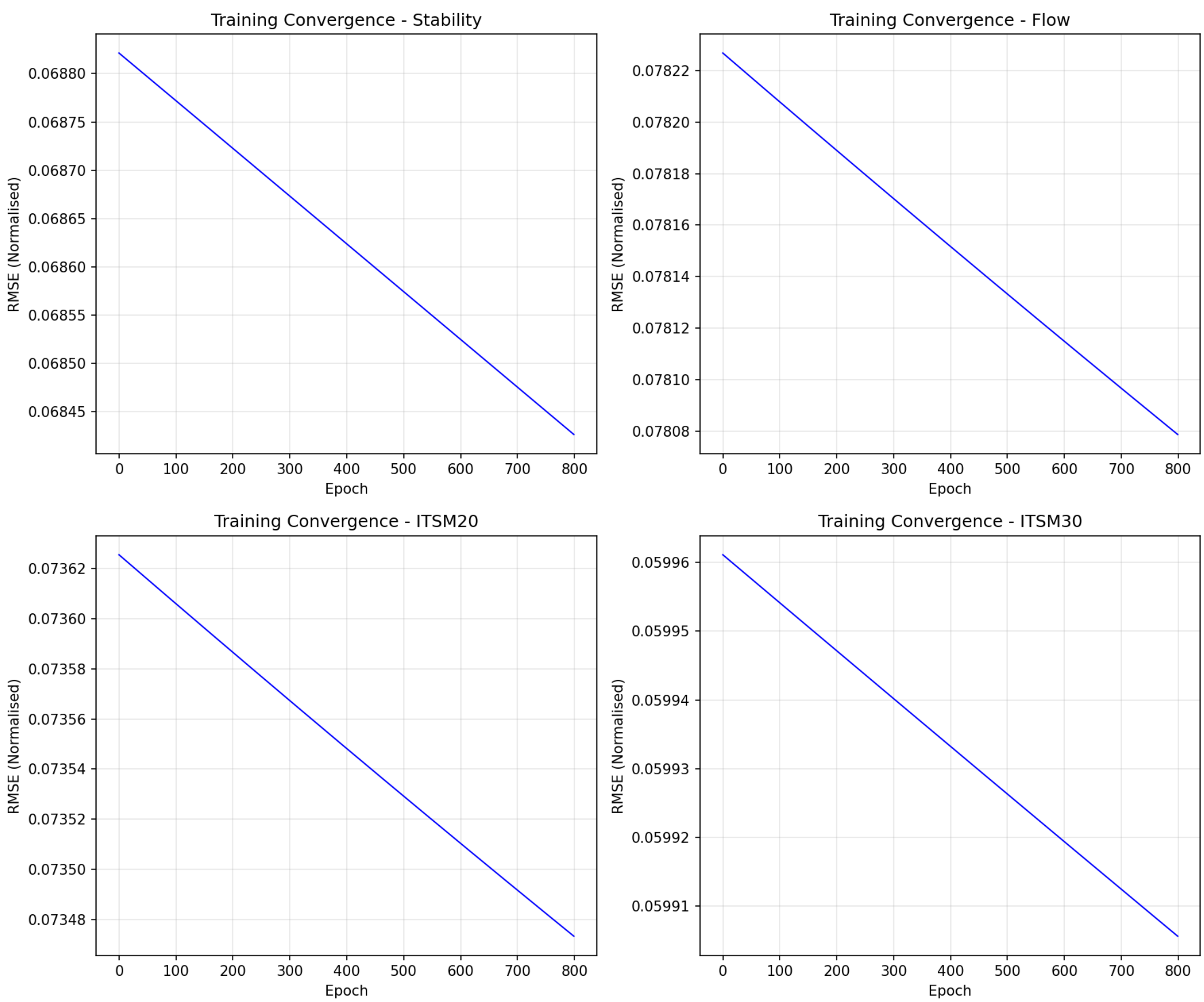

Figure 1: Training Convergence Curves for All Four Output Variables

Examination of the convergence curves reveals several notable characteristics. The Stability system begins at approximately 0.0688 normalized RMSE and decreases to approximately 0.0684 by epoch 800. The Flow system shows similar behavior, starting near 0.0782 and declining to approximately 0.0780. The ITSM20 and ITSM30 systems demonstrate comparable patterns with starting values around 0.0736 and 0.0599 respectively, both showing gradual improvement throughout training.

The smooth, continuous nature of all convergence curves indicates stable gradient descent behavior without oscillations or divergence issues. The relatively slow rate of improvement in later epochs suggests that the systems approached local optima. The learning rate of 0.01 provided an appropriate balance between convergence speed and stability. The absence of early stopping (all systems trained for the full 800 epochs) indicates that the tolerance threshold was not triggered, meaning small but consistent improvements continued throughout training.

4.4 Prediction Accuracy on Test Data

Figure 2 presents scatter plots comparing predicted values against actual values for the test dataset across all four output variables. The diagonal red dashed line represents perfect prediction (predicted = actual). Points clustering closely around this line indicate accurate predictions, while deviations represent prediction errors.

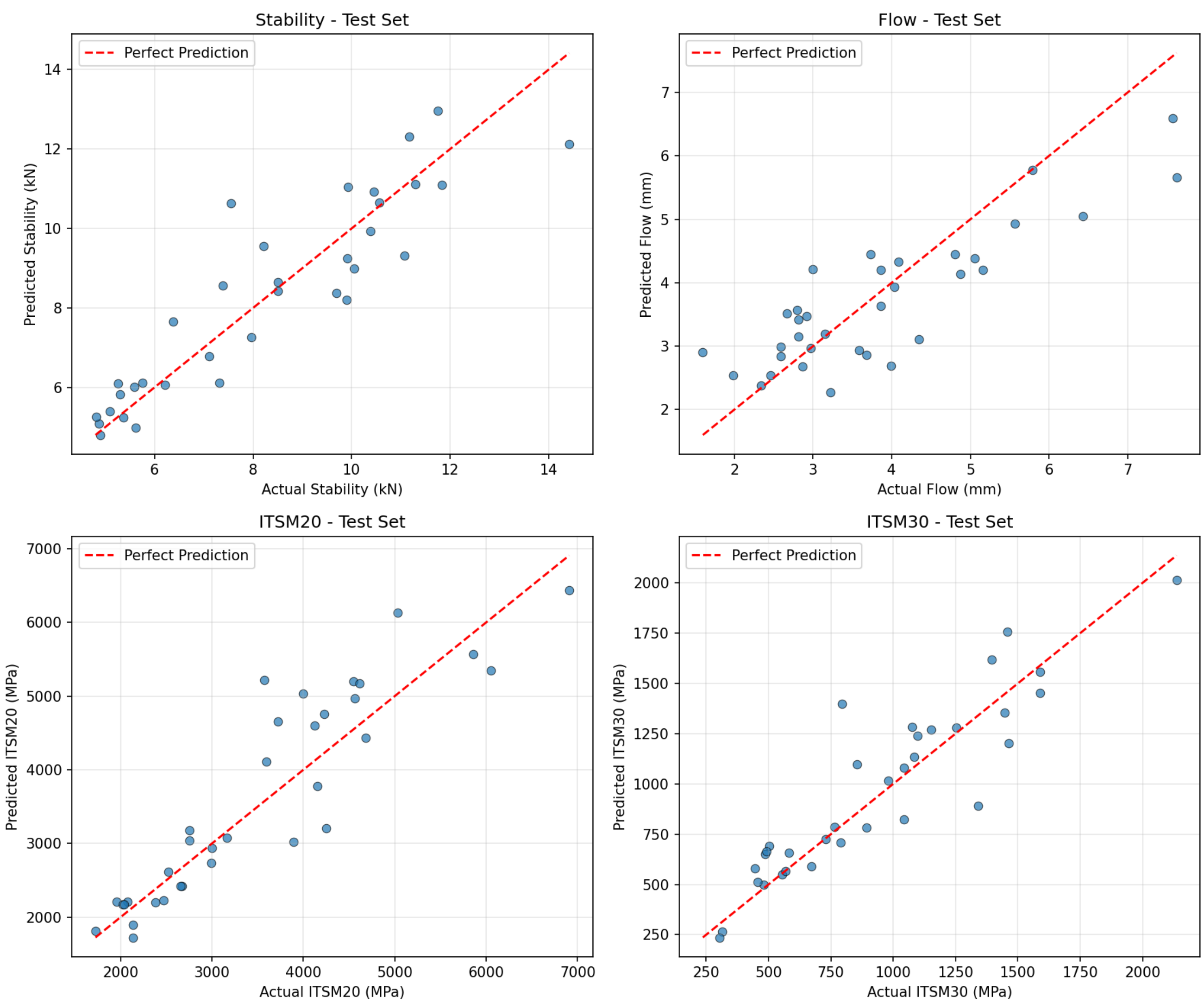

Figure 2: Predicted versus Actual Values on Test Dataset

The Stability plot (upper left) shows predictions ranging from approximately 5 to 13 kN against actual values in a similar range. The points generally follow the diagonal trend, though some scatter is evident, particularly at higher stability values where a few samples show notable under-prediction. This pattern suggests the model captures the overall stability relationship well but may have limited accuracy for extreme values outside the main training distribution.

The Flow plot (upper right) displays predictions versus actuals across a range of approximately 2 to 7 mm. The scatter pattern reveals reasonable prediction accuracy for mid-range values but shows systematic under-prediction for high flow values (above 6 mm). Several data points at high actual flow values cluster well below the diagonal line, indicating the model may be smoothing extreme deformation characteristics.

The ITSM20 plot (lower left) covers a wide range from approximately 1500 to 7000 MPa. The prediction pattern shows a positive correlation with actual values, but with considerable scatter. Several points appear above the diagonal in the mid-range (3000-5000 MPa), indicating over-prediction for some samples. The wide range of ITSM20 values and inherent measurement variability in stiffness testing may contribute to the larger absolute errors observed for this output.

The ITSM30 plot (lower right) spans values from approximately 250 to 2000 MPa. This system demonstrates the tightest clustering around the diagonal line among all four outputs, particularly for lower stiffness values. The proportionally better fit for ITSM30 compared to ITSM20 may reflect more consistent material behavior at the elevated temperature condition, where viscous effects dominate and reduce measurement variability.

4.5 Membership Function Visualization

Figure 3 visualizes the Gaussian membership functions for all 10 input variables in the Stability prediction system, shown as a representative example. Each subplot corresponds to one input variable, with the x-axis representing the normalized input range [0, 1] and the y-axis representing the membership degree [0, 1]. Each colored curve represents one of the 12 fuzzy rules in the Stability system.

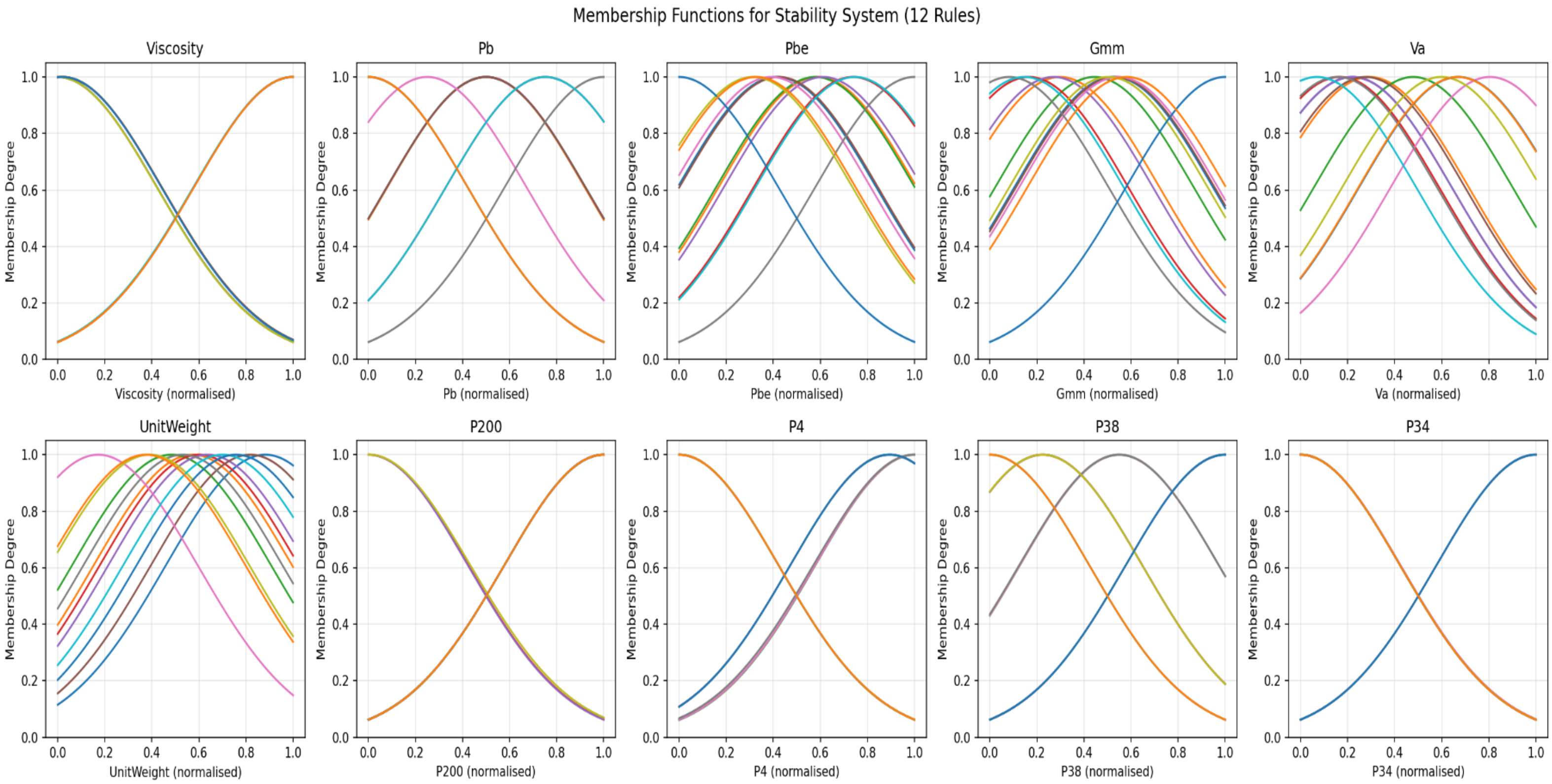

Figure 3: Gaussian Membership Functions for the Stability System (12 Rules)

The Viscosity membership functions (first subplot) reveal a bimodal distribution with cluster centers concentrated near the boundaries (0 and 1 in normalized space). This pattern reflects the dataset composition, which likely contains samples prepared with two distinct binder viscosity levels. The relatively narrow spreads indicate good discrimination between the two viscosity regimes.

The Pb (asphalt content) plot shows membership functions distributed across three main regions: low (near 0), medium (around 0.5), and high (near 1.0). This tri-modal pattern allows the fuzzy system to capture different behavioral regimes associated with under-asphalted, optimally-asphalted, and over-asphalted mixtures. The Pbe (effective asphalt content) functions show broader overlap in the mid-range, indicating a more continuous relationship with stability in this region.

The Gmm (maximum specific gravity) and Va (air voids) membership functions both show distributed coverage across the normalized range with substantial overlap between rules. This overlap is desirable as it ensures smooth interpolation between rules and prevents discontinuous output behavior. The UnitWeight plot demonstrates similar distributed coverage, indicating the importance of compaction quality as a continuous input factor.

The aggregate gradation parameters (P200, P4, P38, P34) exhibit interesting patterns. P200 shows bimodal clustering at extremes, reflecting the dataset's filler content variation. The P4, P38, and P34 plots show more pronounced boundary clustering with membership functions centered near 0 and 1, suggesting the dataset contains samples with distinctly different aggregate gradations (fine-graded versus coarse-graded mixtures). The P34 functions show particularly strong boundary concentration, indicating this largest aggregate fraction varies in a more discrete manner across samples.

5. Interpretation of Results

5.1 Performance Assessment

The RMSE values obtained demonstrate that the TSK fuzzy systems achieve reasonable prediction accuracy across all four output variables. Comparing training and test RMSE values provides insight into model generalization capability. For Stability, the test RMSE (1.057 kN) is approximately 40% higher than the training RMSE (0.757 kN). For Flow, the test RMSE (0.785 mm) exceeds the training RMSE (0.577 mm) by approximately 36%. The ITSM20 and ITSM30 systems show similar generalization gaps of approximately 44% and 51% respectively.

These generalization gaps, while notable, fall within acceptable ranges for engineering applications involving material property prediction. The gaps indicate some degree of overfitting to the training data, which is expected given the relatively small dataset size (134 training samples) and the complexity of the fuzzy systems (11-12 rules with over 100 parameters each). The regularization applied during consequent parameter estimation helped mitigate overfitting but did not eliminate it entirely.

5.2 Engineering Significance

From a practical engineering perspective, the achieved prediction accuracies have meaningful implications. The Stability RMSE of approximately 1 kN corresponds to roughly 10% of the typical stability range (5-15 kN) observed in the dataset. This level of accuracy enables preliminary assessment of mixture adequacy for given traffic loading conditions, though final acceptance decisions should still incorporate laboratory testing.

The Flow prediction error of approximately 0.78 mm is significant relative to the narrow acceptable range (typically 2-4 mm for Marshall design). Predictions should therefore be interpreted with appropriate uncertainty bands, and the model is better suited for identifying potentially problematic mixtures (very high or very low predicted flow) rather than precise flow determination.

The ITSM predictions, while exhibiting larger absolute errors (577 MPa and 185 MPa for ITSM20 and ITSM30 respectively), remain useful for mechanistic-empirical design screening. These errors represent approximately 10-15% of typical ITSM values, which is acceptable for initial pavement structural analysis. The model can effectively identify mixtures with potentially inadequate stiffness characteristics warranting further investigation.

5.3 Model Interpretability

A key advantage of TSK fuzzy systems over black-box machine learning models is their interpretability. Each rule in the system corresponds to a recognizable region in the input space, defined by the antecedent membership functions. The linear consequent functions provide insight into how input variables influence the output within each rule's domain. For instance, positive consequent coefficients indicate that increasing the corresponding input variable increases the predicted output value for samples falling primarily within that rule.

The membership function visualization (Figure 3) reveals meaningful patterns consistent with asphalt mixture theory. The clustering of viscosity membership functions at extreme values reflects the common practice of using discrete binder grades. The distributed coverage of air void and unit weight membership functions captures the continuous nature of compaction quality variations. These interpretable structures enhance confidence in the model's physical basis and facilitate expert validation of the learned relationships.

6. Design Choices

6.1 Feature Selection Approach

Feature selection methods may be employed. In this implementation, all 10 input variables were retained without explicit feature selection or dimensionality reduction. This decision was based on domain knowledge indicating that each input variable captures distinct physical properties relevant to asphalt performance: binder characteristics (Viscosity), mix composition (Pb, Pbe), volumetric properties (Gmm, Va, UnitWeight), and aggregate gradation (P200, P4, P38, P34). The relatively modest input dimensionality (10 features) and the ability of the TSK system to automatically weight feature importance through consequent coefficients justified retaining all features. Future work could explore wrapper or embedded feature selection methods to potentially improve model parsimony.

6.2 How to Use the System as an End User

For end users seeking to make predictions with the trained TSK systems, modify the main.py file to include a prediction function that loads the trained models and accepts new input data. Set save_model=True in the save_all_results call to enable model persistence. After training, load the saved systems and normaliser from the pickle file. Prepare new input data as a numpy array with 10 features in the same order as the training data columns: Viscosity, Pb, Pbe, Gmm, Va, UnitWeight, P200, P4, P38, P34.

Normalize the input data using the normaliser's transform method, then call the predict method on each system to obtain normalized predictions. Apply inverse transformation to convert predictions back to original measurement units. The outputs will be Stability in kN, Flow in mm, ITSM20 in MPa, and ITSM30 in MPa.

Users should note that predictions are most reliable for input values within the ranges represented in the training data.

7. Conclusion

This project successfully designed and implemented a TSK fuzzy inference system for predicting asphalt pavement properties. The methodology employed subtractive clustering in joint input-output space for automatic rule generation, Gaussian membership functions for fuzzy set definition, and a hybrid learning algorithm combining gradient descent with least-squares estimation for parameter optimization.

The resulting systems demonstrate reasonable prediction capabilities with RMSE values of 1.057 kN for Stability, 0.785 mm for Flow, 576.67 MPa for ITSM20, and 184.55 MPa for ITSM30 on the test dataset. The automatic rule generation identified 11-12 rules per output as optimal, with the learned membership functions reflecting meaningful physical patterns in the asphalt mixture data.

The developed system provides a practical computational tool for road construction engineers to assess pavement quality based on mixture composition. The interpretable rule-based structure enables validation against domain knowledge and identification of the input factors most influential for each output property. Future work could explore ensemble methods combining multiple TSK systems, feature selection to identify the most predictive input subsets, or integration with additional data sources such as field performance measurements.

Appendix A: Sample Rule Parameters

This appendix presents the detailed parameters of selected fuzzy rules from the Stability system as a representative example. Each rule comprises antecedent parameters (Gaussian membership function centers and standard deviations for each input variable) and consequent parameters (linear function coefficients including bias term).

Rule 1 (Stability System):

Antecedent Centers: [-0.00002, 0.5, 0.41564, 0.526, 0.22062, 0.88222, 1.0, 0.89474, 1.0, 0.99999]

Antecedent Sigmas: [0.42421, 0.42432, 0.42428, 0.42429, 0.42414, 0.42416, 0.42426, 0.42426, 0.42426, 0.42428]

Consequent Parameters: [-0.02409, 0.19366, 0.45913, -0.51258, 0.36791, 0.25431, 0.03183, -0.02427, -0.02, -0.03019, -0.03787]

Rule 2 (Stability System):

Antecedent Centers: [0.02057, 0.49951, 0.58866, 0.29849, 0.29337, 0.57429, 1.00016, -0.00002, -0.00005, 0.0]

Antecedent Sigmas: [0.42369, 0.42145, 0.42334, 0.42478, 0.42348, 0.42297, 0.42389, 0.42422, 0.42423, 0.42426]

Consequent Parameters: [-0.02527, 1.35493, 0.44426, 0.26859, 0.72908, 0.61185, -0.06973, -0.09303, -0.00567, 0.01191, -0.00043]

Sample Rule from Flow System (Rule 1):

Antecedent Centers: [0.02027, 0.50055, 0.59762, 0.29723, 0.23053, 0.65012, 1.00003, -0.00003, -0.00003, -0.00001]

Antecedent Sigmas: [0.42367, 0.42402, 0.42418, 0.4243, 0.42383, 0.42381, 0.4242, 0.42419, 0.42421, 0.42423]

Consequent Parameters: [-0.22732, -0.59817, 0.53991, 0.02865, 0.70179, 0.13016, 0.37354, -0.22929, -0.06929, -0.03788, -0.00042]

Sample Rule from ITSM20 System (Rule 1):

Antecedent Centers: [0.02027, 0.50015, 0.32756, 0.63619, 0.25851, 0.94056, 1.0, 0.89474, 1.00001, 1.00001]

Antecedent Sigmas: [0.42368, 0.42401, 0.42421, 0.42424, 0.42406, 0.42408, 0.42426, 0.42426, 0.42425, 0.42423]

Consequent Parameters: [0.01527, 0.0343, -0.11867, -0.00506, 0.06417, 0.08254, 0.11673, 0.01509, 0.01207, 0.02015, 0.02652]

Sample Rule from ITSM30 System (Rule 1):

Antecedent Centers: [0.02039, 0.49997, 0.32746, 0.6363, 0.25868, 0.94045, 1.0, 0.89473, 1.00001, 1.00002]

Antecedent Sigmas: [0.42397, 0.42423, 0.42423, 0.42424, 0.42417, 0.42416, 0.42426, 0.42426, 0.42425, 0.42421]

Consequent Parameters: [0.0317, 0.15284, 0.19068, -0.38149, 0.07755, 0.03314, 0.11877, 0.03162, 0.02947, 0.02984, 0.0268]

The consequent parameters represent, in order: bias (p_0), and coefficients for Viscosity (p_1), Pb (p_2), Pbe (p_3), Gmm (p_4), Va (p_5), UnitWeight (p_6), P200 (p_7), P4 (p_8), P38 (p_9), and P34 (p_{10}). Interpretation of the coefficients reveals physically meaningful relationships. For instance, in the Stability Rule 2, the strong positive coefficient for Pb (1.35493) indicates that higher asphalt content increases stability within this rule's domain. The positive coefficients for Va (0.61185) and Gmm (0.72908) reflect the importance of proper volumetric properties. Negative coefficients, such as for P4 (-0.09303), suggest that increased coarse aggregate retention slightly reduces stability in certain mixture configurations. The complete rule parameters for all 45 rules (12 for Stability, 11 each for Flow, ITSM20, and ITSM30) are documented in the rule_summary.txt output file.

Appendix B: Configuration Parameters Summary

This appendix summarizes all configurable parameters used in the TSK fuzzy system implementation, enabling reproducibility and facilitating parameter tuning for future research.

Parameter	Value	Description
TEST_RATIO	0.20	Fraction for test set
RANDOM_SEED	42	Random seed for reproducibility
CLUSTER_RADIUS	1.2	Neighborhood radius (r_a)
SQUASH_FACTOR	1.25	Ratio for r_β = factor × r_a
ACCEPT_RATIO	0.5	Cluster acceptance threshold
REJECT_RATIO	0.15	Cluster rejection threshold
LEARNING_RATE	0.01	Gradient descent step size (η)
MAX_EPOCHS	800	Maximum training iterations
TOLERANCE	1e-8	Early stopping threshold
Regularization (λ)	0.01	Ridge regression coefficient