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Introduction

The late Rear Admiral Arthur A. Ageton, USN (1900-1971), described "The Secant-Cosecant Method for Determining the Altitude and Azimuth of a Heavenly Body" in the U.S. Naval Institute *Proceedings* for October 1931 while a Lieutenant and a member of the Line Class of the Postgraduate School, U.S. Naval Academy. His table was published by the then U.S. Navy Hydrographic Office as "H.O. 211, Dead Reckoning Altitude and Azimuth Table" in several editions until discontinued in 1971. It reappeared in 1975 in Volume II of the Defense Mapping Agency Hydrographic Center's "Pub. No.9, American Practical Navigator (Bowditch)," as "Table 35, The Ageton Method." Other applications of the table were discussed in the Naval Institute *Proceedings* for July 1932.

Since Ageton's Table (a) requires but two columns of figures, (b) may be used with the coordinates of the DR position, and (c) requires only simple addition and subtraction, it has been the most durable and probably the most popular of all the "short methods" of sight reduction. It has been reprinted in full in many popular textbooks of navigation.

The present version introduces two significant simplifications:

- (1) Entries for half-minutes of arc have been deleted resulting in precision to the nearest minute of arc (one nautical mile) when used without interpolation. This is in keeping with the variability of the apparent position of the sea horizon and is about the limit of accuracy of the hand-held sextant.
- (2) The numerical duplication of the second half of Ageton's Table has been eliminated, a procedure common to most trigonometric tables. This has required a change in the column headings and a restatement of the rules for the use of the table; namely, the substitution of "less than 90°" or "more than 90°" for Ageton's "top of the table" or "bottom of the table."

The much shorter table (9 pages instead of 36 in the original), improved format, and absence of numerical duplication should result in fewer page openings, less table searching, and less chance for copy error. In addition, precision is consistent with the practical accuracy of celestial observations at sea.

This table is the simplest of all methods for pencil-and-paper calculation, barring multi-volume "inspection tables" such as DMAHC Pub's. 229 and 249, and is the ideal backup method for always fallible electronic instruments and calculators. It is small enough to be slipped between the pages of the *Nautical Almanac* for storage or folded and stuffed into the sextant box.

Though of no practical importance, the table has been recalculated to correct the numerous end-figure errors of the original table due to its calculation by six-place logarithms.

The author would be remiss if he did not acknowledge his indebtedness to O. B. Ellis for his indefatigable efforts without which this project would have been well nigh impossible. Also, he wishes to thank the distinguished Captain Alton B. Moody for his valuable suggestions.

Finally, it should be noted this little booklet is in no sense intended to be a textbook, as it is as compact as possible. Whereas the use of the table is presented as completely and clearly as possible, the theory has been omitted, except for the Appendix. .

The Second Edition

This edition includes an important addition devised by the late D. H. Sadler, former Superintendent of H.M. Nautical Almanac Office and renowned British authority on navigation and navigational tables, an alternative sight reduction technique which makes it possible to salvage otherwise entirely satisfactory sights formerly discarded because they fell in the *forbidden zone* where K is between 82° and 98° , a little less than 9 per cent of sights. This edition also includes a graph devised by Elliot Laidlaw of USPS by means of which these sights can be identified and the appropriate reduction technique selected before the table is opened.

The long-familiar term *GMT* (Greenwich Mean Time) has been replaced by *UT* (Universal Time) in the *Nautical Almanac*, so the new term has been used in this edition of the *Compact Table* as well. There is no change in the meaning or significance of the new term in navigation: $UT = \text{GMT}$.

The only known error has been corrected (rule 2c for great circle distance).

Format of the Table

The table is composed of log secants and log cosecants of angles from 0° to 180° multiplied by 100,000 to avoid the need for a decimal point preceding the mantissa. The column headings are arranged as follows:

Top of each page:

Bottom of each page:

Minutes of arc for angles at the top of the page are read from top to bottom on the left side of the page; minutes for angles at the bottom of the page are read from bottom to top on the right side of the page.

There are paired columns of numbers for each degree of arc, either A and B or B and A. The location of the letter adjacent to the degree heading determines to which column the letter applies.

As it is often necessary to enter the table with an A or B to find the corresponding B or A in the paired columns, note that the numbers decrease from infinitely large at the beginning of the table (left column, $0^\circ/90^\circ$) to 15051 at the end ($45^\circ/135^\circ$), then decrease (right column) to 0 at the beginning. As a consequence, there is no duplication of numbers (except for $45^\circ 00'$) so the corresponding number is easily found.

Sight Reduction

Before entering the table, the four angles listed below must be known. As it is intended that the table be used without interpolation, each should be expressed only to the nearest minute of the arc.

1. The corrected sextant altitude of the body, Ho . This is the sextant angle after the necessary corrections have been applied and is the angle relative to the celestial horizon.
2. The declination of the body, dec. This is found in the Almanac for the UT of the sight.
3. The latitude of the observer, L. This is usually the dead reckoning (DR) latitude from the plot.
4. The meridian angle of the body, t. This is the hour angle of the body measured 0° to 180° east or west of the navigator's meridian and is computed from the navigator's assumed longitude (usually the DR longitude) and the GHA of the body from the Almanac for the time of the sight:

	Body west of navigator:	$t \text{ W} = \text{GHA} - \text{Lo}$
	Body east of navigator, west of Greenwich:	$t \text{ E} = \text{Lo} - \text{GHA}$
Lo W	(Body east of navigator, east of Greenwich:	$t \text{ E} = 360^\circ - (\text{GHA} - \text{Lo})$
	Body east of navigator:	$t \text{ E} = 360^\circ - (\text{GHA} + \text{Lo})$
	Body west of navigator, east of Greenwich	$t \text{ W} = \text{Lo} - (360^\circ - \text{GHA})$
Lo E	(Body west of navigator, west of Greenwich:	$t \text{ W} = \text{GHA} + \text{Lo}$

Were local hour angle, LHA, used instead of t , the table would require twice as many column headings: $t \text{ W} = \text{LHA}$, but $t \text{ E} = 360^\circ - \text{LHA}$.

The Standard Technique

The standard technique for using the table described below is applicable for more than 90 per cent of sights with accuracy of approximately one mile. However, when K approaches 90° , the area where K is between 82° and 98° , large and unpredictable errors may occur. Reduction by Sadler's technique results in no loss of accuracy in this area. These sights may be identified by Laidlaw's graph before the table is used. The technique and graph appear on pages 12-14.

The following detailed description of the standard technique refers to the numbered sequence shown in the box. This is the simplest way to layout the calculation as the mechanics of the operation are more apparent and less duplication is required.

	(1)	E/W	A	(2)							
dee	<u>(3)</u>	N/S	+8	<u>(4)</u>	A	<u>(5)</u>					
					A	<u>(6)</u>	-8	<u>(7)</u>	8	<u>(8)</u>	A <u>(9)</u>
K	<u>(11)</u>	N/S	←	←	←	←	←	A	<u>(10)</u>		
L	<u>(12)</u>	N/S									
K ~ L	<u>(13)</u>	→	→	→	→	+8 <u>(14)</u>
He	<u>(17)</u>	←	←	←	←	←	←	←	←	A <u>(15)</u>	-8 <u>(16)</u>
Ho	<u>(18)</u>	Z	N/S	<u>(21)</u>	E/W	←	←	←	←	A <u>(20)</u>	
a	<u>(19)</u>	mi	T/A		Zn	<u>(22)</u>					

1. Enter the meridian angle, t (1), to the nearest minute of arc, label it East or West, and find A (2) for this angle from the table.
2. Enter the declination of the body, dec (3), to the nearest minute of arc, label it North or South, and find B (4) and the corresponding A (5) for this angle from the table.
3. Add (2) and (4) to find A (6) and enter this same value as A (9).
4. From the table, find B (7) for the tabulated value closest to A (6) and enter this same value as B (8).
5. Subtract (7) from (5) to find A (10).
6. From the table, find the angle K (11) corresponding to A (10); if t is greater than 90° , K is greater as well; if t is less than 90° , K is also less than 90° . Give K the same name as the declination, North or South.
7. Enter the latitude of the observer, L (12), to the nearest minute of arc and label it North or South. Calculate $(K \sim L)$ (13), the sum of (11) and (12) if they are of contrary name, the difference if the same name. [$K = L$ and $(K \sim L) = 0$ if the body is on the prime vertical, that is, where $Z = 90^\circ$.]

L is usually the DR latitude; however, any other position may be used: AP, Ep, or wild guess-with less accuracy the farther the chosen position is from the actual position of the observer. However, the advantages of using the DR coordinates have contributed to the popularity and longevity of the table. If the DR position is reasonably accurate and the sight taken accurately, the intercept will be quite short. This is not only a check on the reliability of the DR and the sight, but plotting is simplified as a result. Also, in the case of a multi-body fix, the same L and La may be used in calculating all the sights. By contrast, "inspection tables" require the use of an AP based on integral degrees of latitude and hour angle; long and unwieldy intercepts often result. Calculator users will be familiar with plotting from the DR position as there is no advantage in doing it any other way.

8. From the table, find B (14) for the angle $(K \sim L)$ (13).
9. Add (8) and (14) to find A (15).
10. From the table, find B (16) corresponding to A (15) and, at the same time, angle Hc (17), the calculated altitude.
11. Enter the observed sextant altitude (corrected sextant altitude), Ho (18), to the nearest minute of arc and calculate the intercept, a (19). The intercept is the difference between (17) and (18) in minutes of arc = nautical miles. If the computed altitude, He, is greater than the observed altitude, Ho, the intercept is measured Away from the direction of the body (the reciprocal of Zn), and if Ho is greater, it is measured Toward the body (in the direction of Zn). A useful mnemonic: "Coast Guard Academy:: CGA = Computed Greater Away." Label the intercept T or A accordingly.

Where the body is very near the horizon, its altitude may be negative due to the effect of refraction and/or dip. Under these circumstances, it is useful to remember the above rule may also be stated: The intercept is the algebraic difference between H_o and H_c and is determined by subtracting H_c from H_o . If the difference is positive (+), the intercept is Toward; if it is negative (-), the intercept is Away. The calculated altitude, H_c , will be negative if K and L have the same name and ($K \sim L$) is greater than 90° or if K and L have contrary names and K is greater than ($90^\circ - L$). In the latter case, Z is less than 90° .

Example of the Complete Reduction of a Sight

The navigator takes a sight on the lower limb of the sun at 14-57-09 on 1 October 1980. At that time, his DR position is $L 25^\circ 37' N$, $Lo 87^\circ 06' W$. The sextant altitude, hs , is $33^\circ 31'$, the index correction $+1'$, the height of eye is 6 metres, and watch time is 5 seconds fast.

	<i>Time</i>		<i>Altitude</i>
WT	14-57-09	1 Oct	
WE	<u>- 05</u>		
ZT	14-57-04		
ZD	<u>+ 6</u>		
UT	<u>20-57-04</u>	10 ct	
			<i>Almanac</i>
hs			$33^\circ 31'$
IC			$+ 01'$
Dip			$- 04'$
ha			<u>$33^\circ 28'$</u>
Alt corr			$+ IS'$
Ho			<u>$33^\circ 43'$</u>

GHA sun 20h	$122^\circ 38'$		dec sun	$3^\circ 29' S$
57m 04s	<u>$14^\circ 16'$</u>		d carr	$+ 01' S$
GHA sun	$136^\circ 54'$		dec sun	<u>$3^\circ 30'$</u> S
La	<u>$87^\circ 06'$</u> W			
t	$49^\circ 48' W$			

Intercept and Azimuth from the Table

t	$49^\circ 48' W$	A 11702			
dec	$3^\circ 30' S$	<u>+B</u> <u>8_1</u>	A 121432		
		A 11783	<u>-B</u> <u>18894</u>	B 18894	A 11783
K	$5^\circ 25' S$	↔	↔	↔	↔ A 102538
L	$25^\circ 37' N$				
$K \sim L$	$31^\circ 02'$	→	→	→	→ +8 6709
H_c	$33^\circ 41'$	↔	↔	↔	↔ A 25603 -B 7982
H_o	$33^\circ 43'$		Z N $114^\circ W$	↔	↔ A 3801
a	2 mi T		Zn 246°		

12. Subtract (16) from (9) to fmd A (20). Occasionally this will be a small negative number, manifestly impossible for a log secant or log cosecant, and is due to the fact that (9) is not a tabulated number. This is most easily corrected by substituting zero for A (20) as it occurs only when the body is on the prime vertical and $Z = 90^\circ$.

13. From the table, find the azimuth angle, Z (21), corresponding to A (20). When K has the same name and is greater than L, Z is less than 90°; otherwise it is greater than 90°. Ordinarily it will suffice to take Z as the nearest whole degree; however, minutes can be converted to tenths of a degree if desired. The prefix for Z is the name of the elevated pole and the same as the name of the latitude, L (12); label it North or South. The suffix is the same as the name of the meridian angle, t (1), and is labeled East or West.

14. Z (21) is converted to Zn (22), azimuth, the direction relative to north, as follows:

14. Z (21) is converted to Zn (22), azimuth, the direction relative to north, as follows:

Z NE, Zn = Z Z SE, Zn = 180° - Z
 Z NW, Zn = 360° - Z Z SW, Zn = 180° + Z

The Sadler Technique

This technique is to be applied to sights where K is between 82° and 98° , the formerly *forbidden zone*. These sights can be identified by Laidlaw's graph before the Compact Table is used.

This technique, illustrated by the example, requires a different arrangement of terms and two new quantities, hI and h_z . Although hI is modified by H_0 , the value for H_c is insensitive to quite large errors in H_0 . The calculation of Z_n is the same as in the standard technique except R is represented by its two components in the rightmost column.

The error in the standard technique fluctuates in a saw-toothed fashion, plus and minus relative to the correct value, so at many points the error is zero, even in the *forbidden zone*. However, the magnitude of the swing increases geometrically as K approaches 90°. Sadler's technique is at least as accurate in this area as the standard technique for sights in other areas, about 1 mile. In the example, K is 90°, but direct trigonometric calculation confirms the value of H_c precisely; by the standard technique, H_c is 30' in error!

Example of Sadler's Technique

A sight was taken and the following quantities found:

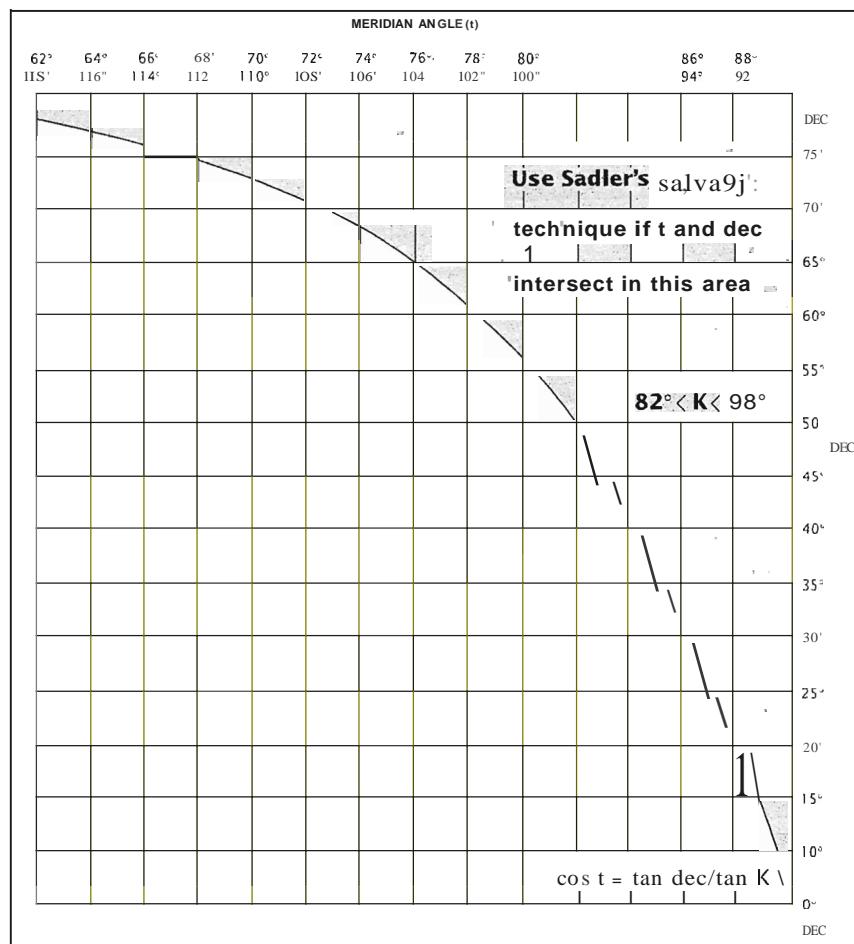
$t = 89^{\circ}06'W$, Dec $53^{\circ}16'S$, L $33^{\circ}19'S$, Ho $26^{\circ}35'$.

t	89°06'W	-----►	B 180390	A 5.4
dec	53°16'S	A 9614	+B 22323	+B 22323
L	33°19'S	+A 26022	+B 7798	
h1	26°07'	◀ - A 35636		
Ho	26°35'			
h1 + Ho	52°42'			
(h1 + Ho)/2	26°21'	-----►	-B 4764	
h2	0°30'	◀ ----- A 205747		
He (h1 + h2)	26°37'	-----►	-B 4865	
Ho	26°35'	Z S 41°59'W	◀ --- A 17463.4	
a	--2mi	A Zn 222°		

Signs: $h J$ is negative if L and dec have contrary names.

$h2$ is positive if is less than 90° ; negative if t is more than 90° , $0^\circ 00'$ if t equals 90° .

The graph below will determine whether the standard or the Sadler technique is required before the table is entered.



If the values for t and dec intersect to the right and above the curve in the shaded area, reduce the sight by Sadler's technique. As the $82^\circ/98^\circ$ boundary is somewhat arbitrary, the standard technique may be used for sights where the intersection is on or very close to the curve though in the shaded area.

Plotting

In the case of a single sight, the intercept is drawn from the observer's assumed position (usually the DR) in the direction of the body, if Toward, or in the reciprocal direction, if Away. The length of the intercept will be the number of miles calculated (19). (The actual line representing the intercept is often omitted from the plot.) The line of position (LOP) is a line drawn perpendicular to the intercept at its terminus.

In the case for a multi-body fix, all the LOP's are calculated for the same position, the DR at the time of the fix. The advancement or retirement of any given sight is measured along the course line before or after the position used for the calculation and the intercept drawn from that point. Alternatively, each sight may be calculated from its own DR and all intercepts plotted from the DR for the time of the fix; this automatically advances or retires the other sights.

BRIEF SUMMARY OF THE STANDARD TECHNIQUE

Rules:

1. Observe the rules at the top of the table pages.
2. Interpolation is unnecessary for practical navigation.
3. If K is between 82° and 98° , use Sadler's Technique.
4. Give K the same name as declination.
- S. The prefix for Z is the same as the name for L; the suffix is the same as the name for t.

Format of the calculation:

t	(7)	EjW	A	(2)								
dec	(3)	N/S	+B	(4)	A	(5)	B	(8)	A	(9)		
K	(71)	N/S	A	(6)	-B	(7)						
L	(72)	N/S	A	(70)								
$K \sim L$	(73)		→	→	→	→	→	→	+B	(74)		
Hc	(77)		←	←	←	←	←	←	A	(75)	-8	(76)
Ho	(78)		Z	N/S	(2/)		E/W	←	←	A	(20)	
a	(79)		Zn	(22)								

Procedure:

1. Enter meridian angle (7) and find (2) from the table.
2. Enter declination (3) and find (4) and (5) from the table.
3. Add (2) and (4) to find (6) and enter (6) as (9) as well.
4. Find (7) and (8) corresponding to (6).
5. Subtract (7) from (5) to find (70).
6. Find angle K (11) from the table corresponding to (70).
7. Enter L (72) and calculate $(K \sim L)$ (13).
8. From the table, find (74) corresponding to (13).
9. Add (8) and (74) to find (75).
10. From the table, find Hc (77) and (16) corresponding to (75).
11. Enter Ho (18) and calculate the intercept (79).
12. Subtract (76) from (9) to find (20).
13. From the table, find Z (21) corresponding to (20).
14. Convert Z (21) to Zn (22):

$$\begin{array}{ll} Z \text{ NE}, & Zn = Z \\ Z \text{ NW}, & Zn = 360^\circ - Z \end{array}$$

$$\begin{array}{ll} Z \text{ SE}, & Zn := 180^\circ - Z \\ Z \text{ SW}, & Zn := 180^\circ + Z \end{array}$$

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°.
Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	0° 90°	B A	A B	1° 91°	B A	A B	2° 92°	B A	A B	3° 93°	B A	A B	4° 94°	B A
	B A	89° 179°	A B	B A	88° 178°	A B	B A	87° 177°	A B	B A	86° 176°	A B	B A	85° 175°	A B
00	-----	0.0	175814	6.6	145718	26.5	128120	59.6	115642	105.9	60				
01	353627	0.0	5097	6.8	5358	26.9	7880	60.2	5461	106.8	59				
02	323524	0.0	4391	7.1	5001	27.4	7641	60.9	5282	107.7	58				
03	305915	0.0	3696	7.3	4646	27.8	7403	61.6	5103	108.6	57				
04	293421	0.0	3012	7.5	4295	28.3	7166	62.2	4925	109.5	56				
05	283730	0.0	172339	7.8	143946	28.7	126931	62.9	114748	110.4	55				
06	275812	0.1	1676	8.0	3600	29.2	6697	63.6	4571	111.3	54				
07	269118	0.1	1023	8.2	3257	29.6	6465	64.3	4395	112.2	53				
08	3318	0.1	0379	8.5	2916	30.1	6233	65.0	4220	113.1	52				
09	258203	0.1	169745	8.7	2579	30.6	6003	65.7	4045	114.0	51				
10	253627	0.2	169121	9.0	142243	31.1	125774	66.4	113872	114.9	50				
11	249488	0.2	8505	9.3	1911	31.5	5546	67.1	3699	115.9	49				
12	5709	0.3	7897	9.5	1581	32.0	5320	67.8	3526	116.8	48				
13	2233	0.3	7298	9.8	1253	32.5	5094	68.5	3355	117.7	47				
14	239015	0.4	6708	10.1	0928	33.0	4870	69.2	3184	118.7	46				
15	236018	0.4	166125	10.3	140605	33.5	124647	69.9	113013	119.6	45				
16	3216	0.5	5550	10.6	0285	34.0	4425	70.6	2844	120.5	44				
17	0583	0.5	4982	10.9	139967	34.5	4205	71.3	2675	121.5	43				
18	228100	0.6	4422	11.2	9651	35.0	3985	72.1	2506	122.4	42				
19	5752	0.7	3869	11.5	9338	35.5	3766	72.8	2339	123.4	41				
20	223525	0.7	163322	11.8	139027	36.0	123549	73.5	112171	124.3	40				
21	1406	0.8	2783	12.1	8718	36.5	3333	74.3	2005	125.3	39				
22	219385	0.9	2250	12.4	8411	37.1	3117	75.0	1839	126.2	38				
23	7455	1.0	1724	12.7	8106	37.6	2903	75.8	1674	127.2	37				
24	5607	1.1	1204	13.0	7804	38.1	2690	76.5	1510	128.2	36				
25	213834	1.1	160690	13.3	137503	38.6	122478	77.3	111346	129.2	35				
26	2130	1.2	0182	13.6	7205	39.2	2267	78.0	1183	130.1	34				
27	0491	1.3	159680	13.9	6909	39.7	2057	78.8	1020	131.1	33				
28	208912	1.4	9184	14.2	6615	40.3	1848	79.5	0858	132.1	32				
29	7388	1.5	8693	14.6	6322	40.8	1640	80.3	0696	133.1	31				
30	205916	1.7	158208	14.9	136032	41.4	121432	81.1	110536	134.1	30				
31	4492	1.8	7728	15.2	5744	41.9	1226	81.9	0375	135.1	29				
32	3113	1.9	7254	15.6	5457	42.5	1021	82.6	0216	136.1	28				
33	1777	2.0	6784	15.9	5173	43.0	0817	83.4	0057	137.1	27				
34	0480	2.1	6320	16.2	4890	43.6	0614	84.2	109898	138.1	26				
35	199221	2.3	155861	16.6	134609	44.2	120412	85.0	109740	139.1	25				
36	7998	2.4	5406	16.9	4330	44.7	0211	85.8	9583	140.1	24				
37	6808	2.5	4956	17.3	4053	45.3	0010	86.6	9426	141.1	23				
38	5650	2.7	4511	17.6	3777	45.9	119811	87.4	9270	142.2	22				
39	4522	2.8	4070	18.0	3503	46.5	9612	88.2	9115	143.2	21				
40	193422	2.9	153634	18.4	133231	47.1	119415	89.0	108960	144.2	20				
41	2350	3.1	3201	18.7	2961	47.6	9218	89.8	8805	145.2	19				
42	1304	3.2	2774	19.1	2692	48.2	9022	90.6	8651	146.3	18				
43	0282	3.4	2350	19.5	2425	48.8	8827	91.4	8498	147.3	17				
44	189283	3.6	1931	19.9	2159	49.4	8633	92.3	8345	148.4	16				
45	188307	3.7	151515	20.3	131896	50.0	118440	93.1	108193	149.4	15				
46	7353	3.9	1104	20.6	1633	50.7	8248	93.9	8041	150.5	14				
47	6419	4.1	0696	21.0	1373	51.3	8056	94.7	7890	151.5	13				
48	5505	4.2	0292	21.4	1114	51.9	7866	95.6	7739	152.6	12				
49	4609	4.4	149892	21.8	0856	52.5	7676	96.4	7589	153.6	11				
50	183732	4.6	149496	22.2	130600	53.1	117487	97.3	107439	154.7	10				
51	2872	4.8	9103	22.6	0346	53.7	7299	98.1	7290	155.8	09				
52	2029	5.0	8713	23.1	0093	54.4	7112	99.0	7141	156.9	08				
53	1202	5.2	8327	23.5	129841	55.0	6925	99.8	6993	157.9	07				
54	0390	5.4	7945	23.9	9591	55.7	6739	100.7	6846	159.0	06				
55	179593	5.6	147566	24.3	129342	56.3	116554	101.6	106699	160.1	05				
56	8811	5.8	7190	24.7	9095	56.9	6370	102.4	6552	161.2	04				
57	8042	6.0	6817	25.2	8849	57.6	6187	103.3	6406	162.3	03				
58	7287	6.2	6448	25.6	8605	58.2	6004	104.2	6260	163.4	02				
59	6544	6.4	6081	26.0	8362	58.9	5823	105.0	6115	164.5	01				
60	175814	6.6	145718	26.5	128120	59.6	115642	105.9	105970	165.6	00				

Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	5° 95°	B A		6° 96°	B A		A B	7° 97°	B A		A B	8° 98°	B A		A B	9° 99°	B A
00	105970	166	-0 7	239	91411	325	85644	425	80567	538	60							
01	5826	167	97 57	240	308	326	555	426	487	540	59							
02	5683	168	837	241	205	328	465	428	408	542	58							
03	5539	169	-- 7	243	103	330	376	430	328	544	57							
04	5397	17	598	244	001	331	286	432	249	546	56							
05	105254	171	97 80	245	90899	333	85197	434	80170	548	55							
06	5113	172	361	247	798	334	109	435	091	550	54							
07	4971	173	3	248	696	336	020	437	012	552	53							
08	4830	175	- 26	249	595	337	84931	439	79933	554	52							
09	4690	176	08	251	494	339	843	441	855	556	51							
10	104550	17	91	252	90394	341	84755	443	79777	558	50							
11	4411	178	- 74	253	293	342	667	444	698	560	49							
12	4272	179	658	255	193	344	579	446	620	562	48							
13	4133	18	542	256	093	345	192	448	542	564	47							
14	3995	181	26	258	89994	347	404	450	465	566	46							
15	103857	18	96310	259	89894	349	84317	452	79387	568	45							
16	3720	184	- 95	260	795	350	230	454	309	571	44							
17	3583	185		262	696	352	143	455	232	573	43							
18	3447	18	95 66	263	598	353	056	457	155	575	42							
19	3311	18	851	264	499	355	83970	459	078	577	41							
20	103175	188	95738	266	89401	357	83884	461	79001	579	40							
21	3040	19	624	267	303	358	797	463	78924	581	39							
22	2905	191	- 0	269	205	360	711	465	847	583	38							
23	2771	19	97	270	107	362	626	467	771	585	37							
24	2637	193	285	272	010	363	540	468	694	587	36							
25	102504	194	951 2	273	88913	365	83455	470	78618	589	35							
26	2371	196	O	274	816	367	369	472	542	591	34							
27	2238	197	- 948	276	719	368	284	474	466	593	33							
28	2106	19	36	277	623	370	199	476	390	596	32							
29	1974	19	25	279	526	371	114	478	315	598	31							
30	101843		946 4	280	88430	373	83030	480	78239	600	30							
31	1712		- 3	282	334	375	82945	482	164	602	29							
32	1581		393	283	239	376	861	483	088	604	28							
33	1451		283	284	143	378	777	485	013	606	27							
34	1321		73	286	048	380	693	487	77938	608	26							
35	101192		94 63	287	87953	382	82609	489	77863	610	25							
36	1063		93954	289	858	383	526	491	789	612	24							
37	0934		5	290	764	385	442	493	714	615	23							
38	0806		36	292	669	387	359	495	639	617	22							
39	0678		28	293	575	388	276	497	565	619	21							
40	100550	213	93519	295	87481	390	82193	499	77491	621	20							
41	0423	21	411	296	388	392	110	501	417	623	19							
42	0296	215	304	298	294	393	027	503	343	625	18							
43	0170	217	96	299	201	395	81945	505	269	628	17							
44	0044	218	89	301	108	397	863	506	195	630	16							
45	99918	219	2982	302	87015	399	81780	508	77122	632	15							
46	793	220	876	304	86922	400	698	510	048	634	14							
47	668	222	69	305	829	402	617	512	76975	636	13							
48	544	223	663	307	737	404	535	514	902	638	12							
49	419	224	558	308	645	405	453	516	829	641	11							
50	99296	225	92452	310	86553	407	81372	518	76756	643	10							
51	172	227	347	311	461	409	291	520	683	645	09							
52	049	2	242	313	370	411	210	522	610	647	08							
53	98926	229	37	314	278	412	129	524	538	649,	07							
54	804	231	032	316	187	414	048	526	465	652	06							
55	98682	232	- 928	317	86096	416	80967	528	76393	654	05							
56	560	233	824	319	006	418	887	530	321	656	04							
57	439	235	720	320	85915	419	807	532	248	658	03							
58	318	236	617	322	825	421	727	534	177	660	02							
59	197	237	514	323	734	423	647	536	105	663	01							
60	98077		1411	325	85644	425	567	538	76033	665	00							
	B	84°			83°	A	B	82°	A	B	81°	A	B	80°	A	B	170°	B
	A	174°			173°	B	A	172°	B		171°	B	A	170°	B			

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°.
Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	10° 100°	B A	A B	11° 101°	B A	A B	12° 102°	B A	A B	13° 103°	B A	A B	14° 104°	B A
	B A	79° 169°	A B	B A	78° 168°	A B	B A	77° 167°	A B	B A	76° 166°	A B	B A	75° 165°	A B
00	76033	565		71940	805		68212	960		64791	1128		61632	1310	60
01	75961	667		875	808		153	962		737	131		582	313	59
02	890	669		810	810		093	965		682	133		531	316	58
03	819	672		746	813		034	968		627	136		481	319	57
04	747	674		681	815		67975	970		573	139		430	322	56
05	75676	676		71616	818		67916	973		64519	1142		61380	1325	55
06	605	678		552	820		857	976		464	145		330	329	54
07	534	681		488	823		798	978		410	148		279	332	53
08	464	683		423	825		739	981		356	151		229	335	52
09	393	685		359	828		681	984		302	154		179	338	51
10	75323	687		71295	830		67622	987		64248	1157		61129	1341	50
11	252	690		231	833		563	989		194	160		079	344	49
12	182	692		167	835		505	992		140	163		029	348	48
13	112	694		104	838		447	995		086	166		60979	351	47
14	042	696		040	840		388	998		032	169		929	354	46
15	74972	699		70976	843		67330	1000		63978	1172		60879	1357	45
16	902	701		913	845		272	003		925	175		830	360	44
17	832	703		850	848		214	006		871	178		780	364	43
18	763	706		786	850		156	009		818	181		730	367	42
19	693	708		723	853		098	011		764	184		681	370	41
20	74624	710		70660	855		67040	1014		63711	1187		60631	1373	40
21	555	712		597	858		66982	017		658	190		582	377	39
22	486	715		534	860		925	020		605	193		533	380	38
23	417	717		471	863		867	022		551	196		483	383	37
24	348	719		409	865		810	025		498	199		434	386	36
25	74279	722		70346	868		66752	1028		63445	1202		60385	1390	35
26	210	724		284	870		695	031		392	205		336	393	34
27	142	726		221	873		638	033		340	208		287	396	33
28	073	729		159	876		580	036		287	211		238	399	32
29	005	731		097	878		523	039		234	214		189	403	31
30	73937	733		70034	881		66466	1042		63181	1217		60140	1406	30
31	869	736		69972	883		409	045		129	220		091	409	29
32	801	738		910	886		353	047		076	223		042	412	28
33	733	740		849	888		296	050		024	226		59994	416	27
34	665	743		787	891		239	053		62972	229		945	419	26
35	73597	745		69725	894		66182	1056		62919	1232		59897	1422	25
36	530	748		664	896		126	059		867	235		848	426	24
37	462	750		602	899		069	062		815	238		800	429	23
38	395	752		541	901		013	064		763	241		751	432	22
39	328	755		479	904		65957	067		711	244		703	435	21
40	73261	757		69418	907		65900	1070		62659	1247		59654	1439	20
41	194	759		357	909		844	073		607	250		606	442	19
42	127	762		296	912		788	076		555	254		558	445	18
43	060	764		235	914		732	079		503	257		510	449	17
44	72993	767		174	917		676	081		451	260		462	452	16
45	72927	769		69113	920		65620	1084		62400	1263		59414	1455	15
46	860	771		503	922		564	087		348	266		366	459	14
47	794	774		68992	925		509	090		297	269		318	462	13
48	727	776		932	928		453	093		245	272		270	465	12
49	661	779		871	930		398	096		194	275		222	469	11
50	72595	781		68811	933		65342	1099		62142	1278		59175	1472	10
51	529	783		750	936		287	102		091	281		127	475	09
52	463	786		690	938		231	104		040	285		079	479	08
53	398	788		630	941		176	107		61989	288		032	482	07
54	332	791		570	944		121	110		938	291		58984	485	06
55	72266	793		68510	946		65066	1113		61887	1294		58937	1489	05
56	201	796		451	949		011	116		836	297		889	492	04
57	136	798		391	952		64956	119		785	300		842	495	03
58	070	800		331	954		901	122		734	303		795	499	02
59	005	803		272	957		846	125		683	306		748	502	01
60	71940	805		68212	960		64791	1128		61632	1310		58700	1506	00

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°.
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	A 8	15° 105°	B A	A B	16° 106°	B A	A B	17° 107°	B A	A B	18° 108°	B A	A B	19° 109°	B A
00	58700	1506		55966	1716		53406	1940		51002	2179		48736	2433	60
01	653	509		922	719		365	944		50963	183		699	437	59
02	606	512		878	723		324	948		924	188		662	442	58
03	559	516		834	727		283	952		885	192		626	446	57
04	512	519		90	730		242	956		847	196		589	450	56
05	58465	1523		55	1734		53200	1960		50803	2200		48553	2455	55
06	418	526		13	738		159	964		769	204		516	459	54
07	372	529		659	741		118	968		731	208		480	464	53
08	325	533		61	745		077	971		692	212		443	468	52
09	278	536		572	749		036	975		653	216		407	472	51
10	58232	1540		5552	1752		52995	1979		50615	2221		48371	2477	50
11	185	543		484	756		955	983		576	225		334	481	49
12	139	547		—	760		914	987		538	229		298	485	48
13	092	550		3	763		873	991		500	233		262	490	47
14	046	553		35	767		832	995		461	237		226	494	46
15	57999	1557		5531	1771		52791	1999		50423	2241		48189	2499	45
16	953	560		267	774		751	2003		385	246		153	503	44
17	907	564		22	78		710	007		346	250		117	508	43
18	860	567		—	782		670	011		308	254		081	512	42
19	814	571		38	785		629	014		270	258		045	516	41
20	57768	1574		55095	89		52589	2018		50232	2262		48009	2521	40
21	722	578		—	793		548	022		194	266		47973	525	39
22	676	581		0	96		508	026		156	271		937	530	38
23	630	58 <i>i</i>		54	65		467	030		118	275		901	534	37
24	584	588		923	84		427	034		080	279		865	539	36
25	57539	1591		54880	1808		52387	2038		50042	2283		47829	2543	35
26	493	595		837	911		346	042		004	287		793	547	34
27	447	598		794	— 5		306	046		49966	292		758	552	33
28	401	602		75	—		266	050		928	296		722	556	32
29	356	605		7	823		226	054		890	300		686	561	31
30	57310	1609		54666	826		52186	2058		49852	2304		47650	2565	30
31	265	612		623	830		146	062		815	309		615	570	29
32	219	616		581	834		106	066		777	313		579	574	28
33	174	619		53	—		066	070		739	317		544	579	27
34	128	623		496	—		026	074		702	321		SOB	583	26
35	57083	1627		54453	—		51986	2078		49664	2326		47473	2588	25
36	038	630		41	—		946	082		626	330		437	592	24
37	56992	634		368	—		906	086		589	334		402	597	23
38	947	637		326	—		867	090		551	338		366	601	22
39	902	641		284	—		827	094		514	343		331	606	21
40	56857	1644		54242	—		51787	2098		49477	23 7		47295	2610	20
41	812	648		199	—		748	102		439	351		260	615	19
42	767	651		15 <i>i</i>	—		708	106		402	355		225	619	18
43	722	655		11	—		668	110		365	36		139	624	17
44	677	658		073	—		629	114		327	364		154	628	16
45	56633	1662		54031	—		51589	2118		49290	2368		47119	2633	15
46	588	666		53989	—		550	122		253	372		084	637	14
47	543	669		947	—		510	126		216	377		049	642	13
48	498	673		905	—		471	130		179	381		014	647	12
49	454	676		864	—		432	134		142	385		46979	651	11
50	56409	1680		53822	—		51393	2139		49104	239		46944	2656	10
51	365	683		780	—		353	143		067	35		908	660	09
52	320	687		738	—		314	147		030	98		874	.665	08
53	276	691		697	—		275	151		48993	3		839	669	07
54	231	694		655	—		236	155		957			804	674	06
55	56187	1698		53614	—		51197	2159		48920	2411		46769	2678	05
56	143	701		572	—		158	163		883	46		734	683	04
57	099	705		531	—		119	167		846	20		699	688	03
58	054	709		489	—		080	171		809	424		664	692	02
59	010	712		448	—		041	175		773	29		630	697	01
60	55966	1716		53406	—		51002	2179		48736	2433		46595	2701	00
	B A	74° 164°	A	B A	73° 163°	A	B A	72° 162°	A	B A	71° 16°	A	B A	70° 160°	A

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°.
Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A 20° B B 110° A	A 21° B B 111° A	A 22° B B 112° A	A 23° B B 113° A	A 24° B B 114° A	
	B 69° A A 159° B	B 68° A A 158° B	B 67° A A 157° B	B 66° A A 156° B	B 65° A A 155° B	
00	46595 2701	44567 2985	42642 3283	40812 3597	39069 3927	60
01	560 706	534 990	611 289	782 603	040 933	59
02	525 711	501 995	580 294	753 608	012 938	58
03	491 715	463 999	549 299	723 613	38984 944	57
04	456 720	436 3004	518 304	693 619	955 950	56
05	46422 2724	44403 3009	42486 3309	40664 3624	38927 3955	55
06	387 729	370 014	455 314	634 630	899 961	54
07	353 734	337 019	424 319	604 635	871 966	53
08	318 738	305 024	393 324	575 640	842 972	52
09	284 743	272 029	362 330	545 646	814 978	51
10	46249 2748	44239 3034	42331 3335	40516 3651	38786 3983	50
11	215 752	207 038	300 340	486 657	758 989	49
12	181 757	174 043	269 345	457 662	730 995	43
13	146 762	142 048	238 350	427 667	702 4000	47
14	112 766	109 053	207 355	398 673	674 006	46
15	46078 2771	44077 3058	42176 3360	40368 3678	38646 4012	45
16	043 776	044 063	145 366	339 684	618 018	44
17	009 780	012 068	115 371	310 689	589 023	43
18	45975 785	43979 073	084 376	280 695	562 029	42
19	941 790	947 078	053 381	251 700	534 035	41
20	45907 2794	43915 3083	42022 3386	40222 3706	38506 4040	40
21	873 799	882 088	41992 392	192 711	478 046	39
22	839 804	850 093	961 397	163 716	450 052	38
23	805 808	818 097	930 402	134 722	422 058	37
24	771 813	785 102	899 407	105 727	394 063	36
25	45737 2818	43753 3107	41869 3412	40076 3733	38366 4069	35
26	703 822	721 112	838 418	046 738	338 075	34
27	669 827	689 117	808 423	017 744	311 080	33
28	635 832	657 122	777 428	39988 749	283 086	32
29	601 837	625 127	747 433	959 755	255 092	31
30	45567 2841	43592 3132	41716 3438	39930 3760	38227 4098	30
31	534 846	560 137	686 444	901 766	200 103	29
32	500 851	528 142	655 449	872 771	172 109	28
33	466 855	496 147	625 454	843 777	144 115	27
34	433 860	464 152	594 459	814 782	117 121	26
35	45399 2865	43432 3157	41564 3465	39785 3788	38089 4127	25
36	365 870	401 162	533 470	756 793	061 132	24
37	332 874	369 167	503 475	727 799	034 138	23
38	298 879	337 172	473 480	698 804	006 144	22
39	265 884	305 177	443 486	669 810	37979 150	21
40	45231 2889	43273 3182	41412 3491	39641 3815	37951 4156	20
41	198 893	241 187	382 496	612 821	924 161	19
42	164 898	210 192	352 502	583 826	896 167	18
43	131 903	178 197	322 507	554 832	869 173	17
44	097 908	146 202	291 512	526 838	841 179	16
45	45064 2913	43114 3207	41261 3517	39497 3843	37814 4185	15
46	031 917	083 212	231 523	468 849	786 190	14
47	44997 922	051 217	201 528	439 854	759 196	13
48	964 927	020 222	171 533	411 860	732 202	12
49	931 932	42988 228	141 539	382 865	704 208	11
50	44898 2937	42956 3233	41111 3544	39354 3811	37677 4214	10
51	864 941	925 238	081 549	325 877	650 220	09
52	831 946	893 243	051 555	296 882	623 225	08
53	798 951	862 248	021 560	268 888	595 231	07
54	765 956	831 253	40991 565	239 893	568 237	06
55	44732 2961	42799 3258	40961 3571	39211 3899	37541 4243	05
56	699 965	768 263	931 576	182 905	514 249	04
57	666 970	736 268	902 581	154 910	487 255	03
58	633 975	705 273	872 587	125 916	459 261	02
59	600 980	674 278	842 592	097 921	432 267	01
60	44567 2985	42642 3283	40812 3597	39069 , 3927	37405 4272	00

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°,
Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	25° 115°	B A	A B	26° 116°	B A	A B	27° 117°	B A	A B	28° 118°	B A	A B	29° 119°	B A
00	37405	4272		35816	4634		34295	5012		32839	5407		31443	5818	60
01	378	278		790	640		271	018		815	413		420	825	59
02	351	284		764	646		246	025		792	420		397	832	58
03	324	290		738	652		221	031		768	427		375	839	57
04	297	296		712	659		196	038		744	433		352	846	56
05	37270	4302		35687	4665		34172	5044		32720	5440		31329	5853	55
06	243	308		661	671		147	051		697	447		306	860	54
07	216	314		635	677		122	057		673	454		284	867	53
08	189	320		609	683		098	064		650	460		261	874	52
09	162	326		583	690		073	070		626	467		238	881	51
10	37135	4332		35558	4696		34048	5077		32602	5474		31216	5888	50
11	108	337		532	702		024	083		579	481		193	895	49
12	082	343		506	708		33999	089		555	487		171	902	48
13	055	349		481	714		975	096		532	494		148	910	47
14	028	355		455	721		950	102		508	501		125	917	46
15	37001	4361		35429	4727		33925	5109		32485	5508		31103	5924	45
16	36974	367		404	733		901	115		461	515		080	931	44
17	948	373		378	739		876	122		438	521		058	938	43
18	921	379		353	746		852	129		414	528		035	945	42
19	894	385		327	752		827	135		391	535		013	952	41
20	36867	4391		35302	4758		33803	5142		32367	5542		30990	5959	40
21	841	397		276	764		779	148		344	549		968	966	39
22	814	403		251	771		754	155		320	555		945	973	38
23	787	409		225	777		730	161		297	562		923	980	37
24	761	415		200	783		705	168		274	569		900	988	36
25	36734	4421		35174	4789		33681	5174		32250	5576		30878	5995	35
26	708	427		149	796		657	181		227	583		856	6002	34
27	681	433		123	802		632	187		204	590		833	009	33
28	655	439		098	808		608	194		180	596		811	016	32
29	628	445		073	815		584	201		157	603		788	023	31
30	36602	4451		35047	4821		33559	5207		32134	5610		30766	6030	30
31	575	457		022	827		535	214		110	617		744	037	29
32	549	463		34997	833		511	220		087	624		721	045	28
33	522	469		971	840		487	227		064	631		699	052	27
34	496	475		946	846		463	233		041	638		677	059	26
35	36469	4481		34921	4852		33438	5240		32018	5645		30655	6066	25
36	443	487		896	859		414	247		31994	651		632	073	24
37	417	493		870	865		390	253		971	658		610	080	23
38	390	500		845	871		366	260		948	665		588	088	22
39	364	506		820	878		342	266		925	672		566	095	21
40	36338	4512		34795	4884		33318	5273		31902	5679		30544	6102	20
41	311	518		770	890		294	280		879	686		521	109	19
42	285	524		745	897		269	286		856	693		499	116	18
43	259	530		719	903		245	293		833	700		477	124	17
44	233	536		694	910		221	300		810	707		455	131	16
45	36206	4542		34669	4916		33197	5306		31787	5714		30433	6138	15
46	180	548		644	922		173	313		763	721		411	145	14
47	154	554		619	929		149	320		740	727		389	153	13
48	128	560		594	935		125	326		717	734		367	160	12
49	102	566		569	941		101	333		695	741		345	167	11
50	36076	4573		34544	4948		33078	5340		31672	5748		30323	6174	10
51	050	579		519	954		054	346		649	755		301	181	09
52	024	585		494	961		030	353		626	762		279	189	08
53	35998	591		469	967		006	360		603	769		257	196	07
54	972	597		444	973		32982	366		580	776		235	203	06
55	35946	4603		34420	4980		32958	5373		31557	5783		30213	6211	05
56	920	609		395	986		934	380		534	790		191	218	04
57	894	616		370	993		910	386		511	797		169	225	03
58	868	622		345	999		887	393		488	804		147	232	02
59	842	628		320	5005		863	400		466	811		125	240	01
60	35816	4634		34295	5012		32839	5407		31443	5818		30103	6247	00
	B A	64° 154°	A B	B A	63° 153°	A B	B A	62° 152°	A B	B A	61° 151°	A B	B A	60° 150°	A B

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°.
/ IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	30° 120°	B A	A B	31° 121°	B A	A B	32° 122°	B A	A B	33° 123°	B A	A B	34° 124°	B A	
00	30103	6247		28816	6693		27579	7158		26389	7641		25244	8143	60	
01	081	254		795	701		559	166		370	649		225	151	59	
02	059	262		774	709		539	174		350	657		206	160	58	
03	037	269		753	716		518	182		331	665		188	168	57	
04	016	276		732	724		498	190		311	674		169	177	56	
05	29994	6283		28711	6731		27478	7197		26292	7682		25150	8185	55	
06	972	291		690	739		458	205		273	690		132	194	54	
07	950	298		669	747		438	213		253	698		113	202	53	
08	928	305		648	754		418	221		234	707		094	211	52	
09	907	313		627	762		398	229		215	715		076	219	51	
10	29885	6320		28607	6770		27378	7237		26195	7723		25057	8228	50	
11	863	327		586	777		357	245		176	731		039	237	49	
12	841	335		565	785		337	253		157	740		020	245	48	
13	820	342		544	793		317	261		137	748		001	254	47	
14	798	350		523	800		297	269		118	756		24983	262	46	
15	29776	6357		28502	6808		27277	7277		26099	7765		24964	8271	45	
16	755	364		481	816		257	285		079	773		946	280	44	
17	733	372		461	823		237	293		060	781		927	288	43	
18	712	379		440	831		217	301		041	789		909	297	42	
19	690	386		419	839		197	309		022	798		890	305	41	
20	29668	6394		28398	6846		27177	7317		26003	7806		24872	8314	40	
21	647	401		378	854		157	325		25983	814		853	323	39	
22	625	409		357	862		137	333		964	823		835	331	38	
23	604	416		336	869		117	341		945	831		816	340	37	
24	582	423		315	877		098	349		926	839		798	349	36	
25	29561	6431		28295	6885		27078	7357		25907	7848		24779	8357	35	
26	539	438		274	892		058	365		887	856		761	366	34	
27	518	446		253	900		038	373		868	864		742	375	33	
28	496	453		233	908		018	381		849	873		724	383	32	
29	475	461		212	916		26998	389		830	881		706	392	31	
30	29453	6468		28191	6923		26978	7397		25811	7889		24687	8401	30	
31	432	475		171	931		959	405		792	898		669	409	29	
32	410	483		150	939		939	413		773	906		650	418	28	
33	389	490		130	947		919	421		754	914		632	427	27	
34	367	498		109	954		899	429		735	923		614	435	26	
35	29346	6505		28089	6962		26879	7437		25716	7931		24595	8444	25	
36	325	513		068	970		860	445		697	940		577	453	24	
37	303	520		048	978		840	454		678	948		559	462	23	
38	282	528		027	986		820	462		659	956		541	470	22	
39	261	535		006	993		800	470		640	965		522	479	21	
40	29239	6543		27986	7001		26781	7478		25621	7973		24504	8488	20	
41	218	550		966	009		761	486		602	982		486	496	19	
42	197	558		945	017		741	494		583	990		467	505	18	
43	176	565		925	024		722	502		564	998		449	514	17	
44	154	573		904	032		702	510		545	8007		431	523	16	
45	29133	6580		27884	7040		26682	7518		25526	8015		24413	8531	15	
46	112	588		863	048		663	527		507	024		395	540	14	
47	091	595		843	056		643	535		488	032		376	549	13	
48	069	603		823	064		623	543		469	041		358	558	12	
49	048	610		802	071		604	551		451	049		340	567	11	
50	29027	6618		27782	7079		26584	7559		25432	8058		24322	8575	10	
51	006	625		762	087		565	567		413	066		304	584	09	
52	28985	633		741	095		545	575		394	075		286	593	08	
53	964	640		721	103		526	584		375	083		267	602	07	
54	942	648		701	111		506	592		356	092		249	611	06	
55	28921	6656		27680	7119		26487	7600		25338	8100		24231	8619	05	
56	900	663		660	126		467	608		319	109		213	628	04	
57	879	671		640	134		448	616		300	117		195	637	03	
58	858	678		619	142		428	624		281	126		177	646	02	
59	837	686		599	150		409	633		263	134		159	655	01	
60	28816	6693		27579	7158		26389	7641		25244	8143		24141	8664	00	
	B A	59° 149°	A B	B A	58° 148°	A B	B A	57° 147°	A B	B A	56° 146°	A B	B A	55° 145°	A B	

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90° .
Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	35° 125°	B A	A B	36° 126°	B A	A B	37° 127°	B A	A B	38° 128°	B A	A B	39° 129°	B A	,
00	24141	8664		23078	9204		22054	9765		21066	10347		20113	10950		60
01	123	672		061	213		037	775		050	357		097	960		59
02	105	681		043	223		020	784		033	367		082	970		58
03	087	690		026	232		003	794		017	376		066	980		57
04	069	699		009	241		21987	803		001	386		050	991		56
05	24051	8708		22991	9250		21970	9813		20985	10396		20035	11001		55
06	033	717		974	259		953	822		969	406		019	011		54
07	015	726		957	269		937	832		953	416		004	022		53
08	23997	734		939	278		920	841		937	426		19988	032		52
09	979	743		922	287		903	851		921	436		973	042		51
10	23961	8752		22905	9296		21887	9861		20905	10446		19957	11052		50
11	943	761		888	306		870	870		889	456		942	063		49
12	925	770		870	315		853	880		872	466		926	073		48
13	907	779		853	324		837	889		856	476		911	083		47
14	889	788		836	333		820	899		840	486		895	094		46
15	23871	8797		819	9343		21803	9909		20824	10496		19880	11104		45
16	854	806		801	352		787	918		808	505		864	114		44
17	836	815		784	361		770	928		792	515		849	125		43
18	818	824		767	370		754	937		776	525		834	135		42
19	800	833		750	380		737	947		760	535		818	145		41
20	23782	8842		22732	9389		21720	9957		20744	10545		19803	11156		40
21	764	851		715	398		704	966		728	555		787	166		39
22	747	859		698	408		687	976		712	565		772	176		38
23	729	868		681	417		671	986		696	575		756	187		37
24	711	877		664	426		654	995		681	585		741	197		36
25	23693	8886		22647	9435		21638	10005		20665	10595		19726	11207		35
26	676	895		630	445		621	015		649	605		710	218		34
27	658	904		613	454		605	024		633	615		695	228		33
28	640	913		595	463		588	034		617	625		680	239		32
29	622	922		578	473		572	044		601	636		664	249		31
30	23605	8931		22561	9482		21555	10053		20585	10646		19649	11259		30
31	587	940		544	491		539	063		569	656		634	270		29
32	569	949		527	501		522	073		553	666		618	280		28
33	552	958		510	510		506	082		537	676		603	291		27
34	534	967		493	520		490	092		522	686		588	301		26
35	23516	8977		22476	9529		21473	10102		20506	10696		19572	11312		25
36	499	986		459	538		457	112		490	706		557	322		24
37	481	995		442	548		440	121		474	716		542	332		23
38	463	9004		425	557		424	131		458	726		527	343		22
39	446	013		408	566		408	141		442	736		511	353		21
40	23428	9022		22391	9576		21391	10151		20427	10746		19496	11364		20
41	410	031		374	585		375	160		411	756		481	374		19
42	393	040		357	595		358	170		395	767		466	385		18
43	375	049		340	604		342	180		379	777		450	395		17
44	358	058		323	614		326	190		364	787		435	406		16
45	23340	9067		22306	9623		21309	10199		20348	10797		19420	11416		15
46	323	076		289	632		293	209		332	807		405	427		14
47	305	085		272	642		277	219		316	817		390	437		13
48	288	094		256	651		261	229		301	827		375	448		12
49	270	104		239	661		244	239		285	838		359	458		11
50	23253	9113		22222	9670		21228	10248		20269	10848		19344	11469		10
51	235	122		205	680		212	258		254	858		329	479		09
52	218	131		188	689		195	268		238	868		314	490		08
53	200	140		171	699		179	278		222	878		299	501		07
54	183	149		154	708		163	288		207	888		284	511		06
55	23165	9158		22138	9718		21147	10298		20191	10899		19269	11522		05
56	148	168		121	727		131	307		175	909		254	532		04
57	130	177		104	737		114	317		160	919		238	543		03
58	113	186		087	746		098	327		144	929		223	553		02
59	096	195		070	756		082	337		128	940		208	564		01
60	23078	9204		22054	9765		21066	10347		20113	10950		19193	11575		00
	B A	54° 144°	A B	B A	53° 143°	A B	B A	52° 142°	A B	B A	51° 141°	A B	8 A	50° 140°	A B	

t AND K ARE BOTH GREATER OR BOTH LESS THAN 90°.
Z IS LESS THAN 90° ONLY WHEN K HAS THE SAME NAME AND IS GREATER THAN L.

	A B	40° 130°	B A	A B	41° 131°	B A	A B	42° 132°	B A	A B	43° 133°	B A	A B	44° 134°	B A	
	B A	49° 139°	A B	B A	48° 138°	A B	B A	47° 137°	A B	B A	46° 136°	A B	B A	45° 135°	A B	
00	19193	11575	18306	12222	17449	12893	16622	13587	15823	14307	60					
01	178	585	291	233	435	904	608	599	810	319	59					
02	163	596	277	244	421	915	595	611	797	331	58					
03	148	606	262	255	407	927	581	623	784	343	57					
04	133	617	248	266	393	938	568	634	771	355	56					
05	19118	11628	18233	12277	17379	12950	16554	13646	15758	14368	55					
06	103	638	219	288	365	961	541	658	745	380	54					
07	088	649	204	299	351	972	527	670	731	392	53					
08	073	660	190	310	337	984	514	682	718	404	52					
09	058	670	175	321	323	995	500	694	705	417	51					
10	19043	11681	18161	12332	17309	13007	16487	13705	15692	14429	50					
11	028	692	146	343	295	018	473	717	679	441	49					
12	013	702	132	354	281	030	460	729	666	453	48					
13	18998	713	118	365	267	041	446	741	653	466	47					
14	983	724	103	376	253	053	433	753	640	478	46					
15	18968	11734	18089	12387	17239	13064	16419	13765	15627	14490	45					
16	953	745	074	399	225	076	406	777	615	503	44					
17	939	756	060	410	212	087	392	789	602	515	43					
18	924	766	045	421	198	098	379	800	589	527	42					
19	909	777	031	432	184	110	366	812	576	540	41					
20	18894	11788	18017	12443	17170	13121	16352	13824	15563	14552	40					
21	879	799	002	454	156	133	339	836	550	564	39					
22	864	809	17988	465	142	145	326	848	537	577	38					
23	849	820	974	476	128	156	312	860	524	589	37					
24	834	831	959	487	115	168	299	872	511	601	36					
25	18820	11842	17945	12499	17101	13179	16285	13884	15498	14614	35					
26	805	852	931	510	087	191	272	896	485	626	34					
27	790	863	916	521	073	202	259	908	472	639	33					
28	775	874	902	532	059	214	245	920	460	651	32					
29	760	885	888	543	045	225	232	932	447	663	31					
30	18746	11895	17874	12554	17032	13237	16219	13944	15434	14676	30					
31	731	906	859	566	018	248	205	956	421	688	29					
32	716	917	845	577	004	260	192	968	408	701	28					
33	701	928	831	588	16990	272	179	980	395	713	27					
34	686	939	816	599	977	283	166	992	382	726	26					
35	18672	11949	17802	12610	16963	13295	16152	14004	15370	14738	25					
36	657	960	788	622	949	306	139	016	357	750	24					
37	642	971	774	633	935	318	126	028	344	763	23					
38	628	982	760	644	922	330	113	040	331	775	22					
39	613	993	745	655	908	341	099	052	318	788	21					
40	18598	12004	17731	12666	16894	13353	16086	14064	15306	14800	20					
41	583	015	717	678	880	365	073	076	293	813	19					
42	569	025	703	689	867	376	060	088	280	825	18					
43	554	036	689	700	853	388	046	100	267	838	17					
44	539	047	674	712	839	400	033	112	255	850	16					
45	18525	12058	17660	12723	16826	13411	16020	14124	15242	14863	15					
46	510	069	646	734	812	423	007	136	229	875	14					
47	495	080	632	745	798	435	15994	149	216	888	13					
48	481	091	618	757	785	446	980	161	204	900	12					
49	466	102	604	768	771	458	967	173	191	913	11					
50	18451	12113	17590	12779	16758	13470	15954	14185	15178	14926	10					
51	437	123	576	791	744	482	941	197	165	938	09					
52	422	134	561	802	730	493	928	209	153	951	08					
53	408	145	547	813	717	505	915	221	140	963	07					
54	393	156	533	825	703	517	902	234	127	976	06					
55	18378	12167	17519	12836	16690	13528	15888	14246	15115	14988	05					
56	364	178	505	847	676	540	875	258	102	15001	04					
57	349	189	491	859	662	552	862	270	089	014	03					
58	335	200	477	870	649	564	849	282	077	026	02					
59	320	211	463	881	635	575	836	294	064	039	01					
60	18306	12222	17449	12893	16622	13587	15823	14307	15051	15051	00					
	B A	49° 139°	A B	B A	48° 138°	A B	B A	47° 137°	A B	B A	46° 136°	A B	B A	45° 135°	A B	

Other Applications of the Table

Although designed primarily for sight reduction, the table may be used for the solution of other navigational problems.

Great Circle Distance and Initial Course

Where L_1 and LO_1 are the coordinates of the point of departure, L_2 and LO_2 those of the destination, and DLo the angular difference (difference in longitude) between the two, the following substitutions are made for the purpose of calculation: L_1 for L , L_2 for dec , DLo for t , C for Z .

As DLo may exceed 90°, the following rules must be observed:

1. L_1 and L_2 have the same name.
 - a. DLo and K are both greater or both less than 90° .
 - b. D is greater than 90° only if DLo and $(K \sim L_1)$ are both greater than 90° .
 - c. C is greater than 90° if K is less than L_1 ; C is less than 90° if K is greater than L_1 .
 2. L_1 and L_2 have contrary names.
 - a. DLo and K are both greater or both less than 90° .
 - b. When $(K \sim L_1)$ is more than 180° , subtract 180° before entering the table.
 - c. D is less than 90° when DLo and $(K \sim L_1)$ are both less than 90° .
 - d. C is greater than 90° when $(K \sim L_1)$ is less than 180° .

The problem of determining rhumb line course and distance between points on the great circle track is lengthy and complex using this table and is best done by calculator or by plotting. See Bowditch, Volume II, §1012 for a discussion of this application of Ageton's Table.

The Time a Body Is on the Prime Vertical

An observation of a body on the prime vertical ($Z = 90^\circ$) will yield an LOP parallel to the meridian and, hence, the navigator's longitude, provided the sight is accurate and the latitude certain. Also, as a body near the prime vertical is changing direction (azimuth) at its slowest rate, it is an excellent time to check the compass by means of a compass bearing on the body. The sun is used almost exclusively for this purpose.

It must be remembered that the body will cross the prime vertical only when its declination is less than the latitude and has the same name.

Example of the Time a Body Is on the PV

On 21 December 1980, your ZT 1600 DR will be L $28^\circ 40' 5$, Lo $21^\circ 44' W$. Using the table, determine the ZT when the sun will be on the afternoon prime vertical, to the nearest minute.

Note in the following calculation the 8 corresponding to the initial A for declination is subtracted from the 8 beneath it to give the final A from which t is derived.

ZT	1600					
ZD	+ 1					
UT	1700					
dec sun 17h		$23^\circ 26' S$	A 40046	-B	3738	
L		$28^\circ 40' S$	<u>-A 31902</u>			
			A 8144	B	25244	
t on PM PV		$37^\circ 33' W$	← ← ← ←	A	<u>21506</u>	
Lo		$21^\circ 44' W$				
GHA sun on PM PV		<u>$59^\circ 17'$</u>				
		$45^\circ 25'$	GHA sun for 15 h from Almanac			
		$13^\circ 52'$	55m 28s from Increments and Corrections in the Almanac			
Sun on PM PV at		UT 15-55-28	UT 1555			
		ZO + <u>1</u> (rev)				
Sun on PM PV		ZT 1455	to the nearest minute			

As it is not normally wanted, altitude is not shown, but it is the angle corresponding to A 8144 in the example and is $56^\circ 00'$.

The procedure is quite straightforward. For the approximate time a body will be on the PV—mid-morning or mid-afternoon is usually close enough—the declination of the body is taken from the Almanac and the coordinates of the vessel's position for that time from the plot. t is found from the table and the more exact time deduced by applying the longitude; the result should be accurate to the nearest minute. [Interpolation of declination using Bowditch's Table 25 will result in the same value for t as will calculation by Ageton's Table.]

The Identification of an Unknown Body

When the sky is largely obscured by clouds, a sight may be possible but the identity of the body in doubt. Identification is possible by means of this table though it is more easily accomplished by a Star Finder if available.

The declination and SHA of the body are found by means of the table and compared with the values given for the various stars and planets in the Almanac. The reliability of the identification depends largely on the accuracy of the bearing taken on the body.

The problem is essentially the reverse of that for sight reduction. Substitutions are: Z for t, Ho for dec. The values for dec and t are derived from the table. t is converted to GHA by means of the longitude and the required SHA from the relationship: SHA body = GHA body - GHA Aries.

Rules:

1. Give K the same name as L.
2. When Z is greater than 90° , K is also greater than 90° ,
3. Declination is always less than 90° and has the same name as L, except when Z and (K ~ L) are both greater than 90° ,
4. t is less than 90° when K is greater than L; greater than 90° when K is less than L.

Example of Unknown Body Identification

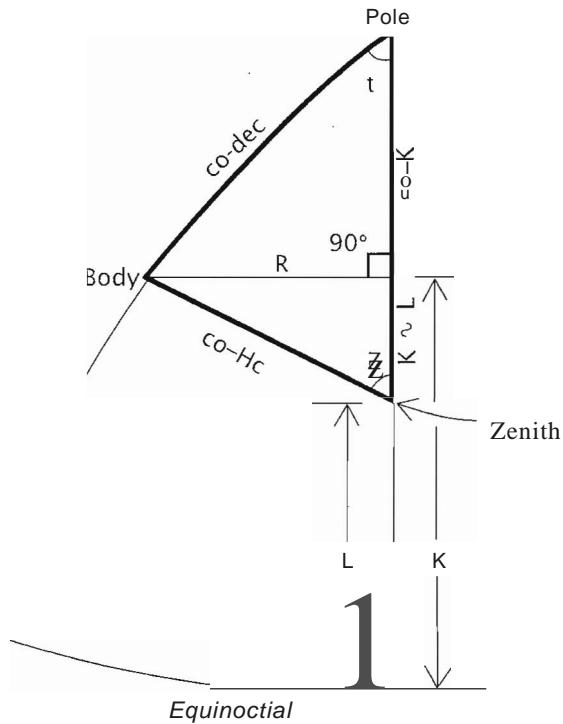
At ZT 05-55-00 on 29 October 1980, an observation of an unknown body results in a corrected sextant altitude, Ho, of $40^\circ 17'$. Its compass bearing is 100° true. The DR position is L $35^\circ 20'N$, Lo $60^\circ 18'W$.

Z	100°	A	665
Ho	$40^\circ 17'$	+B	11756
		A	18939
		A	12421
		-B	<u>18045</u>
		B	18045
		A	12421
K	$101^\circ 35' N$	←	↔ ↔ ↔ ←
L	$35^\circ 20' N$	↔	↔ ↔ ↔ ↔
K ~ L	$66^\circ 15' N$	→	→ → → → →
dec	$15^\circ 25' N$	↔	↔ ↔ ↔ ↔ ↔
		t	$51^\circ 12' E$
		La	$60^\circ 18' W$
		GHA body	$9^\circ 06' = 369^\circ 06'$
		GHA Aries	$186^\circ 38'$
		SHA body	$182^\circ 28'$

From the calculation: dec $15^\circ 25' N$, SHA $182^\circ 28'$
Denebola (Almanac): dec $14^\circ 41' N$, SHA $182^\circ 59'$

So, the unknown body is Denebola.

Appendix: The Ageton Formulas



**The Divided Navigational Triangle (heavy outline)
on the Plane of the Horizon (after Ageton)**

When using the table, the log esc of an angle is A; the log sec of an angle is B.

Sight Reduction-Standard Technique

$$\log \text{esc } R = \log \text{esc } t + \log \sec \text{dec}$$

$$\log \text{esc } K = \log \text{ese dec} - \log \sec R$$

$$\log \text{esc He} = \log \text{see } R + \log \sec (K \sim L)$$

$$\log \text{esc } Z = \log \text{esc } R - \log \sec \text{He}$$

Sight Reduction-Sadler's Technique

$$\log \text{ese } hI = \log \text{esc dec} + \log \text{ese } L$$

$$\log \text{esc } hz = \log \sec t + \log \text{see dec} + \log \sec L - \log \sec ([hI + Ho]/2)$$

$$He = hI + hz$$

$$\log \text{esc } Z = \log \text{esc } t + \log \sec \text{dec} - \log \sec \text{He}$$

Time a Body Is on the Prime Vertical

$Z = 90^\circ$ and $K = L$ or $(K - L) = 0$

$$\begin{aligned}\log \csc H_c &= \log \csc dec - \log \csc L \\ \log \csc t &= \log \sec H_c - \log \sec dec\end{aligned}$$

Great Circle

Substitute point of departure for Zenith and destination for Body, whence
 $L_1 = L$, $L_2 = dec$, $DLo = t$, $C = Z$, $D = co - H_c$.

$$\begin{aligned}\log \csc R &= \log \csc DLo + \log \sec L_2 \\ \log \csc K &= \log \csc L_2 - \log \sec R \\ \log \sec D &= \log \sec R + \log \sec (K - L_1) \\ \log \csc C &= \log \csc R - \log \csc D\end{aligned}$$

Unknown Body Identification

Z , H_o and L are known. To find t and dec :

$$\begin{aligned}\log \csc R &= \log \csc Z + \log \sec H_o \\ \log \csc K &= \log \csc H_o - \log \sec R \\ \log \csc dec &= \log \sec R + \log \sec (K - L) \\ \log \csc t &= \log \csc R - \log \sec dec\end{aligned}$$

Angle R is never used, nor is it ever necessary to find its value from the table; consequently, it is not included in the format of the sight reduction calculation, nor is it shown in any of the examples.

About the Author

Allan E. Bayless, a neurosurgeon, has been an amateur sailor since he taught himself to sail in Newport Bay, California, at the age of sixteen. He has owned several sailboats and has cruised Puget Sound and the coastal waters of Southern California and New England.

Dr. Bayless is a Life Member of the United States Power Squadrons and has been active in the National Educational Department for many years. He has chaired various committees, including the Navigation Committee (1970–1975), and was the Director of Education in 1980. He was a member of the Council (Marine) of the Institute of Navigation for three years and has been on the board of directors of the Navigation Foundation since its inception.

He is the author of numerous articles on nautical subjects and of book reviews for *The Ensign* and other periodicals. He is cited in the prefaces of the 1975 and 1981 editions of *Bowditch*, Volume II; an extract from the Compact Table appears on page 517 and a brief description on page 518 of the latter edition.