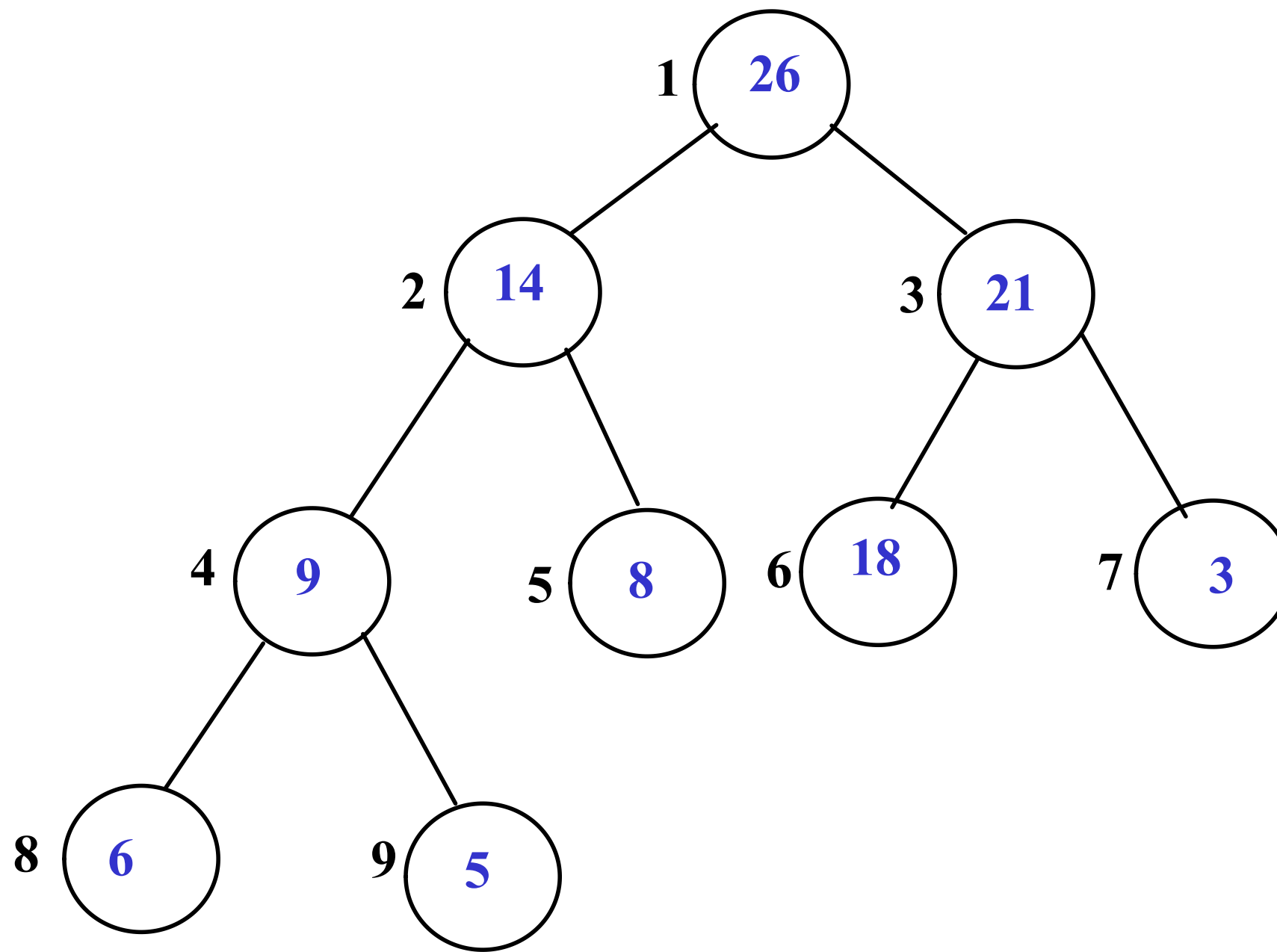


Size = 9  
 $9/2 = 4$

**Heapify**

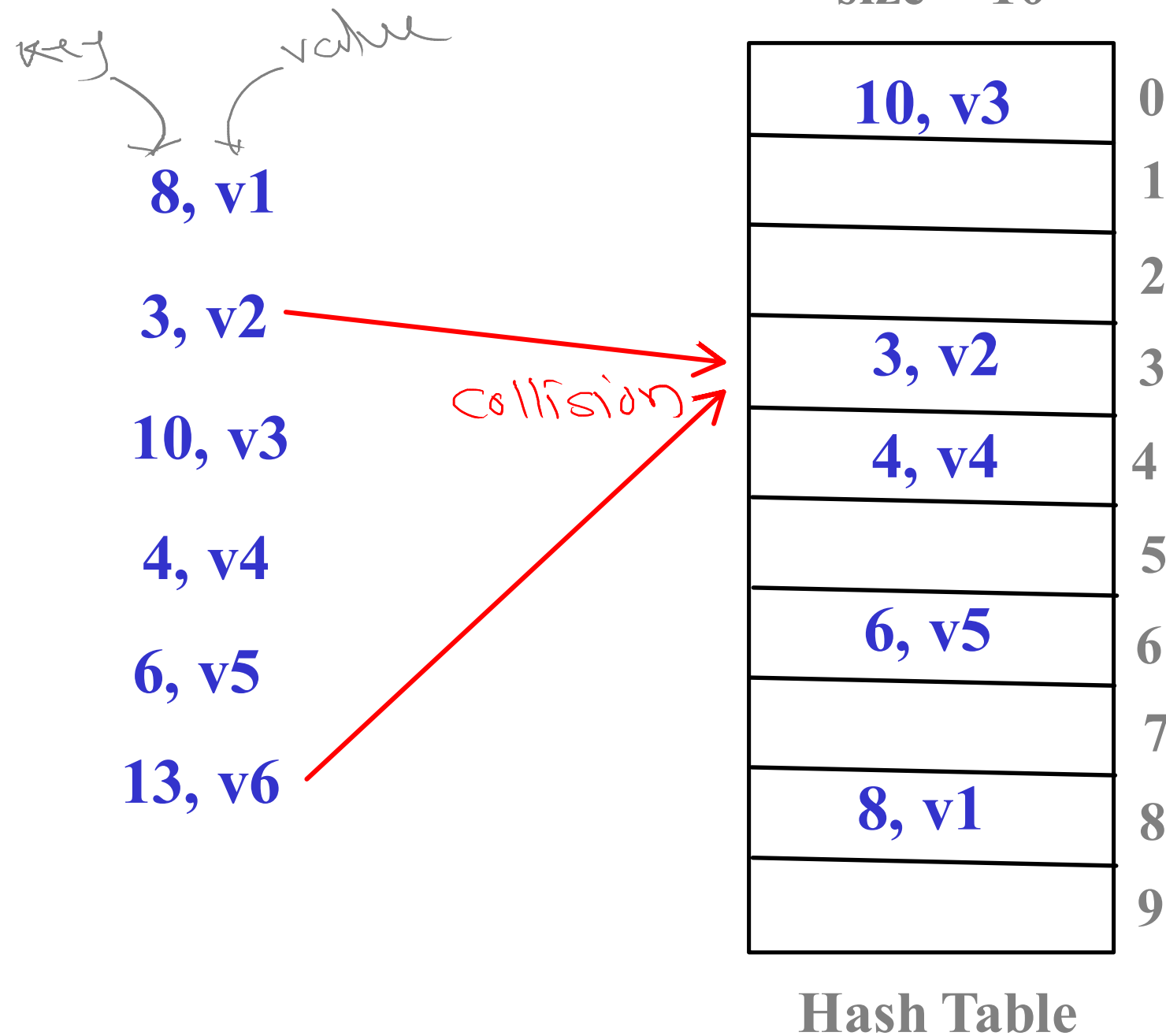
**Time :  $O(n)$**



6	26	21	14	8	18	3	9	5
1	2	3	4	5	6	7	8	9

# Hashing

$$h(k) = k \% \text{size}$$



$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3$$

insert/add :  $O(1)$

slot =  $k \% \text{size}$ ;

arr[slot] = entry;

search :  $O(1)$

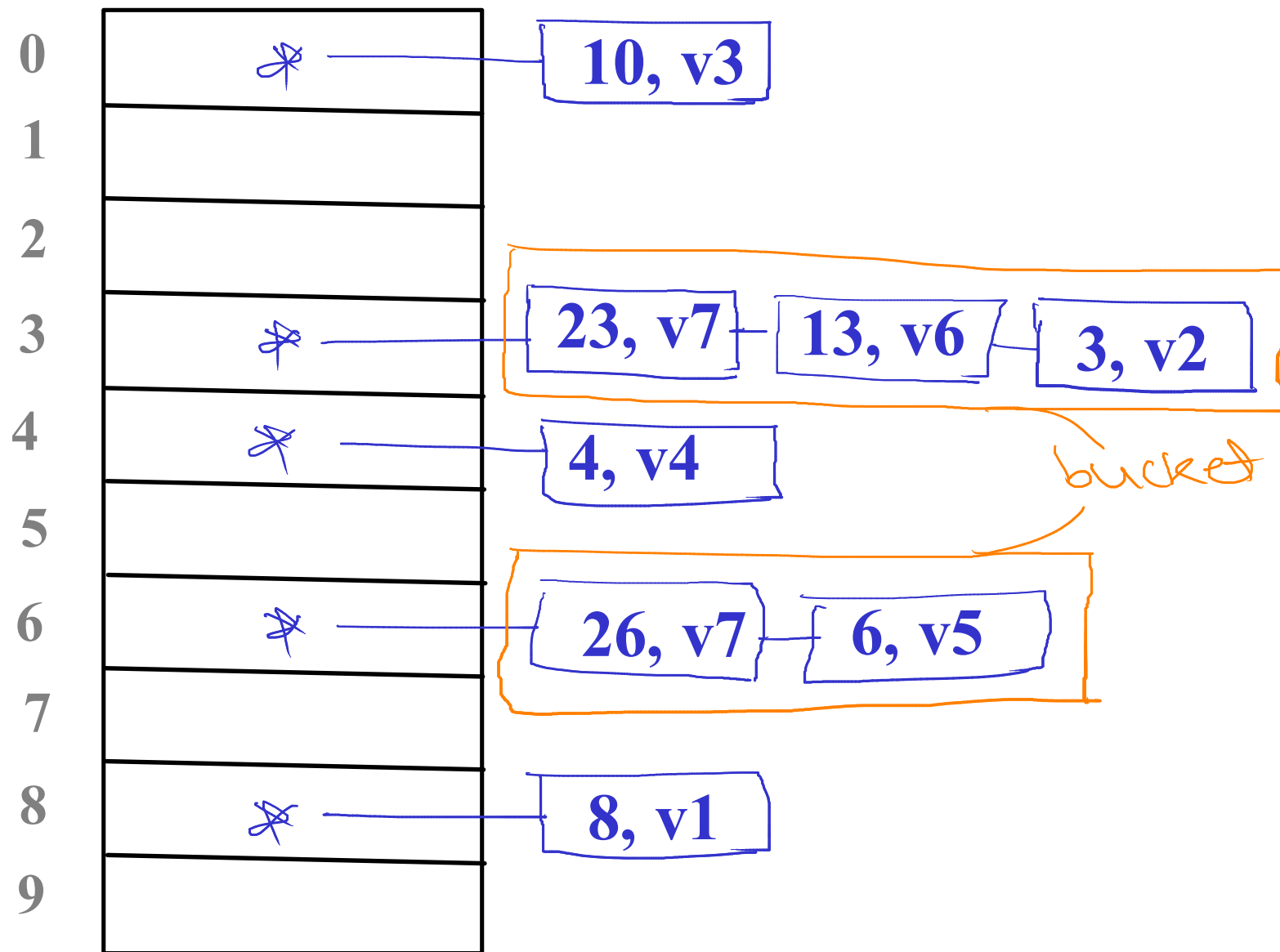
slot =  $k \% \text{size}$

return arr[slot].value

# Closed Addressing/ Seperate Chaining / Chaining

size = 10

8, v1  
3, v2  
10, v3  
4, v4  
6, v5  
13, v6  
23, v7  
26, v7



Hash Table

$$h(k) = k \% \text{size}$$

$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3 \text{ (C)}$$

$$h(23) = 23 \% 10 = 3 \text{ (C)}$$

$$h(26) = 26 \% 10 = 6 \text{ (C)}$$

- extra space is needed
- data(key-value) is kept outside the table
- Worst case :  $O(n)$ 
  - ↳ if all keys yield same slot

# Open Addressing - Linear Probing

size = 10

8, v1		0
3, v2		1
10, v3		2
4, v4		3
6, v5		4
13, v6		5
		6
		7
		8
		9

Hash Table

$$h(k) = \text{key} \% \text{size}$$

$$h(k, i) = [ \underline{h(k)} + \underline{f(i)} ] \% \text{size}$$

$$f(i) = \underline{i}$$

where  $i = 1, 2, 3, \dots$

↳ probe number

$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3 \text{ (collision)}$$

$$h(13, 1) = [3 + 1] \% 10 = 4 \text{ (1st probe) (collision)}$$

$$h(13, 2) = [3 + 2] \% 10 = 5 \text{ (2nd probe)}$$

Probing — find new slot for key when collision is occurred

Primary clustering — take long runs of filled slots "near" key position

# Open Addressing - Quadratic Probing

size = 10

	10, v3	0
		1
		2
8, v1		
3, v2		
10, v3	3, v2	3
	4, v4	4
4, v4		5
6, v5	6, v5	6
	13, v6	7
13, v6	8, v1	8
		9

Hash Table

collision →

→

→

$$h(k) = \text{key \% size}$$

$$h(k, i) = [ h(k) + f(i) ] \% \text{ size}$$

$$f(i) = i^2$$

where  $i = 1, 2, 3, \dots$

↳ probe number

$$h(8) = 8 \% 10 = 8$$

$$h(8) = 8 \% 10 = 8$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3$$

$$h(13, 1) = [ 3 + 1 ] \% 10 = 4 \text{ (1st probe) (collision)}$$

$$h(13, 2) = [ 3 + 4 ] \% 10 = 7 \text{ (2nd probe)}$$

Secondary clustering — take long runs of filled slots "away" key position

# Open Addressing - Quadratic Probing

size = 10

23, v7

33, v8

10, v3	0
	1
23, v7	2
3, v2	3
4, v4	4
	5
6, v5	6
13, v6	7
8, v1	8
33, v8	9

Hash Table

$$h(k) = \text{key \% size}$$

$$h(k, i) = [ h(k) + f(i) ] \% \text{ size}$$

$$f(i) = i^2$$

where  $i = 1, 2, 3, \dots$

$$h(23) = 23 \% 10 = 3 \text{ (C)}$$

$$h(23, 1) = [3 + 1] \% 10 = 4 \text{ (C) (1st)}$$

$$h(23, 2) = [3 + 4] \% 10 = 7 \text{ (C) (2nd)}$$

$$h(23, 3) = [3 + 9] \% 10 = 2 \text{ (5th)}$$

$$h(33) = 33 \% 10 = 3$$

$$h(33, 1) = [3 + 1] \% 10 = 4$$

$$h(33, 2) = [3 + 4] \% 10 = 7$$

$$h(33, 3) = [3 + 9] \% 10 = 2$$

$$h(33, 4) = [3 + 16] \% 10 = 9$$

# Hashing - Double Hashing

size = 11

8, v1

3, v2

10, v3

25, v6

	0
	1
	2
3, v2	3
	4
	5
25, v6	6
	7
8, v1	8
	9
10, v3	10

Hash Table

$$h1(k) = \text{key} \% \text{size}$$

$$h2(k) = 7 - (\text{key} \% 7)$$

$$h(k, i) = [h1(k) + i * h2(k)] \% \text{size}$$

$$h1(8) = 8 \% 11 = 8$$

$$h1(3) = 3 \% 11 = 3$$

$$h1(10) = 10 \% 11 = 10$$

$$h1(25) = 25 \% 11 = 3 \text{ (collision)}$$

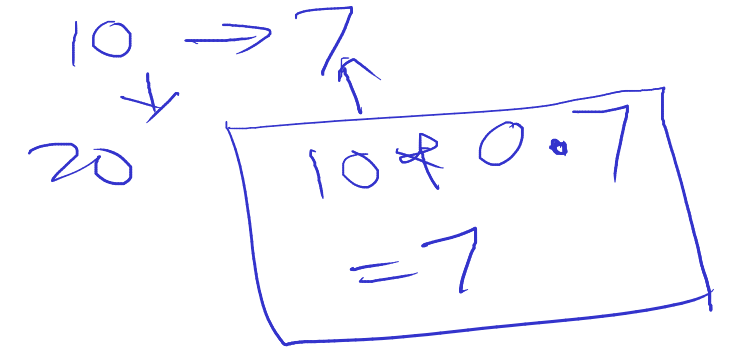
$$h2(25) = 7 - 4 = 3$$

$$h(25, 1) = [3 + 1 * 3] \% 11 \\ = 6 \text{ (1st probe)}$$

## Rehashing

$$\text{Load Factor} = \frac{n}{N} \quad (0 \text{ to } 1)$$

$(\lambda)$



**n - Number of elements (key value pairs) in hash table**

**N - Number of slots in hash table**

<b>if <math>n &lt; N</math></b>	<b>Load factor <math>&lt; 1</math></b>	<b>- free slots are available</b>
<b>if <math>n = N</math></b>	<b>Load factor <math>= 1</math></b>	<b>- no free slots</b>
<b>if <math>n &gt; N</math></b>	<b>Load factor <math>&gt; 1</math></b>	<b>- can not insert at all</b>

**- Rehashing is make the hash table size twice of existing size if hash table is 70 or 75 % full**

**- In rehashing existing key value pairs are again mapped according to new hash table size**