

2011

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**Group - A****( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

 $10 \times 1 = 10$ 

- i) The degree and order of the differential equation

$$y = \frac{x d^2 y}{dx^2} + r \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \text{ are}$$

- |         |                   |
|---------|-------------------|
| a) 2, 2 | b) 2, 1           |
| c) 3, 2 | d) none of these. |

- ii) The geometric series  $1 + r + r^2 + \dots$  is convergent if

- |                 |                   |
|-----------------|-------------------|
| a) $-1 < r < 1$ | b) $r > 1$        |
| c) $r = 1$      | d) none of these. |

- iii) The series  $1 + 1 + 1 + \dots$  is

- |                |                   |
|----------------|-------------------|
| a) convergent  | b) divergent      |
| c) oscillatory | d) none of these. |



- iv) An absolutely convergent series is
  - a) necessarily convergent
  - b) not necessarily convergent
  - c) conditionally convergent
  - d) none of these.
- v) Leibnitz's test is applied to
  - a) a constant series
  - b) an alternating series
  - c) series of positive terms only
  - d) none of these.
- vi) If  $W_1$  and  $W_2$  be two subspaces of a vector space  $V(F)$  then  $W_1 \cap W_2$  is
  - a) necessarily a subspace
  - b) not a subspace
  - c) is a subspace only when one is contained within another
  - d) none of these.
- vii) In the vector space  $R^3$  over the field  $R$  the vectors  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$  are
  - a) linearly independent
  - b) linearly dependent
  - c) none of these.



viii) Integrating factor of differential equation

$$x \log x \frac{dy}{dx} + y = 2 \log x \text{ is}$$

- a)  $\log x$                       b)  $\log (\log x)$
- c)  $e^x$                           d) none of these.

ix) The upper bound of the sequence  $\{(-3)^n\}$  is

- a) 4                      b) 0  
c) -3                     d) none of these.

x) If  $T$  is a linear mapping for  $V$  to  $V$  and  $\alpha, \beta \in V$  and  $a, b$  are scalars, then

- a)  $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$   
 b)  $T(a\alpha + b\beta) = aT(\alpha) - bT(\beta)$   
 c)  $T(a\alpha + b\beta) = aT(\alpha)$   
 d)  $T(a\alpha + b\beta) = bT(\beta)$

xi) The roots of the auxiliary equation of the given differential equation  $\frac{d^2 y}{dx^2} - \frac{4dy}{dx} + 4y = 0$  are

- a) 2, 4                      b) 2, 2  
c) 1, 1                      d) none of these.

xii) Let  $T: R^2 \rightarrow R^2$  be a linear transformation defined by  $T(x, y) = (2x - y, x + y)$ . Then kernel of  $T$  is

- a)  $\{(1, 2)\}$                       b)  $\{(0, 0)\}$   
c)  $\{(1, 2), (1, -1)\}$               d) none of these.



xiii) If  $T : V_2 \rightarrow V_3$  be defined by

$T(v_2) = \{ (x, 0, 0) : x \text{ is a real number} \}$  then rank of  $T$  is

- a) 3                      b) 2  
c) 1                      d) 0.

xiv) If  $T$  is a linear transformation from vector space  $V$  into  $W$ , then

- a)  $\text{rank}(T) + \text{nullity}(T) = \dim(V)$   
b)  $\text{rank}(T) + \text{nullity}(T) = \dim(W)$   
c)  $\text{rank}(T) - \text{nullity}(T) = \dim(V)$   
d) none of these.

### Group – B

**( Short Answer Type Questions )**

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{n!}{n}$

3. Show that the mapping defined by  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$  is linear. Find the value of  $T(1, 2)$ .

4. Solve :

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x.$$

5. The linear transformation  $T: R^3 \rightarrow R^3$  is defined by  $T(x, y, z) = (x - y, x + 2y, y + 3z)$ . Show that  $T$  is invertible and determine  $T^{-1}$ .



6. Prove that the set of vectors  $\{ (1, -2, 3), (2, 3, 1), (-1, 3, 2) \}$  is linearly independent. Also verify whether this set forms a basis of  $V_3$  or not.
7. Let  $S = \{ (x, y, z) : (x, y, z) \in R^3, x + y + z = 0 \}$ . Prove that  $S$  is a subspace. Find the dimension of  $S$ .

**Group – C****( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) Find the order and degree of the differential equation

$$\frac{d^2 y}{dx^2} = \left( y + \left( \frac{dy}{dx} \right)^2 \right)^{1/4}$$

- b) Verify whether the differential equation  $e^y dx + \left( x e^y + 2y \right) dy = 0$  is exact. If so, then solve it.

- c) Solve the differential equation  $x \frac{dy}{dx} - 2y = x y^4$ .

3 + 5 + 7

9. a) Solve  $y = px + \frac{a}{p}$  and also obtain the singular solution.

- b) Solve  $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$ .

- c) Test the convergence of the series  $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$ .

5 + 6 + 4



10. a) State D' Alembert's ratio test. Test the convergence of

the series  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ .

- b) Prove that the series

$\left(1 + \frac{1}{2}\right) - \left(1 + \frac{1}{4}\right) + \left(1 + \frac{1}{8}\right) - \left(1 + \frac{1}{16}\right) + \dots$  is an oscillating series.

- c) Verify for the following example that the converse of the statement "If  $\sum a_n$  be a convergent series then

$\lim_{n \rightarrow \infty} a_n = 0$ " is not true, where  $\sum a_n = \sum_{n=1}^{\infty} \frac{1}{n}$ .

5 + 6 + 4

11. a) Define basis of a vector space.

- b) Show that the vectors  $\alpha_1 = (1, 0, -1)$ ,  $\alpha_2 = (1, 2, 1)$  and  $\alpha_3 = (0, -3, 2)$  form a basis for  $R^3$ .

- c) Determine the value of  $k$  so that the set

$S = \{ (1, 2, 1), (k, 3, 1), (2, k, 0) \}$  is linearly dependent in  $R^3$ .

3 + 6 + 6

12. a) Let  $T: R^3 \rightarrow R^2$  be a linear transformation defined by  $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$  for  $(x, y, z) \in R^3$ . Obtain a matrix representation for the linear transformation  $T$ .



- b) Let  $V =$  set of all  $2 \times 2$  matrices and  $T: V \rightarrow V$  be defined by  $T(X) = AX - XA$  where  $X \in V$  and  $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ .

Find the basis of  $\ker(T)$  and the nullity.

- c) Let  $T: R^3 \rightarrow R^2$  be a linear mapping defined by  $T(x, y, z) = (x + 2y, z)$ . Verify that

dimension (kernel ( $T$ )) + dimension (Image ( $T$ ))

$$= \dim(R^3).$$

$$5 + 6 + 4$$

13. a) Define a subspace of a vector space.

- b) State a necessary and sufficient condition for  $W \subseteq V$  to be a subspace of  $V(F)$ .

- c) Test whether the series

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \dots (0 < x < 1)$$

is convergent or not.

$$3 + 3 + 9$$

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