# 2012
# MATHEMATICS

*Time Allotted* : 3 Hours *Full Marks* : 7

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own word as far as practicable.*

## GROUP – A
## ( Multiple Choice Type Questions )

1. Choose the correct alternative for any *ten* the following: $10 \times 1 =$

i) Integrating factor of the differential equation

$x\,(1-x^2)\,dy + (2x^2y - y - ax^3)\,dx = 0$ is

a) $x/\sqrt{1-x^2}$ b) $x/\sqrt{x(x^2-1)}$

c) $x/\sqrt{x^2-1}$ d) $x^2/\sqrt{1-x^2}$.

ii) The order and degree of the differential equation

$\sqrt{d^2y/dx^2} + dy/dx = y$ are

a) 2, 1 b) 1, 2

c) 2, 2 d) 2, 3.

iii) $\dfrac{1}{(D-2)(D-3)} e^{2x}$ is

a) $-e^{2x}$ b) $xe^{2x}$

c) $-xe^{2x}$ d) $-xe^{3x}$.

iv) If for a sequence $\{U_n\}$, $\lim_{h \to \infty} U_n = 0$, then

a) $\{U_n\}$ is converget

b) $\{U_n\}$ is divergent

c) $\{U_n\}$ is convergent to 0

d) none of these.

v) The infinite series $\sum \dfrac{n}{n+1}$ is

a) divergent b) convergent

c) oscillatory d) none of these.

vi) The value of $a$ for which { (1,2,3), (0,–1,9), (4,0,$a$) } is linearly dependent is

a) – 20 b) – 10

c) – 5 d) None of these.

vii) If the third order square matrix $A$ is diagonalizable, then the number of independent eigenvectors of $A$ is

a) two b) three

c) one d) none of these.

viii) If $S$ and $T$ be two subspaces of a vector space $V$, then which of the following is also a subspace of $V$?

a) $S \cup T$ b) $S - T$

c) $T - S$ d) $S \cap T$.

ix) The dimension of the subspace $\{(x, 0, y, 0) \mid x, y \in R\}$ is

a) 1 b) 2

c) 3 d) 4.

x) Let $V$ and $W$ be two vector spaces over $R$ and $T : V \to W$ is a linear mapping. Then $Im\ T$ is a sub-space of

a) $V$ b) $W$

c) $V \cup W$ d) $V \cap W$.

xi) The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

a) $p \geq 1$ b) $p > 1$

c) $p \leq 1$ d) none of these.

xii) The lower bound of the sequence $\{(-1)^{n-1}/n!\}$ is

a) $-1/2$ b) $1/2$

c) 0 d) none of these.

xiii) Eliminating $A$ and $B$ from $y = A \cos x + B \sin y$, the differential equation is

a) $\frac{d^2 y}{dx^2} = 0$ b) $\frac{d^2 y}{dx^2} - y = 0$

c) $\frac{d^2 y}{dx^2} + y = 0$ d) $\frac{d^2 y}{dx^2} = 1$.

xiv) The particular integral of $(D^2 + 1)\, y = \sin x$ is

a) $x \sin x$ b) $x \cos x$

c) $x \tan x$ d) $-\frac{x}{2} \cos x$.

## GROUP – B

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve : $(px - y)(py + x) = a^2 p$

by using the substitution $x^2 = u,\ y^2 = v$ where $p = \frac{dy}{dx}$

3. Examine the convergence of the sequence

$\left\{\left(1 + \frac{2}{n}\right)^n\right\}$.

4. Examine the convergence of the series

$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots\ldots\ldots\ldots$

5. Show that $W = \{(x_1, x_2, x_3, x_4) \in R^4 \mid x_1 - x_2 + x_3 = x_4\}$ is a subspace of $R^4$.

6. Find the representative matrix of the linear transformation $T : R^3 \to R^3$ defined by

$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$

7. Find the basis of

$S = \{(x, y, z) \in R^3 \mid x + 2y + z = 0,\ 2x + y + 3z = 0\}$

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve : $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\,\dfrac{dy}{dx} = 12\,\dfrac{\log x}{x^2}$

   b) Obtain the general solution and singular solution of the equation $y = px + \sqrt{a^2\,p^2 + b^2}$

   c) Solve : $3\dfrac{dy}{dx} + \dfrac{2y}{x+1} = \dfrac{x^3}{y^2}$ 5 + 6 + 4

9. a) State Leibnitz theorem for Alternating series and test the convergence of the series

   $1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots\ldots\ldots\ldots$

   b) Test the convergence of the following series

   $\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{2^3} + \dfrac{1}{3^3} + \ldots\ldots\ldots\ldots\ldots$

   c) Show that the sequence $\left\{2 + \dfrac{(-1)^n}{n}\right\}$ is convergent.

   6 + 5 + 4

10. a) Solve $\dfrac{d^2y}{dx^2} + 4\,\dfrac{dy}{dx} + 29\,y = 0$

    when $x = 0$ , $y = 0$ , $\dfrac{dy}{dx} = 15$

    b) Show that the sequence

    $\sqrt{2}$ , $\sqrt{2 + \sqrt{2}}$, $\sqrt{2 + \sqrt{2 + \sqrt{2}}}$, ………. converges to 2.

    c) Define basis and dimension of a vector space.

    6 + 6 + 3

11. a) Prove that the vectors $(x_1, y_1)$ and $(x_2, y_2)$ are linearly dependent, if and only if $x_1 y_2 - x_2 y_1 = 0$.

b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis of $R^3$. Express $(1, 0, 0)$ as a linear combination of $\alpha_1$, $\alpha_2$ and $\alpha_3$.

c) If $\alpha_1, \alpha_2, \alpha_3$ form a basis of a vector space $V$, then prove that $\alpha_1 + \alpha_3$, $2\alpha_1 + 3\alpha_2 + 4\alpha_3$ and $\alpha_1 + 2\alpha_2 + 3\alpha_3$ also form a basis of the vector space $V$. 4 + 6 + 5

12. a) Let $T$ be defined by $T(x, y) = (x', y')$ where

$x' = x \cos\theta - y \sin\theta$, $y' = x \sin\theta + y \cos\theta$

Prove that $T$ is a linear transformation.

b) The linear transformation $T : R^3 \to R^3$ transforms the basis vectors $(1, 2, 1)$, $(2, 1, 0)$ & $(1, -1, -2)$ to the basis vectors $(1, 0, 0)$, $(0, 1, 0)$ & $(0, 0, 1)$ respectively. Find $T$. Hence find $T(3, -3, 3)$.

c) Find the Kernel, Image, Nullity and Rank of

$T : R^3 \to R^2$ where

$T(1, 0, 0) = (2, 1)$

$T(0, 1, 0) = (0, 1)$

$T(0, 0, 1) = (1, 1)$ 4 + 7 + 4

13. a) Prove that a subset $S$ of a vector space $V$ over $R$ is a subspace if and only if $\alpha x + \beta y \in S$ for all $\alpha, \beta \in R$ and $x, y \in S$.

b) Show that the family $M_2$ of all real square matrices of order 2 forms a vector space over reals, and find a basis for $M_2$.

c) Let $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a + b = 0 \text{ and } a, b, c, d \in R \right\}$

Prove that $S$ is a subspace of $M_2$. 5 + 6 + 4