ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS

SEMESTER - 2

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $\alpha = (1, 0, 3)$ and $\beta = (-1, 2, 5)$, then $\alpha + 3\beta$ is equal to

a) $(-2, 6, 18)$ b) $(2, -6, -18)$

c) $(2, -6, 18)$ d) $(-1, -3, 5)$. ☐

ii) The basis of a vector space contains

a) linearly independent set of vectors

b) linearly dependent set of vectors

c) scalars only

d) none of these. ☐

iii) Integrating factor of $x\,dx = -y\,dy$ is

a) $1/(xy)$ b) $1/(x^2 + y^2)$

c) $1/y^2$ d) none of these. ☐

iv) The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

a) $p = 1$ b) $p > 1$

c) $p < 1$ d) none of these. ☐

v) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2} \right)^{2/3} - 3\frac{dy}{dx} = 4$ are

a) 2, 2 b) $2, \frac{2}{3}$

c) 2, 1 d) 2, 3. ☐

vi) If the three vectors (5, 2, 3), (7, 3, x) and (9, 4, 5) are linearly dependent, then x is

a) 1 b) 2

c) 3 d) 4. ☐

vii) If $\lim_{n \to \infty} a_n = 0$, then the series $\Sigma (-1)^n a_n$ is

a) convergent b) divergent

c) oscillatory d) none of these. ☐

viii) The family of curves $y = e^x (A \cos x + B \sin x)$ is represented by the differential equation

a) $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - y$ b) $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$

c) $\frac{d^2y}{dx^2} = \frac{dy}{dx} - 2y$ d) $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} + y.$ ☐

ix) The sequence $1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}, \ldots\ldots$ converges to

a) ∞ b) 0

c) 1 d) $\frac{1}{2}$.

x) The four vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0,) and (0, 1, a, b) are linearly independent if

a) $a \neq 0, b \neq 2$ b) $a \neq 2, b \neq 0$

c) $a \neq 0, b \neq -2$ d) $a \neq -2, b \neq 0$.

xi) The solution of $\log\left(\frac{dy}{dx}\right) = ax + by$ is

a) $b\,e^{-ax} + a\,e^{-by} + k = 0$ b) $b\,e^{ax} + a\,e^{-by} + k = 0$

c) $b\,e^{-ax} + a\,e^{by} + k = 0$ d) $b\,e^{ax} + a\,e^{by} + k = 0$.

xii) $\lim_{n \to \infty} \frac{x^n}{\lfloor n}$ is equal to

a) 0 b) 1

c) -1 d) none of these.

xiii) The lower bound of the sequence $\left\{\frac{(-1)^{n-1}}{n!}\right\}$) is

a) $-\frac{1}{2}$ b) $\frac{1}{2}$

c) 1 d) 0.

xiv) The value of $\lim_{n \to \infty} \log(1/n)$ is equal to

a) 0 b) 1

c) $-\infty$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the equation of curve whose slope at any point (x, y) on it is $2y$ and which passes through the point (3, 1).

3. Test for the convergence of the series :

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots, \quad x > 0.$$

4. Examine whether the vectors (1, 2, 3, 0), (2, 1, 0, 3), (1, 1, 1, 1) and (2, 3, 4, 1) are linearly dependent or not. If yes, find among them which are independent.

5. Solve any three :

a) $x \frac{dy}{dx} + y = y^2 \log x$

b) $\cos^2 x \frac{dy}{dx} + y = \tan x$

c) $y = px - \frac{a}{p}$ where $p = \frac{dy}{dx}$

d) $\left(D^2 - 2D + 1 \right) y = x e^x$ where $D = \frac{d}{dx}$.

6. Define the limit of a sequence. Find

$$\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{2}{n^2} + \ldots + \frac{n}{n^2} \right].$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Define basis of a vector space V. Show that $\alpha_1 = (1, 0, 0)$, $\alpha_2 = (0, 1, 0)$ and $\alpha_3 = (0, 0, 1)$ form a basis of the vector space V_3. 5

b) If $\{ \alpha, \beta, \gamma \}$ be a basis of real vector space V and $c \neq 0$ be a real number, examine whether $\{ \alpha + c\beta, \beta + c\gamma, \gamma + c\alpha \}$ is a basis of V or not. 5

c) Find the value of k for which the vectors (1, 2, 1), (k, 1, 1) and (1, 1, 2) in R^3 are linearly dependent. 5

8. Test the convergence of any *three* of the following series : $3 \times 5 = 15$

a) $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^2}{10} + \ldots\ldots$

b) $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots\ldots$

c) $\sum_{n=1}^{\alpha} \left(1 + \frac{1}{\sqrt{n}}\right)^{-\sqrt{n}}$

d) $\sum_{n=1}^{\alpha} \left(\frac{\cos nx}{n^2}\right)$.

9. Solve any *three* of the following : $3 \times 5 = 15$

a) $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y (\log y)^2}{x^2}$

b) $y = 2px - p^2$ where $p = \frac{dy}{dx}$

c) $e^x \sin y \, dx + (e^x + 1) \cos y \, dy = 0$

d) $(D^2 - 2D) y = e^x \sin x$.

10. a) Prove that $s = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)$ is a basis of R^3.

b) Show that $w = \{(x, y, z) \leftarrow R^3 / x + y + z = 0\}$ is a sub-space of R^3 and find a basis of w.

c) Determine K so that the set S is linearly dependent in R^3

$S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$. $5 + 5 + 5$

11. a) Define the linear sum of two sets of vectors S and T.

b) If S and T are two sub-spaces of a vector space V, obtain a relation between rank (S), rank (T) and rank (V) .

c) Let $T : R^2 \to R^2$ be a linear transformation such that $T(1, 1) = (2, -3)$ and $T(1, -1) = (4, 7)$. Find the matrix of T. $3 + 6 + 6$

12. a) Using D' Alembert's ratio test, show that the following series is convergent :

$$x^2 + (2^2 / 3.4)\, x^4 + (2^2.4^2 / 3.4.5.6)\, x^6 + \ldots\ldots \infty$$

b) Prove that every absolutely convergent series is convergent.

c) Show that the following series is convergent :

$$u_n = \sqrt{n^3 + 1} - \sqrt{n^3} \text{ in } n \in [1, \infty].$$

END