1 Introduction to Function Notation

Function notation is a way of representing functions in a mathematical form. A function, in general, is a relation that assigns a single output for each input. It is usually written as $f(x)$, where:

- f denotes the function's name.
- x denotes the input, also called the independent variable.
- $f(x)$ denotes the output, also called the dependent variable.

Example 1: If $f(x) = 2x + 3$, this means for each input value of x, the function f gives an output by multiplying x by 2 and then adding 3.
Example 2: Let $g(x) = x^2 - 4x + 7$. Here, for each input x, the function g outputs the square of x minus four times x plus 7.

Practice Problems

1. Let $f(x) = 3x - 5$. Find $f(2)$.

2. Let $g(x) = x^2 + 1$. Find $g(-3)$.

3. Let $h(x) = \frac{x}{x-1}$. Find $h(3)$.

4. If $j(x) = 4x^2 - 2x + 1$, find $j(0)$.

5. Let $k(x) = \sqrt{x + 4}$. Find $k(5)$.

6. If $f(x) = x^3$, what is $f(1)$?

7. Let $p(x) = 2 - x^2$. Find $p(-2)$.

8. If $q(x) = 5x + 4$, calculate $q(0)$.

9. Let $r(x) = 7 - 2x$. What is $r(4)$?

10. If $s(x) = \frac{1}{x}$, what is $s(2)$?

2 Function Operations

Function operations include adding, subtracting, multiplying, and dividing functions. If $f(x)$ and $g(x)$ are two functions, the operations can be performed as follows:

2.1 Addition and Subtraction

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$

Example: Let $f(x) = 2x + 3$ and $g(x) = x^2$.

(f + g)(x) =

(f - g)(x) =

2.2 Multiplication

- $(f \cdot g)(x) = f(x) \cdot g(x)$

Example: With $f(x) = 2x + 3$ and $g(x) = x^2$.

$(f \cdot g)(x) =$

Practice Problems

1. Let $f(x) = x + 2$ and $g(x) = 3x - 1$. Find $(f + g)(x)$.

2. Let $f(x) = x^2$ and $g(x) = x + 4$. Find $(f - g)(x)$.

3. If $f(x) = 2x$ and $g(x) = x^2 + 1$, what is $(f \cdot g)(x)$?

4. Let $f(x) = x - 1$ and $g(x) = 4x$. Find $(f \cdot g)(x)$.

5. Let $f(x) = 5$ and $g(x) = x^2 - x$. Find $(f + g)(x)$.

6. If $f(x) = 3x + 7$ and $g(x) = 2x^2$, calculate $(f - g)(x)$.

7. Let $f(x) = x^3$ and $g(x) = 3$. Find $(f + g)(x)$.

8. If $f(x) = 4x$ and $g(x) = \frac{x}{2}$, what is $(f \cdot g)(x)$?

9. Let $f(x) = x^2 - 1$ and $g(x) = 2x + 3$. Find $(f + g)(x)$.

10. If $f(x) = x$ and $g(x) = x^2 - x + 1$, calculate $(f - g)(x)$.

3 Composite Functions

A composite function is a function formed by combining two functions. If f and g are functions, then the composite function $f \circ g$ is defined as:

$$(f \circ g)(x) = f(g(x))$$

Example: Let $f(x) = 2x + 1$ and $g(x) = x^2$.

$(f \circ g)(x) =$

Practice Problems

1. Let $f(x) = 3x + 2$ and $g(x) = x - 1$. Find $(f \circ g)(x)$.

2. If $f(x) = x^2$ and $g(x) = 4x + 3$, what is $(f \circ g)(x)$?

3. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. Find $(f \circ g)(x)$.

4. If $f(x) = x - 3$ and $g(x) = 2x$, calculate $(g \circ f)(x)$.

5. Let $f(x) = x^3$ and $g(x) = x + 1$. Find $(g \circ f)(x)$.

6. If $f(x) = 2x + 5$ and $g(x) = \frac{x}{2}$, what is $(f \circ g)(x)$?

7. Let $f(x) = x^2 - 4$ and $g(x) = 3x$. Find $(g \circ f)(x)$.

8. If $f(x) = x + 1$ and $g(x) = x^2$, calculate $(f \circ g)(x)$.

9. Let $f(x) = 2x$ and $g(x) = \sqrt{x}$. What is $(g \circ f)(x)$?

10. If $f(x) = 4x + 1$ and $g(x) = x^2 - 3$, find $(f \circ g)(x)$.

4 Real-World Applications of Linear Functions

Linear functions are useful tools for modeling and solving real-world problems. This section provides examples of such applications and practice problems for students to solve.

4.1 1. Solving Problems with Linear Functions

Linear functions model situations with a constant rate of change. Here are some examples:

Example: A ride-sharing service charges a flat fee of \$3 plus \$0.75 per mile driven. If a customer travels 10 miles, what is the total cost of the ride?

Practice Problems

For each of the following Write a function for the scenario provided and then solve.

1. A painter charges a flat fee of \$50 for supplies plus \$25 per hour of labor. If a job takes 6 hours, what is the total cost?

2. A moving company charges \$100 for the truck rental and \$0.50 per mile driven. If the total cost for a move was \$175, how many miles were driven?

3. A company manufactures widgets. The cost to produce x widgets is given by the linear function $C(x) = 10x + 200$. If the company produces 50 widgets, what is the total cost?

4. A car rental company charges \$30 per day and \$0.20 per mile. If a customer rents the car for 3 days and drives 150 miles, what is the total cost?

Mobile Data Usage and Billing

Imagine you have a mobile data plan with a company. The company charges based on how much data you use in a month, and there is a limit beyond which they apply extra charges.

1. **Data Usage Function:**
   Let $f(x)$ represent your data usage in gigabytes (GB) over a given period based on the number of hours x you spend on different apps. For example, $f(x) = 0.2x$

2. **Billing Function:**
   Let $g(x)$ represent the cost of your data usage in dollars based on the number of gigabytes x you consume. The company charges \$30 plus \$5 per GB. $g(x) =$

3. **Composite Function:**
   Using the expressions for $f(x)$ and $g(x)$, write the composite function $g(f(x))$ which calculates your total bill in terms of the number of hours x spent streaming.

4. **Example:**
   Suppose you stream videos for 10 hours. Calculate $f(10)$, the total data used. Then, calculate the total bill using $g(f(10))$.

   Example: Streaming for 10 hours