

# Unit: Polynomials

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# 1 Polynomials

## Unit Overview

In this unit, students will explore various operations and problem-solving strategies involving polynomials. Topics include multiplication, division, factoring, solving equations, and polynomial functions. The unit culminates in problem-solving applications and graphing polynomial functions.

## 1.1 Multiplying Polynomials

### Objectives

- Multiply a monomial by a polynomial using the distributive property.
- Multiply binomials
- Multiply a binomial by a trinomial
- Multiply any size polynomial by any polynomial

### Steps for Multiplying Polynomials

1. Use the distributive property to multiply terms.
2. Combine like terms to simplify the expression.

Example 1: Multiply  $3x(2x^2 + 5x - 4)$

Example 2: Multiply  $5(x - 5)$

Example 2: Multiply  $(3 + 2)(x - 5)$

Example 2: Multiply  $(x + 3)(x - 5)$

Example 3: Multiply  $(x + 2)(x^2 - 3x + 4)$

### Practice Problems

1.  $2x(x^2 + 3x + 1)$
2.  $(x + 4)(x - 2)$
3.  $(2x + 1)(x^2 - x + 5)$
4.  $3(x + 2)(x - 1)$
5.  $x(x^2 + 5x - 6)$
6.  $(2x - 3)(x + 4)$
7.  $(x + 1)(x^2 + 2x + 3)$
8.  $4(x - 2)(x + 3)$
9.  $2x(x + 1)(x - 5)$
10.  $(x - 3)(2x^2 + x + 4)$
11.  $5x(x^2 - 4x + 3)$
12.  $(3x + 2)(x - 1)(x + 5)$
13.  $x^2(x - 2)(x + 6)$
14.  $3(x + 2)(x^2 - x - 4)$
15.  $(x + 3)(x - 1)(x + 2)$
16.  $6x(x + 5)(x - 4)$
17.  $(x^2 + 4x + 4)(x - 3)$
18.  $2(x + 1)(2x - 3)(x + 5)$
19.  $(x - 2)^2(x + 4)$
20.  $4(x + 3)(x^2 + x - 2)$
21.  $(2x - 1)(x + 5)(x - 3)$
22.  $x(x + 4)(x^2 - 3x + 2)$
23.  $(3x^2 + x - 4)(x - 1)$
24.  $2(x - 5)(x + 2)(x - 1)$

## 1.2 Dividing Polynomials and GCF Factoring

### Objectives

- Divide polynomials by monomials.
- Simplify expressions by reducing coefficients and exponents.
- Factor polynomials using the greatest common factor (GCF).

### Steps for Division

1. Divide each term of the polynomial by the monomial.
2. Simplify by dividing coefficients and subtracting exponents.

Example 1: Divide  $6x^3 + 12x^2 - 9x$  by  $3x$

Example 2: Divide  $9x^4 + 27x^2 - 12x^3$  by  $3x$

### Steps for GCF Factoring

1. Identify the greatest common factor of all terms.
2. Factor out the GCF.

Example: Factor  $4x^3 + 8x^2 + 12x$

Example: Factor  $4x^5 + 12x^3 + 24x$

### Practice Problems

1. Divide  $10x^4 - 15x^3$  by  $5x$ .
2. Factor  $6x^2 + 18x + 12$ .
3. Factor  $3x^4 - 9x^3 + 6x^2$ .
4. Divide  $x^3 + 2x^2 + x$  by  $x$ .
5. Divide  $8x^3 - 12x^2 + 16x$  by  $4x$ .
6. Factor  $5x^2 + 10x + 15$ .
7. Factor  $4x^3 + 8x^2 - 12x$ .
8. Divide  $x^4 + 3x^3 - x^2$  by  $x^2$ .
9. Divide  $6x^5 - 9x^4 + 12x^3$  by  $3x^2$ .
10. Factor  $2x^2 + 14x + 24$ .
11. Factor  $9x^3 - 27x^2 + 18x$ .
12. Divide  $2x^4 + 6x^3 + 4x^2$  by  $2x^2$ .
13. Divide  $12x^3 - 18x^2 + 6x$  by  $6x$ .
14. Factor  $10x^2 + 20x - 30$ .
15. Factor  $6x^3 - 24x^2 + 18x$ .
16. Divide  $x^5 + 2x^4 + x^3$  by  $x^3$ .
17. Divide  $15x^4 - 5x^3 + 10x^2$  by  $5x^2$ .
18. Factor  $8x^2 + 16x + 24$ .
19. Factor  $4x^3 - 16x^2 + 12x$ .
20. Divide  $9x^4 + 27x^3 - 18x^2$  by  $3x^2$ .

## 1.3 Factoring by Grouping

### Objectives

- Factor polynomials with four or more terms by grouping.
- Apply grouping techniques to simplify expressions.

### Steps for Factoring by Grouping

1. Group the terms into two pairs.
2. Factor out the greatest common factor (GCF) from each group.
3. If the remaining binomials are the same, factor them out.

Example: Factor  $x^3 + 3x^2 + 2x + 6$

$$x^3 + 3x^2 + 2x + 6$$

1. Group terms:  $(x^3 + 3x^2) + (2x + 6)$ .
2. Factor out GCF from each group:  $x^2(x + 3) + 2(x + 3)$ .
3. Factor out the common binomial:  $(x + 3)(x^2 + 2)$ .

### Practice Problems

1. Factor  $x^3 + 2x^2 + x + 2$ .
2. Factor  $2x^3 + 4x^2 + 3x + 6$ .
3. Factor  $3x^3 - 9x^2 + 4x - 12$ .
4. Factor  $x^4 - 2x^3 + 3x - 6$ .
5. Factor  $x^3 - x^2 + 2x - 2$ .
6. Factor  $4x^3 + 8x^2 - 2x - 4$ .
7. Factor  $3x^3 + 6x^2 - x - 2$ .
8. Factor  $x^4 + 3x^3 - 2x - 6$ .
9. Factor  $2x^3 + 3x^2 + 4x + 6$ .
10. Factor  $x^3 - 3x^2 + 2x - 6$ .
11. Factor  $3x^3 - x^2 + 6x - 2$ .
12. Factor  $2x^4 + 4x^3 + 6x - 12$ .
13. Factor  $x^3 + 5x^2 - 4x - 20$ .
14. Factor  $2x^3 + 6x^2 + x + 3$ .
15. Factor  $x^3 - 4x^2 - 3x + 12$ .
16. Factor  $3x^3 - 6x^2 + 2x - 4$ .
17. Factor  $4x^4 + 8x^3 - 3x - 6$ .
18. Factor  $x^3 + x^2 - x - 1$ .
19. Factor  $2x^3 + 4x^2 + 3x + 6$ .
20. Factor  $x^4 - 3x^3 + 2x - 6$ .

## 1.4 Factoring Trinomials

### Objectives

- Factor trinomials of the form  $ax^2 + bx + c$ , where:

### Steps for Factoring Trinomials

1. Factor by decomposition. Multiply  $a$  and  $c$ , then find factors of  $ac$  that add to  $b$ .
2. Split the middle term and factor by grouping.

Example: Factor  $2x^2 + 7x + 3$

### Practice Problems

- |                                |                               |
|--------------------------------|-------------------------------|
| 1. Factor $x^2 + 6x + 8$ .     | 11. Factor $x^2 - 5x - 6$ .   |
| 2. Factor $3x^2 + 10x + 8$ .   | 12. Factor $4x^2 + 8x + 3$ .  |
| 3. Factor $x^2 - 4x - 12$ .    | 13. Factor $x^2 + 4x + 3$ .   |
| 4. Factor $5x^2 + 14x + 8$ .   | 14. Factor $5x^2 + 13x + 6$ . |
| 5. Factor $x^2 + 5x + 6$ .     | 15. Factor $x^2 - 7x + 10$ .  |
| 6. Factor $4x^2 + 12x + 9$ .   | 16. Factor $6x^2 + 15x + 9$ . |
| 7. Factor $x^2 - 9x + 20$ .    | 17. Factor $2x^2 + 6x + 4$ .  |
| 8. Factor $2x^2 + 7x + 3$ .    | 18. Factor $3x^2 - 8x - 3$ .  |
| 9. Factor $x^2 - 6x + 8$ .     | 19. Factor $x^2 + 3x - 10$ .  |
| 10. Factor $3x^2 + 11x + 10$ . | 20. Factor $2x^2 + 5x - 3$ .  |



## 1.5 Factoring Special Products

### Objectives

- Recognize and factor difference of squares.
- Recognize and factor perfect square trinomials.

Example: Factor  $x^2 - 9$  (Difference of Squares)

Example: Factor  $x^2 + 6x + 9$  (Perfect Square Trinomial)

### Practice Problems

1. Factor  $x^2 - 16$ .
2. Factor  $4x^2 - 25$ .
3. Factor  $x^2 + 10x + 25$ .
4. Factor  $9x^2 - 12x + 4$ .
5. Factor  $x^2 - 9$ .
6. Factor  $16x^2 - 1$ .
7. Factor  $x^2 + 8x + 16$ .
8. Factor  $25x^2 - 36$ .
9. Factor  $4x^2 - 49$ .
10. Factor  $x^2 + 12x + 36$ .
11. Factor  $9x^2 - 1$ .
12. Factor  $x^2 - 6x + 9$ .
13. Factor  $36x^2 - 49$ .
14. Factor  $4x^2 + 4x + 1$ .
15. Factor  $x^2 - 25$ .
16. Factor  $16x^2 + 24x + 9$ .
17. Factor  $49x^2 - 64$ .
18. Factor  $x^2 + 14x + 49$ .
19. Factor  $81x^2 - 16$ .
20. Factor  $x^2 - 4x + 4$ .

## 1.6 Solving Polynomial Equations (Factoring)

### Objectives

- Solve polynomial equations by factoring.
- Use the zero product property to find solutions.

### The Zero Product Property

The **Zero Product Property** states that if the product of two or more factors is zero, then at least one of the factors must be zero. Mathematically, if:

$$a \cdot b = 0,$$

then either:

$$a = 0 \quad \text{or} \quad b = 0.$$

### Steps for Solving Polynomial Equations

1. Factor the polynomial completely.
2. Set each factor equal to zero.
3. Solve for the variable in each equation.

Example: Solve  $2x^2 - 8x = 0$

Example: Solve  $x^2 + 5x + 6 = 0$

Example: Solve  $9x^2 + 12x - 5 = 3x^2 - 7x + 2$

Example: Solve  $16x^4 + 4x - 200 = 4(x + 14)$

#### Real world Example 1

Imagine you want to create a rectangular flower bed in your garden. The design calls for the length to be 3 meters more than the width, and you need the area to be 70 square meters.

#### Real world Example 2

you're coaching a basketball team and want to know how long a shot stays in the air. Suppose a player releases the ball from a height of 8 feet with an upward force so that its height (in feet) is modeled by:

$$h(t) = -2t^2 + 6t + 8$$

Where  $h$  is height, and  $t$  is time in seconds.

## Practice Problems

1. Solve  $x^2 + 7x + 12 = 0$ .
2. Solve  $3x^2 - 15x = 0$ .
3. Solve  $x^2 - 16 = 0$ .
4. Solve  $4x^2 + 12x + 9 = 0$ .
5. Solve  $x^2 + 5x + 6 = 0$ .
6. Solve  $2x^2 - 8x = 0$ .
7. Solve  $x^2 - 9 = 0$ .
8. Solve  $3x^2 + 6x + 3 = 0$ .
9. Solve  $x^2 + 4x + 3 = 0$ .
10. Solve  $5x^2 - 10x = 0$ .
11. Solve  $x^2 - 25 = 0$ .
12. Solve  $4x^2 + 8x + 4 = 0$ .
13. Solve  $x^2 + 6x + 8 = 0$ .
14. Solve  $2x^2 - 4x = 0$ .
15. Solve  $x^2 - 4 = 0$ .
16. Solve  $9x^2 + 12x + 4 = 0$ .
17. Solve  $x^2 + 3x + 2 = 0$ .
18. Solve  $3x^2 - 9x = 0$ .
19. Solve  $x^2 - 1 = 0$ .
20. Solve  $2x^2 + 4x + 2 = 0$ .

## 1.7 Problem-Solving with Polynomials

### Objectives

- Solve word problems involving polynomial equations.
- Apply polynomial operations to real-world scenarios, such as area, perimeter, and business contexts.

#### Example: Area Problem

The area of a rectangle is  $12x^2 + 18x$ . If the width is  $6x$ , find the length.

#### Example: Revenue Problem

A company sells  $x$  units of a product at a price of  $(50 - 2x)$  dollars each. Find the revenue function and determine the maximum revenue.

### Practice Problems

1. The area of a triangle is  $x^2 + 5x + 6$ . If the base is  $x + 2$ , find the height.
2. The profit function of a business is  $P(x) = -2x^2 + 40x - 100$ . Find the maximum profit.
3. The perimeter of a rectangle is  $4x^2 + 8x$ . If the width is  $x$ , find the length.
4. A farmer's total cost is  $C(x) = 3x^2 - 5x + 10$ . Find the cost when  $x = 4$ .

## 1.8 Completing the Square

### Objectives

- Solve quadratic equations by completing the square.
- Rewrite quadratic equations in vertex form.

### Steps for Completing the Square

1. Rewrite the equation in the form  $ax^2 + bx + c = 0$ .
2. Isolate the constant term on one side.
3. Add and subtract  $\left(\frac{b}{2}\right)^2$  to complete the square.
4. Factor the trinomial and solve.

Example: Solve  $x^2 + 6x + 5 = 0$  by completing the square

### Practice Problems

- |                                      |  |
|--------------------------------------|--|
| 1. Solve $x^2 + 4x + 1 = 0$          | 11. Solve $2x^2 + 12x + 8 = 2x + 7$ .  |
| 2. Solve $x^2 - 10x + 16 = 0$        | 12. Solve $x^2 + 2x + 1 = 4$ .         |
| 3. solve $x^2 + 8x + 15 = 8$         | 13. Solve $x^2 - 12x + 35 = 0$ .       |
| 4. solve $2x^2 + 14x + 14 = 2x + 5$  | 14. Solve $3x^2 + 15x + 12 = 5x + 6$ . |
| 5. Solve $x^2 + 5x + 6 = 0$ .        | 15. Solve $x^2 + 3x - 4 = 0$ .         |
| 6. Solve $x^2 - 6x + 9 = 0$ .        | 16. Solve $x^2 + 11x + 30 = 5$ .       |
| 7. Solve $x^2 + 7x + 10 = 8$ .       | 17. Solve $2x^2 + 8x + 10 = 4x + 3$ .  |
| 8. Solve $2x^2 + 10x + 8 = 3x + 4$ . | 18. Solve $x^2 - 5x + 6 = 0$ .         |
| 9. Solve $x^2 - 4x - 5 = 0$ .        | 19. Solve $x^2 + 6x + 9 = 3x + 4$ .    |
| 10. Solve $x^2 + 9x + 18 = 10$ .     | 20. Solve $3x^2 - 6x + 1 = 2x - 7$ .   |

## 1.9 Quadratic Formula

### Objectives

- Derive and use the quadratic formula to solve equations.
- Analyze the discriminant to determine the nature of the roots.

### Quadratic Formula

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Steps for Using the Formula

1. Identify  $a$ ,  $b$ , and  $c$  from the equation  $ax^2 + bx + c = 0$ .
2. Substitute into the formula and simplify.

Example: Solve  $x^2 + 4x - 5 = 0$  using the quadratic formula

Example: Solve  $3x^2 + 2x - 5 = 0$  using the quadratic formula



## Practice Problems

1. Solve  $x^2 + 6x + 9 = 0$ .
2. Solve  $2x^2 - 4x - 6 = 0$ .
3. Solve  $x^2 + 2x + 5 = 0$ .
4. Solve  $3x^2 - 12x + 9 = 0$ .
5. Solve  $x^2 + 6x + 9 = 5$ .
6. Solve  $2x^2 - 4x - 6 = 8$ .
7. Solve  $x^2 + 2x + 5 = 3x + 7$ .
8. Solve  $3x^2 - 12x + 9 = 15$ .
9. Solve  $x^2 - 4x + 3 = 10$ .
10. Solve  $2x^2 + 5x + 4 = 6x + 9$ .
11. Solve  $x^2 + 8x + 16 = 2x + 7$ .
12. Solve  $4x^2 - 3x - 2 = 1$ .
13. Solve  $x^2 + 10x + 25 = 12$ .
14. Solve  $2x^2 - 6x + 5 = 3x - 4$ .
15. Solve  $x^2 - 7x + 12 = 15$ .
16. Solve  $3x^2 + 9x + 6 = 5x + 4$ .
17. Solve  $x^2 + 3x + 2 = 10$ .
18. Solve  $5x^2 - 8x + 3 = x + 2$ .
19. Solve  $x^2 - 2x - 8 = 7$ .
20. Solve  $4x^2 + 6x + 1 = 2x + 10$ .

## 1.10 Unit Review

### Objectives

- Review all topics in the unit, including operations with polynomials, factoring, solving.

#### Expand the Following:

1.  $2x(x^2 + 3x + 1)$
2.  $(x + 4)(x - 2)$
3.  $(3x + 1)(x^2 - x + 5)$
4.  $4(x + 2)(x - 1)$
5.  $(x - 3)(x^2 + 5x - 4)$
6.  $2x(x + 1)(x - 5)$
7.  $(x + 3)(2x^2 + x - 1)$
8.  $5(x - 2)(x + 4)$
9.  $x^2(x + 3)(x - 1)$
10.  $3(x + 5)(x^2 - 2x + 3)$

#### Divide the Following:

11.  $10x^4 - 15x^3$  by  $5x$
12.  $x^3 + 2x^2 + x$  by  $x$
13.  $6x^5 - 9x^4 + 12x^3$  by  $3x^2$
14.  $x^4 + 3x^3 - x^2$  by  $x^2$
15.  $12x^3 - 18x^2 + 6x$  by  $6x$
16.  $8x^3 + 4x^2 - 16x$  by  $4x$
17.  $2x^4 + 4x^3 - 6x^2$  by  $2x^2$
18.  $15x^4 - 10x^3 + 5x^2$  by  $5x^2$
19.  $4x^4 + 8x^3 - 12x^2$  by  $2x^2$
20.  $9x^3 - 18x^2 + 27x$  by  $3x$

**Factor the Following:**

- 21.  $x^3 + 2x^2 + x + 2$
- 22.  $3x^3 - 9x^2 + 4x - 12$
- 23.  $x^4 - 2x^3 + 3x - 6$
- 24.  $2x^3 + 6x^2 + 4x + 12$
- 25.  $x^3 - 4x^2 + 3x - 12$
- 26.  $4x^3 + 8x^2 - 2x - 4$
- 27.  $3x^3 + 6x^2 - x - 2$
- 28.  $x^4 + 5x^3 - x - 5$
- 29.  $x^3 + 3x^2 - 2x - 6$
- 30.  $6x^3 + 18x^2 + 12x + 36$

**Factor the Following Trinomials:**

- 31.  $x^2 + 6x + 8$
- 32.  $3x^2 + 10x + 8$
- 33.  $x^2 - 4x - 12$
- 34.  $5x^2 + 14x + 8$
- 35.  $x^2 + 5x + 6$
- 36.  $2x^2 - 8x + 6$
- 37.  $x^2 - 9x + 20$
- 38.  $2x^2 + 7x + 3$
- 39.  $x^2 + 8x + 16$
- 40.  $4x^2 + 8x + 3$

**Solve the Following:**

- 41.  $x^2 + 7x + 12 = 0$
- 42.  $3x^2 - 15x = 0$
- 43.  $x^2 - 16 = 0$
- 44.  $4x^2 + 12x + 9 = 0$
- 45.  $x^2 + 5x + 6 = 0$
- 46.  $x^2 - 6x + 9 = 0$
- 47.  $x^2 + 7x + 10 = 8$

48.  $2x^2 + 10x + 8 = 3x + 4$

49.  $x^2 - 12x + 35 = 0$

50.  $3x^2 + 15x + 12 = 5x + 6$

**Factor the Following Special Products:**

51.  $x^2 - 16$

52.  $4x^2 - 25$

53.  $x^2 + 10x + 25$

54.  $9x^2 - 12x + 4$

55.  $x^2 - 25$

56.  $36x^2 - 49$

57.  $x^2 + 14x + 49$

58.  $16x^2 - 9$

59.  $x^2 - 4$

60.  $81x^2 - 16$