Trigonometry: Angles of Elevation and Depression

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1 Right Triangle Trigonometry

1.1 Basic Definitions

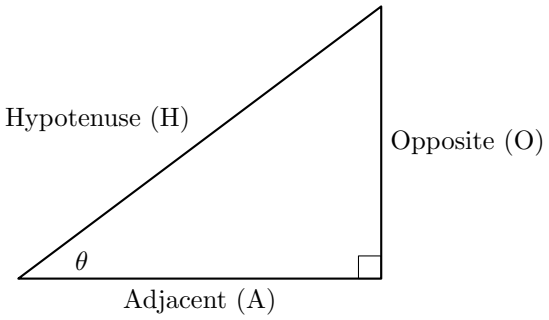
In a right triangle, the sides are referred to relative to one of the acute angles, θ:

- **Opposite (O)**: The side across from angle θ.
- **Adjacent (A)**: The side next to angle θ.
- **Hypotenuse (H)**: The longest side of the right triangle (opposite the right angle).

The three primary trigonometric ratios are:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}.$$

1.2 Diagram of a Right Triangle



1.3 Example Problem

Example: Finding Trig Ratios

Consider a right triangle where θ is one of the acute angles, the length of the opposite side is 3 cm, and the length of the hypotenuse is 5 cm.

1. Determine $\sin\theta$.

2. Determine $\cos\theta$.

3. Determine $\tan\theta$.

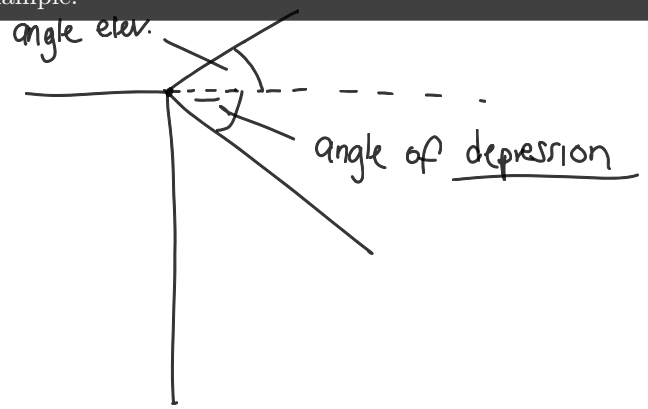
2 Angles of Elevation and Depression

2.1 Definitions

- **Angle of Elevation:** The angle formed between a horizontal line of sight and the line of sight to an object above the horizontal.
- **Angle of Depression:** The angle formed between a horizontal line of sight and the line of sight to an object below the horizontal.

Diagram of Angles of Elevation and Depression

Example:



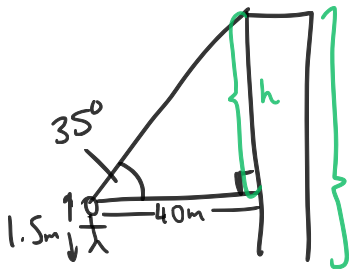
2.2 Example Problem

Example: Angle of Elevation

A person stands on level ground and observes the top of a building. The angle of elevation to the top of the building is 35°. The person's eyes are 1.5 m above the ground, and the horizontal distance from the person to the base of the building is 40 m.

SOHCAH TOA

1. Draw a diagram to represent this situation.



2. Let h be the height of the building. Write an expression using trigonometry to solve for h.

$$\tan(35) = \frac{h}{40} \qquad h = 40\tan(35)$$

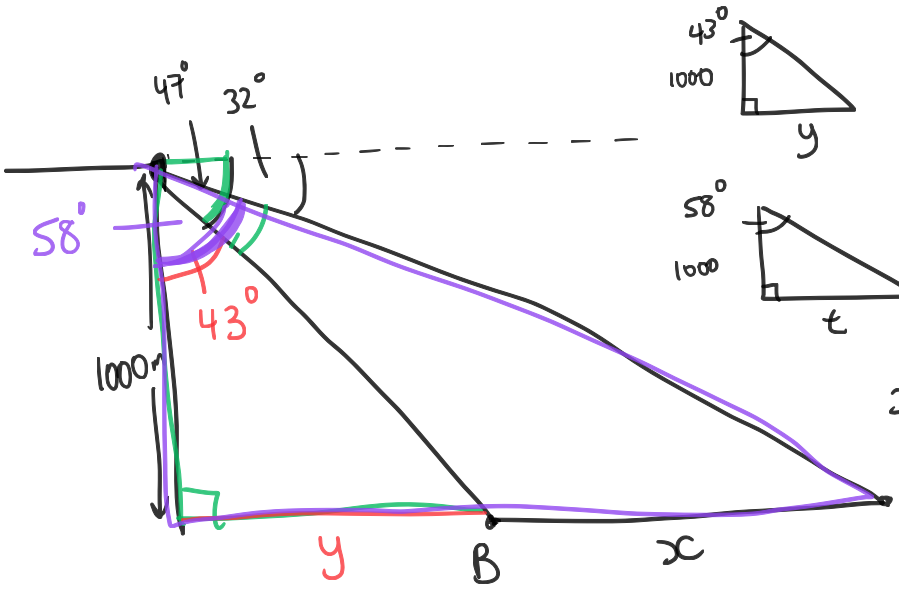
3. Solve for the height of the building.

$$b = h + 1.5$$
$$= 40\tan(35) + 1.5$$

$$\boxed{b = 29.5\text{m}}$$

Example:

An observer on a cliff 1000 m above sea level sights two ships due east. The angles of depression of the ships are 47° and 32°. Find, to the nearest meter, the distance between the two ships



$\tan(43) = \frac{y}{1000}$

$y = 1000\tan(43)$

$\tan(58) = \frac{t}{1000}$

$t = 1000\tan(58)$

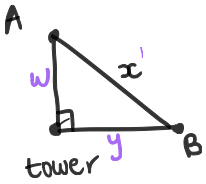
$x = 1000\tan(58) - 1000\tan(43)$

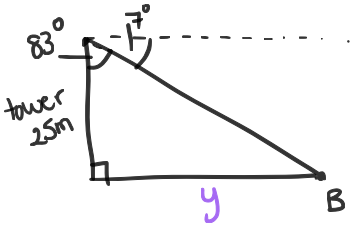
$= 1000(\tan(58) - \tan(43))$

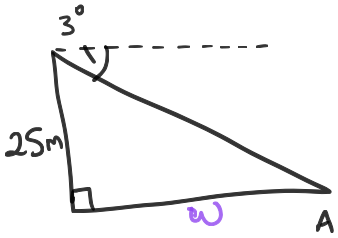
$x = 667.8\text{m}$

Example:

From the top of a 25-m lookout tower, a fire ranger observes one fire due east of the tower at an angle of depression of 7°. She sees another fire due north of the tower at an angle of depression of 3°. How far apart are the fires to the nearest metre?







$\tan(83) = \frac{y}{25}$

$y = 25\tan(83)$

$\tan(87) = \frac{w}{25}$

$w = 25\tan(87)$

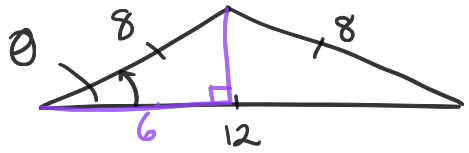
$x^2 = w^2 + y^2$

$x^2 = (25\tan(87))^2 + (25\tan(83))^2$

$x = 518.7\text{m}$

Example:

A roof has the shape of an isosceles triangle with equal sides 8 m long and base 12 m long. What is the measure of the angle of inclination of the roof to the nearest degree?



$$\cos(\theta) = \frac{6}{8}$$

$$\theta = \cos^{-1}\left(\frac{6}{8}\right)$$

$$\theta = 41.4^\circ$$

2.3 Additional Practice

1. A pilot spots a boat at an angle of depression of 12°. If the airplane is flying at an altitude of 1500 m, how far (in a horizontal distance) is the boat from the point on the water directly below the plane?

2. A tower stands on a hill. From a point 120 m down the hill, the angle of elevation to the top of the tower is 28°. If the tower itself is 20 m tall, and the base of the tower is 15 m higher in elevation than the point of observation, find the angle of elevation from the base of the tower to its top. (Hint: Use right triangles to represent each part of the scenario.)