SUPPLEMENTARY TUTORIAL 02 IT 1070 (June Intake)

1) A random variable Y has the following probability mass function defined in tabular form.

| Y | 1 | 2 | -2 |
|------|---|----|----|
| p(Y) | c | 2c | 3c |

- i. Find the value of c.
- ii. Compute p (1), p (2), and p (-2).
- iii. Find E(Y) and Var(Y)
- 2) An experiment takes a random amount of time W, measured in seconds, to complete.

The probability density function of W is,

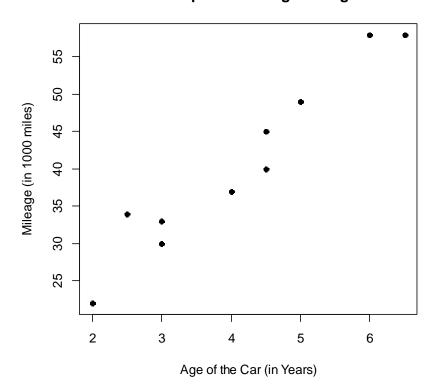
$$\mathbf{f}(W) = \left\{ \begin{array}{ll} \alpha W + 1/2 & \quad ; 0 < W < 1 \\ 0 & \quad ; Otherwise \end{array} \right.$$

Where α is a constant.

- i. Calculate the value of α .
- ii. Calculate the probability that the experiment takes less than 0.5 seconds to complete.
- 3) A second-hand car dealer has 10 cars for sale. She decides to investigate the link between the age of cars (in years) and the mileage (in thousand miles). The data, the scatterplot and the regression outputs are given below.

| Age | 2 | 2.5 | 3 | 4 | 4.5 | 4.5 | 5 | 3 | 6 | 6.5 |
|---------|----|-----|----|----|-----|-----|----|----|----|-----|
| Mileage | 22 | 34 | 33 | 37 | 40 | 45 | 49 | 30 | 58 | 58 |

Scatterplot of Mileage vs. Age



R Output:

Coefficients:

| (Intercept) | Age |
|-------------|-------|
| 8.892 | 7.734 |

Analysis of Variance Table

| Response: Mi | ileage | | | | | |
|-------------------|--------------|------------|--------------|----------|----------|-----|
| | Df | Sum Sq | Mean Sq | F Value | Pr(>F) | |
| Age | T | P | \mathbf{S} | 121.7756 | 4.05e-06 | *** |
| Residuals | \mathbf{U} | Q | 9.7738 | | | |
| Total | 9 | R | | | | |
| | | | | | | |
| Signif. Codes: | 0 '***' | 0.001 '**' | 0.01 '*' | 0.05 '.' | 0.1'' | 1 |

- a) Find values marked P, Q, R, S, T and U in the ANOVA table (Show workings)
- b) State the estimated regression equation.
- c) Test whether the slope of the Regression line is significant and state the conclusions.
- d) Find the estimated mileage when the age of the car is 5 years.
- e) Provide two meaningful conclusions from the above analysis

4) Assume that the continuous random variable x has the probability density function,

$$\mathbf{f}(\mathbf{x}) = \begin{cases} k (9-4\mathbf{x}^2) & \text{; for } 0 \le \mathbf{x} \le 3/2 \\ 0 & \text{;Otherwise} \end{cases}$$

Where k is a constant.

- i. Calculate the value of k.
- ii. Find the mean and variance of x.
- iii. Find the cumulative distribution function of x.
- 5) A car-pooling study shows that the number of passengers, X in a car (excluding the driver) is likely to assume the values 0,1,2,3 and 4 with probabilities given by the table.

| X | 0 | 1 | 2 | 3 | 4 |
|------------|-----|-----|---|------|------|
| $P(X=X_i)$ | 0.7 | 0.1 | c | 0.05 | 0.05 |

- i. Find the value of c.
- ii. Find the cumulative distribution function of X
- iii. Calculate E(X) and Var(X).