Approximation Algorithms: MidTerm

1. Let $G = (V, E)$ be a complete undirected graph with edge costs satisfying triangle inequality and let k be a positive integer. The problem is to partition V into sets $V_1, \ldots, V_k$ so as to minimize the costliest edge between two vertices of the same set, i.e.

$$\min \max_{1 \le i \le k} \max_{u,v \in V_i} cost(u, v)$$

. Give a factor 2 approximation algorithm for this problem together with a tight example. (5)

2. Let x^* be an optimum solution to the linear relaxation of the Integer Program for minimum Vertex cover in $G = (V, E)$. Suppose $x_i^* \in \{0, 1/2, 1\}, 1 \le i \le n$ where x_i is the variable associate with $v_i \in V$. Further assume that the vertices of G are colored with 4 colors such that every pair of adjacent vertices have different colors. How will you round the solution x^* to obtain a 1.5-approximation for the minimum Vertex Cover. Give an algorithm and analyse its guarantee. (5)

3. We are given a directed acyclic graph $G = (V, E)$ vertices $s, t \in V$, edge-costs $c : E \to \Re^+$, edge-lengths $l : E \to \Re^+$ and a length bound L. Give a full polynomial time approximation scheme (FPTAS) to find the minimum cost path from s to t of length at most L. Hint: First give a Dynamic Program to solve the problem assuming the edge-costs are integer. (5)

4. Consider the following problem arising in communication networks. The network consists of a cycle on n nodes. Some set C of calls is given: each call has an originating node and a destination node on this cycle. Each call can be routed either clockwise or anticlockwise around the cycle and the objective is to route the calls so that the maximum load on any link (edge of the cycle) is minimised. The load on a link is the number of calls routed through it. Give a 2-approximation algorithm for this problem. (5)

5. Consider the following problem. There is a set U of n nodes, which we can think of as users (e.g., these are locations that need to access a service, such as a Web server). You would like to place servers at multiple locations. Suppose you are given a set S of possible sites that would be willing to act as locations for the servers. For each site $s \in S$, there is a fee $f_s \ge 0$ for placing a server at that location. Your goal will be to approximately minimize the cost while providing the service to each of the customers. So far this is very much like the Set Cover Problem: The places s are sets, the weight of set s is f_s, and we want to select a collection of sets that covers all users. There is one extra complication: Users $u \in U$ can be served from multiple sites, but there is an associated cost d_{us} for serving user u from site s. When the value d_{us} is very high, we do not want to serve user u from site s; and in general the service cost d_{us} serves as an incentive to serve customers from "nearby" servers whenever possible.

   So here is the question, which we call the *Facility Location Problem*: Given the sets U and S, and costs f and d, you need to select a subset $A \subseteq S$ at which to place servers (at a cost of $\sum_{s \in A} f_s$), and assign each user u to the active server where it is cheapest to be served, $\min_{s \in A} d_{us}$. The goal is to minimize the overall cost $\sum_{s \in A} f_s + \sum_{u \in U} \min_{s \in A} d_{us}$. Give an $H(n)$- approximation for this problem.

   (Note that if all service costs d_{us} are 0 or infinity, then this problem is exactly the Set Cover Problem: f_s is the cost of the set named s, and d_{us} is 0 if node u is in set s, and infinity otherwise.) (5)