Lecture 1

Wednesday, August 11, 2021 11:01 AM

3CNF SAT : Satisfiability formula in 3CNF form → conjunctive normal form, each clause has 3 literals $(x_i, \bar{x}_i)$

NP-hard a boolean formula with $\wedge, \vee, \neg$, we are interested in knowing if there is an assignment of T/F values to variables which makes the formula TRUE

3-CNF max-SAT : Given a 3CNF formula find an assignment of T/F values to variables which maximises the no. of clauses satisfied.

NP-hard

$\phi = (x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee \bar{x}_3 \vee x_4)$

Given a formula $\phi$ with $m$ clauses $\exists$ a polynomial time algorithm which finds a boolean assignment satisfying at least $7/8\,m$ clauses

assign each variable independently a True/False with prob $1/2$. ] randomised algorithm.

Consider a clause $C = (x_i \vee x_j \vee x_k)$

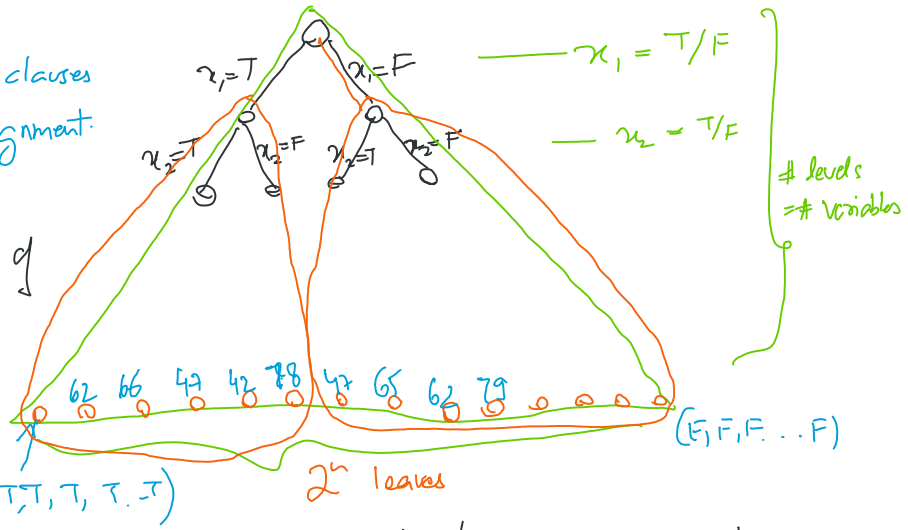
Probability that the clause is not satisfied $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1/8$.

$\Rightarrow$ probability that the clause is satisfied $= 7/8$

Expected no. of clauses satisfied in a random assignment $= 7/8\,m$

$m = 80$

Derandomizing this algorithm.




no. on each leaf is the no. of clauses satisfied in corresponding assignment.

Avg. of no's on leaves is 70

One of the two subtrees has avg of leaves $\geq 70$. Suppose this corresponds to $x_1 = F$. We set $x_1 = F$ and proceed.

At the first step we are looking at all assignment where $x_1 = T$ & compute the avg. no. of clauses satisfied in these.

$(x_1 \vee x_2 \vee x_3)$ if $x_1 = T$ then clause is true & contributes 1