Approximation Algorithms Lecture 18

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1 Recap

Linear programs for approximating vertex cover and set cover.

2 Content

2.1 Lower bounds on approximation guarantees

Suppose we have a minimization problem, say vertex cover.

Let's say optimum vertex cover is **OPT**. Then we write an integer program, relax it to get a linear program, and solve it to get $\mathbf{OPT}_{LP} \leq \mathbf{OPT}$. Then we get a vertex cover by tweaking $\mathbf{OPT}_{LP}$ (using a deterministic rounding in this case), and this is $\geq \mathbf{OPT}$. We then argue that the vertex cover so obtained is at most $2\mathbf{OPT}_{LP}$. We could say that this analysis is weak, but for that we need proof.

Let's look at the gap between **OPT** and $\mathbf{OPT}_{LP}$. If this gap is close to 2, we can't do any better than $2\mathbf{OPT}_{LP}$ using a linear programming approach.

Consider a clique on n vertices (K_n).

Then $\mathbf{OPT}_{VC} = n - 1$, since if we leave 2 vertices, the edge between those won't have been covered, and $\mathbf{OPT}_{LP} = \frac{n}{2}$, since we can assign $\frac{1}{2}$ to each vertex here.

In other words, this is a bad lower bound.

Note 1

Try to come up with a set cover instance whose optimum set cover is $O(\log n)$ times the optimal LP solution for the set cover relaxation.

2.2 An f-approximation for set cover

Let f be the maximum of sets any element belongs to.

Consider the same LP as before. If $x_i^* \geq \frac{1}{f}$, then pick set S_i in the set cover solution.

Claim 0.1

The sets picked form a set cover.

Proof. Suppose there is an element that is not covered. Then we have all sets covering it with weights $< \frac{1}{f}$, which is a contradiction. □

$$
\begin{aligned}
\text{Number of sets picked} &= \left| \left\{ i \mid x_i^* \geq \frac{1}{f} \right\} \right| \\
&\leq f \sum_i x_i^* \\
&= f \cdot \mathbf{OPT}_{LP} \\
&\leq f \cdot \mathbf{OPT}
\end{aligned}
$$

2.3 Primal-dual algorithms

2.4 Steiner forest problem

The steiner tree problem says this: find a minimum cost connected subgraph which includes all terminals (T).

The steiner forest problem says this: Given k pairs of terminals (s_i, t_i), find a minimum cost subgraph in which each pair s_i, t_i is connected.