Approximation Algorithms Lecture 11

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1 Recap

Fully polynomial time approximation scheme (FPTAS) for knapsack problem

2 Content

2.1 APTAS for bin packing

We will exhibit an APTAS for bin packing with the solution $\leq (1+\varepsilon)\mathbf{OPT} + 1/\varepsilon^2 + 2$.

We will first write an integer program for bin packing.

Given n_i objects of size s_i, $1 \leq i \leq k$, and $\sum_i n_i = m$.

A configuration c is a k-tuple $(o_1^c, o_2^c, \ldots, o_k^c)$ such that $\sum_i o_i s_i \leq 1$, and $o_i \in \mathbb{Z}_{\geq 0}$.

Let x_c be the number of bins which have configuration c. Then $x_c \in \mathbb{Z}_{\geq 0}$.

Then we need to minimize $\sum_c x_c$.

There is another constraint: $\sum_c x_c o_i^c \geq n_i$ for all $1 \leq i \leq k$, which is the integer program.

Relax the integrality constraint on x_c to $x_c \geq 0$, and this will give us a linear program.

Note that in a linear program, the optimal answer is at a vertex. Let the number of different configurations be α. Then the optimal solution lies at the intersection of α hyperplanes (inequalities).

We have $\alpha + k$ inequalities in all and at a vertex, we set α inequalities as equalities. At least $\alpha - k$ configurations are set to 0, and so at most k configurations have non-zero x_c.

Our algorithm then becomes:

1. Solve the LP and round up all x_c variables.

Note that solution is $\leq$ LP value + k, which is at most $\mathbf{OPT} + k$.