COL352 HW 1

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1 Problem 1

Let $x, y, z \in \Sigma^*$. We say that z is a shuffle of x and y if the characters in x and y can be interleaved, while maintaining their relative order within x and y, to get z. Formally, if $|x| = m$ and $|y| = n$, then $|z|$ must be $m+n$, and it should be possible to partition the set $\{1, 2, \ldots, m+n\}$ into two increasing sequences, $i_1 < i_2 < \cdots < i_m$ and $j_1 < j_2 < \cdots < j_n$, such that $z[i_k] = x[k]$ and $z[j_k] = y[k]$ for all k.

Given two languages $L_1, L_2 \subseteq \Sigma^*$, define shuffle$(L_1, L_2) = \{z \in \Sigma^* \mid z$ is a shuffle of some $x \in L_1$ and some $y \in L_2\}$. Prove that the class of regular languages is closed under the shuffle operation.

2 Problem 2

Let L_1 be a regular language and L_2 be any language (not necessarily regular) over the same alphabet Σ. Prove that the language $L = \{x \in \Sigma^* \mid x \cdot y \in L_1$ for some $y \in L_2\}$ is regular. Do this by mathematically defining a DFA for L starting from a DFA for L_1 and the language L_2.

3 Problem 3

Prove that the class of regular languages is closed under inverse homomorphisms. That is, prove that if $L \subseteq \Gamma^*$ is a regular language and $f : \Sigma^* \to \Gamma^*$ is a homomorphism, then $f^{-1}(L) = \{x \in \Sigma^* \mid f(x) \in L\}$ is regular. Do this by mathematically defining a DFA for $f^{-1}(L)$ starting from a DFA for L and the function f.

4 Problem 4

Prove that the class of regular languages is closed under homomorphisms. That is, prove that if $L \subseteq \Sigma^*$ is a regular language, then so is $f(L) = \{f(x) \mid x \in L\}$. Here, it is advisable to informally describe how you will turn a DFA for L into an NFA for $f(L)$.

5 Problem 5

Prove that if $L \subseteq \Sigma^*$ is a regular language then the language $L_0 = \{x \subseteq \Sigma^* \mid x \cdot rev(x) \in L\}$ is also regular, where $rev(x)$ is the reverse of string x. Here, instead of constructing an NFA for L_0 directly, it could be more convenient to use the already proven closure properties. For example, it might be better to write L_0 as a union of a finite collection of languages, and then construct an NFA for each language in that collection.

Proof. Since L is regular, there must be a DFA $D = (Q, \Sigma, \delta, q_0, A)$ that recognizes it. Consider the NFA $N = (Q, \Sigma, \Delta, A, \{q_0\})$, where $\Delta = \{(q', a, q) \mid \delta(q, a) = q', q \in Q, a \in A\}$, which recognizes $rev(L)$ as done in class.
Now consider the NFA $N' = (Q \times Q, \Sigma, \Delta', \{q_0\} \times A, \{(q, q) \mid q \in Q\})$, where Δ' is defined as $\{((q, q'), a, (q'', q''')) \mid \delta(q, a) = q'' \wedge (q', a, q''') \in \Delta \wedge q, q', q'', q''' \in Q \wedge a \in \Sigma\}$.

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6 Problem 6

Design an algorithm that takes as input the descriptions of two DFAs, D_1 and D_2, and determines whether they recognize the same language.