COL352 Lecture 24

Contents

1 Recap

Completed PDAs and grammars last time.

2 Definitions

Definition 1

A Deterministic Turing Machine (DTM) is a 8-tuple of the form $(Q, \Sigma, \Gamma, \delta, q_0, \sqcup, q_{acc}, q_{rej})$ where

1. Q is a finite set of states
2. Σ is a finite alphabet
3. $\Gamma \supsetneq \Sigma$ is the tape alphabet
4. $q_0 \in Q$ is the initial state
5. $\sqcup \in \Gamma \setminus \Sigma$ is a blank symbol (tape initially contains input $x \in \Sigma^*$ followed by infinitely many $\sqcup$).
6. $q_{acc} \in Q$ is the accepting state, $q_{rej} \in Q$ is the rejecting state, and $q_{acc} \neq q_{rej}$
7. $\delta : (Q \setminus \{q_{acc}, q_{rej}\}) \times \Gamma \to Q \times \Gamma \times \{L, R\}$. Here the first input is the current state, second input is current tape content, the first component of the output is the new state, second is new tape content, and third is the direction in which we need to move.

If the machine is reading the leftmost tape cell and it makes a leftward move, the location of the head doesn't change.

Definition 2

A DTM M accepts $x \in \Gamma^*$ if $\exists x_1, x_2 \in \Gamma^* : (\epsilon, q_0, x) \vdash_M^* (x_1, q_{acc}, x_2)$.

Definition 3

$L \in \Sigma^*$ is said to be recognised by M if $\forall x \in \Sigma^* : x \in L \iff M$ accepts x.

Definition 4

L is said to be Turing-recognisable if it is recognised by some DTM.

Definition 5

A Turing machine M is called a decider if it halts on every input $x \in \Sigma^*$, i.e.,

$$\forall x \in \Sigma^* : \exists x_1, x_2 \in \Gamma^* : (\epsilon, q_0, x) \vdash^* (x_1, q_{acc}, x_2) \text{ or } (\epsilon, q_0, x) \vdash^* (x_1, q_{rej}, x_2)$$

A language $L \in \Sigma^*$ is said to be decided by M if M is a decider and L is recognised by M.

Definition 6

$L \in \Sigma^*$ is said to be decidable if it is decided by some DTM M.

Definition 7

A 2−tape DTM is $(Q, \Sigma, \Gamma, \delta, q_0, \sqcup, q_{acc}, q_{rej})$, where $Q, \Sigma, \Gamma, q_0, \sqcup, q_{acc}, q_{rej}$ are as before, and $\delta : (Q \setminus \{q_{acc}, Q_{rej}\}) \times \Gamma^2 \to Q \times \Gamma^2 times\{L, R, S\}^2$.

For now, S is used since moving both tape heads preserves parity of sum of locations, except for the borders. The language recognised or decided by 2-tape DTM is defined analogously.

3 Content

Question 1

What is the set of instantaneous descriptions of a DTM?

Answer. Set of instantaneous description $= \Gamma^* \times Q \times \Gamma^*$, and a state (x_1, q, x_2) is the following: x_1 is the part of the tape that is strictly to the left of the current head, and x_2 is the remaining part (leaving out the trailing blanks).

Exercise: Formally define the "changes to" relation $\vdash_M$ on the set of instantaneous descriptions.

$\vdash_M^*$ is the reflexive transitive closure of $\vdash_M$.

Claim 0.1

$\forall L \in \Sigma^*$, L is decidable $\implies$ L is Turing-recognisable.

Proof. Definition. $\square$

Note that we will see later on that the opposite direction is not true.

3.1 2-tape DTM

We now have two tapes.
Observe that

Theorem 1

L recognised by a DTM $\iff$ L recognised by a 2-tape DTM.

L decided by a DTM $\implies$ L decided by a 2-tape DTM.

Proof. The forward implication is obvious (just don't use the second tape).

Consider the instantaneous description of the 2-tape DTM – (x_1, y_1) and (x_2, y_2) being the corresponding strings before and not before the tape head on both tapes, and let $\square$ be the blank symbol of the 2-tape DTM.

Our single tape DTM would have the following: $\triangleright x_1 \to y_1\square \ldots \square \# x_2 \to y_2 \square \ldots \square \sqcup \ldots$

Call the 2-tape one M_2 and the 1-tape one M_1.

$M_1 = (Q, \Sigma, \Gamma, \delta, q_0, \square, q_{acc}, q_{rej})$

$M_2 = (Q', \Sigma, \Gamma \cup \{\triangleright, \to, \#, \sqcup\}, q_0', \sqcup, q_{acc}', q_{rej}')$

Behaviour of M_1

1. Preprocess phase

To establish the invariant, we need to change $x\llcorner\!\lrcorner\ldots$ to $\triangleright \rightarrow x\# \rightarrow \llcorner\!\lrcorner\ldots$

2. Simulation phase

   We shall store (state of M_2, symbol from Γ, symbol from Γ, $L/R/S$, $L/R/S$).

   For each transition of M_2, M_1 makes three passes over the tape.

   (a) Insert $\square$ before the $\#$ and overwrite the leftmost $\llcorner\!\lrcorner$ by $\square$.
   (b) Read the symbols after the two $\rightarrow$s and save them in the state.
   (c) Execute the transitions of M_2 in M_1 (doesn't need a complete pass of its own).
   (d) Implement the transitions on the tape: overwrite the cells next to the $\rightarrow$s and move the $\rightarrow$s.

$\square$