COL352 Lecture 9

Contents

1 Recap

Last class stuff about the summary. Also look at the second reference for a proof via GNFAs.

2 Definitions

Same as last class.

3 Content

We shall do the proof slightly differently.

Claim 0.1

$L_{ijk} = L_{ij(k-1)} \cup L'$, where $L' = \Big(L_{ik(k-1)} \cup (L^*_{kk(k-1)}) L_{kj(k-1)}\Big)$

Proof. Two parts.

1. $L_{ij(k-1)} \cup L' \subseteq L_{ijk}$:

   Proof.

   (a) $x \in L_{ij(k-1)} \subseteq L_{ijk} \implies x \in L_{ijk}$

   (b) $x \in L' \implies x = x_0 x_1 \cdots x_{N-1} x_N$ where $x_0 \in L_{ik(k-1)}$. $\forall l = 1 \dots N-1, x_l \in L_{kk(k-1)}$. Now check that $x \in L$.

   □

2. Suppose $x \in L_{ijk}$. If the run of D on x starting from i never visits k then $x \in L_{ij(k-1)}$. If the run visits k N times, then $x = x_0 \dots x_{N-1} x_N$, where $\forall l = 0 \dots N-1, \hat{\delta}(i, x_0, \dots, x_l) = k$. Then $x_0 \in L_{ik(k-1)}$, and $\forall l = 1 \dots N-1, x_l \in L_{kk(k-1)}$, and $x_N \in L_{kj(k-1)}$. Hence $x \in L'$.

□

Claim 0.2

The set of all regular languages is countably infinite.

Note 1

Proof using regular expressions - easy to see using lexicographical ordering. We shall prove using definition of DFA.

Proof. Wlog, let $\Sigma = \{0, \dots, m-1\}$, and $Q = \{1, \dots, n\}$. We can encode a DFA with a finite length binary string, as follows: n ones, one zero, $|\Sigma|$ ones, one zero, $\lceil \log n \rceil$ bits denoting initial state, then $nm\lceil \log n \rceil$ bits for δ as a table ($Q \times \Sigma$ table, with each entry with size $\lceil \log n \rceil$), and the set of accepting states in n bits.

Now consider the first DFA in this total ordering that accepts a language L. □

Corollary 1

The set of regular languages over Σ is uncountable.

Example 1

$\{\langle P, x\rangle \mid P \text{ halts on } x\}$: not regular else we'll get a contradiction to the halting problem.

Example 2

$\{x \in \{0,1\}^* \mid \#0 = \#1 \text{ in } x\}$

Example 3

Set of all palindromes over Σ with at least 2 characters.

Example 4

$\{0^n 1^n \mid n \in \mathbb{N} \cup \{0\}\}$

Now we show that the last example is indeed not a regular language.

Proof. Suppose $L = \{0^n 1^n \mid n \in \mathbb{N} \cup \{0\}\}$ is regular. Let $D = (Q, \{0,1\}, \delta, q_0, A)$ be a DFA recognizing L. Let $n = |Q|$.

We know $0^n 1^n \in L$. Look at the run of D on $0^n 1^n$ – say $q_0, q_1, \dots, q_n, q_{n+1}, \dots, q_{2n}$.

Some two states of $q_0, \dots, q_n$ are equal. Suppose $q_i = q_j, 0 \leq i < j \leq n$.

Claim 0.3

D also accepts $0^{n+k(j-i)}1^n$ for $k \geq -1$.

Proof. We can construct this inductively. □

Since this constructed string doesn't belong to L for $k \neq 0$. □