



ADVANCED LEVEL

DAY 1

Each problem is worth 7 points.

These problems are to be kept confidential till Monday, 18th May 2020, 1200 hours (GMT).

Problem 1

Let ABC be a triangle with incentre I . The incircle of the triangle ABC touches the sides AC and AB at points E and F , respectively. Let ℓ_B and ℓ_C be the tangents to the circumcircle of BIC at B and C , respectively. Show that there is a circle tangent to EF , ℓ_B and ℓ_C with centre on the line BC .

Problem 2

Geoff has an infinite stock of sweets, which come in n flavours. He arbitrarily distributes some of the sweets amongst n children (a child can get sweets of any subset of all flavours, including the empty set). Call a distribution of sweets k -nice if every group of k children together has sweets in at least k flavours. Find all subsets S of $\{1, 2, \dots, n\}$ such that if a distribution of sweets is s -nice for all $s \in S$, then it is s -nice for all $s \in \{1, 2, \dots, n\}$.

Problem 3

We call a set of integers *special* if it has 4 elements and can be partitioned into 2 disjoint subsets $\{a, b\}$ and $\{c, d\}$ such that $ab - cd = 1$. For every positive integer n , prove that the set $\{1, 2, \dots, 4n\}$ cannot be partitioned into n disjoint special sets.

Problem 4

Prove that, for all sufficiently large integers n , there exist n numbers a_1, a_2, \dots, a_n satisfying the following three conditions:

- Each number a_i is equal to either -1 , 0 or 1 .
- At least $2n/5$ of the numbers a_1, a_2, \dots, a_n are non-zero.
- The sum $a_1/1 + a_2/2 + \dots + a_n/n$ is 0 .

Note: Results with $2/5$ replaced by a constant c will be awarded points depending on the value of c .