Department of Mathematics, IIT Delhi

MTL733: Quiz.

Time: 45 minutes **Date:** 25-10-2021 **Total Marks:** 15

Q.1) Let $B(\cdot)$ be a one-dimensional Brownian motion defined on a filtered probability space $\big(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}\big)$.

a) Find the process $f(t,\omega) \in \mathcal{Y}(0,T)$ such that the random variable $F := \int_0^T B^2(s)\,ds$ can be expressed as

$$F = \mathbb{E}[F] + \int_0^T f(t,\omega)\,dB(t).$$

b) Find the solution $X(t)$ of the SDE

$$dX(t) = 2(3 - \log(X(t)))X(t)\,dt + 4X(t)\,dB(t); \quad X(0) = 1.$$

Determine the distribution of $Y(t) := \log(X(t))$.

4 + (3+2) marks

Q.2) i) Suppose $\mathbf{Y}(t) = \big(Y_1(t), Y_2(t)\big) \in \mathbb{R}^2$ is given by

$$\begin{aligned} dY_1(t) &= 2\,dt + dB_1(t) + 2dB_2(t) + 3dB_3(t) \\ dY_2(t) &= \beta(t)\,dt + dB_1(t) + 2dB_2(t) + 2dB_3(t) \end{aligned}$$

where $\beta(\cdot)$ is bounded adapted process and $\mathbf{B}(t) = \big(B_1(t), B_2(t), B_3(t)\big)$ is 3-dimensional Brownian motion. Show that there are infinitely many equivalent martingale measures Q for $\mathbf{Y}(t)$.

ii) Find the solution of the SDE

$$dX(t) = \frac{1}{3}X^{\frac{1}{3}}\,dt + X^{\frac{2}{3}}\,dB(t); \quad X(0) = 3. \tag{1}$$

Explain whether one can guarantees the existence of solution of (1) from the **existence and uniqueness theorem for SDE** or not.

3+ (2+1) marks

Best of Luck!!!