Assignment - 2 Navneet

Q.1 L.I or L.D

(1) [1 0 0], [1 1 0], [1 1 1]

$\vec{0} = x(1,0,0) + y(1,1,0) + z(1,1,1)$

$(0,0,0) = (x+y+z, y+z, z)$

$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow$ rank = 3

So, infinite solution and ~~set~~ vectors are linearly independent

(2) $[7 \ \underset{\vec{A}}{-3} \ 11 \ -6]$, $[-56 \ \underset{\vec{B}}{24} \ -88 \ 48]$

$\vec{B} = 8\vec{A}$, So, Both are linearly dependent.

(3) [-1 5 0], [16 8 -3], [-64 56 9]

$(0,0,0) = x(-1,5,0) + y(16,8,-3) + z(-64,56,9)$

$= (-x+16y-64z, 5x+8y+56z, -3y+9z)$

$A = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix} \rightarrow$ echelon form: $\begin{bmatrix} 1 & 0 & 16 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

So, The vectors are linearly dependent.

(4) [1 -1 1], [1 1 -1], [-1 1 1], [0 1 0]

↳ As dimension of R^3 is 3 and any set of n vectors ~~with~~ vectors, where $n>3$, then these vectors are linearly dependent.

(5) [2 -4], [1 9], [3 5]

↳ linearly independent.

2) $\lambda = 1-\sqrt{10}$

$$A-\lambda I = \begin{pmatrix} -3+\sqrt{10} & 2 & 3 \\ 2 & \sqrt{10} & -6 \\ -1 & -2 & -1+\sqrt{10} \end{pmatrix} \rightarrow A = \begin{pmatrix} -3+\sqrt{10} & 2 & 3 \\ 2 & \sqrt{10} & -6 \\ 0 & -4+\sqrt{10} & -8+2\sqrt{10} \end{pmatrix}$$

$$R_2 \rightarrow \left(\frac{3-\sqrt{10}}{2}\right) R_2 + R_1$$

$$A = \begin{pmatrix} -3+\sqrt{10} & 2 & 3 \\ 0 & 3/2\sqrt{10}-5 & 3\sqrt{10}-6 \\ 0 & -4+\sqrt{10} & -8+2\sqrt{10} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sqrt{10}-3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$y + 2z = 0$, $x - z(3-\sqrt{10}) = 0$

let $z = k$, $x + (3+\sqrt{10})z = 0$

$y = -2k$ $\quad x = -(3+\sqrt{10})k$

So, $\vec{x} = \cancel{\begin{pmatrix} k \\ -2k \\ -(3+\sqrt{10})k \end{pmatrix}} \rightarrow \vec{x} = \cancel{k\begin{pmatrix} 1 \\ -2 \\ -(3+\sqrt{10}) \end{pmatrix}}$

$$\vec{X} = \begin{pmatrix} -(3+\sqrt{10})k \\ -2k \\ k \end{pmatrix} \rightarrow \boxed{\vec{X} = k\begin{pmatrix} -3+\sqrt{10} \\ -2 \\ 1 \end{pmatrix}}$$

$\lambda = 1+\sqrt{10}$, $A = \begin{pmatrix} -3-\sqrt{10} & 2 & 3 \\ 2 & -\sqrt{10} & -6 \\ -1 & -2 & -\sqrt{10}-1 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 0 & -3+\sqrt{10} \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow A\vec{X} = 0$$

$y + 2z = 0$

$x + (-3+\sqrt{10})z = 0$

let, $z = k$ $\quad y = -2k$, $\quad x = (3-\sqrt{10})k$.

So, $\vec{X} = \begin{pmatrix} (3-\sqrt{10})k \\ -2k \\ k \end{pmatrix} = k\begin{pmatrix} 3-\sqrt{10} \\ -2 \\ 1 \end{pmatrix}$

(2) $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

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$|A - \lambda I| = 0 \rightarrow \begin{vmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)\{\underbrace{(4-\lambda)(1-\lambda)}+2\} = 0 \rightarrow \lambda = 1$

$(\lambda-1)(\lambda-4)+2=0 \rightarrow$

$\lambda^2 - 4\lambda - \lambda + 4 + 2 = 0 \rightarrow \lambda^2 - 5\lambda + 6 = 0.$

$\lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$(\lambda-3)(\lambda-2) = 0 \leftarrow \lambda(\lambda-2) - 3(\lambda-2) = 0$

$\lambda = 3, \lambda = 2$

(1) $\underline{\lambda = 1}$, $A - \lambda I = \begin{pmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix} \rightarrow (A-\lambda I)\vec{x} = 0$

$(A - \lambda I) = \begin{pmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$-2x = 0, \quad 3x + z = 0, \quad y = K$

$x = 0 \qquad z = 0$

So, $\boxed{\vec{X} = K\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$

(2) $\lambda = 2$, $(A - \lambda I) = \begin{pmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{pmatrix} \rightarrow (A - \lambda I) = \begin{pmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$(A - \lambda I) = \begin{pmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

So, $2x + z = 0, \quad -y + z = 0$, let $x = t$

then, $z = -2x$

$z = -2t \qquad y = -2t$

So, $\vec{x} = \begin{pmatrix} t \\ -2t \\ -2t \end{pmatrix} = t\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

(3) $\lambda = 3$, $A - \lambda I = \begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}$

$(A - \lambda I) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix} \rightarrow A - \lambda I = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$x + z = 0$, $-2y + 2z = 0$

$x = -k$ $\quad z = y = k$.

So, $\vec{x} = \begin{pmatrix} -k \\ k \\ k \end{pmatrix} \rightarrow \vec{x} = k \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

(3) $A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{pmatrix} \rightarrow (A - \lambda I) = \begin{pmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{pmatrix}$

$|A - \lambda I| = (5-\lambda)\left((3-\lambda)(-\lambda)\right) = 0$

$\lambda = 5$, $\lambda = 3$, $\lambda = 0$

So, (i) $\lambda = 0$, $A - \lambda I = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{pmatrix}$ $\quad$ ~~-1 0 3~~

$5x = 0 \rightarrow x = 0$

$-x + 3z = 0 \rightarrow z = 0$

$y = 1$

So, $\vec{x} = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} = 1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(ii) $\lambda = 3$, $(A - 3I) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{pmatrix} \rightarrow 2x = 0$

$-3y = 0$, $-z = 0$

$\boxed{\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$

(iii) $\lambda = 5$, $A - 5I = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{pmatrix} \rightarrow -5y = 0, \rightarrow y = 0$

$-x - 2z = 0$

$z = k$, $x = -$

So $\vec{x} = \begin{pmatrix} 2k \\ 0 \\ k \end{pmatrix} = k \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

4. $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{pmatrix} \rightarrow A - \lambda I = \begin{pmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{pmatrix}$

$|A - \lambda I| = (-\lambda)\left[(3-\lambda)(-2-\lambda)\right] \rightarrow \lambda = 0, \ \lambda = -2, \ \lambda = 3$

i) $\lambda = 0$, $(A - 0.I) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{pmatrix}$

$3y + 4z = 0, \quad -2z = 0.$

$y = 0, \ z = 0, \quad x = k$

So, $\boxed{\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$

(ii) $\lambda = -2$, $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow 2x = 0, \ 5y + 4z = 0$

$x = 0, \quad 5k + 4z = 0$

$4z = -5k$

$z = -\frac{5}{4}k$

So, $\vec{x} = \begin{pmatrix} 0 \\ k \\ -5/4k \end{pmatrix} \rightarrow \boxed{\vec{x} = k\begin{pmatrix} 0 \\ 1 \\ -5/4 \end{pmatrix}}$

(iii) $\lambda = 3$, $A = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{pmatrix} \rightarrow -3x = 0, \ 4z = 0$

$-5z = 0.$

So $y = k$

So $\boxed{\vec{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$

5. $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$; $tr(A) = 6$, $det(A) =$

→ because rows and
are same

Now, $\lambda_1 \lambda_2 \lambda_3 = 0$, and

$\lambda_1 = 0$ → repeating

$\lambda_1 + \lambda_2 + \lambda_3 = 6$

$\lambda_2 + \lambda_3 = 6$

so, $\lambda_3 = 6$

Hence, by property of eigenvalues, we can say that $\boxed{\lambda_1 + \lambda_2 + \lambda_3 = 6}$ and λ_1

so, $\lambda = 0$, $\lambda = 0$; $\lambda = 6$

6) [3 -2 0 4], [5 0 0 1], [-6 1 0 1],
[2 0 0 3]

$\rightarrow 0 = x(3,-2,0,4) + y(5,0,0,1) + z(-6,1,0,1) + w(2,0,0,3)$

$0 = 3x+5y-6z+2w, -2x+z, 0, 4x+y+z+3w)$

$$A = \begin{pmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 \end{pmatrix} \rightarrow A = \begin{pmatrix} 3 & 5 & -6 & 2 \\ 4 & 1 & 1 & 3 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 5 & -6 & 2 \\ 4 & 1 & 1 & 3 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 5 & -6 & 2 \\ 0 & -17 & 27 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -9/10 & 2/5 \\ 0 & 0 & 39/10 & 13/5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ $\rightarrow$ infinite solⁿ so, linearly dependent

(7) [3 4 7], [2 0 3], [8 2 3], [5 5 6]

$\Rightarrow$ ~~always~~ linearly dependent.

(8) [6 0 3 1 4 2], [0 -1 2 7 0 5],
[12 3 0 -19 8 -11]

$\Rightarrow 0 = (6x+12z, -y+3z, 3x+2y, x+7y-19z, 4x+8z, 2x+5y-11z)$

$$A = \begin{pmatrix} 6 & 0 & 12 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 & 0 \\ 1 & 7 & -19 & 0 & 0 & 0 \\ 4 & 0 & 8 & 0 & 0 & 0 \\ 2 & 5 & -11 & 0 & 0 & 0 \end{pmatrix} \rightarrow A = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, vector sets are linearly dependent.