Navneet

Linear Algebra - Trisem - II Assignment

Q.1 :- $A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}$ $\rightarrow R_4 \rightarrow R_4 - 2R_3$

$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{pmatrix}$:- $R_3 \rightarrow R_3 - 3R_1$
$R_2 \rightarrow R_2 - 2R_1$

$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{pmatrix}$ $\rightarrow R_4 \rightarrow R_4 + R_3$

$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{pmatrix}$ $R_2 \leftrightarrow R_3$

$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{pmatrix}$ $R_4 \rightarrow R_4 - R_3$

$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $8 \rightarrow$ No of non-zero matrix rows in matrix

$\boxed{\text{Rank, } \rho(A) = 3}$

2 W :- vector space of 2×2 symmetric matrices
$W = \{ A : \text{where } A^T = -A \}$

$\boxed{T : W \rightarrow P_2}$ Where $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-$

rank(T) = dim$(\text{Im } T)$ and nullity(T) = dim(Ker

For $w \in W$ now $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ → symmetric matrix

So, $w = w^T$ → like $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ → $\boxed{b = c}$

and hence the $\boxed{\dim W = 3}$

Now, if we find the rank(T):

standard basis :- $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$\left.\begin{aligned} \text{So, } T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} &= 1 + 0.x + (-1)x^2 = 1 - x^2 \\ T\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} &= -1 + 0.x + 1x^2 = -1 + x^2 \\ T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} &= 0 + 0.x + 0.x^2 = 0 \end{aligned}\right\} \text{Image}$$

So, $M = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ → $M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ → rank$(M) = 1$

So $\boxed{\text{rank}(T) = 1 = \dim(\text{Im } T) = \text{rank}(M)}$

So, $\boxed{\text{rank}(T) = 1}$ and $\boxed{\text{nullity}(T) = 2}$ → ∵ rank + nullity =

3.3 $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ → $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$\lambda^2 - 3\lambda - \lambda + 3$ ← $(2-\lambda)^2 - 1 = 0$

$$\text{Adj. }A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \rightarrow \text{ so, } A^{-1} = \frac{1}{4-(1)} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

n/w, $(A^{-1} - \lambda I) \cdot \vec{x} = 0$

For $\lambda = 1/3$ $\rightarrow \left[\begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} - \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix} \right] \vec{x} = 0$

$$\begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$$

$$\frac{1}{3} \begin{pmatrix} x+y \\ x+y \end{pmatrix} = \vec{0}, \quad x+y=0$$

$x = -y$, $x = -k$, $y = k$

So vector-space $= \begin{pmatrix} -k \\ k \end{pmatrix}$

so, eigenvector for $\lambda = 1/3 \rightarrow$ $\cancel{(-1, 1)}$ $\vec{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

For $\lambda = 1$, $(A^{-1} - I) \cdot \vec{x} = 0$

$$\left(\begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \cdot \vec{x} = \vec{0}$$

$$\begin{pmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \rightarrow \begin{pmatrix} -1/3\, x + 1/3\, y \\ 1/3\, x - 1/3\, y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-x + y = 0, \quad x - y = 0$

$x = y, \quad x = y = k$

so, vector-space $= \begin{pmatrix} k \\ k \end{pmatrix} \rightarrow$ eigen-vector $:\rightarrow \boxed{\vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

(2) Now, eigenvalues of $A + 4I$:- $\lambda + 4$

so, $\boxed{\lambda = 7, 5}$

$$B = A + 4I = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix}$$

$$B = \begin{pmatrix} 6 & -1 \\ -1 & 6 \end{pmatrix}$$

Now, for (1) $\lambda = 7$, $(B - 7I).\vec{x} = \vec{0}$

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \rightarrow \quad x + y = 0$$

$$x = -y$$

So, eigenvector $= (-1, 1)$

(2) $\boxed{\lambda = 5}$, $(B - 5I).\vec{x} = \vec{0}$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \rightarrow \quad x - y = 0$$

$$x = y$$

So eigenvector $= (1, 1)$

Q.4 Eq^n:- $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

$\Rightarrow$ 1st iteration :- $y = 0, z = 0$.

- $3x = 7.85 \rightarrow \boxed{x = 2.61}$
- $0.1x - 0.3z + 19.3 = -7y$

$$y = \frac{0.1 \times 2.61 + 19.3}{-7} \rightarrow \boxed{y = -2.79}$$

- $z = \dfrac{71.4 + 0.2y - 0.3x}{10} = \dfrac{71.4 + 0.2(-2.79) - 0.3 \times 2.61}{10}$

$$z = 7.00$$

So, from 1st iteration :- $\boxed{x = 2.61,\ y = -2.79,\ z = 7.00}$

2nd iteration :- $y = -2.79$, $z = 7.00$

- $x = \dfrac{7.85 + 0.2z + 0.1y}{3} = 2.99$

- $y = \dfrac{0.1x - 0.3z + 19.3}{-7}$, $z = 7$, $x = 2.99$

$$\boxed{y = -2.49}$$

- $z = \dfrac{71.4 + 0.2y - 0.3x}{10}$, $x = 2.99$, $y = -2.49$

$$\boxed{z = 7.0005 = 7}$$

So, from 2nd iteration :- $\boxed{\begin{aligned} x &= 2.99 \\ y &= -2.49 \\ z &= 7 \end{aligned}}$

3rd iteration :- $\rightarrow y = -2.49,\ z = 7$

- $x = \dfrac{7.85 + 0.2z + 0.1y}{3} = 3.00$

- $y = \dfrac{0.1x - 0.3z + 19.3}{-7} \rightarrow x = 3,\ z = 7$

$\boxed{y = -2.5}$

- $z = \dfrac{71.4 + 0.2y - 0.3x}{10} \rightarrow y = -2.5,\ x = 3$

$\boxed{z = 7}$

So, $\boxed{x = 3,\ y = -2.5,\ z = 7}$

Q.5 eqn :- $\left\{\begin{aligned} x + 3y + 2z &= 0 \\ 2x - y + 3z &= 0 \\ 3x - 5y + 4z &= 0 \end{aligned}\right.$ $\quad x + 17y + 4z = 0$

~~3x + 2y + 5z = 0~~ $\quad$ ~~3x + 2y + 5~~ $\quad 3x + 2y + 5z =$

So, $A = \begin{bmatrix} 1 & 17 & 4 \\ 3 & -5 & 4 \\ 3 & 2 & 5 \end{bmatrix} \rightarrow A = \begin{bmatrix} 1 & 17 & 4 \\ 3 & -5 & 4 \\ 0 & 7 & +1 \end{bmatrix}$

$A = \begin{pmatrix} 1 & 17 & 4 \\ 0 & -56 & -8 \\ 0 & 7 & +1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 17 & 4 \\ 0 & -56 & -8 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow$ rank $= 2 < 3$

So, the consistent SOLE have infinite solution.

$x + 17y + 4z = 0, \quad -56y - 8z = 0$

$7y + z = 0$

let, $y = k$, $z = -7k$. $\quad$ ~~let, $z = k$, $y = \frac{-k}{7}$~~

then $x + 17(k) + (-28k) = 0$

$\boxed{x = 11k}$, so, $\boxed{x = 11k}, \boxed{y = k}, \boxed{z = -7k}$; $k \in$

Q. 6 $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

(i) $T(P_1)+T(P_2) = T(P_1+P_2)$

$T(a_1+b_1x+c_1x^2) + T(a_2+b_2x+c_2x^2)$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2 + (a_2+1)+(b_2+1)x+(c_2+1)x^2$

$= (a_1+a_2+2) + (b_1+b_2+2)x + (c_1+c_2+2)x^2$

$T(P_1+P_2) = T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2)$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

As, $\boxed{T(P_1)+T(P_2) \neq T(P_1+P_2)}$

So, T is not a linear transformation

Q. 7 $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$

- For ~~this~~ S to be ~~a~~ basis of R^3, the vectors must be a linear independent.

$x(1,2,3)+y(3,1,0)+z(-2,1,3) = (0,0,0)$

$(x+3y-2z,\ 2x+y+z,\ 3x+3z) = (0,0,0)$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{pmatrix} \rightarrow A = \begin{pmatrix} 1 & 3 & -2 \\ 3 & 0 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -9 & 9 \\ 0 & -5 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -9 & 9 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \text{infinite solution}$$

So, vectors of S are linearly dependent

subspace of $S = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -9 & -9 \end{bmatrix}$ → These two rows vectors are basis for subspace spanning

So, the dimension of subspace is 2.

Q.8 $3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + 7z = 16$.

First iteration :- • $x = \frac{23 + 6y - 2z}{3} = \boxed{7.66}$

• $y = -15 + z + 4x = \boxed{15.64}$ ← $x = 7.66$

• $z = \frac{16 + 3y - x}{7}$ ← $x = 7.66,\ y = 15.64$

$\boxed{z = 6.46}$

So, values :- $x = 7.66,\ y = 15.64,\ z = 6.46$

Second iteration :- • $x = \frac{23 + 6y - 2z}{3}$; $y = 15.64,\ z = 6.46$

$x = 34.64$

• $x = 34.64,\ z = 6.46$

then $y = 4x + z - 15 = 130.02$

• $y = 130.02,\ x = 34.64$

$z = \frac{16 + 3y - x}{7} = 53.06$.

values :- $x = 34.64,\ y = 130.02,\ z = 53.06$

Third iteration :- • $x = \frac{23 + 6y - 2z}{3}$ → $y = 130.02,\ z = 53.06$

$\boxed{x = 232.33}$

• $x = 232.33$ and $z = 53.06$ → $y = 4x + z - 15$
$= 967.38$

• $y = 967.38,\ x = 232.33$

then, $z = \frac{16 + 3y - x}{7} = 383.68$

So, soln :- $\boxed{x = 232.33,\ y = 967.38,\ z = 383.68}$

Q.9 Matrix operations are extensively used in image processing. ~~e~~. Like transpose of matrix is used to rotate the image in various direction and the blur matrix is used to blur certain ~~types~~ area of image.

~~Apart We have~~

Apart from this, images are made up of matrix itself. Images are made up of pixels which are arrang in grid to produce image.

Q.10 . Linear transformation ~~is~~ plays very important role in computer vision. ~~In~~ ~~to~~ Linear transformation is extensively used in manipulating image for various purpose.

One example is rotating image with θ angle. about ~~z-axis~~ x-axis.

For this purpose we use famous rotation matrix in 2D to do this task.

~~so~~ Here, $T: V \to W$,

$$\text{When } T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

If we have to rotate (x, y) about θ, then new x' and y' are:-

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this way we perform this basic operation for each pixel of the image and find the rotated image.

This transformation is also used in image registration, object detection and image alignment.