Assignment - Maths. Navneet

(a)

(i) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$.

$AX=B$; $A=\begin{pmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$; $B=\begin{pmatrix}5\\ 13\\ 32\end{pmatrix}$

Augmented Matrix :-

$$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 3 & 1 & -3 & 13\\ 2 & 19 & -47 & 32\end{pmatrix} \quad \{R_2\to R_2-\tfrac{3}{2}R_1\}$$

$$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & 11/2 & -27/2 & 11/2\\ 2 & 19 & -47 & 32\end{pmatrix} \quad \{R_3\to R_3-R_1$$

$$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & 11/2 & -27/2 & 11/2\\ 0 & 22 & -54 & 27\end{pmatrix} \quad \{R_3\to R_3-4R_2\}$$

$$(A:B)=\begin{pmatrix}2 & -3 & 7 & 5\\ 0 & 11/2 & -27/2 & 11/2\\ 0 & 0 & 0 & 5\end{pmatrix}$$

$\boxed{\rho(A:B)\neq\rho(A)}$ also, $A=\begin{pmatrix}2 & -3 & 7\\ 0 & 11/2 & -37/2\\ 0 & 0 & 0\end{pmatrix}$, $C=\begin{pmatrix}5\\ 11/2\\ 5\end{pmatrix}$

$\rho(A:B)=3$ And $\rho(A)=2$, so No solution exist. (inconsi

(b) (ii) $2x-y+3z=8$, $-x+2y+z=4$, $3x+y-4z=0$

$AX=B$, $A:B=\begin{pmatrix}2 & -1 & 3 & 8\\ -1 & 2 & 1 & 4\\ 3 & 1 & -4 & 0\end{pmatrix}$ $\{R_2\to 2R_2\}$

$$A:B=\begin{pmatrix}2 & -1 & 3 & 8\\ -2 & 4 & 2 & 8\\ 3 & 1 & -4 & 0\end{pmatrix} \quad \{R_2\to R_2+R_1\}$$

$$A:B=\begin{pmatrix}2 & -1 & 3 & 8\\ 0 & 3 & 5 & 16\\ 3 & 1 & -4 & 0\end{pmatrix} \quad \{R_3\to R_3-\tfrac{3}{2}R_1\}$$

$$A:B = \begin{pmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 5/2 & -17/2 & -12 \end{pmatrix} \quad \{R_3 \to 2R_3\}$$

$$A:B = \begin{pmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 5 & -17 & -24 \end{pmatrix} \quad \{R_3 \to R_3 - 5/3 R_2\}$$

$$A:B = \begin{pmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 0 & -76/3 & \cancel{-152} -152/3 \end{pmatrix}$$

$\rho(A:B) = \rho(A) = 2$

So, $AX = B$,

$$\begin{pmatrix} 2 & -1 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -76/3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \\ -152/3 \end{pmatrix}$$

$2x - y + 3z = 8$

$3y + 5z = 16$

$\frac{-76}{3} z = \frac{-152}{3}$ $\longrightarrow$ $z = 2$, $y = 2$, $\cancel{x=2}$ $x = 2$

Hence, Sol^n are $\boxed{(x,y,z) = (2,2,2)}$

(iii) $4x - y = 12$, $\quad -x + 5y - 2z = 0$, $\quad -2x + 4z = -8$

$$\begin{pmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ -8 \end{pmatrix}$$

Augmented matrix, $A:B = \begin{pmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{pmatrix} \quad \{R_2 \to 4R_2 + R_1\}$

$$A:B = \begin{pmatrix} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ -2 & 0 & 4 & -8 \end{pmatrix} \quad \{R_3 \to 2R_3 + R_1\}$$

$$A:B = \begin{pmatrix} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ 0 & -1 & 8 & -4 \end{pmatrix} \quad \{R_3 \to 19R_3 + R_2\}$$

$$A:B = \begin{pmatrix} 4 & -1 & 0 & 12 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & 144 & -64 \end{pmatrix}$$

As, $\rho(A:B) = \rho(A)$, so consistent.
$= 3$. So, unique sol^n

$$\begin{pmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 0 & 144 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ -64 \end{pmatrix}$$

$144z = -64 \rightarrow z = -\frac{64}{144} \rightarrow \boxed{z = -\frac{4}{9}}$

And $19y - 8z = 12 \xrightarrow{z} \boxed{y = \frac{4}{9}}$

And $4x - y = 12 \rightarrow x = \frac{28}{9}$

(b) Eq^n:- $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z = \mu$.

(i) $A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{pmatrix}$ Since, the SOE has no sol^n.
So, $\rho(A:B) \neq \rho(A)$

$(R_2 \rightarrow R_2 - R_1)$ $A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 1 & 2 & \lambda & \mu \end{pmatrix}$ $\{R_3 \rightarrow R_3 - R_1\}$.

$A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{pmatrix}$ $\{R_3 \rightarrow R_3 - R_2\}$

$A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{pmatrix}$ (Now, this is echlon form but as $\rho(A:B) = \rho(A)$. So,

$\lambda - 3 = 0 \rightarrow \boxed{\lambda = 3}$

$\mu - 10 \neq 0 \rightarrow \boxed{\mu \neq 10}$

(ii) For unique sol^n:- $\rho(A:B) = \rho(A) = 3$

Echlon form:- $A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ \mu-10 \end{pmatrix} \rightarrow (\lambda-3)z = \mu - 10$$

$\boxed{\lambda \neq 3, \ \mu \neq 10}$

(iii) for infinitely many sol^n, $\rho(A:B)$

$$A:B = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{pmatrix}$$

So, $\lambda-3=0$, $\mu-10=0$

$\boxed{\lambda=3, \mu=10}$

(c) $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$

$$A:B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{pmatrix} \quad \{R_2 \to R_2 - R_1\}$$

$$A:B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 1 & 4 & 10 & \lambda^2 \end{pmatrix} \quad \{R_3 \to R_3 - R_1\} \longrightarrow A:B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{pmatrix}$$

$\{R_3 \to R_3 - 3R_2\}$

$$A:B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{pmatrix}$$

$\rho(A:B)=3$, $\rho(A)=2$ } but since it have sol^n

So, $\lambda^2-3\lambda+2=0 \rightarrow (\lambda-1)(\lambda-2)=0 \rightarrow \boxed{\lambda=1,2}$

(d) $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$

$AX=0$, since the system of Eq^n is homogenous so sol^n will ~~either~~ to be consistent.

~~augm~~ matrix, $A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{pmatrix}$ $\rightarrow \{R_2 \to R_2 - 2R_1\}$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 1 & -11 & 14 \end{pmatrix} \quad \{R_3 \to R_3 - R_1\}$$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{pmatrix} \quad \{R_3 \to R_3 - 2R_2\}$$

$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{pmatrix}$$

{ since in echlon form

$\therefore$ no. of Eq^n > no. of variable,

So z is free variable

Now $x + 3y - 2z = 0$

$-7y + 8z = 0$

let us set, $\boxed{z = 7}$, then $-7y = -56$

$\boxed{y = 8}$

& $x + 3(8) - 2(7) = 0 \rightarrow x + 24 - 14 = 0 \rightarrow \boxed{x = -10}$

(e) Given Eq^{n}s :- $3x + y - \lambda z = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y + \lambda z = 0$

Possesses non-trivial sol^n.

$$A = \begin{pmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{pmatrix} \quad \{R_2 \rightarrow 3R_2 - 4R_1\}$$

$$A = \begin{pmatrix} 3 & 1 & -\lambda \\ 0 & -10 & 4\lambda - 9 \\ 2\lambda & 4 & \lambda \end{pmatrix}$$

CASE: 1, $\boxed{\lambda = 0}$, then $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & -10 & -9 \\ 0 & 4 & 0 \end{pmatrix}$, But now, if we make element (3,2) to zero then element (3,3) will be non-zero & Hence the condition is not satisfying &, $\boxed{\lambda \neq 0}$

~~or~~

Now for finding λ, we need to work out $|A| = 0$

$$|A| = \begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = \begin{vmatrix} 3 + 2\lambda & 5 & 0 \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix}$$

$$= (3 + 2\lambda)(-2\lambda + 12) - 5(16 + 4\lambda)$$

$$0 = -6\lambda + 36 - 4\lambda^2 + \cancel{12\lambda}\ 24\lambda - 80 - 20\lambda$$

$$0 = -4\lambda^2 - 2\lambda - 44 \rightarrow 2\lambda^2 + \lambda + 22 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{1 - 4(2)(22)}}{4(2)(\cancel{22})}$$

since, $\boxed{D < 0}$, so No real sol^n of λ exist.