Unit-3

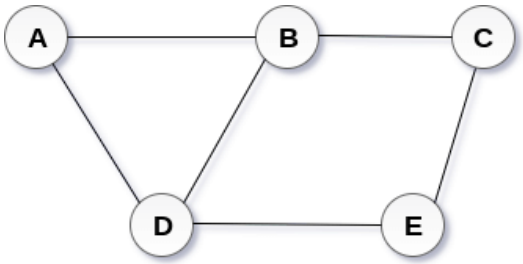
Graph

A graph can be defined as group of vertices and edges that are used to connect these vertices. A graph can be seen as a cyclic tree, where the vertices (Nodes) maintain any complex relationship among them instead of having parent child relationship.

Definition

A graph G can be defined as an ordered set G(V, E) where V(G) represents the set of vertices and E(G) represents the set of edges which are used to connect these vertices.

A Graph G(V, E) with 5 vertices (A, B, C, D, E) and six edges ((A,B), (B,C), (C,E), (E,D), (D,B), (D,A)) is shown in the following figure.



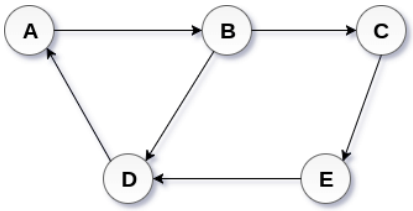
Undirected Graph

Directed and Undirected Graph

A graph can be directed or undirected. However, in an undirected graph, edges are not associated with the directions with them. An undirected graph is shown in the above figure since its edges are not attached with any of the directions. If an edge exists between vertex A and B then the vertices can be traversed from B to A as well as A to B.

In a directed graph, edges form an ordered pair. Edges represent a specific path from some vertex A to another vertex B. Node A is called initial node while node B is called terminal node.

A directed graph is shown in the following figure.



Directed Graph

Graph Terminology

Path

A path can be defined as the sequence of nodes that are followed in order to reach some terminal node V from the initial node U.

Closed Path

A path will be called as closed path if the initial node is same as terminal node. A path will be closed path if $V_0=V_N$.

Simple Path

If all the nodes of the graph are distinct with an exception $V_0=V_N$, then such path P is called as closed simple path.

Cycle

A cycle can be defined as the path which has no repeated edges or vertices except the first and last vertices.

Connected Graph

A connected graph is the one in which some path exists between every two vertices (u, v) in V. There are no isolated nodes in connected graph.

Complete Graph

A complete graph is the one in which every node is connected with all other nodes. A complete graph contain n(n-1)/2 edges where n is the number of nodes in the graph.

Weighted Graph

In a weighted graph, each edge is assigned with some data such as length or weight. The weight of an edge e can be given as w(e) which must be a positive (+) value indicating the cost of traversing the edge.

Digraph

A digraph is a directed graph in which each edge of the graph is associated with some direction and the traversing can be done only in the specified direction.

Loop

An edge that is associated with the similar end points can be called as Loop.

Adjacent Nodes

If two nodes u and v are connected via an edge e, then the nodes u and v are called as neighbours or adjacent nodes.

Degree of the Node

A degree of a node is the number of edges that are connected with that node. A node with degree 0 is called as isolated node.

Graph representation

By Graph representation, we simply mean the technique to be used to store some graph into the computer's memory.

A graph is a data structure that consist a sets of vertices (called nodes) and edges. There are two ways to store Graphs into the computer's memory:

- **Sequential representation** (or, Adjacency matrix representation)
- **Linked list representation** (or, Adjacency list representation)

In sequential representation, an adjacency matrix is used to store the graph. Whereas in linked list representation, there is a use of an adjacency list to store the graph.

Now, let's start discussing the ways of representing a graph in the data structure.

Sequential representation

In sequential representation, there is a use of an adjacency matrix to represent the mapping between vertices and edges of the graph. We can use an adjacency matrix to represent the undirected graph, directed graph, weighted directed graph, and weighted undirected graph.

If adj[i][j] = w, it means that there is an edge exists from vertex i to vertex j with weight w.

An entry A_{ij} in the adjacency matrix representation of an undirected graph G will be 1 if an edge exists between V_i and V_j. If an Undirected Graph G consists of n vertices, then the adjacency matrix for that graph is n x n, and the matrix A = [aij] can be defined as -

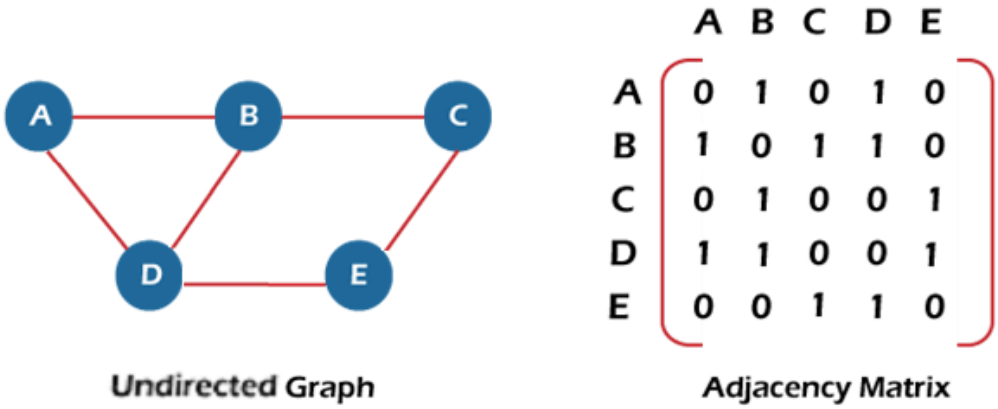
a_{ij} = 1 {if there is a path exists from V_i to V_j}

a_{ij} = 0 {Otherwise}

It means that, in an adjacency matrix, 0 represents that there is no association exists between the nodes, whereas 1 represents the existence of a path between two edges.

If there is no self-loop present in the graph, it means that the diagonal entries of the adjacency matrix will be 0.

Now, let's see the adjacency matrix representation of an undirected graph.



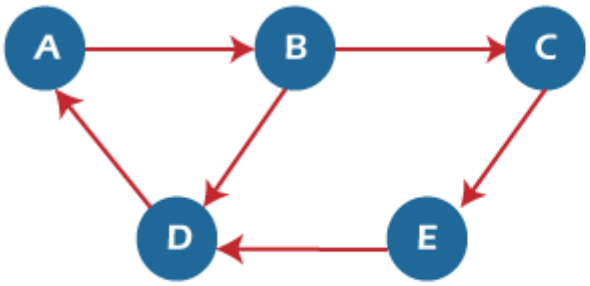
In the above figure, an image shows the mapping among the vertices (A, B, C, D, E), and this mapping is represented by using the adjacency matrix.

There exist different adjacency matrices for the directed and undirected graph. In a directed graph, an entry A_{ij} will be 1 only when there is an edge directed from V_i to V_j.

Adjacency matrix for a directed graph

In a directed graph, edges represent a specific path from one vertex to another vertex. Suppose a path exists from vertex A to another vertex B; it means that node A is the initial node, while node B is the terminal node.

Consider the below-directed graph and try to construct the adjacency matrix of it.



Directed Graph

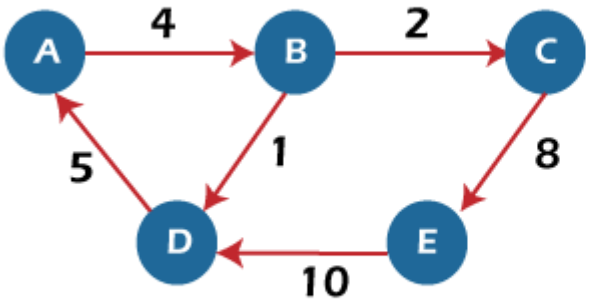
	A	B	C	D	E
A	0	1	0	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	1	0	0	0	0
E	0	0	0	1	0

Adjacency Matrix

In the above graph, we can see there is no self-loop, so the diagonal entries of the adjacent matrix are 0.

Adjacency matrix for a weighted directed graph

It is similar to an adjacency matrix representation of a directed graph except that instead of using the '1' for the existence of a path, here we have to use the weight associated with the edge. The weights on the graph edges will be represented as the entries of the adjacency matrix. We can understand it with the help of an example. Consider the below graph and its adjacency matrix representation. In the representation, we can see that the weight associated with the edges is represented as the entries in the adjacency matrix.



weighted Directed Graph

	A	B	C	D	E
A	0	4	0	0	0
B	0	0	2	1	0
C	0	0	0	0	8
D	5	0	0	0	0
E	0	0	0	10	0

Adjacency Matrix

In the above image, we can see that the adjacency matrix representation of the weighted directed graph is different from other representations. It is because, in this representation, the non-zero values are replaced by the actual weight assigned to the edges.

Adjacency matrix is easier to implement and follow. An adjacency matrix can be used when the graph is dense and a number of edges are large.

Though, it is advantageous to use an adjacency matrix, but it consumes more space. Even if the graph is sparse, the matrix still consumes the same space.

Linked list representation

An adjacency list is used in the linked representation to store the Graph in the computer's memory. It is efficient in terms of storage as we only have to store the values for edges.

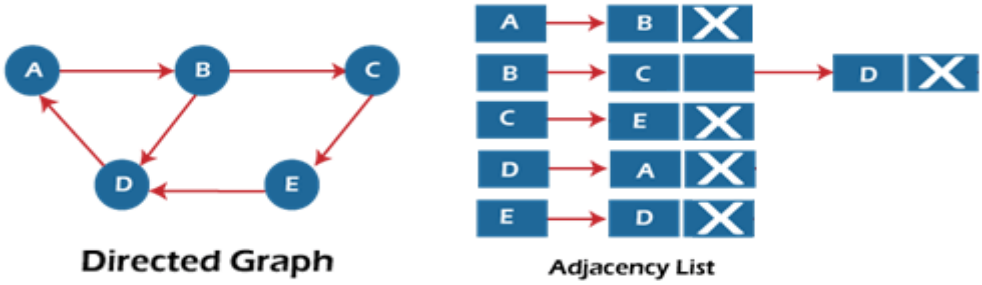
Let's see the adjacency list representation of an undirected graph.

In the above figure, we can see that there is a linked list or adjacency list for every node of the graph. From vertex A, there are paths to vertex B and vertex D. These nodes are linked to nodes A in the given adjacency list.

An adjacency list is maintained for each node present in the graph, which stores the node value and a pointer to the next adjacent node to the respective node. If all the adjacent nodes are traversed, then store the NULL in the pointer field of the last node of the list.

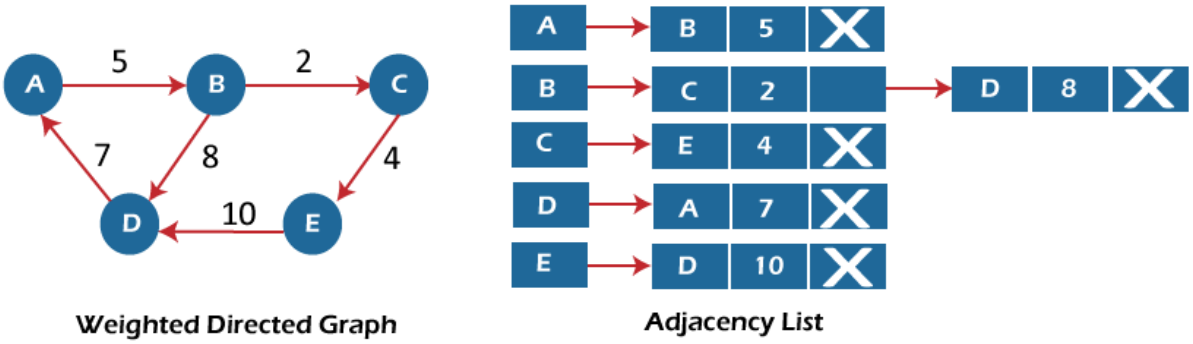
The sum of the lengths of adjacency lists is equal to twice the number of edges present in an undirected graph.

Now, consider the directed graph, and let's see the adjacency list representation of that graph.



For a directed graph, the sum of the lengths of adjacency lists is equal to the number of edges present in the graph.

Now, consider the weighted directed graph, and let's see the adjacency list representation of that graph.



In the case of a weighted directed graph, each node contains an extra field that is called the weight of the node.

In an adjacency list, it is easy to add a vertex. Because of using the linked list, it also saves space.

Advantages of Graphs:

1. Versatility: Graphs can represent a wide range of relationships and connections between entities, making them versatile for modeling various real-world scenarios.

2. Complex Relationships: They are suitable for representing complex relationships such as networks, social connections, and transportation routes.

3. Traversal and Search: Graph algorithms enable efficient traversal and search operations, facilitating tasks like finding paths, determining connectivity, and exploring relationships.

4. Data Representation: Graphs provide a compact and efficient way to represent data structures like trees, maps, and networks.

Disadvantages of Graphs:

1. Complexity: Graphs can be complex to implement and analyze, especially in large-scale systems with many nodes and edges.

2. Memory Usage: Graphs may require more memory compared to other data structures, especially if storing additional information (e.g., weights) for each edge or node.

3. Algorithm Complexity: Some graph algorithms, such as finding the shortest path or detecting cycles, can have high time complexity, making them computationally expensive for large graphs.

4. Maintaining Consistency: Ensuring the consistency and integrity of graph data structures, especially in dynamic environments with frequent updates, can be challenging.

Applications of Graph Data Structure

1. Social Networks: Graphs are used to represent social networks, where users are represented as nodes, and connections between users (friendships, followers) are represented as edges.

2. Transportation Networks: Graphs model transportation networks such as road networks, railway lines, and flight routes, where nodes represent cities or locations, and edges represent connections or routes between them.

3. Internet Routing: In the internet, routers and servers are represented as nodes, and the connections between them are represented as edges. Graphs help in modeling and optimizing internet routing protocols.

4. Recommendation Systems: Graphs are used in recommendation systems to model relationships between users and items (such as movies, books, or products). Users and items are represented as nodes, and edges represent interactions or preferences.

5. Dependency Management: Graphs represent dependencies between modules or components in software engineering. Nodes represent modules, and edges represent dependencies between them, helping in managing dependencies and building software systems.

6. Biological Networks: Graphs are used to represent biological networks such as protein-protein interaction networks, gene regulatory networks, and metabolic pathways. Nodes represent biological entities, and edges represent interactions or relationships between them, aiding in understanding biological systems.

Graph traversal

Graph traversal in data structures is the process of systematically visiting all the nodes (vertices) and edges of a graph. It's akin to exploring a map to understand its layout and connections. There are two main methods of graph traversal: Depth-First Search (DFS) and Breadth-First Search (BFS).

Depth-First Search (DFS):
In DFS, the traversal starts at a chosen vertex and explores as deeply as possible along each branch before backtracking. It's like exploring a maze by choosing a path and following it until hitting a dead end, then backtracking to try another path. DFS is often implemented using recursion or a stack.

Breadth-First Search (BFS):
In BFS, the traversal starts at a chosen vertex and explores all of its neighbors before moving on to the next level of vertices. It's like exploring a maze by checking all adjacent paths before moving to the next set of paths. BFS is often implemented using a queue.

Both DFS and BFS have their own advantages and applications. DFS is useful for tasks such as topological sorting and cycle detection, while BFS is useful for tasks such as finding shortest paths and determining connected components.

Overall, graph traversal is a fundamental operation in graph theory and is used in various applications such as pathfinding, network analysis, and recommendation systems. It helps in understanding the structure and connectivity of a graph and extracting valuable information from it.

Breadth First Search or BFS for a Graph

Breadth First Search (BFS) is a fundamental graph traversal algorithm. It involves visiting all the connected nodes of a graph in a level-by-level manner.
Breadth First Search (BFS) is a graph traversal algorithm that explores all the vertices in a graph at the current depth before moving on to the vertices at the next depth level. It starts at a specified vertex and visits all its neighbors before moving on to the next level of neighbors. BFS is commonly used in algorithms for pathfinding, connected components, and shortest path problems in graphs.
Breadth First Search (BFS) for a Graph Algorithm:
Let's discuss the algorithm for the BFS:

1. Initialization: Enqueue the starting node into a queue and mark it as visited.
2. Exploration: While the queue is not empty:
    a. Dequeue a node from the queue and visit it (e.g., print its value).
    b. For each unvisited neighbor of the dequeued node:
        i. Enqueue the neighbor into the queue.
        ii. Mark the neighbor as visited.
3. Termination: Repeat step 2 until the queue is empty.

This algorithm ensures that all nodes in the graph are visited in a breadth-first manner, starting from the starting node.

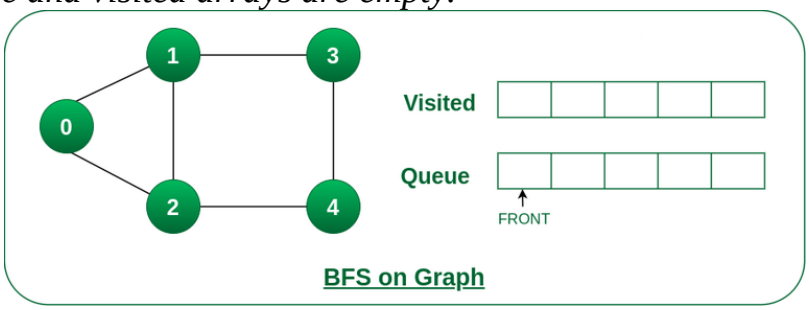
How Does the BFS Algorithm Work?
Starting from the root, all the nodes at a particular level are visited first and then the nodes of the next level are traversed till all the nodes are visited.

To do this a queue is used. All the adjacent unvisited nodes of the current level are pushed into the queue and the nodes of the current level are marked visited and popped from the queue.

Illustration:

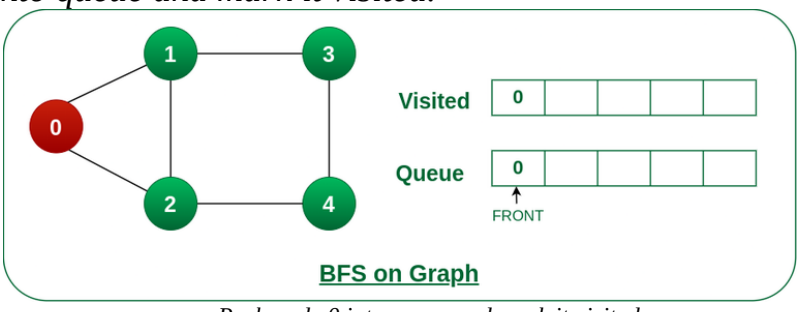
Let us understand the working of the algorithm with the help of the following example.

Step1: *Initially queue and visited arrays are empty.*



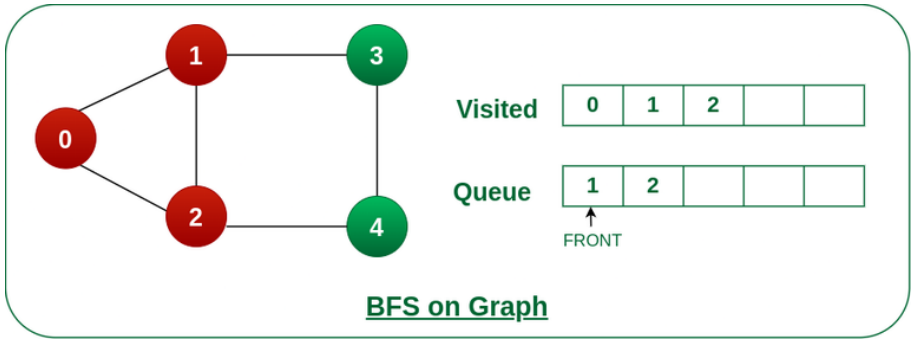
Queue and visited arrays are empty initially.

Step2: *Push node 0 into queue and mark it visited.*



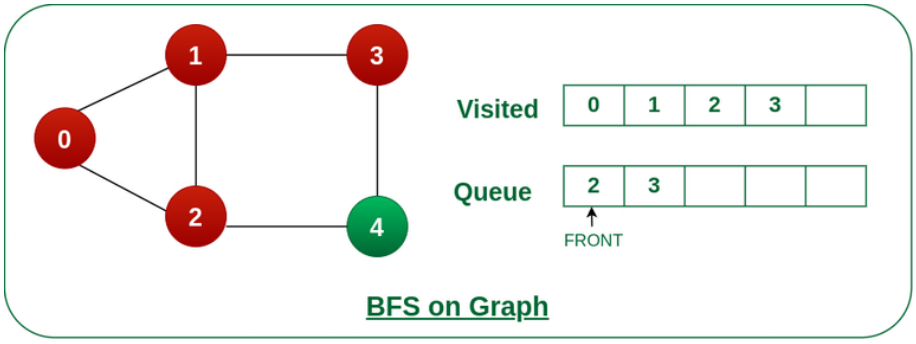
Push node 0 into queue and mark it visited.

Step 3: *Remove node 0 from the front of queue and visit the unvisited neighbours and push them into queue.*



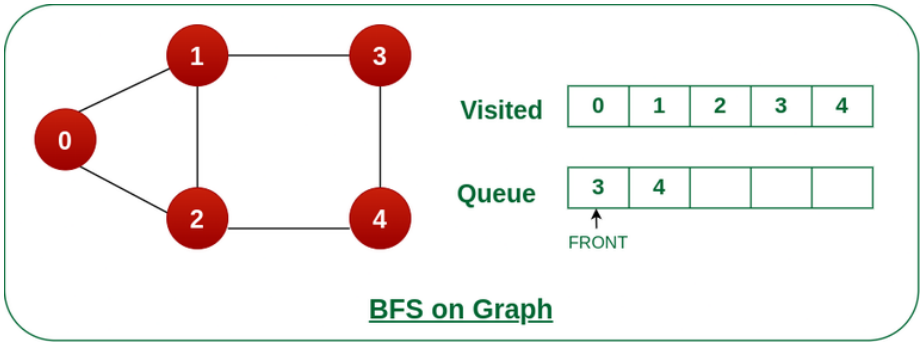
Remove node 0 from the front of queue and visited the unvisited neighbours and push into queue.

Step 4: *Remove node 1 from the front of queue and visit the unvisited neighbours and push them into queue.*



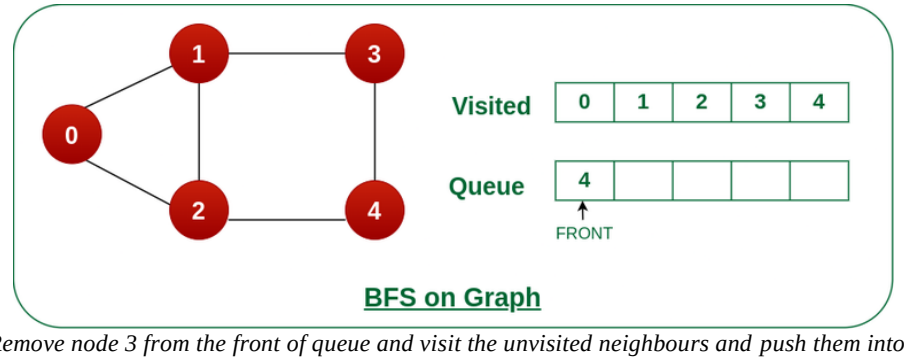
Remove node 1 from the front of queue and visited the unvisited neighbours and push

Step 5: *Remove node 2 from the front of queue and visit the unvisited neighbours and push them into queue.*



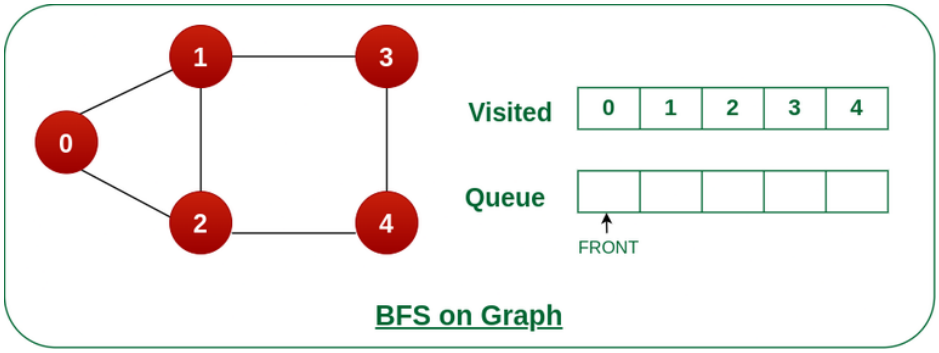
Remove node 2 from the front of queue and visit the unvisited neighbours and push them into queue.

Step 6: *Remove node 3 from the front of queue and visit the unvisited neighbours and push them into queue.*
As we can see that every neighbours of node 3 is visited, so move to the next node that are in the front of the queue.



Remove node 3 from the front of queue and visit the unvisited neighbours and push them into queue.

Steps 7: *Remove node 4 from the front of queue and visit the unvisited neighbours and push them into queue.*
As we can see that every neighbours of node 4 are visited, so move to the next node that is in the front of the queue.



Remove node 4 from the front of queue and visit the unvisited neighbours and push them into queue.

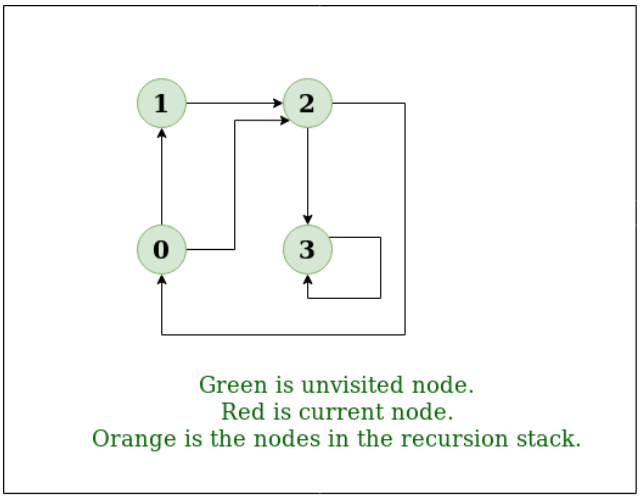
Now, Queue becomes empty, So, terminate these process of iteration.

Depth First Traversal (or DFS)

Depth First Traversal (or DFS) for a graph is similar to Depth First Traversal of a tree. The only catch here is, that, unlike trees, graphs may contain cycles (a node may be visited twice). To avoid processing a node more than once, use a boolean visited array. A graph can have more than one DFS traversal.

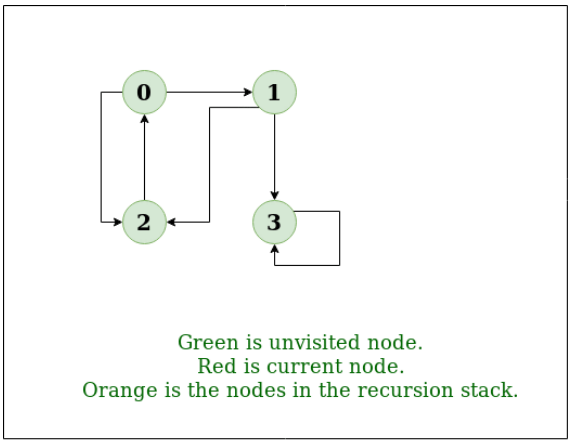
Example:

Input: *n = 4, e = 6*
0 -> 1, 0 -> 2, 1 -> 2, 2 -> 0, 2 -> 3, 3 -> 3
Output: *DFS from vertex 1 : 1 2 0 3*
Explanation:
DFS Diagram:



Example 1

Input: *n = 4, e = 6*
2 -> 0, 0 -> 2, 1 -> 2, 0 -> 1, 3 -> 3, 1 -> 3
Output: *DFS from vertex 2 : 2 0 1 3*
Explanation:
DFS Diagram:

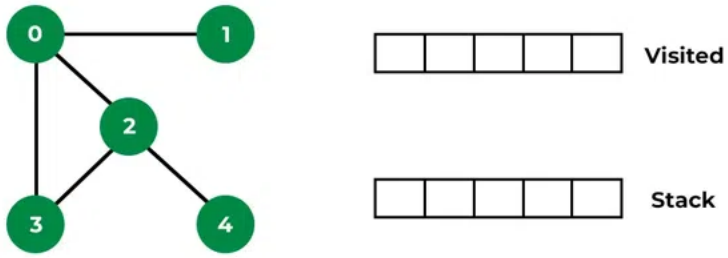


How does DFS work?

Depth-first search is an algorithm for traversing or searching tree or graph data structures. The algorithm starts at the root node (selecting some arbitrary node as the root node in the case of a graph) and explores as far as possible along each branch before backtracking.

Let us understand the working of **Depth First Search** with the help of the following illustration:

Step1: *Initially stack and visited arrays are empty.*



DFS on Graph

Stack and visited arrays are empty initially.

Step 2: *Visit 0 and put its adjacent nodes which are not visited yet into the stack.*

Visit node 0 and put its adjacent nodes (1, 2, 3) into the stack

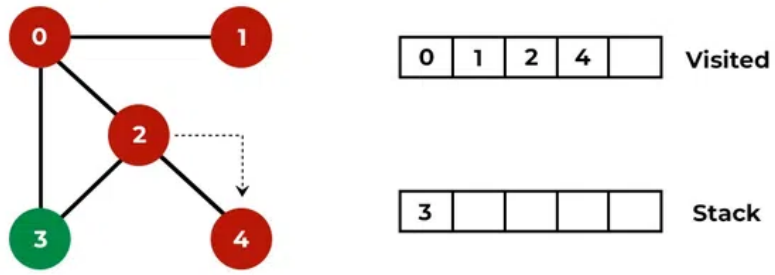
Step 3: *Now, Node 1 at the top of the stack, so visit node 1 and pop it from the stack and put all of its adjacent nodes which are not visited in the stack.*

Visit node 1

Step 4: *Now, Node 2 at the top of the stack, so visit node 2 and pop it from the stack and put all of its adjacent nodes which are not visited (i.e, 3, 4) in the stack.*

Visit node 2 and put its unvisited adjacent nodes (3, 4) into the stack

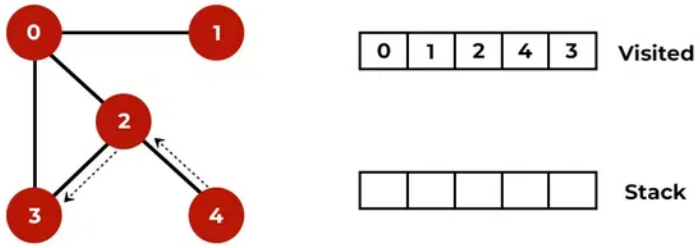
Step 5: *Now, Node 4 at the top of the stack, so visit node 4 and pop it from the stack and put all of its adjacent nodes which are not visited in the stack.*



DFS on Graph

Visit node 4

Step 6: *Now, Node 3 at the top of the stack, so visit node 3 and pop it from the stack and put all of its adjacent nodes which are not visited in the stack.*



DFS on Graph

Visit node 3

Now, Stack becomes empty, which means we have visited all the nodes and our DFS traversal ends.

Depth-First Search (DFS) Question:
Question: Perform Depth-First Search (DFS) on the following graph starting from vertex A.

```
    A
   / \
  B   C
 / \   \
D   E   F
```

Solution:
The traversal order for DFS starting from vertex A is: A -> B -> D -> E -> C -> F

Breadth-First Search (BFS) Question:
Question: Perform Breadth-First Search (BFS) on the following graph starting from vertex A.

```
    A
   / \
  B   C
 / \   \
D   E   F
```

Solution:
The traversal order for BFS starting from vertex A is: A -> B -> C -> D -> E -> F

Comparison of Breadth-First Search (BFS) and Depth-First Search (DFS) with examples:

Breadth-First Search (BFS):

1. BFS explores all the nodes at the present depth before moving on to the nodes at the next depth.
2. It uses a queue data structure to maintain the order of exploration.
3. BFS is useful for finding the shortest path in an unweighted graph.
4. Example: Exploring all the rooms on each floor of a building before moving to the next floor.

Depth-First Search (DFS):

1. DFS explores as far as possible along each branch before backtracking.
2. It uses a stack data structure or recursion to maintain the order of exploration.
3. DFS is useful for tasks like topological sorting and cycle detection.
4. Example: Searching through a maze by exploring a path until reaching a dead end, then backtracking and trying another path.

In summary, BFS explores nodes level by level, while DFS explores as deeply as possible along each branch. Each algorithm has its own advantages and applications depending on the problem at hand.

Spanning tree

A spanning tree can be defined as the subgraph of an undirected connected graph. It includes all the vertices along with the least possible number of edges. If any vertex is missed, it is not a spanning tree. A spanning tree is a subset of the graph that does not have cycles, and it also cannot be disconnected.

A spanning tree consists of (n-1) edges, where 'n' is the number of vertices (or nodes). Edges of the spanning tree may or may not have weights assigned to them. All the possible spanning trees created from the given graph G would have the same number of vertices, but the number of edges in the spanning tree would be equal to the number of vertices in the given graph minus 1.

A complete undirected graph can have $\mathbf{n^{n-2}}$ number of spanning trees where **n** is the number of vertices in the graph. Suppose, if **n = 5**, the number of maximum possible spanning trees would be $\mathbf{5^{5-2} = 125.}$

Applications of the spanning tree

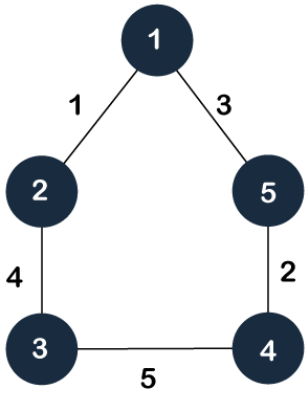
Basically, a spanning tree is used to find a minimum path to connect all nodes of the graph. Some of the common applications of the spanning tree are listed as follows -

- Cluster Analysis
- Civil network planning
- Computer network routing protocol

Now, let's understand the spanning tree with the help of an example.

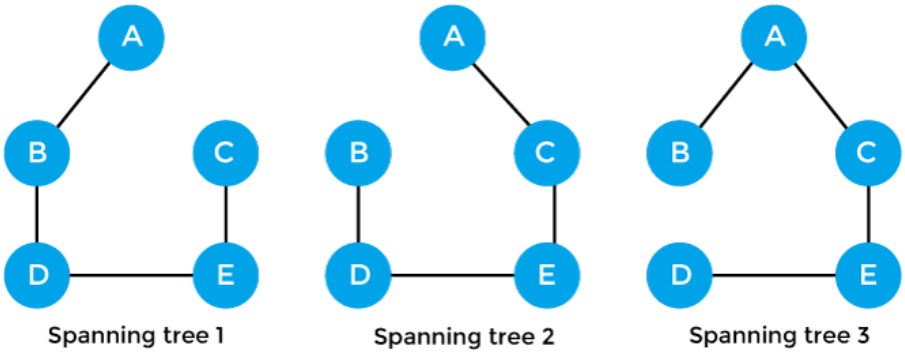
Example of Spanning tree

Suppose the graph be -



As discussed above, a spanning tree contains the same number of vertices as the graph, the number of vertices in the above graph is 5; therefore, the spanning tree will contain 5 vertices. The edges in the spanning tree will be equal to the number of vertices in the graph minus 1. So, there will be 4 edges in the spanning tree.

Some of the possible spanning trees that will be created from the above graph are given as follows -



Properties of spanning-tree

Some of the properties of the spanning tree are given as follows -

- There can be more than one spanning tree of a connected graph G.
- A spanning tree does not have any cycles or loop.
- A spanning tree is **minimally connected,** so removing one edge from the tree will make the graph disconnected.
- A spanning tree is **maximally acyclic,** so adding one edge to the tree will create a loop.
- There can be a maximum $\mathbf{n^{n-2}}$ number of spanning trees that can be created from a complete graph.
- A spanning tree has **n-1** edges, where 'n' is the number of nodes.
- If the graph is a complete graph, then the spanning tree can be constructed by removing maximum (e-n+1) edges, where 'e' is the number of edges and 'n' is the number of vertices.

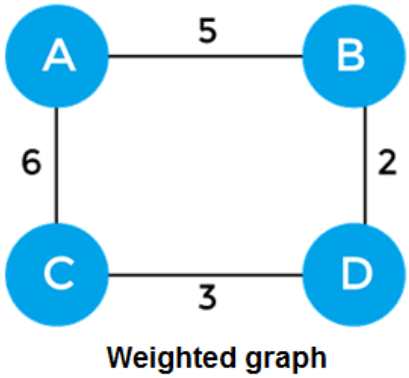
So, a spanning tree is a subset of connected graph G, and there is no spanning tree of a disconnected graph.

Minimum Spanning tree

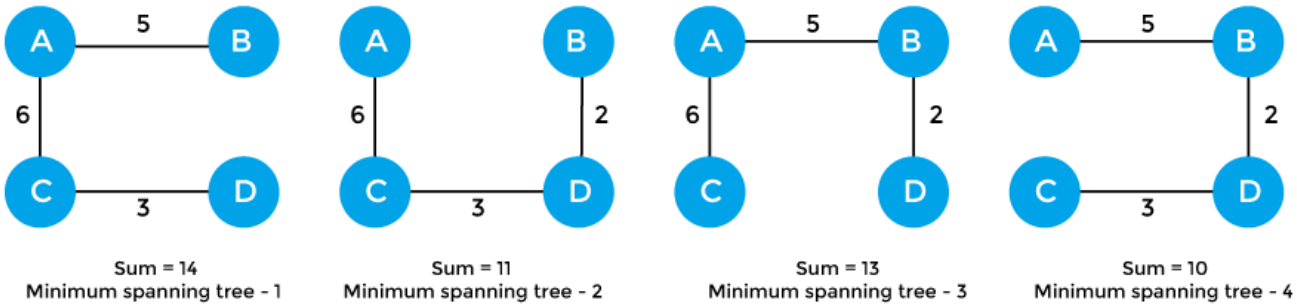
A minimum spanning tree can be defined as the spanning tree in which the sum of the weights of the edge is minimum. The weight of the spanning tree is the sum of the weights given to the edges of the spanning tree. In the real world, this weight can be considered as the distance, traffic load, congestion, or any random value.

Example of minimum spanning tree

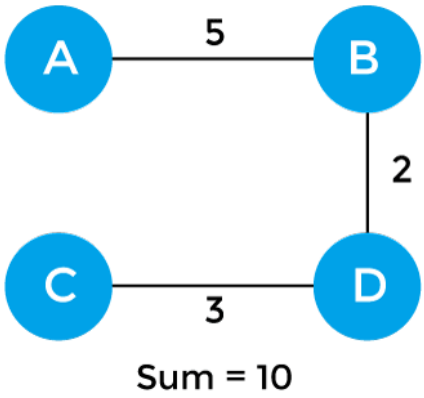
Let's understand the minimum spanning tree with the help of an example.



The sum of the edges of the above graph is 16. Now, some of the possible spanning trees created from the above graph are -



So, the minimum spanning tree that is selected from the above spanning trees for the given weighted graph is -



Applications of minimum spanning tree

The applications of the minimum spanning tree are given as follows -

- Minimum spanning tree can be used to design water-supply networks, telecommunication networks, and electrical grids.
- It can be used to find paths in the map.

Algorithms for Minimum spanning tree

A minimum spanning tree can be found from a weighted graph by using the algorithms given below -

- Prim's Algorithm
- Kruskal's Algorithm

Let's see a brief description of both of the algorithms listed above.

Prim's algorithm - It is a greedy algorithm that starts with an empty spanning tree. It is used to find the minimum spanning tree from the graph. This algorithm finds the subset of edges that includes every vertex of the graph such that the sum of the weights of the edges can be minimized.

Kruskal's algorithm - This algorithm is also used to find the minimum spanning tree for a connected weighted graph. Kruskal's algorithm also follows greedy approach, which finds an optimum solution at every stage instead of focusing on a global optimum.

Prim's Algorithm

Spanning tree - A spanning tree is the subgraph of an undirected connected graph.

Minimum Spanning tree - Minimum spanning tree can be defined as the spanning tree in which the sum of the weights of the edge is minimum. The weight of the spanning tree is the sum of the weights given to the edges of the spanning tree.

Prim's Algorithm is a greedy algorithm that is used to find the minimum spanning tree from a graph. Prim's algorithm finds the subset of edges that includes every vertex of the graph such that the sum of the weights of the edges can be minimized.

Prim's algorithm starts with the single node and explores all the adjacent nodes with all the connecting edges at every step. The edges with the minimal weights causing no cycles in the graph got selected.

How does the prim's algorithm work?

Prim's algorithm is a greedy algorithm that starts from one vertex and continue to add the edges with the smallest weight until the goal is reached. The steps to implement the prim's algorithm are given as follows -

- First, we have to initialize an MST with the randomly chosen vertex.
- Now, we have to find all the edges that connect the tree in the above step with the new vertices. From the edges found, select the minimum edge and add it to the tree.
- Repeat step 2 until the minimum spanning tree is formed.

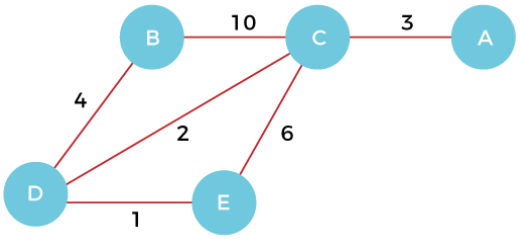
The applications of prim's algorithm are -

- Prim's algorithm can be used in network designing.
- It can be used to make network cycles.
- It can also be used to lay down electrical wiring cables.

Example of prim's algorithm

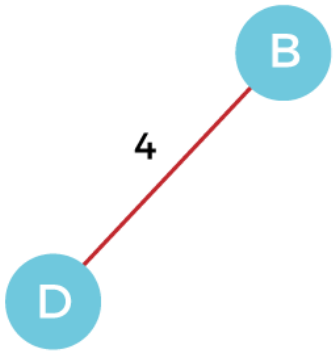
Now, let's see the working of prim's algorithm using an example. It will be easier to understand the prim's algorithm using an example.

Suppose, a weighted graph is -

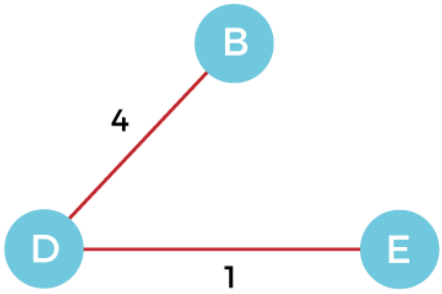


Step 1 - First, we have to choose a vertex from the above graph. Let's choose B.

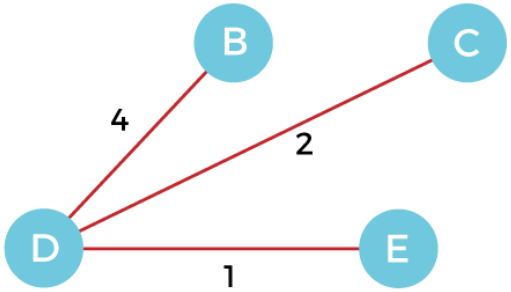
Step 2 - Now, we have to choose and add the shortest edge from vertex B. There are two edges from vertex B that are B to C with weight 10 and edge B to D with weight 4. Among the edges, the edge BD has the minimum weight. So, add it to the MST.



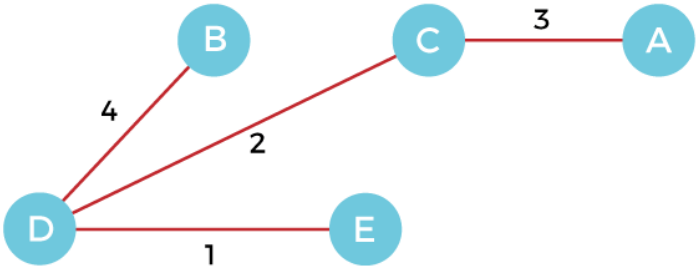
Step 3 - Now, again, choose the edge with the minimum weight among all the other edges. In this case, the edges DE and CD are such edges. Add them to MST and explore the adjacent of C, i.e., E and A. So, select the edge DE and add it to the MST.



Step 4 - Now, select the edge CD, and add it to the MST.



Step 5 - Now, choose the edge CA. Here, we cannot select the edge CE as it would create a cycle to the graph. So, choose the edge CA and add it to the MST.



So, the graph produced in step 5 is the minimum spanning tree of the given graph. The cost of the MST is given below -

Cost of MST = 4 + 2 + 1 + 3 = 10 units.

Algorithm

1. Step 1: Select a starting vertex
2. Step 2: Repeat Steps 3 and 4 until there are fringe vertices
3. Step 3: Select an edge 'e' connecting the tree vertex and fringe vertex that has minimum weight
4. Step 4: Add the selected edge and the vertex to the minimum spanning tree T
5. [END OF LOOP]

6. Step 5: EXIT

Kruskal's Algorithm

Spanning tree - A spanning tree is the subgraph of an undirected connected graph.

Minimum Spanning tree - Minimum spanning tree can be defined as the spanning tree in which the sum of the weights of the edge is minimum. The weight of the spanning tree is the sum of the weights given to the edges of the spanning tree.

Kruskal's Algorithm is used to find the minimum spanning tree for a connected weighted graph. The main target of the algorithm is to find the subset of edges by using which we can traverse every vertex of the graph. It follows the greedy approach that finds an optimum solution at every stage instead of focusing on a global optimum.

How does Kruskal's algorithm work?

In Kruskal's algorithm, we start from edges with the lowest weight and keep adding the edges until the goal is reached. The steps to implement Kruskal's algorithm are listed as follows -

- First, sort all the edges from low weight to high.
- Now, take the edge with the lowest weight and add it to the spanning tree. If the edge to be added creates a cycle, then reject the edge.
- Continue to add the edges until we reach all vertices, and a minimum spanning tree is created.

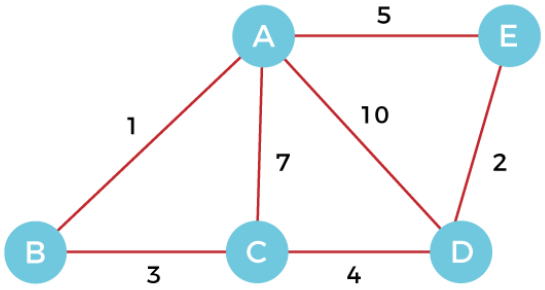
The applications of Kruskal's algorithm are -

- Kruskal's algorithm can be used to layout electrical wiring among cities.
- It can be used to lay down LAN connections.

Example of Kruskal's algorithm

Now, let's see the working of Kruskal's algorithm using an example. It will be easier to understand Kruskal's algorithm using an example.

Suppose a weighted graph is -



The weight of the edges of the above graph is given in the below table -

Edge	AB	AC	AD	AE	BC	CD	DE
Weight	1	7	10	5	3	4	2

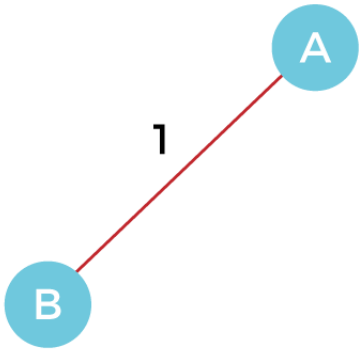
Now, sort the edges given above in the ascending order of their weights.

Edge	AB	DE	BC	CD	AE	AC	AD

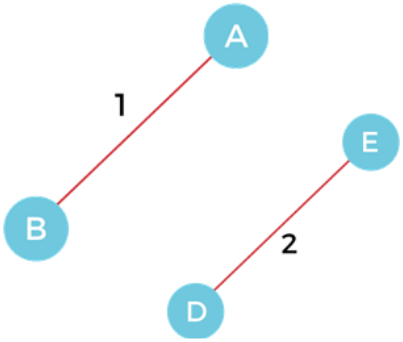
Weight	1	2	3	4	5	7	10

Now, let's start constructing the minimum spanning tree.

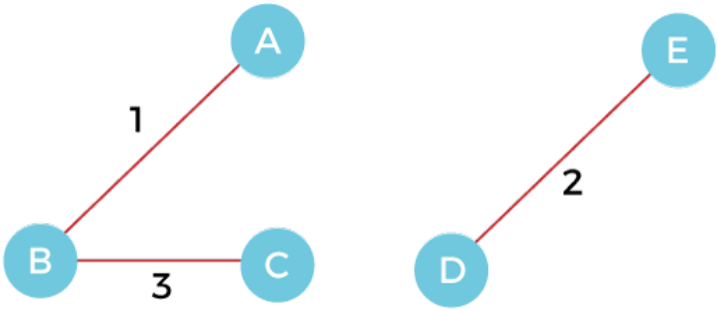
Step 1 - First, add the edge **AB** with weight **1** to the MST.



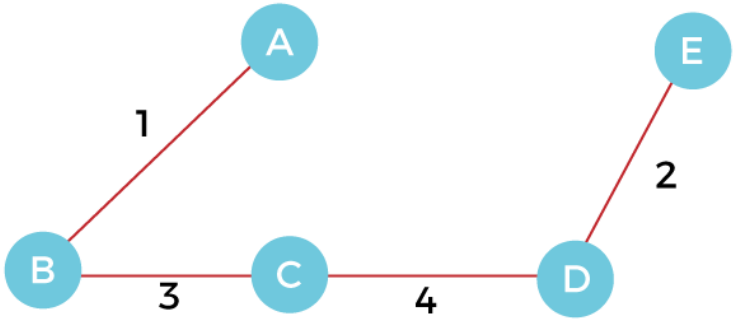
Step 2 - Add the edge **DE** with weight **2** to the MST as it is not creating the cycle.



Step 3 - Add the edge **BC** with weight **3** to the MST, as it is not creating any cycle or loop.



Step 4 - Now, pick the edge **CD** with weight **4** to the MST, as it is not forming the cycle.

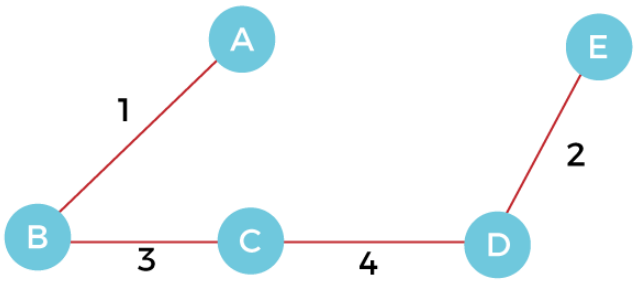


Step 5 - After that, pick the edge **AE** with weight **5.** Including this edge will create the cycle, so discard it.

Step 6 - Pick the edge **AC** with weight **7.** Including this edge will create the cycle, so discard it.

Step 7 - Pick the edge **AD** with weight **10.** Including this edge will also create the cycle, so discard it.

So, the final minimum spanning tree obtained from the given weighted graph by using Kruskal's algorithm is -



The cost of the MST is = AB + DE + BC + CD = 1 + 2 + 3 + 4 = 10.

Now, the number of edges in the above tree equals the number of vertices minus 1. So, the algorithm stops here.

Algorithm

1. Step 1: Create a forest F in such a way that every vertex of the graph is a separate tree.
2. Step 2: Create a set E that contains all the edges of the graph.
3. Step 3: Repeat Steps 4 and 5 **while** E is NOT EMPTY and F is not spanning
4. Step 4: Remove an edge from E with minimum weight
5. Step 5: IF the edge obtained in Step 4 connects two different trees, then add it to the forest F
6. (**for** combining two trees into one tree).
7. ELSE
8. Discard the edge
9. Step 6: END