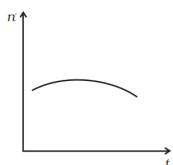


* Choose The Right Answer From The Given Options.[1 Marks Each]

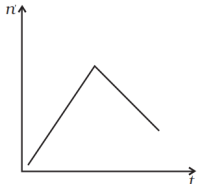
[28]

1. A train whistling at constant frequency is moving towards a station at a constant speed V . The train goes past a stationary observer on the station. The frequency n' of the sound as heard by the observer is plotted as a function of time t Identify the expected curve:

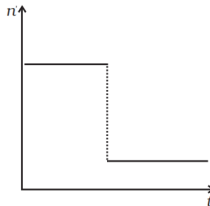
(A)



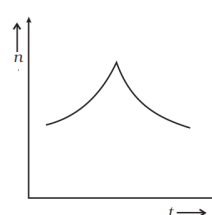
(B)



(C)

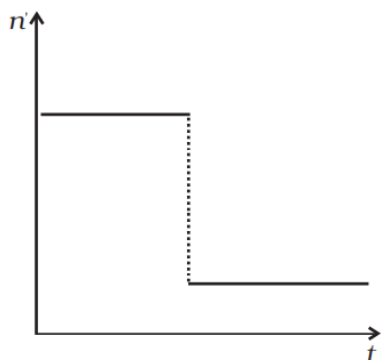


(D)



Ans. :

C.



Explanation:

When observer is at rest and source of sound is moving towards observer then observed frequency n' .

$$n' = \left(\frac{v}{v - v_s} \right) n_0$$

Where n_0 original frequency of source of sound

v = speed of sound in medium

$\therefore n' > n_0$ v_s = speed of source

When source is moving away from observer

$$n' = \frac{v}{(v + v_s)} n_0 \quad n'' < n_0$$

Hence, the frequencies in both cases are same and $n' > n''$. so graph (c) verifies the answer.

2. Equation of a plane progressive wave is given by $y = 0.6 \sin 2\pi \left(t - \frac{x}{2} \right)$. On reflection from a denser medium its amplitude becomes $2/3$ of the amplitude of the incident

wave. The equation of the reflected wave is:

(A) $y = 0.6 \sin 2\pi \left(t + \frac{x}{2} \right)$

(B) $y = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$

(C) $y = 0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$

(D) $y = -0.4 \sin 2\pi \left(t - \frac{x}{2} \right)$

Ans. :

b. $y = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$

Explanation:

After reflection of wave changes by phase 180°

$$y_i = 0.6 \sin 2\pi \left[t + \frac{x}{2} \right]$$

$$y_r \left(\frac{2}{3} \times 0.6 \right) \sin 2\pi \left[\pi + t + \frac{x}{2} \right]$$

$$y_r = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right). \text{ Hence verifies the option (b).}$$

3. A particle has displacement y given by $y = 3 \sin(5\pi t + \phi)$, where y is in metre and t is in second. What are frequency and period of motion?

(A) 0.4Hz, 2.5s

(B) 2.5Hz, 0.4s

(C) 2.5Hz, 2.5s

(D) 0.4Hz, 0.4s

Ans. :

b. 2.5Hz, 0.4s

Explanation:

Comparison with the standard equation shows that

$$\frac{2\pi}{T} = 5\pi, T = \frac{2\pi}{5\pi} = 0.4s$$

$$n = \frac{1}{T} = \frac{1}{.4} = 2.5Hz$$

4. The transverse displacement of a string (clamped at its both ends) is given by $y(x, t) = 0.06 \sin(1\pi x / 3) \cos(120\pi t)$. All the points on the string between two consecutive nodes vibrate with

(A) Same frequency.

(B) Same phase.

(C) Same energy.

(D) Different amplitude.

Ans. :

a. Same frequency.

b. Same phase.

d. Different amplitude.

Explanation:

The frequencies of all particles are same, verifies the option (a).

particles between any two consecutive nodes vibrates either upside or downside having same phase $120\pi t$ at any time, verifies the option (b)

particles have different energies. so rejects the option (c)

As the amplitude of different particles are different between two nodes energy $(E) \propto A^2$. verifies the option (d)

5. A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compressions and rarefactions:
- (A) Density remains constant. (B) Boyle's law is obeyed.
(C) Bulk modulus of air oscillates. (D) There is no transfer of heat.

Ans. :

- d. There is no transfer of heat.

Explanation:

- The density of medium particles are maximum and minimum at compression and rarefaction point, so rejects option (a).
 - Also density changes very rapidly, so temperature of medium increases. So, rejects option (b).
 - Bulk modulus of air remains constant, rejects option (c).
 - The time of compressions and rarefaction is very small so heat does not transfer.
6. The whistle of a railway engine is heard in winter at much longer distances. This is due to:
- (A) Decrease in velocity of sound in winter.
(B) Decrease in the density of air w.r.t. height from the surface of the earth.
(C) Cold air absorbs much smaller energy from sound waves.
(D) Increase in the density of air w.r.t. height from the surface of the earth.

Ans. :

- a. Decrease in velocity of sound in winter.

7. If equation of sound wave is $y = 0.0015 \sin(62.4x + 316t)$, then its wavelength will be:
- (A) 0.2 unit (B) 0.3 unit (C) 0.1 unit (D) 2 unit

Ans. :

- c. 0.1 unit

Explanation:

The given equation is $y = 0.0015 \sin(62.4x + 316t)$, Compare it with the standard equation.

$$y = r \sin \left(\frac{2\pi}{\lambda} x + \frac{2\pi t}{T} \right)$$

$$\frac{2\pi}{\lambda} = 62.4$$

$$\lambda = \frac{2\pi}{62.4} = \frac{2 \times 3.14}{62.4} = 0.1 \text{ unit}$$

8. A wave equation is given by $y = 4 \sin \left[\pi \left(\frac{t}{5} - \frac{x}{9} + \frac{1}{6} \right) \right]$ where, x is in cm and t is in second. The wavelength of the wave is:
- (A) 18cm (B) 9cm (C) 36cm (D) 6cm

Ans. :

- a. 18cm

9. A transverse wave propagating along X-axis is represented by

$$y(x,t) = 8.0 \sin \left(0.5\pi x - 4\pi t - \frac{\pi}{4} \right) \text{ where } x \text{ is in metre and } t \text{ is in seconds. The}$$

speed of the wave is:

- (A) 8m/s (B) $4\pi\text{m/s}$ (C) $0.5\pi\text{m/s}$ (D) $\frac{\pi}{4}\text{m/s}$

Ans. :

a. 8m/s

Explanation:

Comparing with the standard wave equation

$$y = r \sin \left[\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} - \phi \right], \text{ we get}$$

$$\frac{2\pi}{\lambda} = 0.5\pi, \lambda = 4\text{m}, \frac{2\pi}{T} = 4\pi, T = 0.5\text{s}$$

$$\text{Speed of wave} = 8\text{m/s}$$

10. To increase the frequency from 100Hz to 400Hz the tension in the string has to be changed by:

- (A) 4 times. (B) 16 times.
(C) 2 times. (D) None of these.

Ans. :

b. 16 times.

11. A transverse wave propagating along X-axis is represented by $y(x, t) = 8.0 \sin(0.5\pi x - 4\pi t - \frac{\pi}{4})$ where x is in metre and t is in seconds. The speed of the wave is:

- (A) 8m/s (B) $4\pi\text{m/s}$ (C) $0.5\pi\text{m/s}$ (D) $\frac{\pi}{4}\text{m/s}$

Ans. :

d. $\frac{\pi}{4}\text{m/s}$

12. The time period of mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be:

- (A) $\frac{T}{4}$ (B) T (C) $\frac{T}{2}$ (D) 2T

Ans. :

d. 2T

13. The displacement of the wave given by equation $y(x, t) = a \sin(kx - \omega t + \phi)$, where $\phi = 0$ at point x and t = 0 is same as that at point:

- (A) $x + 2n\pi$ (B) $x + \frac{2n\pi}{k}$
(C) $kx + 2n\pi$ (D) Both (a) and (b)

Ans. :

b. $x + \frac{2n\pi}{k}$

Explanation:

$$y(x, 0) = a \sin kx = a \sin(kx + 2n\pi)$$

$$= a \sin k \left(x + \frac{2n\pi}{k} \right)$$

$$\Rightarrow \text{The displacement at points } x \text{ and } \left(x + \frac{2n\pi}{k} \right)$$

are the same where, $n = 1, 2, 3, \dots$

14. A steel wire has linear mass density $6.9 \times 10^{-3} \text{ kg m}^{-1}$. If the wire is under a tension of 60N, then the speed of the transverse waves on the wire is:

(A) 63 ms^{-1} (B) 75 ms^{-1} (C) 73 ms^{-1} (D) 93 ms^{-1}

Ans. :

d. 93 ms^{-1}

Explanation:

Linear mass density = $6.9 \times 10^{-3} \text{ kg m}^{-1}$

Tension, $T = 60 \text{ N}$ Thus, speed of wave on the wire is given by

$$\begin{aligned} \nu &= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60 \text{ N}}{6.9 \times 10^{-3} \text{ kg m}^{-1}}} \\ &= 93 \text{ ms}^{-1} \end{aligned}$$

15. A string of mass 2.5kg is under a tension of 200N. The length of the stretched string is 20.0m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in:

(A) One second (B) 0.5 second
(C) 2 seconds (D) Data given is insufficient.

Ans. :

b. 0.5 second

Explanation:

$M = \text{mass string } 2.5 \text{ kg}, l = 20 \text{ m}$

$M = \text{mas per unit length} = \frac{M}{l} = \frac{2.5}{20} = 0.125 \text{ kg/ m}$

$$\nu = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/ s}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{20 \text{ m}}{40 \text{ m/ s}} = \frac{1}{2} \text{ sec} = 0.5 \text{ sec.}$$

16. A siren placed at a railway platform is emitting sound of frequency 5kHz. A passenger sitting in a moving train A records a frequency of 5.5kHz, while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0kHz. while approaching the same siren. The ratio of the velocity of train B to that of train A is:

(A) $\frac{242}{252}$ (B) 2 (C) $\frac{5}{6}$ (D) $\frac{11}{6}$

Ans. :

b. 2

Explanation:

When listener alone is moving towards the source.

$$n' = \frac{(\nu + \nu_L)n}{\nu}$$

$$\therefore 5.5 = \left(\frac{\nu + \nu_A}{\nu} \right) 5$$

$$\text{and } 6.0 = \left(\frac{\nu + \nu_B}{\nu} \right) 5$$

Solving (i) and (ii), we get

$$\frac{\nu_B}{\nu_A} = 2$$

17. Two sound waves of slightly different frequencies propagating in the same direction produce beats due to:

(A) Interference. (B) Diffraction.
(C) Reflection. (D) Refraction.

Ans. :

a. Interference.

Explanation:

Beats are produced on account of interference of sound waves of slightly different frequencies.

18. Two pulses having equal and opposite displacements moving in opposite directions overlap at $t = t_1$ s. The resultant displacement of the wave at $t = t_1$ s is:

(A) Twice the displacement of each pulse. (B) Half the displacement of each pulse.
(C) Zero. (D) Either (a) or (c).

Ans. :

c. Zero.

Explanation:

The displacement due to two pulses will exactly cancel out each other. Thus, there will be no displacement throughout.

19. Two sound waves with wavelength 5.0m and 5.5m respectively, each propagate in a gas with velocity 330m/s. We expect the following number of beats/sec:

(A) 6 (B) 12 (C) 0 (D) 1

Ans. :

a. 6

20. In a longitudinal wave, the elastic property of the constituents of the medium that determines the stress under compressional strain is:

(A) Young's modulus (Y). (B) Bulk modulus (B).
(C) Shear modulus (S). (D) Either (b) or (C).

Ans. :

b. Bulk modulus (B).

21. Water waves produced by a motor boat sailing in water are:

(A) Neither longitudinal nor transverse.
(B) Both longitudinal and transverse.
(C) Only longitudinal.
(D) Only transverse.

Ans. :

b. Both longitudinal and transverse.

Explanation:

As the waves are produced by motor boat on surface as well as inside water, the waves are both, transverse as well as longitudinal.

22. Which of the following statements is true?
- (A) Both light and sound waves can travel in vacuum.
 - (B) Both light and sound waves in air are transverse.
 - (C) The sound waves in air are longitudinal, while the light waves are transverse.
 - (D) Both light and sound waves in air are longitudinal.

Ans. :

- d. Both light and sound waves in air are longitudinal.

23. Speed of sound waves in a fluid depends upon:

- (A) Directly on density of the medium.
- (B) Square of Bulk modulus of the medium.
- (C) Inversely on the square root of density.
- (D) Directly on the square root of bulk modulus of the medium.

Ans. :

- c. Inversely on the square root of density.
- d. Directly on the square root of bulk modulus of the medium.

Explanation:

Speed of sound wave in fluid of bulk modulus k and density ρ is given by $v = \sqrt{\frac{k}{\rho}}$

so $v = \sqrt{k}$ (if ρ is constant)

And $v = \sqrt{\frac{1}{\rho}}$ (if k is constant)

so verifies the option (c) and (d).

24. Which of the following statements are true for a stationary wave?

- (A) Every particle has a fixed amplitude which is different from the amplitude of its nearest particle.
- (B) All the particles cross their mean position at the same time.
- (C) There is no net transfer of energy across any plane.
- (D) There are some particles which are always at rest.

Ans. :

- a. Every particle has a fixed amplitude which is different from the amplitude of its nearest particle
- b. All the particles cross their mean position at the same time.
- d. There is no net transfer of energy across any plane.
- e. There are some particles which are always at rest.

Explanation:

In stationary waves $[y(x, t) = a \sin kx \cos \omega t]$ the particles between two nodes vibrate with different amplitude which increases from nodes.

The amplitude of a particle will remain constant $a \cos kx$, but varies with λ

$\therefore k = \frac{2\pi}{\lambda}$. Hence verifies the option (a)

particles between two nodes are in same phase i.e., motion of particles between two nodes will be either upward or downward and crosses the mean position at same time. Hence it verifies option (b)

Hence the reject option (c)

As the particles at nodes are rest so energy does not transfer verifies option (d)

The amplitude of particles at nodes has amplitude zero verifies option (e)

25. Two sine waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120° . The resultant amplitude will be:

(A) A (B) 2A (C) 4A (D) $\sqrt{2}A$

Ans. :

a. A

26. The frequency of a sound wave is n and its velocity is v. If the frequency is increased to 4n, the velocity of the wave will be:

(A) ν (B) 2ν (C) 4ν (D) $\frac{\nu}{4}$

Ans. :

a. ν

Explanation:

Velocity of sound is independent of frequency. Therefore, it is same (ν) for frequency n and 4n.

27. Equation of progressive wave is $y = a \sin \left(10\pi x + 11\pi t + \frac{\pi}{3} \right)$ The wavelength of the wave is:

(A) 0.2 unit (B) 0.1 unit (C) 0.5 unit (D) 1 unit

Ans. :

a. 0.2 unit

28. The displacement y of a wave travelling in x-direction is given by

$y = 10^{-4} \sin \left(600t - 2x + \frac{\pi}{3} \right)$ where x and y are in metre and t is in seconds. The speed of wave motion in s^{-1} is:

(A) 300 (B) 600 (C) 1200 (D) 200

Ans. :

a. 300

Explanation:

Here, $y = 10^{-4} \sin \left(600t - 2x + \frac{\pi}{3} \right)$

Compare it with the standard equation of a travelling wave

$$y = r \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right) \neq \phi$$

$$\frac{2\pi}{T} = 600, T = \frac{2\pi}{600} = \frac{\pi}{300} \text{ s}$$

$$\frac{2\pi}{\lambda} = +2\lambda = \frac{2\pi}{2} = \pi \text{ m}$$

$$\nu = \frac{\lambda}{T} = \frac{\pi}{\frac{\pi}{300}} = 300 \text{ m/s}$$

* Given Section consists of questions of 3 marks each.

[108]

29. A travelling harmonic wave on a string is described by
 $y(x, t) = 7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right)$ Locate the points of the string which have the same transverse displacements and velocity as the $x = 1\text{cm}$ point at $t = 2\text{s}$, 5s and 11s .

Ans. : Propagation constant is related to wavelength as:

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{0.0050} \\ = 1256\text{cm} = 12.56$$

Therefore, all the points at distances $n\lambda$, ($n = \pm 1, \pm 2, \dots$ and so on) i.e. $\pm 12.56\text{m}$, $\pm 25.12\text{m}$, ... and so on for $x = 1\text{cm}$, will have the same displacement as the $x = 1\text{cm}$ points at $t = 2\text{s}$, 5s , and 11s .

30. A stone dropped from the top of a tower of height 300m splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340m s^{-1} ? ($g = 9.8\text{m s}^{-2}$)

Ans. : Here, $h = 300\text{m}$, $g = 9.8\text{m s}^{-2}$ and velocity of sound, $v = 340\text{m s}^{-1}$ Let t_1 be the time taken by the stone to reach at the surface of pond.

$$\text{Then using } s = ut + \frac{1}{2}at^2 \Rightarrow h = 0 \times t + \frac{1}{2}gt_1^2$$

$$\therefore t_1 = \sqrt{\frac{2 \times 300}{9.8}} = 7.82\text{s}$$

Also if t_2 is the time taken by the sound to reach at a height h , then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88\text{s}$$

$$\therefore \text{Total time after which sound of splash is heard} = t_1 + t_2 \\ = 7.82 + 0.88 = 8.7\text{s}$$

31. A string of mass 2.50kg is under a tension of 200N . The length of the stretched string is 20.0m . If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Ans. :

Mass of the string, $M = 2.50\text{kg}$

Tension in the string, $T = 200\text{N}$

Length of the string, $l = 20.0\text{m}$

$$\text{Mass per unit length, } \mu = \frac{M}{L} = \frac{2.50}{20} = 0.125\text{kg m}^{-1}$$

The velocity (v) of the transverse wave in the string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}} \\ = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40\text{m/s}$$

Time taken by the disturbance to reach the other end, $t = l/v = 20/40 = 0.50\text{s}$

32. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz . An enemy submarine moves towards the SONAR with a speed of 360km h^{-1} . What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450m s^{-1} .

Ans. : The frequency of SONAR received by enemy submarine will be further reflected back to SONAR which it will receive again with a different frequency.

$$\text{SONAR frequency}(V_s) = 40\text{kHz} = 40 \times 10^3\text{Hz}$$

$$\text{Speed of enemy submarine}(V_o) = 360\text{km/h}$$

$$= 360 \times \frac{5}{18} = 100\text{m/s}$$

$$\text{Speed of sound in water} = 1450\text{m/s}$$

Apparent frequency received by submarine is

$$f' = \left\{ \frac{V+V_o}{V} \right\} f = \left\{ \frac{1450+100}{1450} \right\} \times 40 = 42.76\text{kHz}$$

Now, the reflected wave have a different frequency,

$$f'' = \left\{ \frac{V}{V-V_o} \right\} f' = \left\{ \frac{1450}{1450-100} \right\} \times 42.76 = 45.93\text{kHz}$$

33. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7km s^{-1} ? The operating frequency of the scanner is 4.2 MHz .

Ans. : Speed of sound in the tissue, $v = 1.7\text{km/s} = 1.7 \times 10^3\text{m/s}$

Operating frequency of the scanner, $\nu = 4.2\text{ MHz} = 4.2 \times 10^6\text{Hz}$

The wavelength of sound in the tissue is given as:

$$\begin{aligned} \lambda &= \frac{v}{\nu} \\ &= \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4}\text{m} \end{aligned}$$

34. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340m s^{-1} and in water 1486m s^{-1} .

Ans. : Frequency of the ultrasonic sound, $\nu = 1000\text{ kHz} = 10^6\text{Hz}$

Speed of sound in air, $v_a = 340\text{m/s}$

The wavelength (λ_r) of the reflected sound is given by the relation:

$$\begin{aligned} \lambda_r &= \frac{v}{\nu} \\ &= \frac{340}{10^6} = 3.4 \times 10^{-4}\text{m} \end{aligned}$$

Frequency of the ultrasonic sound, $\nu = 1000\text{ kHz} = 10^6\text{Hz}$

Speed of sound in water, $v_w = 1486\text{m/s}$

The wavelength of the transmitted sound is given as:

$$\lambda_r = \frac{1486}{10^6} = 1.49 \times 10^{-3}\text{m}$$

35. A steel wire has a length of 12.0m and a mass of 2.10kg . What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at $20^\circ\text{C} = 343\text{m s}^{-1}$.

Ans. : Length of the steel wire, $l = 12\text{m}$

Mass of the steel wire, $m = 2.10\text{kg}$

Velocity of the transverse wave, $v = 343\text{m/s}$

$$\text{Mass per unit length, } \mu = \frac{m}{l} = \frac{2.10}{12} = 0.175\text{kg m}^{-1}$$

For tension T, velocity of the transverse wave can be obtained using the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$\therefore T = v^2 \mu$$

$$= (343)^2 \times 0.175 = 20588.575 \approx 2.06 \times 10^4 \text{ N}$$

36.

- If the successive overtones of vibrating string are 280Hz and 350Hz, what is the frequency of the fundamental note?
- If the amplitude of a sound wave is tripled, by how many dB will the intensity level increases?

Ans. :

$$\text{i. Here } nv = 280\text{Hz}$$

$$\text{and } (n + 1)v = 350\text{Hz}$$

$$\therefore (n + 1)v - nv$$

$$= 350 - 280 = 70$$

$$v = 70\text{Hz}$$

$$\text{ii. Here } \frac{a_2}{a_1} = 3$$

$$\therefore \frac{I_2}{I_1} = \left(\frac{a_2}{a_1}\right)^2 = 9$$

$$\text{Now, } \log_{10} \left(\frac{I_2}{I_1}\right) = 10 \log_{10} \left(\frac{I_2}{I_1}\right)$$

$$= 10 \log_{10}(9) = 10 \log_{10} 3^2$$

$$= 20 \log_{10} 3 = \log_{10} 3^{20}$$

$$\therefore \frac{I_2}{I_1} = 3^{20}$$

$$I_2 = 3^{20} I_1.$$

37. A policeman on duty detects a drop of 15% in the pitch of the horn of a motor car as it crosses him. If the velocity of sound is 330m/sec, calculate the speed of the car.

Ans. : Before crossing source is moving towards listener,

$$\therefore V' = \frac{uV}{\nu - \nu_s} \dots (i)$$

After crossing, source is moving away from listener

$$\therefore V'' = \frac{uV}{\nu + \nu_s} \dots (ii)$$

Dividing (ii) by (i), we get

$$\frac{V'}{V''} = \frac{\nu - \nu_s}{\nu + \nu_s}$$

Drop of 15% means

$$\frac{V'}{V''} = \frac{85}{100}$$

$$\frac{85}{100} = \frac{330 - \nu_s}{330 + \nu_s}$$

$$\nu_s = 26.7 \text{ m/s}$$

38. A stone dropped from the top of a tower of height 300m splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the

speed of sound in air is 340m s^{-1} ? ($g = 9.8\text{m s}^{-2}$)

Ans. : Here, $h = 300\text{m}$, $g = 9.8\text{m s}^{-2}$ and velocity of sound, $v = 340\text{m s}^{-1}$ Let t_1 be the time taken by the stone to reach at the surface of pond.

Then using $s = ut + \frac{1}{2}at^2 \Rightarrow h = 0 \times t + \frac{1}{2}gt_1^2$

$$\therefore t_1 = \sqrt{\frac{2 \times 300}{9.8}} = 7.82\text{s}$$

Also if t_2 is the time taken by the sound to reach at a height h , then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88\text{s}$$

$$\therefore \text{Total time after which sound of splash is heard} = t_1 + t_2 \\ = 7.82 + 0.88 = 8.7\text{s}$$

39. A string of mass 2.50kg is under a tension of 200N . The length of the stretched string is 20.0m . If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Ans. :

Mass of the string, $M = 2.50\text{kg}$

Tension in the string, $T = 200\text{N}$

Length of the string, $l = 20.0\text{m}$

Mass per unit length, $\mu = \frac{M}{L} = \frac{2.50}{20} = 0.125\text{kg m}^{-1}$

The velocity (v) of the transverse wave in the string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}} \\ = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40\text{m/s}$$

Time taken by the disturbance to reach the other end, $t = l/v = 20/40 = 0.50\text{s}$

40. A standing wave is represented by $y = 2A \sin kx \cos \omega t$. If one of the component waves is $y_1 = A \sin(\omega t - kx)$, what is the equation of the second component wave?

Ans. : As $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$$y = 2A \sin kx \cos \omega t$$

$$A \sin(kx + \omega t) + A \sin(kx - \omega t)$$

According to superposition principle,

$$y = y_1 + y_2;$$

$$\text{and } y_1 = A \sin(\omega t - kx) = -A \sin(kx - \omega t)$$

$$\therefore y_2 = y - y_1 = 2A \sin kx \cos \omega t + A \sin(kx - \omega t)$$

$$= A \sin(kx + \omega t) + 2A \sin(kx - \omega t)$$

$$= A \sin(kx + \omega t) - 2A \sin(\omega t - kx).$$

41. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7km s^{-1} ? The operating frequency of the scanner is 4.2MHz .

Ans. : Speed of sound in the tissue, $v = 1.7\text{km/s} = 1.7 \times 10^3\text{m/s}$

Operating frequency of the scanner, $\nu = 4.2\text{MHz} = 4.2 \times 10^6\text{Hz}$

The wavelength of sound in the tissue is given as:

$$\lambda = \frac{v}{f}$$

$$= \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4} \text{ m}$$

42. A progressive and a stationary wave have frequency 300Hz and the same wave velocity 360m/s. Calculate.

- The phase difference between two points on the progressive wave which are 0.4m apart,
- The equation of motion of progressive wave if its amplitude is 0.02m,
- The equation of the stationary wave if its amplitude is 0.01m and
- The distance between consecutive nodes in the stationary wave.

Ans. : Wave velocity $v = 360 \text{ m/s}$

Frequency $n = 300 \text{ Hz}$

$$\therefore \text{wavelength } \lambda = \frac{v}{f} = \frac{360}{300} = 1.2 \text{ m}$$

- The phase difference between two points at a distance one wavelength apart is 2π . Phase difference between points 0.4m apart is given by:

$$= \frac{2\pi}{\lambda} \times 0.4 = \frac{2\pi}{1.2} \times 0.4 = \frac{2\pi}{3} \text{ radians}$$

- The equation of motion of a progressive wave is:

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

In the case given

$$y = 0.02 \sin 2\pi \left(300t - \frac{x}{1.2} \right)$$

- The equation of the stationary wave is:

$$y = 2A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

$$\text{Here } 2A = 2 \times 0.01 = 0.02$$

$$\lambda = 1.2 \text{ m}$$

$$\frac{1}{T} = 300 \text{ Hz}$$

$$\therefore y = 0.02 \cos \frac{2\pi x}{1.2} \sin 600\pi t$$

- The distance between the two consecutive nodes in the stationary wave is given by:

$$\frac{\lambda}{2} = \frac{1.2}{2} \text{ m} = 0.6 \text{ m}$$

43. A transverse harmonic wave travelling on a string is described by

$$y(x, t) = 3.0 \sin \left[(36t + 0.018x) + \frac{\pi}{4} \right] \text{ where } x \text{ and } y \text{ are in cm and } t \text{ in sec. The positive direction of } x \text{ is from left to right.}$$

- What is its amplitude and frequency?
- What is the initial phase at the origin?
- What is the least distance between to successive crest in the wave?

Ans. :

- Travelling wave speed $= \frac{\omega}{k} = \frac{36}{0.018}$
 $= \frac{36}{18} \times 10^3$
 $= 2 \times 10^3 \text{ cms}^{-1}$

It travels along the negative x-axis or from right to left.

ii. Amplitude = 3cm

$$\text{Frequency} = \frac{\omega}{2\pi} = \frac{36}{2\pi} \\ = \frac{18}{\pi} \text{ Hz}$$

iii. Initial phase = $\frac{\pi}{4}$

iv. Distance between successive crests = $\lambda = \frac{2\pi}{k}$
 $= \frac{2\pi}{0.018} = 3.5\text{m}$

44. What is the nature of sound waves in air? How is the speed of sound waves in atmosphere affected by the:

i. Humidity.

ii. Temperature?

Ans. : Sound waves are longitudinal in nature velocity of sound,

$$v = \sqrt{\frac{\lambda P}{\rho}}$$

i. For moist air, values of both λ and ρ are less than the corresponding values for dry air.

\therefore speed of sound in moist air tends to increase due to effect of density

$\left(v \propto \frac{1}{\sqrt{\rho}}\right)$ but tends to decrease due to effect of λ ($v \propto \sqrt{\lambda}$).

However the effect of λ is more than that of ρ . Hence speed of sound increases with humidity.

ii. Speed of sound increases with increase of temperature ($v \propto \sqrt{T}$).

45. An underwater swimmer sends a sound signal to the surface. It produces 5 beats per second when compared with the fundamental note of a pipe 20cm long closed at one end. What is the wavelength of sound in water? Given velocities of sound in air and water are 360ms^{-1} and 1500ms^{-1} respectively.

Ans. : The frequency of the fundamental tone of the pipe is

$$n = \frac{v_a}{4L} \\ = \frac{360}{4 \times 0.02} = 450\text{Hz}$$

\therefore The frequency of sound signal = $450 \pm 5 = 445\text{Hz}$ or 455Hz Since the frequency remains unchanged when sound travels from water to air, the frequency of the sound signal in water is either 445 Hz or 455 Hz. Hence the wavelength in water is either

$$\frac{1500}{445} = 3.371\text{m}$$

$$\frac{1500}{455} = 3.297\text{m}$$

46. Calculate the velocity of sound in a gas, in which two wavelengths 2.04m and 2.08m produce 20 beats in 6 seconds.

Ans. : Here, wavelength of one wave, $\lambda_1 = 2.04\text{m}$

Wavelength of the second wave $\lambda_2 = 2.08\text{m}$

Let velocity of sound in the gas = $v\text{ms}^{-1}$

Frequency of one wave = V_1

Frequency of second wave = V_2

$$\therefore V_1 = \frac{\nu}{\lambda_1} = \frac{\nu}{2.04}$$

$$\text{and } V_2 = \frac{\nu}{\lambda_2} = \frac{\nu}{2.08}$$

No. of beats produced per second

$$n = \frac{20}{6}$$

$$\text{As } V_1 - V_2 = n$$

$$\therefore \frac{\nu}{2.04} - \frac{\nu}{2.08} = \frac{20}{6}$$

$$\frac{\nu(2.08-2.04)}{2.04 \times 2.08} = \frac{20}{6}$$

$$\nu = 353.6 \text{ ms}^{-1}.$$

47. A sound wave travelling along a string is described by $y(x, t) = 5 \times 10^{-3} \sin(80x - 3t)$ in which numerical constants are in S.I. unit. Calculate.
- The amplitude.
 - The wave length.
 - The period and frequency of the wave.

$$\text{Ans. : } y(x, t) = 5 \times 10^{-3} \sin(80x - 3t)$$

On comparing the equation with

$$y(x, t) = A \sin(kx - \omega t)$$

$$\text{Amplitude} = 5 \times 10^{-3} \text{ m}$$

$$k = 80, \therefore \lambda = \frac{2\pi}{80} \text{ m} = \frac{\pi}{40} \text{ m}$$

$$\omega = 3 \text{ (i.e.) } 2\pi\nu = \frac{2\pi}{T} = 3$$

$$\therefore \text{Time period} = T = \frac{2\pi}{3} \text{ seconds}$$

$$\text{Frequency, } \nu = \frac{3}{2\pi} \text{ Hz}$$

48. The following equation represents standing wave set up in medium, $y = 4 \cos \frac{\pi x}{5} \sin 40\pi t$, where x and y are in cm and t in sec. Find out the amplitude and the velocity of the two component waves and calculate the distance between adjacent nodes. What is the velocity of a medium particle at $x = 3 \text{ cm}$ at time $\frac{1}{8} \text{ sec}$?

Ans. : The given equation of stationary wave is

$$y = 4 \cos \frac{\pi x}{5} \sin 40\pi t$$

$$\text{or } y = 2 \times 2 \cos \frac{2\pi x}{6} \sin \frac{2x(120)t}{6} \dots (i)$$

$$\text{We know that } y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2xvt}{\lambda} \dots (i)$$

By comparing tow equations, we get

$$a = 2 \text{ cm}, \lambda = 6 \text{ cm and } v = 220 \text{ cm/sec.}$$

The component waves are:

$$y_1 = a \sin \frac{2\pi}{\lambda} (vt - x) \text{ and } y_2 = a \sin \frac{2\pi}{\lambda} (vt + x)$$

$$\text{Distance between two adjacent nodes} = \frac{\lambda}{2} = \frac{6}{2} = 3 \text{ cm.}$$

$$\begin{aligned} \text{Particle velocity } \frac{dy}{dt} &= 4 \cos \frac{\pi x}{5} \cos(40\pi t) \cdot 40\pi \\ &= 160 \cos \frac{\pi x}{5} \cos 40\pi t \end{aligned}$$

$$= 160\pi \cos \frac{\pi x}{3} \cos \left(40\pi \times \frac{1}{8} \right) = 160\pi [\because \cos \pi = \cos 5\pi = -1]$$

Hence, particle velocity = 160cm/ sec.

49. Two sound waves originating from the same source, travel along different paths in air and then meet at a point. If the source vibrates at a frequency of 1kHz and one path is 83cm longer than the other, what will be the nature of interference? The speed of sound in air is 332m/s.

Ans. : Wavelength of sound wave is

$$\lambda = \frac{v}{\nu} = \frac{332}{1 \times 10^3} = 0.332\text{m}$$

Phase difference between the waves arriving at point of observation is

$$\begin{aligned} \phi &= \frac{2\pi}{\lambda} \Delta x \\ &= \frac{2\pi \times 0.83}{0.332} = 5\pi \end{aligned}$$

Since phase difference is an odd multiple of π , the interference is destructive.

50. Find the temperature at which the speed of sound in oxygen will be the same as that in nitrogen at 20°C. Given that molar masses of oxygen and nitrogen are 32 and 28 respectively. Both gases are assumed to be ideal.

Ans. : We know that both oxygen and nitrogen are diatomic gases having same value of constant

$$\gamma = 1.40$$

We know that $v = \sqrt{\frac{\gamma RT}{M}}$. As speed of sound in oxygen at T K is same as the speed of sound in nitrogen at T' = 20 °C = 293K, hence

$$\begin{aligned} v &= \sqrt{\frac{\gamma RT}{M_{N_2}}} = \sqrt{\frac{\gamma RT'}{M_{N_2}}} \\ \Rightarrow T &= T' \frac{M_{O_2}}{M_{N_2}} = \frac{293 \times 32}{28} \\ &= 33\text{K or } 60^\circ\text{C}. \end{aligned}$$

51. A steel wire has a length of 12.0m and a mass of 2.10kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343m s⁻¹.

Ans. : Length of the steel wire, l = 12m

Mass of the steel wire, m = 2.10kg

Velocity of the transverse wave, v = 343m/ s

$$\text{Mass per unit length, } \mu = \frac{m}{l} = \frac{2.10}{12} = 0.175\text{kg m}^{-1}$$

For tension T, velocity of the transverse wave can be obtained using the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$\therefore T = v^2 \mu$$

$$= (343)^2 \times 0.175 = 20588.575 \approx 2.06 \times 10^4\text{N}$$

52. List the differences between a progressive and a stationary wave.

Ans. :

S. No.	Progressive Wave	Stationary Wave
1.	All particles have same phase and amplitude.	Amplitude varies with position.
2.	Speed of motion is same.	Speed varies with position.
3.	Energy is transported.	Energy is not transported.
4.	Same change in pressure and density is with every point.	Pressure and density varies with point.

53. A set of 65 tuning forks is so arranged that each gives 3 beats per second with the previous one and the last sounds the octave of first. Find the frequency of first and last forks?

Ans. : According to the given problem the frequency of the last fork is the octave of the first, i.e., if the frequency of first fork is n , then the frequency of last fork is $2n$. This shows that the forks are in increasing frequency order. As each fork gives 3 beats with previous are, hence frequencies are:

$$n, (n + 3), (n + 2 \times 3), (n + 3 \times 3), \dots, 2n$$

In forms an A.P.

$$\therefore a_n = a + (n - 1)d \text{ or } 2n = n + (65 - 1)3$$

By solving we get

$$n = 192 \text{ and } 2n = 384$$

Frequency of first and last forks are 192 and 384 respectively.

54. A bat emits ultrasonic sound of frequency 100KHz in air. If this sound meets a water surface, what is the wave length of (a) the reflected sound, (b) transmitted sound?
Speed of sound in air = 340ms^{-1} and in water = 1486ms^{-1} .

Ans. :

i. $V = 100\text{kHz}$

$$= 100 \times 10^3 \text{Hz},$$

$$\nu = 340\text{ms}^{-1}$$

$$\text{But } \lambda = \frac{\nu}{V} = \frac{330}{100 \times 10^3} = 3.30 \times 10^{-3} \text{m}$$

ii. $\nu \text{ in water} = 1486\text{ms}^{-1}$

$$\therefore \lambda = \frac{\nu}{V} = \frac{1486}{10^5}$$

$$= 1.486 \times 10^{-2} \text{m}$$

55. A set of 25 tuning forks is arranged in order of decreasing frequency. Each fork gives 3 beats with succeeding one. The first fork is octave of the last. Calculate the frequency of the first and 16th fork.

Ans. : Let frequency of last tuning fork be n

$$\therefore \text{Frequency of 1st tuning fork} = 2n;$$

$$\text{Frequency of 2nd tuning fork} = [2n - 3]$$

$$\text{Frequency of 3rd tuning fork} = (2n - 6) = 2n - 3(3 - 1) \text{ and}$$

$$\text{Sum of 25 tuning fork frequencies}$$

$$= 2n - 3(25 - 1) = n$$

$$\therefore n = 72$$

Hence, frequency of 1st tuning fork = $2n = 144\text{Hz}$

and frequency of 16th fork = $144 - 3(16 - 1) = 99\text{Hz}$

56. Calculate the speed of sound in a gas in which two waves of wavelengths 1.00m and 1.01m produce 10 beats in 3 seconds.

Ans. : Let v = speed of sound in a gas

Frequencies of two waves is

$$v_1 = \frac{v}{\lambda_1} = \frac{v}{1.00}$$

$$v_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$$

$$\text{Given } v_1 - v_2 = \frac{10}{3}$$

$$\frac{v}{1.00} - \frac{v}{1.01} = \frac{10}{3}$$

$$\frac{0.01v}{1 \times 1.01} = \frac{10}{3}$$

$$\therefore v = \frac{10 \times 1 \times 1.01}{3 \times 0.01} = 336.7\text{ms}^{-1}$$

57. An open pipe is suddenly closed at one end with the result that the frequency of sound harmonic of the closed pipe is found to be higher by 100Hz than the fundamental frequency of the open pipe. Calculate the fundamental frequency of the open pipe.

Ans. : Fundamental frequency of open pipe is

$$v_0 = \frac{v}{2l}$$

Frequency of second harmonic of closed pipe of same length is

$$v_c = \frac{3v}{4l} = \frac{3}{2} \left(\frac{v}{2l} \right)$$

$$= \frac{3}{2} v_0$$

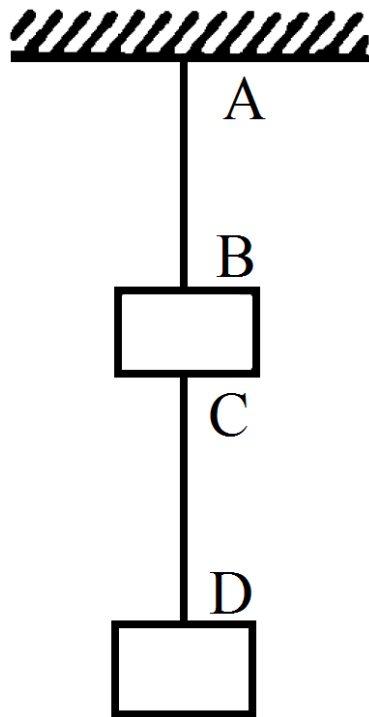
$$\text{Given } v_c = v_0 + 100 \text{ or } \frac{3}{2} v_0 = v_0 + 100$$

$$\Rightarrow \frac{3}{2} v_0 - v_0 = 100$$

$$\Rightarrow \frac{v_0}{2} = 100$$

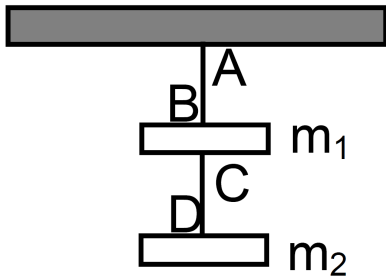
$$\therefore v_0 = 200\text{Hz.}$$

58. Two blocks each having a mass of 3.2kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB figure, The linear mass delity of the wire AB is 10g/m and that of CD is 8g/m. Find the speed of a transverse wave pulse produced



in AB and in CD.

Ans. :



$$m_1 = m_2 = 3.2\text{kg}$$

$$\text{mass per unit length of AB} = 10\text{g/mt} = 0.01\text{kg/mt}$$

$$\text{mass per unit length of CD} = 8\text{g/mt} = 0.008\text{kg/mt}$$

$$\text{for the string CD, } T = 3.2 \times g$$

$$\Rightarrow v = \sqrt{\left(\frac{T}{m}\right)} = \sqrt{\left(\frac{3.2 \times 10}{0.008}\right)} = \sqrt{\frac{(32 \times 10^3)}{8}}$$

$$= 2 \times 10\sqrt{10} = 20 \times 3.14 = 63\text{m/s}$$

$$\text{for the string AB, } T = 2 \times 3.2g = 6.4 \times g = 64\text{N}$$

$$\Rightarrow v = \sqrt{\left(\frac{T}{m}\right)} = \sqrt{\left(\frac{64}{0.01}\right)}$$

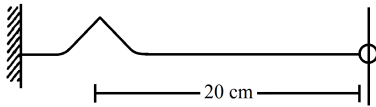
$$= \sqrt{6400}$$

$$= 80\text{m/s}$$

59. A string of linear mass density 0.5g/cm and a total length 30cm is tied to a fixed wall at one end and to a frictionless ring at the other end figure, The ring can move on a vertical rod. A wave pulse is produced on the string which moves towards the ring at a speed of 20cm/s . The pulse is symmetric about its maximum which is located at a distance of 20cm from the end joined to the ring.

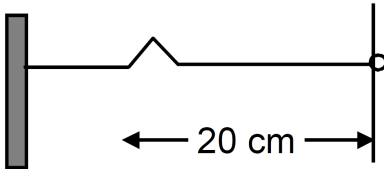
- Assuming that the wave is reflected from the ends without loss of energy, find the time taken by the string to regain its shape.

- b. The shape of the string changes periodically with time. Find this time period.
c. What is the tension in the string?



Ans. : The crest reflects as a crest here, as the wire is traveling from denser to rarer medium.

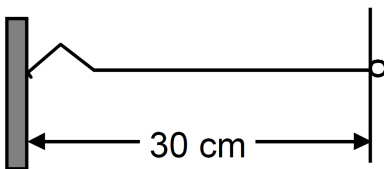
\Rightarrow phase change = 0



- a. To again original shape distance travelled by the wave $S = 20 + 20 = 40\text{cm}$.

Wave speed, $v = 20\text{m/s}$

$$\begin{aligned}\Rightarrow \text{Times} &= \frac{S}{v} \\ &= \frac{40}{20} \\ &= 2\text{sec}\end{aligned}$$



- b. The wave regains its shape, after traveling a periodic distance $= 2 \times 30 = 60\text{cm}$

$$\therefore \text{Time period} = \frac{60}{20} = 3\text{sec}.$$

- c. Frequency, $n = \left(\frac{1}{3} = 3\text{sec}^{-1}\right)$

$$n = \left(\frac{1}{2l}\right) \sqrt{\frac{T}{m}}$$

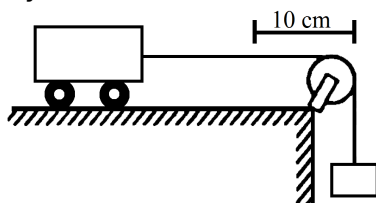
$m = \text{mass per unit length} = 0.5\text{g/cm}$

$$\Rightarrow \frac{1}{3} = \frac{1}{(2 \times 30)} \sqrt{\left(\frac{T}{0.5}\right)}$$

$$\Rightarrow T = 400 \times 0.5 = 200 \text{ dyne}$$

$$= 2 \times 10^{-3} \text{ Newton}$$

60. A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in figure, The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120Hz. If the movable support is pushed to the right by 10cm so that the joint is placed on the pulley, what will be the minimum frequency at which the heavy string can



resonate?

Ans. : Initially because the end A is free, an antinode will be formed.

$$\text{So, } l = \frac{Ql_1}{4}$$

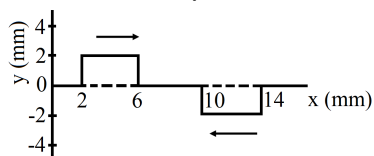
Again, if the movable support is pushed to right by 10m, so that the joint is placed on the pulley, a node will be formed there.

$$\text{So, } l = \frac{\lambda_2}{2}$$

Since, the tension remains same in both the cases, velocity remains same.

As the wavelength is reduced by half, the frequency will become twice as that of 120Hz i.e. 240Hz.

61. Figure shows two wave pulses at $t = 0$ travelling on a string in opposite directions with



the same wave speed 50cm/s. Sketch the shape of the string at $t = 4\text{ms}$, 6ms , 8ms , and 12ms .

Ans. : The distance travelled by the pulses are shown below.

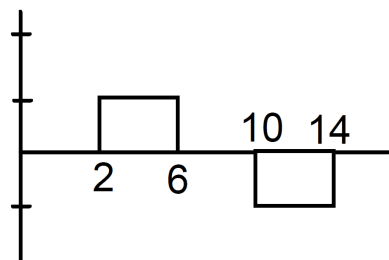
$$t = 4\text{ms} = 4 \times 10^{-3}\text{s}, s = vt = 50 \times 10 \times 4 \times 10^{-3} = 2\text{mm}$$

$$t = 8\text{ms} = 8 \times 10^{-3}\text{s}, s = vt = 50 \times 10 \times 8 \times 10^{-3} = 4\text{mm}$$

$$t = 6\text{ms} = 6 \times 10^{-3}\text{s}, s = 3\text{mm}$$

$$t = 12\text{ms} = 12 \times 10^{-3}\text{s}, s = 50 \times 10 \times 12 \times 10^{-3} = 6\text{mm}$$

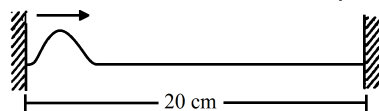
The shape of the string at different times are shown in the figure.



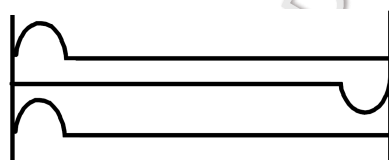
62. A string of length 20cm and linear mass density 0.40g/cm is fixed at both ends and is kept under a tension of 16N. A wave pulse is produced at $t = 0$ near an end as shown in figure, which travels towards the other end.

a. When will the string have the shape shown in the figure again

b. Sketch the shape of the string at a time half of that found in part (a).



Ans. :



a. Velocity of the wave, $V = \sqrt{\frac{T}{\mu}}$

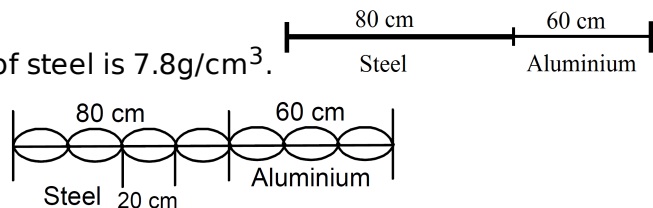
$$= \sqrt{\frac{(16 \times 10^5)}{0.4}} = 2000\text{cm/sec}$$

$$\therefore \text{Time taken to reach to the other end} = \frac{20}{2000} = 0.01\text{sec}$$

Time taken to see the pulse again in the original position = $0.01 \times 2 = 0.02\text{sec}$.

b. At $t = 0.01\text{s}$, there will be a 'though' at the right end as it is reflected.

63. Figure, shows an aluminium wire of length 60cm joined to a steel wire of length 80cm and stretched between two fixed supports. The tension produced is 40N. The cross-sectional area of the steel wire is 1.0mm^2 and that of the aluminium wire is 3.0mm^2 . What could be the minimum frequency of a tuning fork which can produce standing waves in the system with the joint as a node? The density of aluminium is 2.6g/cm^3 and that of steel is 7.8g/cm^3 .



Ans. :

$$\rho_s = 7.8\text{g/cm}^3, \rho_A = 2.6\text{g/cm}^3$$

$$m_s = \rho_s A_s = 7.8 \times 10^{-2}\text{g/cm} \text{ (m = mass per unit length)}$$

$$m_A = \rho_A A_A = 2.6 \times 10^{-2} \times 3\text{g/cm}$$

$$= 7.8 \times 10^{-3}\text{kg/m}$$

A node is always placed in the joint. Since aluminium and steel rod has same mass per unit length, velocity of wave in both of them is same.

$$\Rightarrow v = \sqrt{\frac{T}{m}}$$

$$\Rightarrow \frac{500}{7}\text{m/s}$$

For minimum frequency there would be maximum wavelength for maximum wavelength minimum no of loops are to be produced.

$$\therefore \text{maximum distance of a loop} = 20\text{cm}$$

$$\Rightarrow \text{wavelength} = \lambda = 2 \times 20 = 40\text{cm} = 0.4\text{m}$$

$$\therefore f = \frac{v}{\lambda} = 180\text{Hz}.$$

64. The equation of a wave travelling on a string is:

$$y = (0.10\text{mm}) \sin [(31.4\text{m}^{-1})x + (314\text{s}^{-1})t].$$

- In which direction does the wave travel?
- Find the wave speed, the wavelength and the frequency of the wave.
- What is the maximum displacement and the maximum speed of a portion of the string?

Ans. : The equation of the wave is given by

$$y = (0.10\text{mm}) \sin [(31.4\text{m}^{-1})x + (314\text{s}^{-1})t]. \left(y = r \sin \left\{ \left(\frac{2\pi x}{\lambda} \right) \right\} + \omega t \right)$$

- Negative x-direction

$$\text{b. } k = 31.4\text{m}^{-1}$$

$$\Rightarrow \frac{2\pi}{\lambda} = 31.4$$

$$\Rightarrow \lambda = \frac{2\pi}{31.4} = 0.2\text{m} = 20\text{cm}$$

$$\text{Again, } \omega = 314\text{s}^{-1}$$

$$\Rightarrow 2\pi f = 314$$

$$\Rightarrow f = \frac{314}{2\pi} = \frac{314}{\left(2 \times \frac{3}{14}\right)} = 50 \text{sec}^{-1}$$

$$\therefore \text{ wave speed, } v = \lambda f = 20 \times 50 = 1000 \text{cm/s}$$

$$\text{c. Max. displacement} = 0.10 \text{mm}$$

$$\text{Max. velocity} = a\omega = 0.1 \times 10^{-1} \times 314 = 3.14 \text{cm/sec}$$

*** Given Section consists of questions of 5 marks each.**

[210]

65. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45Hz. The mass of the wire is $3.5 \times 10^{-2} \text{kg}$ and its linear mass density is $4.0 \times 10^{-2} \text{kg m}^{-1}$. What is

- The speed of a transverse wave on the string,
- The tension in the string?

Ans. :

$$\text{a. Mass of the wire, } m = 3.5 \times 10^{-2} \text{kg}$$

$$\text{Linear mass density, } \mu = \frac{m}{l} = 4.0 \times 10^{-2} \text{kg m}^{-1}$$

$$\text{Frequency of vibration, } v = 45 \text{Hz}$$

$$\therefore \text{ length of the wire, } l = \frac{m}{\mu} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875 \text{m}$$

The wavelength of the stationary wave (λ) is related to the length of the wire by the relation:

$$\lambda = \frac{2l}{n}$$

where,

n = Number of nodes in the wire

For fundamental node, $n = 1$:

$$\lambda = 2l$$

$$\lambda = 2 \times 0.875 = 1.75 \text{m}$$

The speed of the transverse wave in the string is given as:

$$v = v\lambda = 45 \times 1.75 = 78.75 \text{m/s}$$

- The tension produced in the string is given by the relation:

$$T = v^2 \mu$$

$$= (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{N}$$

66. A travelling harmonic wave on a string is described by

$$y(x, t) = 7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right)$$

What are the displacement and velocity of oscillation of a point at $x = 1 \text{cm}$, and $t = 1 \text{s}$? Is this velocity equal to the velocity of wave propagation?

Ans. : The given harmonic wave is:

$$y(x, t) = 7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right)$$

For $x = 1 \text{cm}$ and $t = 1 \text{s}$,

$$y = (1, 1) = 7.5 \sin \left[0.0050 + 12 + \frac{\pi}{4} \right]$$

$$= 7.5 \sin \left[12.0050 + \frac{\pi}{4} \right]$$

$$= 7.5 \sin \theta$$

$$\text{Where, } \theta = 12.0050 + \frac{\pi}{4} = 12.0050 + \frac{3.14}{4} = 12.79 \text{ rad}$$

$$= \frac{180}{3.14 \times 12.79} = 732.81^\circ$$

$$\therefore y = (1, 1) = 7.5 \sin[732.81^\circ]$$

$$= 7.5 \sin(90 \times 8 + 12.81^\circ)$$

$$= 7.5 \sin(12.81^\circ)$$

$$= 7.5 \times 0.2217$$

$$= 1.6629 \approx 1.663 \text{ cm}$$

The velocity of the oscillation at a given point and time is given as:

$$v = \frac{d}{dt} y(x, t) = \frac{d}{dt} \left[7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right) \right]$$

$$= 7.5 \times 12 \cos \left(0.0050x + 12t + \frac{\pi}{4} \right)$$

At $x = 1 \text{ cm}$ and $t = 1 \text{ s}$:

$$v = y(1, 1) = 90 \cos \left(12.005 + \frac{\pi}{4} \right)$$

$$= 90 \cos(732.81^\circ) = 90 \cos(90 \times 8 + 12.81^\circ)$$

$$= 90 \cos(12.81^\circ)$$

$$= 90 \times 0.975 = 87.75 \text{ cm/s}$$

Now, the equation of a propagating wave is given by:

$$y(x, t) = a \sin(kx + \omega t + \phi)$$

Where,

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{2\pi}{k}$$

$$\text{And } \omega = 2\pi v$$

$$\therefore v = \frac{\omega}{2\pi}$$

$$\text{Speed} = v = v\lambda = \frac{\omega}{k}$$

Where

$$\omega = 12 \text{ rad/s}$$

$$k = 0.0050 \text{ m}^{-1}$$

$$\therefore v = \frac{12}{0.0050} = 2400 \text{ cm/s}$$

\therefore Hence, the velocity of the wave oscillation at $x = 1 \text{ cm}$ and $t = 1 \text{ s}$ is not equal to the velocity of the wave propagation.

67. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is $3.5 \times 10^{-2} \text{ kg}$ and its linear mass density is $4.0 \times 10^{-2} \text{ kg m}^{-1}$. What is
- The speed of a transverse wave on the string,
 - The tension in the string?

Ans. :

a. Mass of the wire, $m = 3.5 \times 10^{-2} \text{ kg}$

Linear mass density, $\mu = \frac{m}{l} = 4.0 \times 10^2 \text{ kg m}^{-1}$

Frequency of vibration, $\nu = 45 \text{ Hz}$

\therefore length of the wire, $l = \frac{m}{\mu} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875 \text{ m}$

The wavelength of the stationary wave (λ) is related to the length of the wire by the relation:

$$\lambda = \frac{2l}{n}$$

where,

n = Number of nodes in the wire

For fundamental node, $n = 1$:

$$\lambda = 2l$$

$$\lambda = 2 \times 0.875 = 1.75 \text{ m}$$

The speed of the transverse wave in the string is given as:

$$v = \nu \lambda = 45 \times 1.75 = 78.75 \text{ m/s}$$

b. The tension produced in the string is given by the relation:

$$T = v^2 \mu$$

$$= (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{ N}$$

68. A pipe 20cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

Ans. : First (Fundamental); No

Length of the pipe, $l = 20 \text{ cm} = 0.2 \text{ m}$

Source frequency = n^{th} normal mode of frequency, $\nu_n = 430 \text{ Hz}$

Speed of sound, $v = 340 \text{ m/s}$

In a closed pipe, the n^{th} normal mode of frequency is given by the relation:

$$\nu_n = (2n - 1) \frac{v}{4l} \quad n \text{ is an integer} = 0, 1, 2, 3$$

$$430 = (2n - 1) \frac{340}{4 \times 0.2}$$

$$2n - 1 = \frac{430 \times 4 \times 0.2}{340} = 1.01$$

$$2n = 2.01$$

$$n \sim 1$$

Hence, the first mode of vibration frequency is resonantly excited by the given source. In a pipe open at both ends, the n^{th} mode of vibration frequency is given by the relation:

$$\nu_n = \frac{n v}{2l}$$

$$n = \frac{2l \nu_n}{v}$$

$$= \frac{2 \times 0.2 \times 430}{340} = 0.5$$

69. A travelling harmonic wave on a string is described by

$$y(x, t) = 7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right) \quad \text{What are the displacement and velocity of}$$

oscillation of a point at $x = 1\text{cm}$, and $t = 1\text{s}$? Is this velocity equal to the velocity of wave propagation?

Ans. : The given harmonic wave is:

$$y(x, t) = 7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right)$$

For $x = 1\text{cm}$ and $t = 1\text{s}$,

$$\begin{aligned} y &= (1, 1) = 7.5 \sin \left[0.0050 + 12 + \frac{\pi}{4} \right] \\ &= 7.5 \sin \left[12.0050 + \frac{\pi}{4} \right] \\ &= 7.5 \sin \theta \end{aligned}$$

$$\text{Where, } \theta = 12.0050 + \frac{\pi}{4} = 12.0050 + \frac{3.14}{4} = 12.79 \text{ rad}$$

$$= \frac{180}{3.14 \times 12.79} = 732.81^\circ$$

$$\therefore y = (1, 1) = 7.5 \sin[732.81^\circ]$$

$$= 7.5 \sin(90 \times 8 + 12.81^\circ)$$

$$= 7.5 \sin(12.81^\circ)$$

$$= 7.5 \times 0.2217$$

$$= 1.6629 \approx 1.663\text{cm}$$

The velocity of the oscillation at a given point and time is given as:

$$v = \frac{d}{dt} y(x, t) = \frac{d}{dt} \left[7.5 \sin \left(0.0050x + 12t + \frac{\pi}{4} \right) \right]$$

$$= 7.5 \times 12 \cos \left(0.0050x + 12t + \frac{\pi}{4} \right)$$

At $x = 1\text{cm}$ and $t = 1\text{s}$:

$$v = y(1, 1) = 90 \cos \left(12.005 + \frac{\pi}{4} \right)$$

$$= 90 \cos(732.81^\circ) = 90 \cos(90 \times 8 + 12.81^\circ)$$

$$= 90 \cos(12.81^\circ)$$

$$= 90 \times 0.975 = 87.75\text{cm/s}$$

Now, the equation of a propagating wave is given by:

$$y(x, t) = a \sin(kx + \omega t + \phi)$$

Where,

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{2\pi}{k}$$

$$\text{And } \omega = 2\pi v$$

$$\therefore v = \frac{\omega}{2\pi}$$

$$\text{Speed} = v = v\lambda = \frac{\omega}{k}$$

Where

$$\omega = 12 \text{ rad/s}$$

$$k = 0.0050\text{m}^{-1}$$

$$\therefore v = \frac{12}{0.0050} = 2400\text{cm/s}$$

∴ Hence, the velocity of the wave oscillation at $x = 1\text{cm}$ and $t = 1\text{s}$ is not equal to the velocity of the wave propagation.

70. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0km s^{-1} , and that of P wave is 8.0km s^{-1} . A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Ans. : Let v_S and v_P be the velocities of S and P waves respectively.

Let L be the distance between the epicentre and the seismograph.

We have:

$$L = v_S t_S \dots (i)$$

$$L = v_P t_P \dots (ii)$$

Where,

t_S and t_P are the respective times taken by the S and P waves to reach the seismograph from the epicentre

It is given that:

$$v_P = 8\text{km/s}$$

$$v_S = 4\text{km/s}$$

From equations (i) and (ii), we have:

$$v_S t_S = v_P t_P$$

$$4t_S = 8t_P$$

$$t_S = 2 t_P \dots (iii)$$

It is also given that:

$$t_S - t_P = 4 \text{ min} = 240\text{s}$$

$$2t_P - t_P = 240$$

$$t_P = 240$$

$$\text{And } t_S = 2 \times 240 = 480\text{s}$$

From equation (ii), we get:

$$L = 8 \times 240$$

$$= 1920\text{km}$$

Hence, the earthquake occurs at a distance of 1920km from the seismograph.

71. The earth has a radius of 6400km. The inner core of 1000km radius is solid. Outside it, there is a region from 1000km to a radius of 3500km which is in molten state. Then again from 3500km to 6400km the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of 8km s^{-1} in solid parts and of 5km s^{-1} in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter?

Ans. : $r_1 = 1000\text{km}$

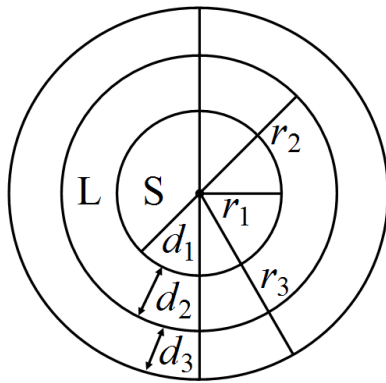
$$r_2 = 3500\text{km}$$

$$r_3 = 6400\text{km}$$

$$d_1 = 1000\text{km}$$

$$d_2 = 3500 - 1000 = 2500\text{km}$$

$$d_3 = 6400 - 3500 = 2900\text{km}$$



Solid distance diametrically

$$= 2(d_1 + d_3) = (1000 + 2900)$$

$$2 \times 3900\text{km}$$

Time taken by wave produced by earthquake in solid part

$$= \frac{3900 \times 2}{8} \text{sec}$$

Liquid part along diametrically $2d_2 = 2 \times 2500$

$$\therefore \text{Time taken by seismic wave in liquid part} = \frac{2 \times 2500}{5}$$

$$\text{Total time } \frac{2 \times 3900}{8} + \frac{2 \times 2500}{5} = 2 \left[\frac{3900}{8} + \frac{2500}{5} \right]$$

$$= 2[487.5 + 500] = 2 \times 987.5 = 1975 \text{ sec.}$$

$$= 32 \text{ min } 55 \text{ sec.}$$

72. A progressive wave is given by $y(x, t) = 8 \cos(300t - 0.15x)$ where x in m, y in cm and t in second. What is the-

- Direction of propagation?
- Wavelength?
- Frequency?
- Wave speed?
- Phase difference between two points 0.2m apart?

Ans. : $y(x, t) = 8 \cos(300t - 0.15x)$

On comparing it with $y = a \cos 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$

- Direction of propagation is + x-axis.

$$\text{ii. } \frac{2\pi}{\lambda} = 0.15$$

$$\Rightarrow \lambda = \frac{2\pi}{0.15}$$

$$= 41.87\text{m}$$

$$\text{iii. } \frac{2\pi}{T} = 300$$

$$2\pi\nu = 300$$

$$\nu = \frac{300}{2\pi}$$

$$= 47.78\text{Hz}$$

$$\text{iv. } \nu = \lambda\nu$$

$$= \frac{2\pi}{0.15} \times \frac{300}{2\pi}$$

$$= 2000\text{m/s}$$

$$\begin{aligned} \text{v. } \Delta\phi &= \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{\lambda} \times 0.2 \\ &= \frac{2\pi \times 0.2 \times 0.15}{2\pi} = 0.03 \text{ radian} \end{aligned}$$

73. A source of frequency 250Hz produces sound waves of wavelength 1.32m in a gas at STP. Calculate the change in the wavelength, when temperature of the gas is 40°C.

Ans. : We have, $\nu_0 = 250\text{Hz}$, $T_0 = 273\text{K}$

$$T_1 = 273 + 40 = 313\text{K}; \lambda_0 = 1.32\text{m}$$

$$\begin{aligned} \therefore \text{Speed of sound, } \nu_0 &= \nu_0 \cdot \lambda_0 = 250 \times 1.32 \\ &= 330\text{m/s} \end{aligned}$$

As we know that,

$$\text{Speed of sound, } \nu \propto \sqrt{T}$$

$$\text{Thus, } \frac{\nu_1}{\nu_0} = \sqrt{\frac{T_1}{T_0}}$$

$$\begin{aligned} \nu_1 &= \nu_0 \sqrt{\frac{T_1}{T_0}} \\ &= 330 \sqrt{\frac{313}{273}} = 353.34\text{m/s} \end{aligned}$$

$$\therefore \nu_1 = \nu_0 \lambda_1$$

$$\lambda_1 = \frac{353.34}{250} = 1.41\text{m}$$

\therefore Change in the wavelength,

$$\begin{aligned} \Delta\lambda &= \lambda - \lambda_0 \\ &= 1.41 - 1.32 = 0.09\text{m} \end{aligned}$$

74. A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased, and he finds that the echo is not heard distinctly, when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 metres, and finds what the echo is again not heard when the drumming rates becomes 60 per minute. Calculate
- The distance between the mountain and the initial position of the man.
 - The velocity of sound.

Ans. : Let d be the distance between the man and the mountain and ν be the velocity of sound.

$$\therefore \text{Distance covered by the echo} = 2d$$

$$\text{Time} = \frac{2d}{\nu}$$

$$\text{Interval between the successive beats} = \frac{60}{40} = 1.5/\text{sec}$$

$$[\therefore \text{Drumming rate} = 40 \text{ per minute}]$$

According to the conditions given,

$$\frac{2d}{\nu} = 1.5 \dots (i)$$

$$\text{Again, } 2(d - 90) = \text{Distance covered by the echo}$$

$$\text{Or } \frac{2d - 180}{\nu} = 1$$

$$[\therefore \text{Drumming rate} = 60 \text{ per minute}] \dots (ii)$$

From (i) and (ii), we have

$$1.5 - \frac{180}{\nu} = 1$$

$$\nu = \frac{180}{0.5} = 360 \text{ m/s}$$

$$\therefore d = 170 \text{ m}$$

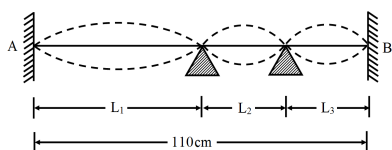
So,

- i. Distance = 270m
- ii. Velocity = 360m/s

75. Discuss the various factors influencing velocity of sound. A sonometer wire of length 110cm is stretched with a tension T and fixed at its ends. The wire is divided into three segments by placing two bridges below it. Where should the bridges be placed so that the fundamental frequencies of the segments are in the ratio 1: 2: 3?

Ans. : For factors influencing velocity of sound, see text.

Numerical: Let L_1 , L_2 , and L_3 be the lengths of the segments of wire AB (Fig.).



Then

$$L_1 + L_2 + L_3 = 110 \text{ cm} \dots (1)$$

Let n_1 , n_2 and n_3 be their respective fundamental frequencies, Thus

$$n_1 = \frac{1}{2L_1} \sqrt{\frac{T}{m}}$$

$$n_2 = \frac{1}{2L_2} \sqrt{\frac{T}{m}} \text{ and } n_3 = \frac{1}{2L_3} \sqrt{\frac{T}{m}}$$

$$\text{Hence } n_1 L_1 = n_2 L_2 = n_3 L_3 \dots (2)$$

$$\text{But } n_1 : n_2 : n_3 = 1 : 2 : 3$$

$$\therefore n_2 = 2n_1 \text{ and } n_3 = 3n_1 \dots (3)$$

From (2) and (3) we have

$$L_1 = 2L_2 = 3L_3 \dots (4)$$

Substituting (4) in (1) we get

$$L_1 + \frac{1}{2}L_1 + \frac{1}{3}L_1 = 110$$

$$L_1 = 60 \text{ cm}$$

$$\text{Hence } L_2 = 30 \text{ cm and } L_3 = 20 \text{ cm}$$

Thus, the bridges should be placed at distances of 60cm and 90cm from end A.

76. A simple harmonic wave is expressed by equation:

$$y = 7 \times 10^{-6} \sin \left(800\pi t - \frac{\pi}{42.5} x \right) \text{ where } y \text{ and } x \text{ are in cm. and } t \text{ in seconds.}$$

Calculate the following:

- i. Amplitude.
- ii. Frequency.
- iii. Wavelength.
- iv. Wave velocity.
- v. Phase difference between two particles separated by 17.0cm.

Ans. : Comparing the given equation with

$Y = A \sin(\omega t - kx)$, we get

- i. Amplitude = $A = 7 \times 10^{-6} \text{ cm}$
- ii. Frequency = $\nu = \frac{\omega}{2\pi} = \frac{800\pi}{2\pi} = 400 \text{ Hz}$
- iii. Wavelength = $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\left(\frac{\pi}{42.5}\right)} = 85 \text{ cm}$
- iv. Wave velocity = $\nu = \frac{\omega}{k} = \frac{800\pi}{\left(\frac{\pi}{42.5}\right)}$
 $= 3400 \text{ cm s}^{-1}$
 $= 340 \text{ ms}^{-1}$
- v. Using $= \frac{\phi}{2\pi} = \frac{x}{\lambda}$, we get
Phase difference = $\phi = \frac{2\pi}{85} \times 17$
 $= \frac{2\pi}{5} \text{ radian}$

77. An incident wave and a reflected wave are represented by: $\xi_1 = a \sin \frac{2\pi}{\lambda}(\nu t - x)$
 $\xi_2 = a \sin \frac{2\pi}{\lambda}(\nu t + x)$ Derive the equation of the stationary wave and calculate the position of the nodes and antinodes.

Ans. : $\xi_1 = a \sin \frac{2\pi}{\lambda}(\nu t - x)$ [Incident wave]

$\xi_2 = a \sin \frac{2\pi}{\lambda}(\nu t + x)$ [Reflected wave]

As there is a phase change of a radian on reflection at the rigid boundary, then

$$\xi_2 = a \sin \left[\frac{2\pi}{\lambda}(\nu t + x) + \pi \right]$$

$$\therefore \xi_2 = -a \sin \frac{2\pi}{\lambda}(\nu t + x)$$

According to the superposition principle, the resultant displacement y at time t and position x is given by

$$\xi = \xi_1 + \xi_2$$

$$\xi = a \sin \frac{2\pi}{\lambda}(\nu t - x) - a \sin \frac{2\pi}{\lambda}(\nu t + x)$$

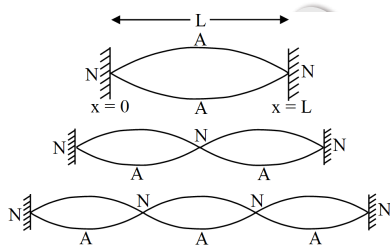
$$\therefore \xi = 2 \cos \frac{2\pi}{\lambda} \nu t \cdot \sin \frac{2\pi}{\lambda}(-x)$$

$$\xi = -2 \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} \nu t$$

Here $2 \sin \frac{2\pi}{\lambda} x = \text{Amplitude}$

Position of Nodes:

At nodes, amplitude = 0



From the figures, we see that there are two nodes in the first normal mode of vibration, then three nodes in the second normal mode; and so on, therefore in the n th normal mode of vibration, there will be $(n + 1)$ nodes.

These nodes are located at $x = 0, \frac{L}{n}, \frac{3L}{n}, \dots L$.

Position of Antinodes: At antinodes, displacement is maximum. As antinodes are located in between the nodes; therefore, their position will be given by

$$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \dots, \frac{(2n-1)L}{2n}$$

where $n = 1, 2, \dots$

78. A drop of water, 2mm in diameter, falling from a height of 50cm in a bucket generates sound which can be heard from the 5m distance. Take all the gravitational energy difference as going into the sound form, the transformation being spread in time over 0.2s. Deduce the average intensity and the amplitude of vibration at the listener's end. Given: density of air = 1.3 kg m^{-3} , frequency of wave = 1000Hz and $c = 350 \text{ ms}^{-1}$?

Ans. : Mass of drop, $m = \frac{4}{3}\pi r^3 \times \rho$, where ρ is the density of water.

Loss in gravitational energy when the drop falls through a height h ,

$$E = \frac{4}{3}\pi r^3 \times \rho \times gh$$

If t be the time during which this energy is fully converted into sound energy, then the intensity of sound at a distance R is given by

$$I = \frac{E}{4\pi R^2 \times t} = \frac{\frac{4}{3}\pi r^3 \times \rho gh}{4\pi R^2 t}$$

$$I = \frac{r^3 \rho gh}{3R^2 t}$$

Now, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$,

$r = 10^{-3} \text{ m}$

$\rho = 10^3 \text{ kg m}^{-3}$,

$g = 9.8 \text{ m s}^{-2}$,

$h = 50 \text{ cm} = 0.5 \text{ m}$,

$R = 5 \text{ m}$ and $t = 0.2 \text{ s}$

$$\therefore I = \frac{(10^{-3})^3 \times 1000 \times 9.8 \times 0.5}{3 \times (5)^2 \times 0.2} \text{ W m}^{-2}$$

$$= 3.267 \times 10^{-7} \text{ W m}^{-2}$$

$$\Rightarrow \text{amp} = \sqrt{I} = \sqrt{3.267 \times 10^{-7}}$$

$$= 5.71 \times 10^{-4} \text{ m}$$

79. Explain Doppler effect in sound. Obtain an expression for apparent frequency of sound when source moves and listener is at rest.

Ans. :

- Doppler effect:** The phenomena of apparent change in pitch of sound caused due to relative motion between a source and an observer is called Doppler effect.
- Let S and O be the source and observer. If v is the frequency of sound with velocity v released by the source, then a number of waves will be received by the observer at rest.
 - When the source approaches the stationary listener, the number of waves received, increases due to the apparent shortening of the wavelength.

$$\text{Wavelength perceived } \lambda' = \frac{\text{velocity of sound w.r. to moving source}}{\text{frequency}}$$

$$\therefore \lambda' = \frac{\nu - \nu_s}{V}$$

Using $\lambda' = \frac{\nu}{V'}$ we get,

$$\text{and } \frac{\nu}{V'} = \frac{\nu - \nu_s}{V}$$

$$V' = V \left(\frac{\nu}{\nu - \nu_s} \right)$$

- b. When the source is moving away from the listener who is at rest, then velocity of source is negative.

$$\therefore V' = \frac{\nu}{\nu - (-\nu_s)} V = \frac{\nu}{\nu + \nu_s} V$$

80. The displacement of an elastic wave is given by the function $y = 3 \sin \omega t + 4 \cos \omega t$ where y is in cm and t is in second. Calculate the resultant amplitude.

Ans. $\because y = 3 \sin \omega t + 4 \cos \omega t \dots (i)$

Let $3 = a \cos \phi \dots (ii)$

$4 = a \sin \phi \dots (iii)$

Then $y = a \cos \phi \sin \omega t + a \sin \phi \cos \omega t$

$y = a \sin(\omega t + \phi)$

From (ii) and (iii)

$$\tan \phi = \frac{4}{3} \text{ or } \phi = \tan^{-1} \frac{4}{3}$$

On squaring and adding (ii) and (iii) equations

$$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$$

$$a^2 (\cos^2 \phi + \sin^2 \phi) = 9 + 16$$

$$a^2 = 25 \Rightarrow a = 5$$

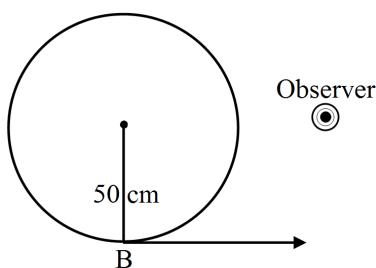
$$y' = 5 \sin(\omega t + \phi) \text{ when } \phi = \tan^{-1} \frac{4}{3}$$

Hence, New amplitude is 5 cm.

81.

i. What is beat phenomenon?

ii.



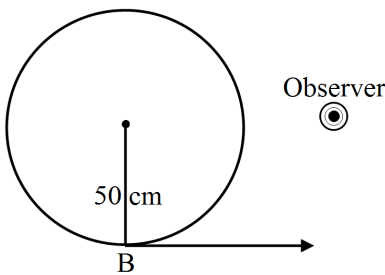
A whistle revolve in a circle with angular velocity of $\omega = 20 \text{ rad s}^{-1}$. If the frequency of the sound is 385 Hz and speed is 340 ms^{-1} , then find the frequency heard by the observer when the whistle is at B.

Ans. :

- i. **Beat phenomenon:** When two sound waves of nearly same frequencies and amplitudes travelling in a medium along the same direction, super impose on each other, the intensity of resultant sound at a particular position rises and falls alternatively with time.

This phenomenon of alternate variation in the intensity of sound with time at a particular position, when two sound waves of nearly same frequencies and amplitudes superimpose on each other is called beats.

ii.



$$W = 20 \text{ rad s}^{-1}$$

$$\text{Frequency of sound } \gamma = 385 \text{ Hz}$$

$$v = \text{Speed} = 340 \text{ ms}^{-1}$$

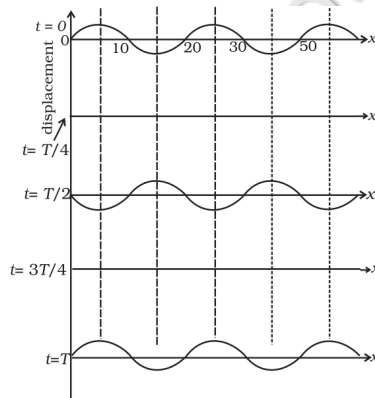
$$v_s = rw = 0.50 \times 20 = 10 \text{ m/s}$$

Using formula,

$$\begin{aligned} \gamma' &= \frac{v}{v - v_s} \times \gamma \\ &= \frac{340 \times 385}{340 - 10} = \frac{340 \times 385}{330} \\ &= 395 \text{ Hz} \end{aligned}$$

Hence, the frequency heard by an observer is 397 Hz when the whistle is at B.

82. The wave pattern on a stretched string is shown in Interpret what kind of wave this is



and find its wavelength.

Ans. : The displacement of medium particles at distance 10, 20, 30, 40, and 50 cm are always rest which is the property of nodes in stationary wave.

At $t = \frac{T}{4}$ and $\frac{3T}{4}$ all particles are at rest which is in stationary wave when the particle crosses its mean position.

so that graph of wave shows stationary wave.

The wave at $x = 10, 20, 30, 40$ cm there are nodes and distance between successive nodes is $\frac{\lambda}{2}$

$$\therefore \frac{\lambda}{2} = (30 - 20) \text{ or } \lambda = 20 \text{ cm.}$$

83. The transverse displacement of a string (clamped at its both ends) is given by

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right) \cos(120\pi t) \text{ where } x \text{ and } y \text{ are in m and } t \text{ in s. The length of}$$

the string is 1.5m and its mass is 3.0×10^{-2} kg. Answer the following: Determine the tension in the string.

Ans. : The velocity of a transverse wave travelling in a string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}} \dots (i)$$

Where,

Velocity of the transverse wave, $v = 180\text{m/s}$

Mass of the string, $m = 3.0 \times 10^{-2}\text{kg}$

Length of the string, $l = 1.5\text{m}$

Mass per unit length of the string, $\mu = \frac{m}{l}$

$$= \frac{3.0}{1.5} \times 10^{-2}$$

$$= 2 \times 10^{-2}\text{kg m}^{-1}$$

Tension in the string = T

From equation (i), tension can be obtained as:

$$T = v^2 \mu$$

$$= (180)^2 \times 2 \times 10^{-2}$$

$$= 648\text{N}$$

84. A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium.

(a) Does the pulse have a definite,

- i. frequency,
- ii. wavelength,
- iii. speed of propagation?

(b) If the pulse rate is 1 after every 20s, (that is the whistle is blown for a split of second after every 20s), is the frequency of the note produced by the whistle equal to $1/20$ or 0.05Hz ?

Ans. : (a)

- i. No
- ii. No
- iii. Yes

(b) No

Explanation:

- a. The narrow sound pulse does not have a fixed wavelength or frequency. However, the speed of the sound pulse remains the same, which is equal to the speed of sound in that medium.
- b. The short pip produced after every 20s does not mean that the frequency of the whistle is $1/20$ or 0.05Hz . It means that 0.05Hz is the frequency of the repetition of the pip of the whistle.

85. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6Hz . The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3Hz . If the original frequency of A is 324Hz , what is the frequency of B?

Ans. : Frequency of string A, $f_A = 324\text{Hz}$

Frequency of string B = f_B

Beat's frequency, $n = 6\text{Hz}$

Beat's frequency is given as:

$$n = |f_A \pm f_B|$$

$$6 = 324 \pm f_B$$

$$f_B = 330\text{Hz or } 318\text{Hz}$$

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension. It is given as:

$$v \propto \sqrt{T}$$

Hence, the beat frequency cannot be 330Hz

$$\therefore f_B = 318\text{Hz}$$

86. Find at what temperature the velocity of sound in air will be $1\frac{1}{2}$ times the velocity at 11°C .

Ans. : Suppose velocity of sound in air at $t^\circ\text{C}$ $1\frac{1}{2}$ times the velocity at 11°C .

$$\text{i.e., } v_t = \frac{3}{2}v_{11}$$

$$\text{As } v_t = v_0 \sqrt{\frac{273+t}{273}}$$

$$\therefore \text{from (i), } v_0 = \sqrt{\frac{273+t}{273}} = \frac{3}{2}v_{11}$$

$$v_0 = \sqrt{\frac{273+t}{273}} = \frac{3}{2}v_0 \frac{284}{273}$$

Squaring both sides, we get

$$\sqrt{\frac{273+t}{273}} = \frac{9}{4} \times \frac{284}{273}$$

$$1092 + 4t = 2556$$

$$4t = 2556 - 1092 = 1464$$

$$t = \frac{1464}{4} = 366^\circ\text{C}$$

87. In the given progressive wave $y = 5 \sin(100\pi t + 0.4\pi x)$ where y and x are in m, t is in s. What is the: Particle velocity amplitude.

Ans. : Standard form of progressive wave travelling in $+x$ direction (kx and ωt have opposite sign is given)

$$\text{Eqn. is } y = a \sin(\omega t - kx + \phi)$$

$$y = 5 \sin(100\pi t - 0.4\pi x + 0)$$

Particle (medium) velocity in the direction of amplitude at a distance x from source.

$$y = 5 \sin(100\pi t - 0.4\pi x)$$

$$\frac{dy}{dt} = 5 \times 100\pi \cos(100\pi t - 0.4\pi x)$$

For maximum velocity of particle is at its mean position

$$\cos(100\pi t - 0.4\pi x) = 1$$

$$\Rightarrow 100\pi t - 0.4\pi x = 0$$

$$\therefore \left(\frac{dy}{dt}\right)_{\max} = 5 \times 100\pi \times 1$$

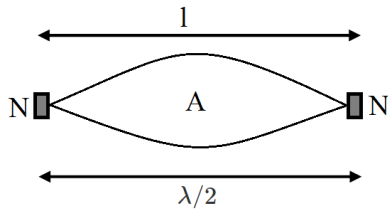
$$v_{\max} \text{ of medium particle} = 500\pi \text{ m/s}$$

88. A steel rod 100cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?

Ans. : Length of the steel rod, $l = 100\text{cm} = 1\text{m}$

Fundamental frequency of vibration, $\nu = 2.53\text{ kHz} = 2.53 \times 10^3\text{Hz}$

When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) are formed at its two ends, as shown in the given figure.



The distance between two successive nodes is $\frac{\lambda}{2}$.

$$\therefore l = \frac{\lambda}{2}$$

$$\lambda = 2l = 2 \times 1 = 2\text{m}$$

The speed of sound in steel is given by the relation:

$$v = \nu \lambda$$

$$= 5.06 \times 10^3\text{m/s}$$

$$= 2.53 \times 10^3 \times 2$$

$$= 5.06\text{km/s}$$

89. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air: Is independent of pressure,

Ans. : Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}} \dots (i)$$

where,

$$\text{Density, } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$$

M = Molecular weight of the gas

V = Volume of the gas

Hence, equation (i) reduces to:

$$v = \sqrt{\frac{\gamma PV}{M}} \dots (ii)$$

Now from the ideal gas equation for $n = 1$:

$$PV = RT$$

For constant T , $PV = \text{Constant}$

Since both M and γ are constants, $v = \text{Constant}$

Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

90. A sitar wire is under a tension of 40N and the length between the bridges is 70cm. A 5m sample of the wire has a mass of 1.0g. Deduce the speed of transverse waves on the wire, frequency of the fundamental and the frequency of the first two harmonics.

Ans. : $T = 40\text{N}$,

$$\mu = \frac{1.0\text{gm}}{5\text{m}} = \frac{10^{-3}\text{kg}}{5\text{m}}$$

$$= 0.0002\text{kg/m}$$

$$l = 70\text{cm} = 0.7\text{m}$$

$$c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{40}{0.0002}} = 447.2\text{ms}^{-1}$$

$$v_1 = \frac{1}{2l} \sqrt{\frac{T}{\mu}} = \frac{447.2}{2 \times 0.7} = \frac{447.2}{1.4}$$

$$= 319.4\text{Hz}$$

$$v_2 = \frac{2}{2l} \sqrt{\frac{T}{\mu}} = 638.8\text{Hz}$$

$$v_3 = \frac{3}{2l} \sqrt{\frac{T}{\mu}} = 958.2\text{Hz}$$

91. For the harmonic travelling wave $y = 2 \cos 2\pi(10t - 0.0080x + 3.5)$ where x and y are in cm and t is second. What is the phase difference between the oscillatory motion at two points separated by a distance of: What is the phase difference between the oscillation of a particle located at $x = 100\text{cm}$ at $t = T$ and $t = 5\text{s}$?

Ans. : $y = 2 \cos 2\pi(10t - 0.0080x + 3.5)$

$$y = 2 \cos(20\pi t - 0.0016\pi x + 7.0\pi)$$

Wave is propagated in $+x$ direction because ωt and kx are in with opposite sign standard equation $y = a \cos(\omega t - kx + \phi)$

$$a = 2 \quad \omega = 20\pi, \quad k = 0.016\pi \text{ and } \phi = 7\pi$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10}\text{sec}$$

$$x = 100\text{cm}$$

$$\text{At } x = 100, \quad t = T$$

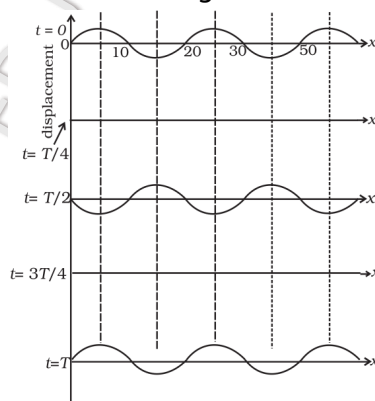
$$\phi_1 = 20\pi T - 0.016\pi(100) + 7\pi = 20\pi \times \frac{1}{10} - 1.6\pi + 7\pi = 7.4\pi$$

$$\text{At } t = 5\text{s}$$

$$\phi_2 = 20\pi(5) - 0.016\pi(100) + 7\pi = 100\pi - 1.6\pi + 7\pi = 105.4\pi$$

$$\phi_2 - \phi_1 = 105.4\pi - 7.4\pi = 98\pi \text{ radian}$$

92. The wave pattern on a stretched string is shown in Interpret what kind of wave this is



and find its wavelength.

Ans. : The displacement of medium particles at distance 10, 20, 30, 40, and 50cm are always rest which is the property of nodes in stationary wave.

At $t = \frac{T}{4}$ and $\frac{3T}{4}$ all particles are at rest which is in stationary wave when the particle crosses its mean position.

so that graph of wave shows stationary wave.

The wave at $x = 10, 20, 30, 40\text{cm}$ there are nodes and distance between successive nodes is $\frac{\lambda}{2}$

$$\therefore \frac{\lambda}{2} = (30 - 20) \text{ or } \lambda = 20\text{cm}.$$

93. Given below are some functions of x and t to represent the displacement of an elastic wave. $y = 100 \cos(100\pi t + 0.5x)$

Ans. : $y = 4 \sin\left(5x - \frac{t}{2}\right) + 3 \cos\left(5x - \frac{t}{2}\right)$

Let $4 = a \cos \phi \dots (ii)$ and $3 = a \sin \phi \dots (iii)$

$$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 4^2 + 3^2 \text{ Squaring and adding (ii), (iii)}$$

$$a^2 = 25 \Rightarrow a = 5$$

Substituting (ii), (iii) in (i)

$$y = a \cos \phi \sin\left(5x - \frac{t}{2}\right) + a \sin \phi \cos\left(5x - \frac{t}{2}\right)$$

$$y = a \sin\left(5x - \frac{t}{2} + \phi\right)$$

$$y = 5 \sin\left(5x - \frac{t}{2} + \phi\right)$$

Which represents the progressive wave in $+x$ direction as the sign of Kx (or $5x$) and $\omega t\left(\frac{1}{2}t\right)$ are opposite so it travels in $+x$ direction. So (d) (ii)

94. The displacement of an elastic wave is given by the function $y = 3 \sin \omega t + 4 \cos \omega t$ where y is in cm and t is in second. Calculate the resultant amplitude.

Ans. : $y = 3 \sin \omega t + 4 \cos \omega t \dots (i)$

Let $3 = a \cos \phi \dots (ii)$

$4 = a \sin \phi \dots (iii)$

Then $y = a \cos \phi \sin \omega t + a \sin \phi \cos \omega t$

$$y = a \sin(\omega t + \phi)$$

From (ii) and (iii)

$$\tan \phi = \frac{4}{3} \text{ or } \phi = \tan^{-1} \frac{4}{3}$$

On squaring and adding (ii) and (iii) equations

$$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$$

$$a^2 (\cos^2 \phi + \sin^2 \phi) = 9 + 16$$

$$a^2 = 25 \Rightarrow a = 5$$

$$y' = 5 \sin(\omega t + \phi) \text{ when } \phi = \tan^{-1} \frac{4}{3}$$

Hence, New amplitude is 5 cm.

95. If c is r.m.s. speed of molecules in a gas and v is the speed of sound waves in the gas, show that c/v is constant and independent of temperature for all diatomic gases.

Ans. : We know that $c\sqrt{\frac{3p}{\rho}}$ for molecules.

$$c = \sqrt{\frac{3RT}{M}}$$

$$\therefore \frac{p}{\rho} = \frac{PT}{M} \therefore \frac{p}{\rho} = \frac{RT/V}{M/V}$$

M = molar mass of gas

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore RV = nRT$$

$$n = 1$$

$$p \frac{PT}{V}$$

$$\frac{c}{v} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{\gamma RT}{M}}} = \sqrt{\frac{3}{\gamma}}$$

$$\gamma = \frac{C_p}{C_v} = \text{adiabatic constant for diatomic gas}$$

$$\gamma = \frac{7}{5}$$

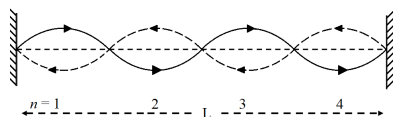
$$\therefore \frac{c}{v} = \sqrt{\frac{3}{7/5}} = \sqrt{\frac{15}{7}} = \text{constant.}$$

96. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1 : 2 : 3 : 4.

Ans. : Let n be the number of loop in the string.

The length of each loop is $\frac{\lambda}{2}$

$$\therefore L = \frac{n\lambda}{2} \text{ or } \lambda = \frac{2L}{n}$$



$$v = v\lambda \text{ and } \lambda = \frac{v}{v}.$$

$$\text{so } \frac{v}{v} = \frac{2L}{n}$$

$$v = \frac{n}{2L} \cdot v \text{ } v \text{ is stretch string} = \sqrt{\frac{T}{m}}$$

$$\therefore v = \frac{n}{2L} \sqrt{\frac{T}{m}}$$

$$\text{For } n=1, v_1 = \frac{1}{2L} \sqrt{\frac{T}{m}} = v_0$$

$$\text{If } n=2 \text{ then } v_2 = \frac{2}{2L} \sqrt{\frac{T}{m}} = 2v_0$$

$$n=3 \text{ then } v_3 = \frac{3}{2L} \sqrt{\frac{T}{m}} = 3v_0$$

$$\therefore v_1 : v_2 : v_3 : v_4 := n_1 : n_2 : n_3 : n_4 := 1 : 2 : 3 : 4$$

97. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, by what factor will the frequency change?

Ans. : The wire is stretched both and so frequency of stretched wire is $v = \frac{n}{2L} \sqrt{\frac{T}{m}}$

As number of harmonic n , length L and tension (T) are kept same in both cases.

$$\therefore v \propto \frac{1}{\sqrt{m}}$$

$$\frac{v_1}{v_2} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \dots (i)$$

$$\text{Mass per unit length} = \frac{\text{mass of wire}}{\text{length}} = \frac{(\pi r^2 l) \rho}{l}$$

$$m = \pi r^2 \rho$$

As material of wire is same.

$$\frac{m_2}{m_1} = \frac{\pi r_2^2 \rho}{\pi r_1^2 \rho} = \frac{(3r)^2}{r^2} = \frac{9}{1}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$$

$$\therefore v_2 = \frac{1}{3} v_1$$

So the frequency of sitar reduced by $\frac{1}{2}$ of previous value.

98. The earth has a radius of 6400km. The inner core of 1000km radius is solid. Outside it, there is a region from 1000km to a radius of 3500km which is in molten state. Then again from 3500km to 6400km the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of 8km s^{-1} in solid parts and of 5km s^{-1} in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter?

Ans. : $r_1 = 1000\text{km}$

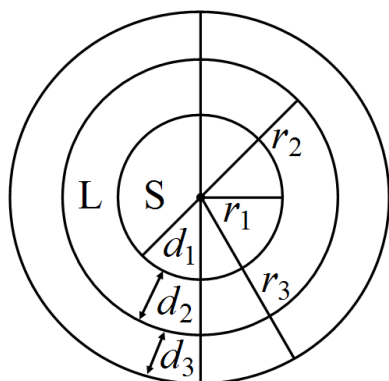
$$r_2 = 3500\text{km}$$

$$r_3 = 6400\text{km}$$

$$d_1 = 1000\text{km}$$

$$d_2 = 3500 - 1000 = 2500\text{km}$$

$$d_3 = 6400 - 3500 = 2900\text{km}$$



Solid distance diametrically

$$= 2(d_1 + d_3) = (1000 + 2900)$$

$$2 \times 3900\text{km}$$

Time taken by wave produced by earthquake in solid part

$$= \frac{3900 \times 2}{8} \text{sec}$$

Liquid part along diametrically $2d_2 = 2 \times 2500$

\therefore Time taken by seismic wave in liquid part = $\frac{2 \times 2500}{5}$

$$\text{Total time } \frac{2 \times 3900}{8} + \frac{2 \times 2500}{5} = 2 \left[\frac{3900}{8} + \frac{2500}{5} \right]$$

$$= 2[487.5 + 500] = 2 \times 987.5 = 1975 \text{ sec.}$$

$$= 32 \text{ min } 55 \text{ sec.}$$

99. Two wires are kept tight between the same pair of supports. The tensions in the wires are in the ratio 2 : 1, the radii are in the ratio 3 : 1 and the densities are in the ratio 1 : 2. Find the ratio of their fundamental frequencies.

Ans. : Frequency $f = \frac{1}{lD} \sqrt{\frac{T}{\pi\rho}}$

$$\Rightarrow f_1 = \frac{1}{l_1 D_1} \sqrt{\frac{T_1}{\pi\rho_1}}$$

$$\Rightarrow f_2 = \frac{1}{l_2 D_2} \sqrt{\frac{T_2}{\pi\rho_2}}$$

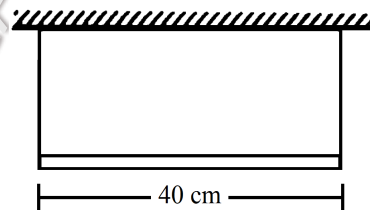
Given that, $\frac{T_1}{T_2} = 2$, $\frac{r_1}{r_2} = 3 = \frac{D_1}{D_2}$

$$\frac{\rho_1}{\rho_2} = \frac{1}{2}$$

So, $\frac{f_1}{f_2} = \frac{l_2 D_2}{l_1 D_1} \sqrt{\frac{T_1}{T_2}} \sqrt{\frac{\pi\rho_2}{\pi\rho_1}}$ ($l_1 = l_2 = \text{length of string}$)

$$\Rightarrow f_1 : f_2 = 2 : 3$$

100. A uniform horizontal rod of length 40cm and mass 1.2kg is supported by two identical wires as shown in figure, Where should a mass of 4.8kg be placed on the rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on



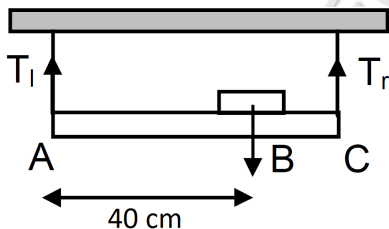
right into its first overtone? Take $g = 10 \text{ m/s}^2$.

Ans. : Length of the rod = $L = 40 \text{ cm} = 0.4 \text{ m}$

Mass of the rod $m = 1.2 \text{ kg}$

Let the 4.8kg mass be placed at a distance 'x' from the left end.

Given that, $f_l = 2f_r$

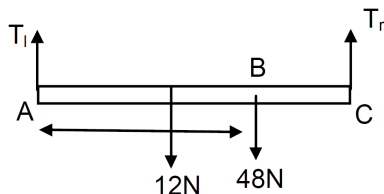


$$\therefore \frac{1}{2l} \sqrt{\frac{T_l}{m}} = \frac{2}{2l} \sqrt{\frac{T_r}{m}}$$

$$\Rightarrow \sqrt{\frac{T_l}{T_r}} = 2$$

$$\Rightarrow \frac{T_l}{T_r} = 4 \dots (1)$$

From the freebody diagram,



$$T_l + T_r = 60\text{N}$$

$$\Rightarrow 4T_r + T_r = 60\text{N}$$

$$\therefore T_r = 12\text{N and } T_l = 48\text{N}$$

Now taking moment about point A,

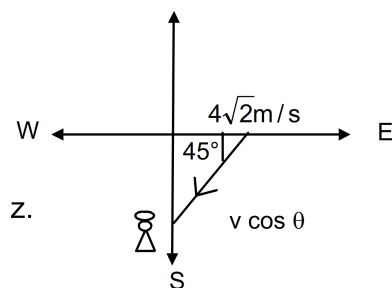
$$T_r \times (0.4) = -48x + 12(0.2)$$

$$\Rightarrow x = 5\text{cm}$$

So, the mass should be placed at a distance 5cm from the left end.

101. A boy riding on his bike is going towards east at a speed. of $4\sqrt{2}\text{m/s}$ At a certain point he produces a sound pulse of frequency 1650Hz that travels in air at a speed of 334m/s. A second boy stands on the ground 45° south of east from him. Find the frequency of the pulse as received by the second boy.

Ans. :



$$u = 334\text{m/s}, v_b = 4\sqrt{2}\text{m/s}, v_0 = 0$$

$$\text{So, } v_s = V_b \cos \theta = 4\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right) = 4\text{m/s}$$

$$\text{So, the apparent frequency } f' = \left(\frac{u+0}{u-v_b \cos \theta}\right) f = \left(\frac{334}{334-4}\right) \times 1650 = 1670\text{Hz.}$$

102. A train running at 108km/h towards east whistles at a dominant frequency of 500Hz. Speed of sound in air is 340m/s.
- What frequency will a passenger sitting near the open window hear?
 - What frequency will a person standing near the track hear whom the train has just passed?
 - A wind starts blowing towards east at a speed of 36km/h. Calculate the frequencies heard by the passenger in the train and by the person standing near the track.

Ans. :

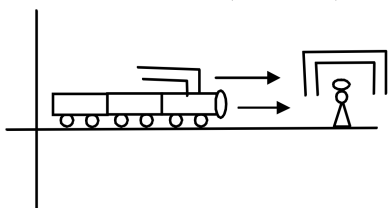
- a. The frequency by the passenger sitting near the open window is 500Hz. The speed of sound in air $V = 340\text{m/s}$.

$$\text{Speed of the source } u = 180\text{km/h}$$

$$= \frac{108000}{3600}\text{m/s} = 30\text{m/s.}$$

- a. After the train has passed the apparent frequency heard by a person standing near the track will be,

$$\text{So, } f'' = \left(\frac{340+0}{340+30} \right) \times 500 = 459\text{Hz}$$



- c. The person inside the source will listen the original frequency of the train.

Here, given $V_m = 10\text{m/s}$

For the person standing near the track

$$\text{Apparent frequency} = \frac{u+V_m+0}{u+V_m-(-V_s)} \times 500 = 458\text{Hz}.$$

103. A source of sound emitting a 1200Hz note travels along a straight line at a speed of 170m/s. A detector is placed at a distance of 200m from the line of motion of the source.

- Find the frequency of sound received by the detector at the instant when the source gets closest to it.
- Find the distance between the source and the detector at the instant it detects the frequency 1200Hz. Velocity of sound in air = 340m/s.

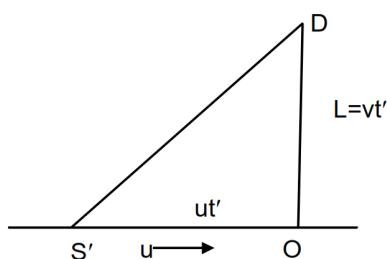
Ans. :

- a. Given that, $f = 1200\text{Hz}$, $u = 170\text{m/s}$, $L = 200\text{m}$, $v = 340\text{m/s}$

From Doppler's equation (as in problem no.84)

$$f' = f \left(\frac{v^2}{v^2 - u^2} \right) = 1200 \times \frac{340^2}{340^2 - 170^2} = 1600\text{Hz}.$$

(Detector)



- b. v = velocity of sound, u = velocity of source let, t be the time taken by the sound to reach at D

$$DO = vt' = L, \text{ and } S'O = ut'$$

$$t' = \frac{L}{v}$$

$$S'D = \sqrt{S'O^2 + DO^2} = \sqrt{u^2 \frac{L^2}{v^2} + L^2} = \frac{L}{v} \sqrt{u^2 + v^2}$$

Putting the values in the above equation, we get

$$S'D = \frac{220}{340} \sqrt{170^2 + 340^2} = 223.6\text{m}.$$

104. Calculate the frequency of beats produced in air when two sources of sound are activated, one emitting a wavelength of 32cm and the other of 32.2cm. The speed of sound in air is 350m/s.

Ans. : Group - I

Given $V = 350$

$$\lambda_1 = 32\text{cm}$$

$$= 32 \times 10^{-2}\text{m}$$

So, $\eta_1 = \text{frequency} = 1093\text{Hz}$

Group - II

$$v = 350$$

$$\lambda_2 = 32.2\text{cm}$$

$$= 32.2 \times 10^{-2}\text{m}$$

$$\eta_2 = \frac{V}{\lambda} = \frac{350}{32.2 \times 10^{-2}} = 1086.96\text{Hz}$$

So beat frequency $= 1093 - 1086 = 7\text{Hz}$.

105. A sound wave of frequency 100Hz is travelling in air. The speed of sound in air is 350m/s.

- By how much is the phase changed at a given point in 2.5ms?
- What is the phase difference at a given instant between two points separated by a distance of 10.0cm along the direction of propagation?

Ans. :

a. Here given $n = 100$, $v = 350\text{m/s}$

$$\Rightarrow \lambda = \frac{v}{n}$$

$$= \frac{350}{100} = 3.5\text{m}$$

In 2.5ms, the distance travelled by the particle is given by

$$\Delta x = 350 \times 2.5 \times 10^{-3}$$

So, phase difference

$$\phi = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \frac{2\pi}{\left(\frac{350}{100}\right)} \times 350 \times 2.5 \times 10^{-3} = \left(\frac{\pi}{2}\right).$$

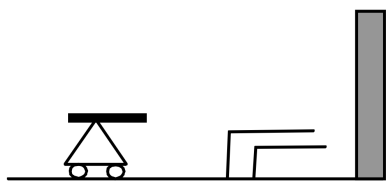
b. In the second case, Given $\Delta x = 10\text{cm} = 10^{-1}\text{m}$

$$\text{So, } \phi = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi \times 10^{-1}}{\left(\frac{350}{100}\right)} = \frac{2\pi}{35}.$$

106. A boy riding on a bicycle going at 12km/h towards a vertical wall whistles at his dog on the ground. If the frequency of the whistle is 1600Hz and the speed of sound in air is 330m/s, find

- The frequency of the whistle as received by the wall.
- The frequency of the reflected whistle as received by the boy.

Ans. : To find out the apparent frequency received by the wall,



a. $V_s = 12\text{km/h} = \frac{10}{3} = \text{m/s}$

$$V_0 = 0, u = 330\text{m/s}$$

$$\text{So, the apparent frequency is given by } f' = \left(\frac{330}{\frac{330-10}{3}} \right) \times 1600 = 1616\text{Hz}$$

- b. The reflected sound from the wall whistles now act as a sources whose frequency is 1616Hz.

$$\text{So, } u = 330\text{m/s, } V_s = 0, V_0 = \frac{10}{3}\text{m/s}$$

So, the frequency by the man from the wall,

$$\Rightarrow f'' = \left(\frac{330 + \frac{10}{3}}{330} \right) \times 1616 = 1632\text{m/s.}$$

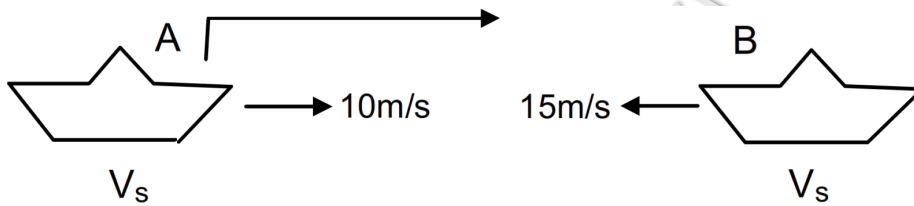
* Case study based questions

[4]

107. Two submarines are approaching each other in a calm sea. The first submarine travels at a speed of 36km/h and the other at 54km/h relative to the water. The first submarine sends a sound signal (sound waves in water are also called sonar) at a frequency of 2000Hz.

- At what frequency is this signal received by the second submarine?
- The signal is reflected from the second submarine. At what frequency is this signal received by the first submarine. Take the speed of the sound wave in water to be 1500m/s.

Ans. :



- According to the questions, $v = 1500\text{m/s}$, $f = 2000\text{Hz}$, $v_s = 10\text{m/s}$, $v_o = 15\text{m/s}$

So, the apparent frequency heard by the submarine B,

$$= \left(\frac{1500+15}{1500-10} \right) \times 2000 = 2034\text{Hz.}$$

- Apparent frequency received by submarine A,

$$= \left(\frac{1500+10}{1500-15} \right) \times 2034 = 2068\text{Hz.}$$

----- खुदी को कर बुलंद इतना कि हर तकदीर से पहले खुदा बंदे से खुद पूछे बता तेरी रज़ा क्या है -----