

* Choose the right answer from the given options. [1 Marks Each]

[64]

Ans. :

c. 60

Solution:

Given, $a_3 = 6$ and $a_5 = 12$.

$$\Rightarrow a + 2d = 6 \text{ and } a + 4d = 12$$

$$\Rightarrow 2d = 6 \Rightarrow d = 3$$

2. The sum of first three terms of a G.P. is $\frac{21}{2}$ and their product is 27. Find the common ratio.

(A) 2

c. 2 or $\frac{1}{2}$

Solution:

Let three terms be $\frac{a}{r}$, a , $a \times r$.

$$\text{Product} = 27 \Rightarrow \left(\frac{a}{r}\right) (a) (a \times r) = 27 \Rightarrow a^3 = 27$$

$$\Rightarrow a = 3.$$

$$\text{sum} = \frac{21}{2} \Rightarrow \frac{(a)}{r+a \times r} = \frac{21}{2} \Rightarrow a \left(\frac{1}{r+1+1 \times r} \right) = \frac{21}{2}$$

$$\Rightarrow \left(\frac{1}{r+1+1 \times r} \right)$$

$$\Rightarrow (r^2 + r + 1) = \left(\frac{7}{2}\right) \Rightarrow r^2 - \left(\frac{5}{2}\right)r + 1 = 0$$

$$\Rightarrow r = 2 \text{ and } \frac{1}{2}.$$

3. If in an infinite G.P., first term is equal to 10 times the sum of all successive terms, the its common ratio is:

$$(\wedge) \stackrel{1}{=}$$

b. $\frac{1}{11}$

11 Solution:

Let the first term of the G.P. be a .

Let its common ratio be r .

According to the question, we have:

First term = 10 [Sum of all successive terms]

$$a = 10 \left(\frac{ar}{1-r} \right)$$

$$\Rightarrow a - ar = 10ar$$

$$\Rightarrow 11ar = a$$

$$\Rightarrow r = \frac{a}{11a} = \frac{1}{11}$$

4. If an A.P. is 1, 7, 13, 19, Find the sum of 22 terms.

(A) 127

(B) 1204

(C) 1408

(D) 1604

Ans. :

c. 1408

Solution:

From the given A.P., $a = 1$ and $d = 7 - 1 = 6$.

$$\text{We know, } s_n = \frac{n}{2}(2a + (n-1)d) \quad s_{22} = \frac{22}{2}(2 \times 1 + (22-1)6) \\ = 11(2 + 126) = 11 \times 128 = 1408.$$

5. After striking the floor, a certain ball rebounds $\frac{4}{5}$ th of height from which it has fallen. Then, the total distance that it travels before coming to rest, if it is gently dropped from a height of 120 m is:

(A) 1260 m

(B) 600 m

(C) 1080 m

(D) None of these

Ans. :

c. 1080 m

6. If a, b, c are in G.P. is 2 and x, y are AM's between a, b and b, c respectively, then:

$$(A) \frac{1}{x} + \frac{1}{y} = 2$$

$$(B) \frac{1}{x} + \frac{1}{y} = \frac{1}{2}$$

$$(C) \frac{1}{x} + \frac{1}{y} = \frac{2}{a}$$

$$(D) \frac{1}{x} + \frac{1}{y} = \frac{2}{b}$$

Ans. :

d. $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$

Solution:

a, b and c are in G.P.

$$\therefore b^2 = ac \dots (i)$$

a, x and b are in A.P.

$$\therefore 2x = a + b \dots (ii)$$

Also, b, y and c are in A.P.

$$\therefore 2y = b + c$$

$$\Rightarrow 2y = b + \frac{b^2}{a} \quad [\text{Using (i)}]$$

$$\begin{aligned}
 \Rightarrow 2y &= b + \frac{b^2}{(2x-b)} \quad [\text{Using (ii)}] \\
 \Rightarrow 2y &= b + \frac{b(2x-b) + b^2}{(2x-b)} \\
 \Rightarrow 2y &= \frac{2bx - b^2 + b^2}{(2x-b)} \\
 \Rightarrow 2y &= \frac{2bx}{(2x-b)} \\
 \Rightarrow y &= \frac{bx}{(2x-b)} \\
 \Rightarrow y(2x-b) &= bx \\
 \Rightarrow 2xy - by &= bx \\
 \Rightarrow bx + by &= 2xy
 \end{aligned}$$

Dividing both the sides by xy :

$$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{2}{b}$$

7. Let $s = \frac{8}{5} + \frac{16}{65} + \dots + \frac{128^{18}}{2} + 1$:

$$\begin{array}{llll}
 \text{(A)} \ s = \frac{1088}{545} & \text{(B)} \ s = \frac{1088}{545} & \text{(C)} \ s = \frac{1056}{545} & \text{(D)} \ s = \frac{545}{1056}
 \end{array}$$

Ans. :

$$\text{a. } s = \frac{1088}{545}$$

8. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \dots + \frac{1}{\log_2^n 4}$ is:
- $$\begin{array}{llll}
 \text{(A)} \ \frac{n(n+1)}{2} & \text{(B)} \ \frac{n(n+1)(2n+1)}{12} & \text{(C)} \ \frac{n(n+1)}{4} & \text{(D) none of these.}
 \end{array}$$

Ans. :

$$\text{c. } \frac{n(n+1)}{4}$$

Solution:

$$\begin{aligned}
 \text{Let } S_n &= \frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \dots + \frac{1}{\log_2^n 4} \\
 \Rightarrow S_n &= \frac{\log 2}{\log 4} + \frac{\log 4}{\log 4} + \frac{\log 8}{\log 4} + \dots + \frac{\log 2^n}{\log 4} \\
 \Rightarrow S_n &= \frac{\log 2}{\log 4} + \frac{\log 2^2}{\log 4} + \frac{\log 2^3}{\log 4} + \dots + \frac{\log 2^n}{\log 4} \\
 \Rightarrow S_n &= \frac{\log 2}{\log 4} + \frac{2\log 2}{\log 4} + \frac{3\log 2}{\log 4} + \dots + \frac{n\log 2}{\log 4} \\
 \Rightarrow S_n &= \frac{\log 2}{\log 4} (1 + 2 + 3 + \dots + n) \\
 \Rightarrow S_n &= \frac{\log 4^{\frac{1}{2}}}{\log 4} (1 + 2 + 3 + \dots + n) \\
 \Rightarrow S_n &= \frac{\frac{1}{2}\log 4}{\log 4} (1 + 2 + 3 + \dots + n) \\
 \Rightarrow S_n &= \frac{1}{2} (1 + 2 + 3 + \dots + n)
 \end{aligned}$$

$$\Rightarrow S_n = \frac{n(n+1)}{4}$$

9. Find the sum of squares of first n terms.

(A) $\frac{n(n+1)}{2}$

(B) $\left(\frac{n(n+1)}{2}\right)^3$

(C) $\frac{n(n+1)(2n+1)}{6}$

(D) $\left(\frac{n(n+1)}{2}\right)^2$

Ans. :

c. $\frac{n(n+1)(2n+1)}{6}$

Solution:

$$\text{Sum of squares of first } n \text{ terms} = 1^2 + 2^2 + 3^2 + \dots + n^2$$

$$k^3 - (k-1)^3 = 3k^2 - 3k + 1$$

On substituting $k = 1, 2, 3, \dots, n$ and adding we get,

$$n^3 = \sum_{i=1}^n k^3 = 0 \quad k^2 = \sum_{i=1}^n k^2 = 0 \quad k + n$$

$$n^3 = 3 \sum_{i=1}^2 k^2 = 0 \quad k^2 - 3 \frac{n(n+1)}{2} + n$$

$$\sum_{i=1}^n k^2 = \frac{n(n+1)(n+2)}{6}.$$

10. The n th term of a G.P. is 128 and the sum of its n terms is 225. If its common ratio is 2, then its first term is:

(A) 1

(B) 3

(C) 8

(D) None of these.

Ans. :

a. 1

Solution:

Let the first term of the geometric progression = x

Common ratio = 2

∴ 2nd term of the G.P. = $2x$

∴ 3rd term = $(2^2)x \dots$

N th term can be written as = $(2^n)x$

Sum of the n terms $S = 255$

as we can see, except x , all other terms in the G.P. are multiples of 2

and sum of all the terms is an odd number.

∴ x must be an odd number.

now n^{th} term

$$(2^n)x = 128 = (2^7) \times 1$$

There are no factors of odd numbers in 128, except 1

$$\therefore x = 1$$

Series of G.P. is:

1, 2, 4, 8, 16, 32, 64, 128

Checking the sum of the n terms,

$$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255$$

∴ First term of the G.P. = 1

11. If first term of a G.P. is 20 and common ratio is 4. Find the 5th term.
(A) 10240 (B) 40960 (C) 5120 (D) 2560

Ans. :

c. 5120

solution:

Given, $a = 20$ and $r = 4$.

We know, $a_n = ar^{n-1}$

$$\Rightarrow a_5 = 20 \times 4^4 = 20 \times 256 = 5120.$$

12. If A.M. of two numbers is $\frac{15}{2}$ and their G.M. is 6, then find the two numbers.
(A) 6 and 8 (B) 12 and 3 (C) 24 and 6 (D) 27 and 3

Ans. :

b. 12 and 3

Solution:

We know, A.M. of two numbers a and b is

$$\frac{(a+b)}{2}$$

$$\Rightarrow \frac{(a+b)}{2} = \frac{15}{2} \Rightarrow a+b = 15.$$

Also, G.M. of two numbers a and b is \sqrt{ab}

$$\Rightarrow \sqrt{ab} = 6 \Rightarrow ab = 36.$$

$$\Rightarrow a(15-a) = 36 \Rightarrow a=3 \text{ or } 12.$$

For $a=3$, $b=12$.

For $a=12$, $b=3$.

So, the two numbers are 3 and 12.

13. If $S_n = \sum_{r=1}^n \frac{1+2+2^2+\dots \text{ Sum to r terms}}{2^r}$, then S_n is equal to:

Ans. :

$$c. \quad n-1 - \frac{1}{2^n}$$

Solution:

We have,

$$S_n = \sum_{r=1}^n \frac{1+2+2^2+\dots \text{ Sum to } r \text{ terms}}{2^r}$$

$$\begin{aligned}
 \Rightarrow S_n &= \sum_{r=1}^n \frac{1(2^r - 1)}{2^r} \\
 \Rightarrow S_n &= \sum_{r=1}^n \left(1 - \frac{1}{2^r}\right) \\
 \Rightarrow S_n &= n - \sum_{r=1}^n \left(\frac{1}{2^r}\right) \\
 \Rightarrow S_n &= n - \left[\frac{\left(\frac{1}{2}\right) \left\{ 1 - \left(\frac{1}{2}\right)^n \right\}}{1 - \frac{1}{2}} \right] \\
 \Rightarrow S_n &= n - \left[1 - \left(\frac{1}{2}\right)^n \right] \\
 \Rightarrow S_n &= n - 1 + \frac{1}{2^n}
 \end{aligned}$$

14. Find the sum of series $1^2 + 3^2 + 5^2 + \dots + 11^2$.

(A) 279

b 286

solution:

$$\begin{aligned}
 & 1^2 + 3^2 + 5^2 + \dots + 11^2 \\
 &= (1^2 + 2^2 + 3^2 + \dots + 11^2) - (2^2 + 4^2 + 6^2 + 8^2 + 10^2) \\
 &= (1^2 + 2^2 + 3^2 + \dots + 11^2) - 2^2(1^2 + 2^2 + 3^2 + 4^2 + 5^2) \\
 &= \frac{16 \times 12 \times 23}{6} - \frac{4 \times 5 \times 6 \times 11}{6} \\
 &= 506 - 220 = 286
 \end{aligned}$$

15. Find the sum of series $6^2 + 7^2 + \dots + 15^2$.

(A) 55

108 •

1185

Solution:

$$\begin{aligned}
 & 6^2 + 7^2 + \dots + 15^2 \\
 &= (1^2 + 2^2 + 3^2 + \dots + 15^2) - (1^2 + 2^2 + 3^2 + 4^2 + 5^2) \\
 &= \frac{15 \times 16 \times 31}{6} - \frac{5 \times 6 \times 11}{6} \\
 &= 1240 - 55 = 1185
 \end{aligned}$$

16. The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is 92. The common ratio of original G.P. is:

(A) 1

2
ns. :

Solution:

$$\frac{a}{1-r} = 3$$

Sum of square terms of G.P. is $\frac{a^2}{1-r^2} = 3$

$$\Rightarrow \frac{a}{1-r} = \frac{a^2}{1-r^2}$$

$$\text{or } a = 1 + r \dots (2)$$

Solving (1) and (2),

$$a = \frac{3}{2} \text{ and } r = \frac{1}{2}$$

$$17. \quad \frac{1}{56} + \frac{1}{144} + \frac{1}{288} + \frac{1}{576} + \frac{1}{1152} + \frac{1}{2304} + \frac{1}{4608} + \frac{1}{9216} + \frac{1}{18432} + \frac{1}{36864} + \frac{1}{73728} + \frac{1}{147456} + \frac{1}{294912} + \frac{1}{589824} + \frac{1}{1179648} + \frac{1}{2359296} + \frac{1}{4718592} + \frac{1}{9437184} + \frac{1}{18874368} + \frac{1}{37748736} + \frac{1}{75497472} + \frac{1}{150994944} + \frac{1}{301989888} + \frac{1}{603979776} + \frac{1}{120795952} + \frac{1}{241591904} + \frac{1}{483183808} + \frac{1}{966367616} + \frac{1}{1932735232} + \frac{1}{3865470464} + \frac{1}{7730940928} + \frac{1}{15461881856} + \frac{1}{30923763712} + \frac{1}{61847527424} + \frac{1}{123695054848} + \frac{1}{247390109696} + \frac{1}{494780219392} + \frac{1}{989560438784} + \frac{1}{1979120877568} + \frac{1}{3958241755136} + \frac{1}{7916483510272} + \frac{1}{15832967020544} + \frac{1}{31665934041088} + \frac{1}{63331868082176} + \frac{1}{126663736164352} + \frac{1}{253327472328704} + \frac{1}{506654944657408} + \frac{1}{1013309889314816} + \frac{1}{2026619778629632} + \frac{1}{4053239557259264} + \frac{1}{8106479114518528} + \frac{1}{16212958229037056} + \frac{1}{32425916458074112} + \frac{1}{64851832916148224} + \frac{1}{129603665832296448} + \frac{1}{259207331664592896} + \frac{1}{518414663329185792} + \frac{1}{1036829326658371584} + \frac{1}{2073658653316743168} + \frac{1}{4147317306633486336} + \frac{1}{8294634613266972672} + \frac{1}{16589269226533945344} + \frac{1}{33178538453067890688} + \frac{1}{66357076906135781376} + \frac{1}{132714153812271562752} + \frac{1}{265428307624543125504} + \frac{1}{530856615249086251008} + \frac{1}{1061713230498172502016} + \frac{1}{2123426460996345004032} + \frac{1}{4246852921992690008064} + \frac{1}{8493705843985380016128} + \frac{1}{16987411687970760032256} + \frac{1}{33974823375941520064512} + \frac{1}{67949646751883040128024} + \frac{1}{135899293503766080256048} + \frac{1}{271798587007532160512096} + \frac{1}{543597174015064321024192} + \frac{1}{1087194348030128642048384} + \frac{1}{2174388696060257284096768} + \frac{1}{4348777392120514568193536} + \frac{1}{8697554784241029136387072} + \frac{1}{17395109568482058272774144} + \frac{1}{34790219136964116545548288} + \frac{1}{69580438273928232591096576} + \frac{1}{139160876547856465182193152} + \frac{1}{278321753095712930364386304} + \frac{1}{556643506191425860728772608} + \frac{1}{1113287012382851721457545216} + \frac{1}{2226574024765703442915090432} + \frac{1}{4453148049531406885830180864} + \frac{1}{8906296099062813771660361728} + \frac{1}{17812592198125627543320735456} + \frac{1}{35625184396251255086641470912} + \frac{1}{71250368792502510173282941824} + \frac{1}{142500737585005020346565883648} + \frac{1}{285001475170010040693131767296} + \frac{1}{570002950340020081386263534592} + \frac{1}{1140005850680040162772527069184} + \frac{1}{2280011701360080325545054138368} + \frac{1}{4560023402720160651090108276736} + \frac{1}{9120046805440321302180216553472} + \frac{1}{18240093610880642604360433106944} + \frac{1}{36480187221761285208720866213888} + \frac{1}{72960374443522570417441732427776} + \frac{1}{14592074888704514083483466485552} + \frac{1}{29184149777409028166966932971104} + \frac{1}{58368299554818056333933865942208} + \frac{1}{116736599109636112667867731884416} + \frac{1}{233473198219272225335735463768832} + \frac{1}{466946396438544450671470927537664} + \frac{1}{933892792877088901342941855075328} + \frac{1}{1867785585754177802685883710150656} + \frac{1}{3735571171508355605371767420301312} + \frac{1}{7471142343016711210743534840602624} + \frac{1}{1494228468603342242148706968120528} + \frac{1}{2988456937206684484297413936241056} + \frac{1}{5976913874413368968594827872482112} + \frac{1}{11953827748826737937186655744964224} + \frac{1}{23907655497653475874373311489928448} + \frac{1}{47815310995306951748746622979856896} + 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\frac{1}{225801422011584245111914685542092354898855689133408} + \frac{1}{451602844023168490223829371084184709797711378266816} + \frac{1}{903205688046336980447658742168369419595422756533632} + \frac{1}{1806411376092673960895317484336738839185855132667264} + \frac{1}{3612822752185347921785634968673477678371710265334528} + \frac{1}{7225645504370695843571269937346955356743420530669056} + \frac{1}{1445129100874139168714253987469391071348684106138112} + \frac{1}{2890258201748278337428507974938782142697368212276224} + \frac{1}{5780516403496556674857015949877564285394736424552448} + \frac{1}{11561032806993113349714031899755128506789472849104896} + \frac{1}{23122065613986226699428063799510257013578945698209792} + \frac{1}{46244131227972453398856127598720514027157891396419584} + \frac{1}{92488262455944906797712255197441028054315782792839168} + \frac{1}{184976524911889813595424510394882056108631565585678336} + \frac{1}{369953049823779627190849020789764112217263131171356672} + 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\frac{1}{19861703125607400773696005904571794687095239937942144024} + \frac{1}{39723406251214801547392011808143589374190479875884288048} + \frac{1}{79446812502429603094784023616287787548380959751768576096} + \frac{1}{15889362504855920618956804723257557509676191950353715216} + \frac{1}{31778725009711841237913609446515115019352383900707430432} + \frac{1}{63557450019423682475827218893030230038704767801414860864} + \frac{1}{12711490003884736495165443778606046007740953560282972176} + \frac{1}{25422980007769472990330887557212092015481907120565944352} + \frac{1}{50845960015538945980661775114424184030963814241131888704} + \frac{1}{10169192003067789196132355022884836806192762848226377744} + \frac{1}{20338384006135578392264710045769673612385525696452755488} + \frac{1}{4067676801227115678452942009153934722477075139290551096} + \frac{1}{8135353602454231356905884018307869444954150278581102192} + \frac{1}{1627070720490846271381176803661578988988230055716220432} + \frac{1}{3254141440981692542762353607323157977976460111432440864} + \frac{1}{6508282881963385085524707214646359555952920222864881728} + \frac{1}{13016565763926770171049414429289791111905840445729763456} + \frac{1}{26033131527853540342098828858579582223811680891459526912} + \frac{1}{52066263055707080684197657717159164447623361782919053824} + \frac{1}{104132526111414161368395315434318328952467123565838067648} + \frac{1}{208265052222828322736790630868636657854934247131676135296} + \frac{1}{416530104445656645473581261737273315709868494263352270592} + \frac{1}{833060208891313290947162523474546631419736988526704541184} + \frac{1}{1666120417782626581894325046948593262839473776553409083768} + \frac{1}{3332240835565253163788650093897186525678947553106818167536} + \frac{1}{6664481671130506327577300187794372551357891106213636335072} + \frac{1}{13328963342261012655154600375588745102715782212427272670144} + \frac{1}{26657926684522025310309200751177490205431564424854545340288} + \frac{1}{53315853369044050620618401502354880410863128849709090680576} + \frac{1}{106631706738088101241236803004709760821726257699418181361152} + \frac{1}{213263413476176202482473606009419521643452515398836362722304} + \frac{1}{426526826952352404964947212018839043286855030797672725444608} + \frac{1}{853053653904704809929894424037678086573710061595345450889216} + \frac{1}{1706107307809409619859788848075356173147420123190690901778432} + \frac{1}{3412214615618819239719577696150712346284840246381381803556864} + \frac{1}{6824429231237638479439155392301424692568880492762763607113728} + \frac{1}{13648858462475276958878310784602849385137760985325527214227456} + \frac{1}{27297716924950553917756621568605698770275521970651054428454912} + \frac{1}{54595433849851107835513343137211397540551043941302108856909824} + \frac{1}{10919086769970221567062668627442279508110208788260421771381968} + \frac{1}{21838173539940443134125337254884595016220417576520843542763936} + \frac{1}{43676347079880886268250674509769190032440835153041687085527872} + \frac{1}{87352694159761772536501349019538380064881670306083374171055744} + \frac{1}{174705388319523545073002688039076760012816340612166748342111488} + \frac{1}{349410776638547090146005376078153520025632681224333496684222976} + \frac{1}{69882155327709418029200573215630720051265136244866699336844592} + \frac{1}{139764310655418836058400576431261600102530272489333398673689184} + \frac{1}{279528621310$$

If a, b, c are in G.P. and $a \bar{x} = b \bar{y} = c \bar{z}$, then xyz are in:

Ans. :

a. AP

Solution:

a, b and c are in G.P.

$$\therefore b^2 = ac$$

Taking log on both the sides:

$$2\log b = \log a + \log c \dots (i)$$

$$\text{Now, } a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$$

Taking log on both the sides:

$$\frac{\log a}{x} = \frac{\log b}{y} = \frac{\log c}{z} \dots \text{(ii)}$$

Now, comparing (i) and (ii):

$$\frac{\log a}{x} = \frac{\log a + \log c}{2y} = \frac{\log c}{z}$$

$$\Rightarrow \frac{\log a}{x} = \frac{\log a + \log c}{2y} \text{ and } \frac{\log a}{x} = \frac{\log c}{z}$$

$$\Rightarrow \log a(2y - x) = x \log c \text{ and } \frac{\log a}{\log c} = \frac{x}{z}$$

$$\Rightarrow \frac{\log a}{\log c} = \frac{x}{(2y-x)} \text{ and } \frac{\log a}{\log c} = \frac{x}{z}$$

$$\Rightarrow \frac{x}{2y-x} = \frac{x}{z}$$

$$\Rightarrow 2y = x + z$$

Thus, x, y and z are in A.P.

18. The two geometric means between the numbers 1 and 64 are:

- (A) 1 and 64 (B) 4 and 16 (C) 2 and 16 (D) 8 and 16.

Ans. :

b. 4 and 16

Solution:

Let the two G.M.s between 1 and 64 be G_1 and G_2 .

Thus, 1, G_1 , G_2 and 64 are in G.P.

$$64 = 1 \times r^3$$

$$\Rightarrow r = \sqrt[3]{64}$$

$$\Rightarrow r = 4$$

$$\Rightarrow G_1 = ar = 1 \times 4 = 4$$

$$\text{And, } G_2 = ar^2 = 1 \times 4^2 = 16$$

Thus, 4 and 16 are the required G.M.s.

19. If S be the sum, P the product and R be the sum of the reciprocals of n terms of a G.P. then P^2 is equal to:

(A) $\frac{S}{R}$

(B) $\frac{R}{S}$

(C) $\left(\frac{R}{S}\right)^n$

(D) $\left(\frac{S}{R}\right)^n$

Ans. :

d. $\left(\frac{S}{R}\right)^n$

Solution:

$$\text{Sum of } n \text{ terms of the G.P., } S = \frac{a(r^n - 1)}{(r - 1)}$$

$$\text{Product of } n \text{ terms of the G.P., } P = a^n r^{\left[\frac{n(n-1)}{2}\right]}$$

$$\text{Sum of the reciprocals of } n \text{ terms of the G.P., } R = \frac{\left[\frac{1}{r^n} - 1\right]}{a\left(\frac{1}{r} - 1\right)} = \frac{(r^n - 1)}{ar^{(n-1)}(r - 1)}$$

$$\therefore P^2 = \left\{ a^2 r^{\frac{2(n-1)}{2}} \right\}^n$$

$$\Rightarrow P^2 = \left\{ \frac{\frac{a(r^n - 1)}{(r - 1)}}{\frac{(r^n - 1)}{ar^{(n-1)}(r - 1)}} \right\}^n$$

$$\Rightarrow P^2 = \left\{ \frac{S}{R} \right\}^n$$

Let the first term of the G.P. be a and the common ratio be r .

$$\text{Sum of } n \text{ terms, } S = \frac{a(r^n - 1)}{r - 1}$$

$$\text{Product of the G.P., } P = a^n r^{\frac{n(n+1)}{2}}$$

$$\text{Sum of the reciprocals of } n \text{ terms, } \Rightarrow R = \frac{\left(\frac{1}{r^{n-1}}\right)}{a\left(\frac{1}{r-1}\right)} = \frac{\left(\frac{1-r^n}{r^n}\right)}{a\left(\frac{1}{r-1}\right)}$$

$$P^2 = \left\{ a^2 r^{\frac{(n+1)}{2}} \right\}^n$$

$$P^2 = \left\{ \frac{\frac{a(r^n - 1)}{r - 1}}{\frac{(1 - r^n)}{a(\frac{1 - r}{r})}} \right\} = \left\{ \frac{S}{R} \right\}^n$$

20. The sum to n terms of the series $\frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{7}} + \dots$ is:
- (A) $\sqrt{2n + 1}$ (B) $\frac{1}{2}\sqrt{2n + 1}$ (C) $\sqrt{2n + 1} - 1$ (D) $\frac{1}{2}\{\sqrt{2n + 1} - 1\}$

Ans. :

d. $\frac{1}{2}\{\sqrt{2n + 1} - 1\}$

Solution:

Let T_n be the n th term of the given series.

Thus, we have

$$\begin{aligned} T_n &= \frac{1}{\sqrt{2n-1} + \sqrt{2n+1}} \\ &= \frac{\sqrt{2n+1} - \sqrt{2n-1}}{2} \end{aligned}$$

Now,

Let S_n be the sum n terms of the given series.

Thus, we have

$$\begin{aligned} S_n &= \sum_{k=1}^n T_k \\ &= \sum_{k=1}^n \left(\frac{\sqrt{2k+1} - \sqrt{2k-1}}{2} \right) \\ &= \frac{1}{2} \sum_{k=1}^n (\sqrt{2k+1} - \sqrt{2k-1}) \\ &= \frac{1}{2} [(\sqrt{3} - \sqrt{1}) + (\sqrt{5} - \sqrt{3}) + (\sqrt{7} - \sqrt{5}) + \dots + (\sqrt{2n+1} - \sqrt{2n-1})] \\ &= \frac{1}{2} \{(-1) + \sqrt{2n+1}\} \\ &= \frac{1}{2}\{\sqrt{2n+1} - 1\} \end{aligned}$$

21. If $\sum n = 210$, then $\sum n^2 =$
- (A) 2870 (B) 2160 (C) 2970 (D) none of these.

Ans. :

a. 2870

Solution:

Given,

$$\sum n = 210$$

$$\Rightarrow n \left(\frac{n+1}{2} \right) = 210$$

$$\Rightarrow n^2 + n - 420 = 0$$

$$\Rightarrow (n - 20)(n + 21) = 0$$

$$\Rightarrow n = 20 \text{ (} \because n > 0 \text{)}$$

Now,

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow \frac{n(n+1)}{2} \times \frac{(2n+1)}{3}$$

$$\Rightarrow (210) \times \left(\frac{41}{3} \right)$$

$$\Rightarrow (70) \times (41)$$

$$\Rightarrow 2870$$

22. The consecutive digits of a three digit number are in GP. If the middle digit be increased by 2, then they form an AP. If 792 is subtracted from this, then we get the number constituting of same three digits but in reverse order. Then, number is divisible by:

(A) 7

(B) 49

(C) 19

(D) None of these

Ans.:

a. 7

23. The sum of the series $1^2 + 3^2 + 5^2 + \dots$ to n terms is:

(A) $\frac{n(n+1)(2n+1)}{2}$

(B) $\frac{n(2n-1)(2n+1)}{3}$

(C) $\frac{(n-1)^2(2n+1)}{6}$

(D) $\frac{(2n+1)^3}{3}$

Ans.:

b. $\frac{n(2n-1)(2n+1)}{3}$

Solution:

Let T_n be the n^{th} term of the given series.

Thus, we have

$$T_n = (2n - 1)^2$$

$$= 4n^2 + 1 - 4n$$

Now, let S_n be the sum of n terms of the given series.

Thus, we have

$$S_n = \sum_{k=1}^n (4k^2 + 1 - 4k)$$

$$\begin{aligned}
 \Rightarrow S_n &= 4 \sum_{k=1}^n k^2 + \sum_{k=1}^n 1 - 4 \sum_{k=1}^n k \\
 \Rightarrow S_n &= \frac{4n(n+1)(2n+1)}{6} + n - \frac{4n(n+1)}{2} \\
 \Rightarrow S_n &= \frac{2n(n+1)(2n+1)}{3} + n - 2n(n+1) \\
 \Rightarrow S_n &= n \left[\frac{2(n+1)(2n+1)}{3} + 1 - 2(n+1) \right] \\
 \Rightarrow S_n &= \frac{n}{3} [(2n+2)(2n+1) + 3 - 6(n+1)] \\
 \Rightarrow S_n &= \frac{n}{3} [4n^2 - 1] \\
 \Rightarrow S_n &= \frac{n(2n-1)(2n+1)}{3}
 \end{aligned}$$

24. The product $(32), (32)\frac{1}{6}(32)\frac{1}{36} \dots$ to ∞ is equal to:
(A) 64 (B) 16 (C) 32 (D) 0

Ans. :

a. 64

Solution:

$$\begin{aligned}
 & 32 \times 32 \frac{1}{6} \times 32 \frac{1}{36} \times \dots \infty \\
 & = 32 \left(1 + \frac{1}{6} + \frac{1}{36} + \dots \infty \right) \\
 & = 32 \left(\frac{1}{1 - \frac{1}{6}} \right) [\because \text{it is a G.P.}] \\
 & = 32 \left(\frac{6}{5} \right) \\
 & = (2^5) \left(\frac{6}{5} \right) \\
 & = 2^6 \\
 & = 64
 \end{aligned}$$

Ans. :

b. $\frac{2}{3}$

Solution:

Let three terms be $\frac{a}{r}, a, x$ ar.

$$\text{product} = 27 \Rightarrow \left(\frac{a}{r}\right)(a)(a \times r) = 27 \Rightarrow a^3 = 27 \Rightarrow a = 3.$$

$$\text{sum} = \frac{21}{2} \Rightarrow \left(\frac{a}{r+a+a \times r} \right) = \frac{21}{2} \Rightarrow a \left(\frac{1}{r+1+1 \times r} \right) = \frac{21}{2}$$

$$\begin{aligned}
 & \Rightarrow \left(\frac{1}{r+1+1 \times r} \right) = \left(\frac{\frac{21}{2}}{3} \right) = \frac{7}{2} \\
 & \Rightarrow \left(r^2 + r + 1 = \right) \left(\frac{7}{2} \right) \Rightarrow r^2 - \left(\frac{5}{2} \right)r + 1 = 0 \\
 & \Rightarrow r = 2 \text{ and } \frac{1}{2}.
 \end{aligned}$$

Terms are $\frac{3}{2}, 3, 3 \times 2$ i.

e. $\frac{3}{2}, 3, 6$.

26. If 100 times the 100th term of an AP with non-zero common difference equals the 50 times its 50th term, then the 150th term of this AP is:

- (A) -150 (B) 150 times its 50th term
(C) 150 (D) zero

Ans. :

d. zero

27. The AM, HM and GM between two numbers are $\frac{144}{15}, 15$ and 12, but not necessarily in this order. Then, HM, GM and AM respectively are:

- (A) 15, 12, $\frac{144}{15}$ (B) $\frac{144}{15}, 12, 15$ (C) 15, 12, $\frac{144}{15}$ (D) $\frac{144}{15}, 15, 12$

Ans. : $\frac{144}{15}, 12, 15$

28. Choose the correct answer.

If the third term of G.P. is 4, then the product of its first 5 terms is:

- (A) 4^3 (B) 4^4 (C) 4^5 (D) None of these.

Ans. :

c. 4^5 .

Solution:

Given that:

$$T_3 = 4$$

$$\Rightarrow ar^{3-1} = 4 \quad [\because T_n = ar^{n-1}]$$

$$\Rightarrow ar^2 = 4$$

$$\text{Product of first 5 terms} = a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$$

$$= a^5 r^{10} = (ar^2)^5 = (4)^5$$

Hence, the correct option is (c).

29. If second term of a G.P. is 2 and the sum of its infinite terms is 8, then its first terms is:

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) 4.

Ans. :

d. 4.

Solution:

$$a_2 = 2$$

$$\therefore ar = 2 \dots (i)$$

$$\text{Also, } S_{\infty} = 8$$

$$\Rightarrow \frac{a}{(1-r)} = 8$$

$$\Rightarrow \frac{a}{\left(1 - \frac{2}{a}\right)} = 8 \text{ [Using (i)]}$$

$$\Rightarrow a^2 = 8(a - 2)$$

$$\Rightarrow a^2 - 8a + 16 = 0$$

$$\Rightarrow (a - 4)^2 = 0$$

$$\Rightarrow a = 4$$

30. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then, the common ratio of this progression is equal to:

(A) $\frac{1}{2}(1 - \sqrt{5})$

(B) $\frac{1}{2} \cdot \sqrt{5}$

(C) $\sqrt{5}$

(D) $\frac{1}{2}(\sqrt{5} - 1)$

Ans. :

c. $\sqrt{5}$

31. The sum of n terms of two arithmetic progressions are in the ratio $(2n + 3) : (7n + 5)$. Find the ratio of their 9th terms.

(A) 4 : 5

(B) 5 : 4

(C) 9 : 31

(D) 31 : 9

Ans. :

c. 9 : 31

Solution:

xplanation: Let a, a' be the first terms and d, d' be the common differences of 2 A.P.'s respectively.

$$\text{Given, } \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']} = \frac{2n+3}{7n+5}$$
$$\Rightarrow \frac{a + (n-1)\frac{d}{2}}{a' + (n-1)\frac{d'}{2}} = \frac{2n+3}{7n+5}$$

If we have to find ratio of 9th terms then

$$\frac{(n-1)}{2} = 8 \Rightarrow n = 17$$

$$\Rightarrow \frac{a + 8d}{a' + 8d'} = \frac{2 \times 17 + 3}{3 \times 17 + 5} = \frac{34 + 3}{119 + 5} = \frac{36}{124} = \frac{9}{31}.$$

32. The sum of the series $\frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81} + \dots$ to n terms is:

(A) $n - \frac{1}{2}(3^{-n} - 1)$ (B) $n - \frac{1}{2}(1 - 3^{-n})$ (C) $n + \frac{1}{2}(3^n - 1)$ (D) $n - \frac{1}{2}(3^n - 1)$

Ans. :

$$b. \quad n - \frac{1}{2}(1 - 3^{-n})$$

Solution:

Let T_n be the n^{th} term of the given series.

Thus, we have

$$T_n = \frac{3^n - 1}{3^n} = 1 - \frac{1}{3^n}$$

Now,

Let S_n be the sum of n terms of the given series.

Thus, we have

$$\begin{aligned}
 S_n &= \sum_{k=1}^n T_k \\
 &= \sum_{k=1}^n \left[1 - \frac{1}{3^k} \right] \\
 &= \sum_{k=1}^n 1 - \sum_{k=1}^n \frac{1}{3^k} \\
 &= n - \left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n} \right] \\
 &= n - \frac{1}{3} \left[\frac{1 - \left(\frac{1}{3}\right)^n}{1 - \frac{1}{3}} \right] \\
 &= n - \frac{1}{2} \left[1 - \left(\frac{1}{3}\right)^n \right] \\
 &= n - \frac{1}{3} \left[1 - 3^{-n} \right]
 \end{aligned}$$

33. Choose the correct answer.

The lengths of three unequal edges of a rectangular solid block are in G.P. If the volume of the block is 216cm^3 and the total surface area is 252cm^2 , then the length of the longest edge is:

- (A) 12cm (B) 6cm (C) 18cm (D) 3cm

Ans. :

- a. 12cm.

Solution:

Aerobic exercise & heart

Let the length, breadth and height of rectangular solid block be $\frac{l}{r}$, a and ar , respectively.

$$\therefore \text{Volume} = \frac{\pi}{r} \times a \times ar = 216\text{cm}^3$$

$$\Rightarrow a^3 = 216 = 6^3 \Rightarrow a = 6$$

$$\text{Also, Surface area} = 2 \left(\frac{a}{r} \cdot a + a \cdot ar + \frac{a}{r} \cdot ar \right) = 252$$

$$\Rightarrow 2a^2 \left(\frac{1}{r} + r + 1 \right) = 252$$

$$\Rightarrow 2 \times 36 \left(\frac{1+r^2+r}{r} \right) = 252$$

$$\Rightarrow 2(1+r^2+r) = 7r$$

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$$\Rightarrow (2r-1)(r-2) = 0$$

$$\therefore r = \frac{1}{2}, 2$$

$$\text{For } r = \frac{1}{2}: \text{Length} = \frac{a}{r} = \frac{6 \times 2}{1} = 12, \text{ Breadth} = a = 6$$

$$\text{Height} = ar = 6 \times \frac{1}{2} = 3$$

$$\text{For } r = 2: \text{Length} = \frac{a}{r} = \frac{6}{2} = 3, \text{ Breadth} = a = 6$$

$$\text{Height} = ar = 6 \times 2 = 12$$

34. $9\frac{1}{3} \cdot 9\frac{1}{9} \cdot 9\frac{1}{27} \dots$ to ∞ , is:

(A) 1

(B) 3

(C) 9

(D) None of these.

Ans.:

b. 3

Solution:

$$9\frac{1}{3} \times 9\frac{1}{9} \times 9\frac{1}{27} \times \dots \infty$$

$$= 9 \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty \right)$$

Here, it is a G.P. with $a = \frac{1}{3}$ and $r = \frac{1}{3}$.

$$\therefore 9 \left(\frac{\frac{1}{3}}{1 - \frac{1}{3}} \right)$$

$$= 9 \left(\frac{1}{2} \right) = 3$$

35. Find the sum to n terms of the series whose n th term is $n(n-2)$.

$$(A) \frac{n(n-1)(2n+4)}{6}$$

$$(B) \frac{n(n-1)(2n-5)}{6}$$

$$(C) \frac{(n-1)(2n-5)}{3}$$

$$(D) \frac{n(n-1)(2n-5)}{3}$$

Ans.:

b. $\frac{n(n-1)(2n-5)}{6}$

Solution:

Given, n^{th} term is $n(n-2)$ So, $a_k = k(k-2)$

Taking summation from $k=1$ to $k=n$ on both sides, we get

$$\sum_{i=1}^n a_k = \sum_{i=1}^n k^2 - 2 \sum_{i=1}^n k = \frac{n(n+1)(2n+1)}{6} - 2 \frac{n(n+1)}{2} = \frac{n(n+1)(2n-5)}{6}.$$

36.

The value of $\sum_{r=1}^n \{(2r-1)a + \frac{1}{b^r}\}$ is equal to:

- (A) $an^2 + \frac{b^{n-1} - 1}{b^{n-1}(b-1)}$ (B) $an^2 + \frac{b^n - 1}{b^n(b-1)}$ (C) $an^3 + \frac{b^{n-1} - 1}{b^n(b-1)}$ (D) none of these.

Ans. :

b. $an^2 + \frac{b^n - 1}{b^n(b-1)}$

Solution:

We have,

$$\begin{aligned}
 & \sum_{r=1}^n \left\{ (2r-1)a + \frac{1}{b^r} \right\} \\
 &= \sum_{r=1}^n \left\{ 2ra - a + \frac{1}{b^r} \right\} \\
 &= \sum_{r=1}^n 2ar - \sum_{r=1}^n a + \sum_{r=1}^n \frac{1}{b^r} \\
 &= an(n+1) - an + \frac{(1-b^n)}{(1-b)b^n} \\
 &= an^2 + \frac{b^n - 1}{b^n(b-1)}
 \end{aligned}$$

37.

Given that $x > 0$, the sum $\sum_{n=1}^{\infty} \left(\frac{x}{x+1}\right)^{n-1}$ equals:

- (A) x (B) $x+1$ (C) $\frac{x}{2x+1}$ (D) $\frac{x+1}{2x+1}$

Ans. :

b. $x+1$

Solution:

$$\begin{aligned}
 & \sum_{n=1}^{\infty} \left(\frac{x}{x+1}\right)^{(n-1)} \\
 &= 1 + \left(\frac{x}{x+1}\right) + \left(\frac{x}{x+1}\right)^2 + \left(\frac{x}{x+1}\right)^3 + \left(\frac{x}{x+1}\right)^4 + \dots \infty \\
 &= \frac{1}{1 - \left(\frac{x}{x+1}\right)} \left[\because \text{it is a G.P. with } a = 1 \text{ and } r = \left(\frac{x}{x+1}\right) \right] \\
 &= \frac{(x+1)}{(x+1-x)} \\
 &= \frac{(x+1)}{1} = (x+1)
 \end{aligned}$$

38. Choose the correct answer.

If in an A.P., $S_n = qn^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the AP, then S_q equals:

- (A) $\frac{q^3}{2}$ (B) mnq (C) q^3 (D) $(m+n)q^2$

Ans. :

c. q^3 .

Solution:

Given,

$$S_n = qn^2 \text{ and } S_m = qm^2$$

$$\therefore S_1 = q, S_2 = 4q, S_3 = 9q \text{ and } S_4 = 16q$$

Now, $t_1 = q$

$$\therefore t_2 = S_2 - S_1 = 4q - q = 3q$$

$$t_3 = S_3 - S_2 = 9q - 4q = 5q$$

$$t_4 = S_4 - S_3 = 16q - 9q = 7q$$

So, the A.P. is: $q, 3q, 5q, 7q, \dots$

Thus, first term is q and common difference is $3q - q = 2q$.

$$\begin{aligned}\therefore S_q &= \frac{q}{2}[2 \times q + (q - 1)2q] = \frac{q}{2} \times [2q + 2q^2 - 2q] \\ &= \frac{q}{2} \times 2q^2 = q^3\end{aligned}$$

39. Jairam purchased a house in Rs. 15000 and paid Rs. 5000 at once. Rest money he promised to pay in annual instalment of Rs. 1000 with 10% per annum interest. How much money is to be paid by Jairam?

(A) Rs. 21555 (B) Rs. 20475 (C) Rs. 20500 (D) Rs. 20700

Ans.:

c. Rs. 20500

40. Let x be the A.M. and y, z be two G.M.s between two positive numbers. Then,

$\frac{y^3 + z^3}{xyz}$ is equal to:

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) None of these.

Ans.:

b. 2

Solution:

Let the two numbers be a and b .

a, x and b are in A.P.

$$\therefore 2x = a + b \dots(i)$$

Also, a, y, z and b are in G.P.

$$\therefore \frac{y}{a} = \frac{z}{y} = \frac{b}{z}$$

$$\Rightarrow y^2 = az, yz = ab, z^2 = by \dots(ii)$$

Now, $\frac{y^3 + z^3}{xyz}$

$$= \frac{y^2}{xz} + \frac{z^2}{xy}$$

$$= \frac{1}{x} \left(\frac{y^2}{z} + \frac{z^2}{y} \right)$$

$$\begin{aligned}
 &= \frac{1}{x} \left(\frac{az}{z} + \frac{by}{y} \right) [\text{Using (ii)}] \\
 &= \frac{1}{x} (a + b) \\
 &= \frac{2}{(a+b)} (a + b) [\text{Using (i)}] \\
 &= 2
 \end{aligned}$$

41. Find the sum $1^3+2^3+3^3+\dots+8^3$.

Ans. :

- d. 1296

Solution:

We know, sum of cubes of first n terms is given by

$$\left(\frac{n(n+1)}{2}\right)^2.$$

Here, $n = 8$ so, sum = $\left(\frac{8 \times 9}{2}\right)^2 + 1296$.

42. Find the sum to 6 terms of each of the series $2*3+4*6+6*11+8*18+\dots$

Ans. :

- d. 966

Solution:

General term of above series is $a_k = 2k*(k^2+2) = 2k^3+4k$

Taking summation from $k=1$ to $k=n$ on both sides, we get

$$\begin{aligned}
 \sum_{i=1}^n a_k &= 2 \sum_{i=1}^n k^3 + 4 \sum_{i=1}^n k = 2 \left(\frac{(n(n+1))^2}{2} \right) + 4 \frac{n(n+1)}{2} \\
 &= n^2 \frac{(n+1)^2}{2} + 2n(n+1) \\
 &= \frac{36 \times 49}{2} + 2 \times 6 \times 7 = 966.
 \end{aligned}$$

43. Choose the correct answer.

If $x, 2y$ and $3z$ are in A.P. where the distinct numbers x, y and z are in G.P., then the common ratio of the G.P. is:

- (A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

Ans. :

- b. $\frac{1}{2}$.

Solution:

Since, x , $2y$ and $3z$ are in A. P., we get

$$2y = \frac{x + 3z}{2}$$

$$\Rightarrow 4y = x + 3z$$

Also, x, y and z are ibn G.P

Therefore, $y = xr$ and $z = xr^2$.

Where 'r' is the common ratio.

$$\therefore 4xr = x + 3xr^2 \text{ [Using (1)]}$$

$$\Rightarrow 4r = 1 + 3r^2$$

$$\Rightarrow 3r^2 - 4r + 1 = 0$$

$$\Rightarrow (3r - 1)(r - 1) = 0$$

$$\Rightarrow r = \frac{1}{3}$$

(For $r = 1$; x, y, z are not distinct)

44. The ratio of the A.M. and G.M. of two positive numbers a and b is 5 : 3. Find the ratio of a to b .

(A) 9 : 1

(B) 3 : 5

(C) 1 : 9

(D) 3 : 1

Ans. :

a. 9 : 1

Solution:

$$\frac{(\text{A.M.})}{(\text{G.M.})} = \frac{5}{3}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{3}$$

Applying componendo and dividendo rule, we get

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{8}{2}$$

$$\Rightarrow \left(\frac{\sqrt{a+b}}{\sqrt{a-b}} \right)^2 = 4$$

$$\Rightarrow \left(\frac{\sqrt{a+b}}{\sqrt{a-b}} \right)^1 = 2$$

$$\left(\frac{\sqrt{a}}{\sqrt{b}} \right)^1 = 3$$

Again applying componendo and dividendo rule, we get

$$\frac{a}{b} = 3 \left(\frac{3}{1} \right)^2 = 9. \text{ so, } a:b = 9:1$$

45. Find the sum of series $1^3 + 3^3 + 5^3 + \dots + 11^3$.

(A) 2556

(B) 5248

(C) 6589

(D) 9874

Ans. :

a. 2556

Solution:

$$1^3 + 3^3 + 5^3 + \dots + 11^3$$

$$= (1^3 + 2^3 + 3^3 + \dots + 11^3) - (2^3 + 4^3 + 6^3 + 8^3 + 10^3)$$

$$= (1^3 + 2^3 + 3^3 + \dots + 11^3) - 2^3(1^3 + 2^3 + 3^3 + 4^3 + 5^3)$$

$$\left(\frac{11 \times 12}{2}\right)^2 - 8\left(\frac{5 \times 6}{2}\right)^2 = 66^2 - 8 \times 15^2 \\ 4356 - 1800 = 2556.$$

46. Let S be the sum, P be the product and R be the sum of the reciprocals of 3 terms of a G.P. then $P^2R^3 : S^3$ is equal to:

Ans. :

- a. 1 : 1

Solution:

Let the three terms of the G.P. be $\frac{a}{r}$, a , ar . Then

$$\begin{aligned}
 S &= \frac{a}{r} + a + ar \\
 &= a\left(\frac{1}{r} + 1 + r\right) \\
 &= a\left(\frac{1+r+r^2}{r}\right) \\
 &= \frac{a(r^2+r+1)}{r}
 \end{aligned}$$

Also,

$$P = \frac{a}{r} \times a \times ar = a^3$$

And,

$$R = \frac{r}{a} + \frac{1}{a} + \frac{1}{ar}$$

$$= \frac{1}{a} \left(\frac{r^2 + r + 1}{r} \right)$$

Now,

$$\frac{P^2 R^2}{S^3} = \frac{(a^3)^2 \times \left[\frac{1}{a} \left(\frac{r^2 + r + 1}{r} \right) \right]^3}{\left[a \left(\frac{r^2 + r + 1}{r} \right) \right]^3}$$

$$= \frac{a^6 \times \frac{1}{a^3} \left(\frac{r^2 + r + 1}{r} \right)^3}{a^3 \left(\frac{r^2 + r + 1}{r} \right)^3}$$

$$= \frac{1}{1}$$

So, the ratio is 1 : 1.

Hence, the correct alternative is option (a).

Ans. :

c. 12

Solution:

We know, $a = 20$, $d = 2$, $a_n = 42$.

$$a + (n - 1)d = 42 \quad 20 \Rightarrow + 2(n - 1) = 42$$

$$\Rightarrow 2(n - 1) = 42 - 20 = 22 \Rightarrow n - 1 = 11 \Rightarrow n = 12.$$

48. 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is:

(A) 15

(B) 20

(C) 25

(D) 30

Ans. :

c. 25

49. Find the sum of series $6^3 + 7^3 + \dots + 20^3$.

(A) 43875

(B) 83775

(C) 43775

(D) 43975

Ans. :

a. 43875

Solution:

$$6^3 + 7^3 + \dots + 20^3$$

$$= (1^3 + 2^3 + 3^3 + \dots + 20^3) - (1^3 + 2^3 + 3^3 + 4^3 + 5^3)$$

$$= \left(\frac{20 \times 21}{2}\right)^2 - \left(\frac{5 \times 6}{2}\right)^2$$

$$= (210)^2 - (15)^2$$

$$= 225 \times 195$$

$$= 43875$$

50. Find the sum of cubes of first n terms

(A) $\frac{n(n+1)}{2}$

(B) $\left(\frac{n(n+1)}{2}\right)^3$

(C) $\frac{n(n+1)(2n+1)}{6}$

(D) $\left(\frac{n(n+1)}{2}\right)$

Ans. :

c. $\frac{n(n+1)(2n+1)}{6}$

Solution:

Sum of cubes of first n terms = $1^3 + 2^3 + 3^3 + \dots + n^3$

$$(k+1)^4 - k^4 = 4k^3 + 6k^2 + 4k + 1.$$

On substituting $k = 1, 2, 3, \dots, n$ and adding we get,

$$4n^3 + n^4 + 6n^2 + 4n = 4$$

$$n = 4 \sum_{i=0}^n k^3 + 6 \sum_{i=0}^n k^2 + 4 \sum_{i=0}^n k + n$$

$$4n^3 + n^4 + 6n^2 + 4n = 4$$

$$= 4 \sum_{i=1}^n k_i^3 + 6 \frac{(n(n+1)(2n+1))}{6} + 4 \frac{n(n+1)}{2} + n \sum_{i=1}^n k_i^n = 0 k_i^n = 0 k^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

51.

The value of $\sum_{r=1}^n \log \left(\frac{a^r}{b^{r-1}} \right)$ is

- (A) $\frac{n}{2} \log \left(\frac{a^n}{b^n} \right)$ (B) $\frac{n}{2} \log \left(\frac{a^{n+1}}{b^n} \right)$ (C) $\frac{n}{2} \log \left(\frac{a^{n+1}}{b^{n-1}} \right)$ (D) $\frac{n}{2} \log \left(\frac{a^{n+1}}{b^{n+1}} \right)$

Ans. : c

(c) The given series is

$$\log a + \log \left(\frac{a^2}{b} \right) + \log \left(\frac{a^3}{b^2} \right) + \log \left(\frac{a^4}{b^3} \right) + \dots + \log \left(\frac{a^n}{b^{n-1}} \right)$$

This is an A. P. with first term $\log a$

$$\text{and the common difference } \log \left(\frac{a^2}{b} \right) - \log a = \log \left(\frac{a}{b} \right)$$

Therefore the sum of n terms is

$$\frac{n}{2} \left[\log a + \log \left(\frac{a^n}{b^{n-1}} \right) \right] = \frac{n}{2} \log \left(\frac{a^{n+1}}{b^{n-1}} \right).$$

Trick : Check for $n = 1, 2$.

52. The sum of all two digit numbers which, when divided by 4, yield unity as a remainder is

- (A) 1190 (B) 1197 (C) 1210 (D) None of these

Ans. : c

(c) The given numbers are 13, 17, ..., 97.

This is an AP with first term 13 and common difference 4.

Let the number of terms be n .

$$\text{Then } 97 = 13 + (n-1)4$$

$$\Rightarrow 4n = 88$$

$$\Rightarrow n = 22$$

Therefore the sum of the numbers

$$= \frac{22}{2} [13 + 97] = 11(110) = 1210.$$

53. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is

- (A) 2489 (B) 4735 (C) 2317 (D) 2632

Ans. : d

(d) Let $S = 1 + 2 + 3 + \dots + 100$

$$= \frac{100}{2} (1 + 100) = 50(101) = 5050$$

$$\begin{aligned}
 \text{Let } S_1 &= 3 + 6 + 9 + 12 + \dots + 99 \\
 &= 3(1 + 2 + 3 + 4 + \dots + 33) \\
 &= 3 \cdot \frac{33}{2} (1 + 33) = 99 \times 17 = 1683 \\
 \text{Let } S_2 &= 5 + 10 + 15 + \dots + 100 \\
 &= 5(1 + 2 + 3 + \dots + 20) \\
 &= 5 \cdot \frac{20}{2} (1 + 20) = 50 \times 21 = 1050 \\
 \text{Let } S_3 &= 15 + 30 + 45 + \dots + 90 \\
 &= 15(1 + 2 + 3 + \dots + 6) \\
 &= 15 \cdot \frac{6}{2} (1 + 6) = 45 \times 7 = 315 \\
 \text{Required sum} &= S - S_1 - S_2 + S_3 \\
 &= 5050 - 1683 - 1050 + 315 = 2632.
 \end{aligned}$$

Ans. : c

(c) Let the first term of A. P. is a and common difference is d .

11th term of A.P. = $a + 10d$

21st term of $A.P.$ = $a + 20d$

$$2(a + 10d) = 7(a + 20d)$$

$$\Rightarrow 2a + 20d = 7a + 140d$$

$$5a + 120d = 0$$

$$\Rightarrow a + 24d = 0$$

Hence 25th term is 0.

55. If $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in H. P., then

(A) a^2b, c^2a, b^2c are in A. P. (B) a^2b, b^2c, c^2a are in H. P.
 (C) a^2b, b^2c, c^2a are in G. P. (D) None of these

Ans. : a

(a) $\frac{b}{a}, \frac{c}{b}, \frac{a}{c}$ are in A. P.

$$\implies \frac{2c}{b} = \frac{b}{a} + \frac{a}{c}$$

$$\Rightarrow \frac{2c}{b} = \frac{bc + a^2}{ac}$$

$$\Rightarrow 2ac^2 = b^2c + ba^2$$

∴ a^2b , c^2a and b^2c are in A. P.

56. If a^2, b^2, c^2 be in A.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in

(A) A. P.

(B) G. P.

(C) H. P.

(D) None of these

Ans. : a

(a) Since a^2, b^2, c^2 be in A. P.

Then $b^2 - a^2 = c^2 - b^2$

$$\Rightarrow (b-a)(b+a) = (c-b)(c+b)$$

$$\Rightarrow \frac{b-a}{c+b} = \frac{c-b}{b+a}$$

$$\Rightarrow \frac{(b-a)(a+b+c)}{(c+a)(b+c)} = \frac{(c-b)(a+b+c)}{(a+b)(c+a)}$$

$$\Rightarrow \frac{b^2+bc-ac-a^2}{(c+a)(b+c)} = \frac{c^2+ac-ab-b^2}{(a+b)(c+a)}$$

$$\Rightarrow \frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a}$$

Hence $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ be in A. P.

57. If the angles of a quadrilateral are in A. P. whose common difference is 10° , then the angles of the quadrilateral are

(A) $65^\circ, 85^\circ, 95^\circ, 105^\circ$

(B) $75^\circ, 85^\circ, 95^\circ, 105^\circ$

(C) $65^\circ, 75^\circ, 85^\circ, 95^\circ$

(D) $65^\circ, 95^\circ, 105^\circ, 115^\circ$

Ans. : b

(b) Suppose that $\angle A = x^\circ$, then $\angle B = x + 10^\circ$,

$\angle C = x + 20^\circ$ and $\angle D = x + 30^\circ$

So, we know that $\angle A + \angle B + \angle C + \angle D = 2\pi$

Putting these values, we get

$$(x^\circ) + (x^\circ + 10^\circ) + (x^\circ + 20^\circ) + (x^\circ + 30^\circ) = 360^\circ$$

$$\Rightarrow x = 75^\circ$$

Hence the angles of the quadrilateral are $75^\circ, 85^\circ, 95^\circ, 105^\circ$.

Trick : In these type of questions, students should satisfy the conditions through options.

Here (b) satisfies both the conditions

i. e. angles are in A. P. with common difference 10° and sum of angles is 360° .

58. Jairam purchased a house in Rs. 15000 and paid Rs. 5000 at once. Rest money he promised to pay in annual installment of Rs. 1000 with 10% per annum interest. How much money is to be paid by Jairam Rs.

(A) 21555

(B) 20475

(C) 20500

(D) 20700

Ans. : c

(c) It will take 10 years for Jairam to pay off Rs. 10000 in 10 yearly installments.

He pays 10% annual interest on remaining amount

\therefore Money given in first year

$$= 1000 + \frac{10000 \times 10}{100} = \text{Rs.} 2000$$

Money given in second year = $1000 + \text{interest of}(10000 - 1000)$ with interest rate 10% per annum = $1000 + \frac{9000 \times 10}{100}$ = Rs. 1900

Money paid in third year = Rs. 1800 etc.

So money given by Jairam in 10 years will be Rs. 2000, Rs. 1900, Rs. 1800, Rs. 1700,....,

which is in arithmetic progression, whose first term $a = 2000$ and $d = -100$

Total money given in 10 years = sum of 10 terms of arithmetic progression

$$= \frac{10}{2} [2(2000) + (10 - 1)(-100)] = \text{Rs. } 15500$$

Therefore, total money given by Jairam

$$= 5000 + 15500 = \text{Rs. } 20500.$$

59. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A. P., then their common difference will be

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

Ans. : c

(c) Let $a - d, a, a + d$ be the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$

Then $(a - d) + a + (a + d) = 12$ and $(a - d)a(a + d) = 28$

$$\Rightarrow 3a = 12 \text{ and } a(a^2 - d^2) = 28$$

$$\Rightarrow a = 4 \text{ and } a(a^2 - d^2) = 28$$

$$\Rightarrow 16 - d^2 = 7$$

$$\Rightarrow d = \pm 3.$$

60. The A. M. of a 50 set of numbers is 38. If two numbers of the set, namely 55 and 45 are discarded, the A. M. of the remaining set of numbers is

(A) 38.5 (B) 37.5 (C) 36.5 (D) 36

Ans. : b

(b) Given, $\frac{\sum x_i}{50} = 38$, $\therefore \sum x_i = 1900$

New value of $\sum x_i = 1900 - 55 - 45 = 1800$, $n = 48$

New mean = $\frac{1800}{48} = 37.5$.

61. 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is

(A) 15 (B) 20 (C) 25 (D) 30

Ans. : c

(c) Let the number of days be n .

Hence a worker can do $\left(\frac{1}{150n}\right)^{th}$ part of the work in a day.

Accordingly,

$$[150 + 146 + 142 + \dots + \text{upto } (n+8) \text{ terms}] \times \frac{1}{150n} = 1$$

$$\Rightarrow n = 17$$

Therefore number of total days in completion = $17 + 8 = 25$.

62. Given that n A.M.'s are inserted between two sets of numbers $a, 2b$ and $2a, b$, where $a, b \in R$. Suppose further that m^{th} mean between these sets of numbers is same, then the ratio $a:b$ equals
- (A) $n-m+1:m$ (B) $n-m+1:n$ (C) $n:n-m+1$ (D) $m:n-m+1$

Ans. : d

(d) m^{th} mean between $a, 2b$ is $a + \frac{m(2b-a)}{n+1}$ (i)

and m^{th} mean between $2a, b$ is $2a + \frac{m(b-2a)}{n+1}$ (ii)

Accordingly, $a + \frac{m(2b-a)}{n+1} = 2a + \frac{m(b-2a)}{n+1}$

$$\Rightarrow m(2b-a) = a(n+1) + m(b-2a)$$

$$\Rightarrow a(n-m+1) = bm$$

$$\Rightarrow \frac{a}{b} = \frac{m}{n-m+1}.$$

63. If α, β, γ are the geometric means between $ca, ab; ab, bc; bc, ca$ respectively where a, b, c are in A.P., then $\alpha^2, \beta^2, \gamma^2$ are in

- (A) A. P. (B) H. P.
 (C) G. P. (D) None of the above

Ans. : a

(a) By hypothesis, $\alpha^2 = a^2bc, \beta^2 = b^2ca, \gamma^2 = c^2ab$ and $2b = a + c$. Hence $2^{n-1} > 100$ are in A.P.

64. Maximum value of sum of arithmetic progression 50, 48, 46, 44, ... is :-
 (A) 325 (B) 648 (C) 652 (D) 650

Ans. : d

For maximum sum $\Rightarrow T_n = 0$

$$a + (n-1)d = 0$$

$$\Rightarrow 50 + (n-1)(-2) = 0 \Rightarrow n = 26$$

$$\text{So } S_{26} = \frac{26}{2}[2 \times 50 + 25 \times (-2)] = 650$$

* Given section consists of questions of 2 marks each.

[34]

65. A manufacturer reckons that the value of a machine, which cost him ₹ 15625 will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Ans. : Present value of the machine = ₹ 15625

Rate of depreciation = 20%

After 1 year value of machine = $15625 - 15625 \times \frac{20}{100} = 15625 - 3125 = ₹ 12500$

After 2 year value of machine = $12500 - 12500 \times \frac{20}{100} = 12500 - 2500 = ₹ 10000$

After 3 year value of machine = $10000 - 10000 \times \frac{20}{100} = 10000 - 2000 = ₹ 8000$

∴ Sequence of values of machine after depreciation is 12500, 10000, 8000, ... is a G.P.

Here $a = 12500$, $r = \frac{10000}{12500} = \frac{4}{5}$

∴ $a_5 = ar^4 = 12500 \times \left(\frac{4}{5}\right)^4 = 12500 \times \frac{256}{625} = ₹ 5120$

Therefore, the value of machine at the end of 5 years is ₹ 5120

66. Find the indicated terms of the sequence, whose n th term is $a_n = \frac{n(n-2)}{n+3}$; a_{20}

Ans. : Given: $a_n = \frac{n(n-2)}{n+3}$

∴ $a_{20} = \frac{20(20-2)}{20+3} = \frac{20 \times 18}{23} = \frac{360}{23}$

Therefore, 20th term is $\frac{360}{23}$.

67. Write the first five terms of the sequence whose n th term is $a_n = n(n+2)$

Ans. : Given: $a_n = n(n+2)$

Putting $n = 1, 2, 3, 4$ and 5 , we get,

$$a_1 = 1(1+2) = 1 \times 3 = 3$$

$$a_2 = 2(2+2) = 2 \times 4 = 8$$

$$a_3 = 3(3+2) = 3 \times 5 = 15$$

$$a_4 = 4(4+2) = 4 \times 6 = 24$$

$$a_5 = 5(5+2) = 5 \times 7 = 35$$

Therefore, the first five terms are 3, 8, 15, 24 and 35.

68. If a, b, c are in G.P., Prove that $\log a, \log b, \log c$ are in A.P.

Ans. : Here, a, b, c are in G.P.

$$b^2 = ac \dots (i)$$

$$\text{Now, } 2\log b = \log b^2$$

$$= \log ac$$

$$2\log b = \log a + \log c$$

$$\log b - \log a = \log c - \log b$$

⇒ $\log a, \log b, \log c$, are in A.P.

69. If a is the G.M. of 2 and $\frac{1}{4}$, find a .

Ans. : a is the G.M. between 2 and $\frac{1}{4}$,

Then,

$$a = \sqrt{2 \times \frac{1}{4}}$$

$$a = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

70. If a, b, c are in G.P., Prove that $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in A.P.

Ans. : Here, a, b, c are in G.P., so

$$b^2 = ac$$

$$\text{Now, } \frac{2}{\log_b m} = 2\log_m b$$

$$= \log_m b^2$$

$$= \log_m ac$$

$$= \log_m a + \log_m c$$

$$\frac{2}{\log_b m} = \frac{1}{\log_a m} + \frac{1}{\log_c m}$$

$$\Rightarrow \frac{1}{\log_b m} - \frac{1}{\log_a m} = \frac{1}{\log_c m} - \frac{1}{\log_b m}$$

$$\Rightarrow \frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m} \text{ are in A.P.}$$

71. Find the sum of the following series to infinit:

$$10 - 9 + 8.1 - 7.29 + \dots \infty$$

Ans. : This infinity G.P. has first term $a = 10$ and common ratio $r = -\frac{9}{10} = -0.9$

Thus the sum of the infinity G.P. will be:

$$\begin{aligned} 10 - 9 + 8.1 - 7.29 + \dots \infty &= \frac{a}{1-r} \quad [\text{Since } |r| < 1] \\ &= \frac{10}{1 - (-0.9)} \\ &= \frac{10}{1.9} \\ &= \frac{100}{19} \\ &= 5.263 \end{aligned}$$

72. Find:

The 8th term of the G.P. 0.3, 0.06, 0.012, ...

Ans. : Here,

First term, $a = 0.3$

$$\text{Common ratio, } r = \frac{a_2}{a_1} = \frac{0.06}{0.3} = 0.2$$

$$\therefore 8^{\text{th}} \text{ term} = a_8 = ar^{(8-1)} = 0.3 (0.2)^7$$

Thus, the 8th term of the given GP is $0.3 (0.2)^7$.

73. Find the sum of the following series to infinity:

$$\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots \infty$$

$$\begin{aligned}\text{Ans. : } S_{\infty} &= \frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots \\ &= \left(\frac{2}{5} + \frac{3}{5^3} + \dots \right) + \left(\frac{3}{5^2} + \frac{3}{5^4} + \dots \right)\end{aligned}$$

$$S_{\infty} = S'_{\infty} + S''_{\infty}$$

For

$$S'_{\infty} = \frac{a}{1-r}$$

$$= \frac{\frac{2}{5}}{1 - \frac{1}{25}}$$

$$= \frac{1}{2} \times \frac{25}{24}$$

$$S'_{\infty} = \frac{5}{12}$$

$$S''_{\infty} = \frac{\frac{3}{25}}{1 - \frac{1}{25}}$$

$$= \frac{3}{25} \times \frac{25}{24}$$

$$= \frac{3}{24}$$

$$S_{\infty} = S'_{\infty} + S''_{\infty}$$

$$= \frac{5}{12} + \frac{3}{24}$$

$$= \frac{13}{24}$$

$$S_{\infty} = \frac{13}{24}$$

74. Find:

The 12th term of the G.P. $\frac{1}{a^3 x^3}, ax, a^5 x^5 \dots$

Ans. : 12th term of the G.P. $\frac{1}{a^3 x^3}, ax, a^5 x^5 \dots$

$$a = \frac{1}{a^3 x^3}$$

$$r = \frac{t_n}{t_{n-1}} = \frac{t_2}{t_1} = \frac{ax}{\frac{1}{a^3 x^3}} = a^4 x^4$$

$$t_n = ar^{n-1}$$

$$t_{12} = ar^{11}$$

$$= \left(\frac{1}{a^3 x^3} \right) (a^4 x^4)^{11}$$

$$= (ax)^{41}$$

75. Find:

The ninth term of the G.P. 1, 4, 16, 64, ...

Ans. : 9th term of G.P. 1, 4, 16, 64, ...

$$t_1 = 1 = a$$

$$t_2 = 4$$

Because it is G.P.

$$\frac{t_2}{t_1} = \text{common ratio} = r$$

$$r = \frac{4}{1} = 4$$

$$t_n = ar^{n-1}$$

$$t_9 = ar^8 = 1(4)^8 = 4^8$$

76. Find the sum of the following series to infinity:

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} + \dots \infty$$

$$\text{Ans. : } S_{\infty} = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$$

$$\Rightarrow a = 1, r = -\frac{1}{3}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1 + \frac{1}{3}}$$

$$S_{\infty} = \frac{3}{4}$$

77. How many terms of the series 2 + 6 + 18 + ... must be make the sum equal to 728₹

Ans. : 2 + 6 + 18 + ...

$$S_n = 728$$

Now,

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$a = 2, r = \frac{6}{2} = 3$$

$$728 = \frac{2(3^n - 1)}{3 - 1}$$

$$728 = \frac{2(3^n - 1)}{2} = (3^n - 1)$$

$$728 + 1 = 3^n$$

$$729 = 3^n$$

$$(3)^6 = 3n$$

$$\Rightarrow n = 6$$

78. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 and the third is increased by 1, we obtain three consecutive terms of a G.P. Find the numbers.

Ans. : Let three numbers in A.P. be $a - d, a + d$

$$\text{Here, } a - d + a + a + d = 21$$

$$3a = 21$$

$$a = 7$$

And,

$(7 - d), (7 - 1), (7 + d) + 1$ are in G.P.

$(7 - d), 6, (8 + d)$ are in G.P.

$$(6)^2 = (7 - d)(8 + d)$$

$$36 = 56 + 7d - 8d - d^2$$

$$d^2 + d - 20 = 0$$

$$(d + 5)(d - 4) = 0$$

$$d = 4, -5$$

So, Numbers are 3, 7, 11 or 12, 7, 2.

79. Find the sum of the following geometric progressions:

1, 3, 9, 27, ... to 8 terms

Ans. : 1, 3, 9, 27, ... to 8 terms

$$a = 1, r = \frac{3}{1} = 3, n = 8$$

$$S_n = a \frac{(r^{n-1})}{r-1}$$

$$S_8 = 1 \frac{(3^8 - 1)}{3 - 1} = 3280$$

80. The sum of interior angles of a triangle is 180° . Show that the sum of the interior angles of polygons with 3, 4, 5, 6, ... sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.

Ans. : We know that, sum of interior angles of a polygon of side n is $(n - 2) \times 180^\circ$.

$$\text{Let } t_n = (n - 2) \times 180^\circ$$

Since, t_n is linear in n , it is n^{th} term of some A.P.

$$t_3 = a = (3 - 2) \times 180^\circ = 180^\circ$$

$$\text{Common difference } d = 180^\circ$$

Sum of the interior angles for a 21 sided polygon is:

$$t_{21} = (21 - 2) \times 180^\circ = 3420^\circ$$

81. The first term of an A.P. is a and the sum of the first p terms is zero, show that the sum of its next q term is $\frac{-a(p+q)q}{p-1}$.
 [Hint: Required sum = $S_{p+q} - S_p$]

Ans. : Let the common difference of the given A.P be d .

Given that $S_p = 0$

$$\Rightarrow \frac{p}{2}[2a + (p-1)d] = 0$$

$$\Rightarrow 2a + (p-1)d = 0$$

$$\Rightarrow d = \frac{-2a}{p-1}$$

Now, sum of next q terms,

$$= S_{p+q} - S_p = S_{p+q} - 0$$

$$= \frac{p+q}{2}[2a + (p+q-1)d]$$

$$= \frac{p+q}{2}[2a + (p-1)d + qd]$$

$$= \frac{p+q}{2} \left[0 + \frac{q-2a}{p-1} \right]$$

$$= \frac{-a(p+q)q}{p-1}$$

* Given section consists of questions of 3 marks each.

[129]

82. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2 respectively. Find the last term and the number of terms.

Ans. : Given: $a = 15$, $r = 2$ and $S_n = 315$

$$\therefore S_n = \frac{a(r^n - 1)}{r-1}$$

$$\Rightarrow 315 = \frac{5(2^n - 1)}{2-1}$$

$$\Rightarrow \frac{315}{5} = 2^n - 1$$

$$\Rightarrow 2^{n-1} = 63$$

$$\Rightarrow 2^n = 64 = 2^6$$

$$\Rightarrow n = 6$$

$$\therefore a_6 = ar^{6-1} = 5 \times 2^5 = 5 \times 32 = 160$$

Hence the number of terms=6 and the last term =160

83. If f is a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{N}$ such that $f(1) = 3$

and $\sum_{x=1}^n f(x) = 120$ find the value of n .

Ans. : $f(1) = 3$

$$f(1+2) = f(1) f(2) = 3 \times 9 = 27$$

$$f(1+3) = f(1) f(3) = 3 \times 27 = 81$$

L.H.S.

$$= f(1) + f(2) + f(3) + \dots + f(n)$$

$$= 3 + 9 + 27 + 81 + \dots + n \text{ terms}$$

$$= \frac{3(3^n - 1)}{3-1} = \frac{3}{2}(3^n - 1)$$

$$= f(1) + f(2) + f(3) + \dots + f(n)$$

$$= 3 + 9 + 27 + 81 + \dots + n \text{ terms}$$

$$= \frac{3(3^n - 1)}{3-1} = \frac{3}{2}(3^n - 1)$$

ATQ

$$\frac{3}{2}(3^n - 1) = 120$$

$$3^n - 1 = 80$$

$$3^n = 81$$

$$n = 4$$

84. Shamshad Ali buys a scooter for ₹ 22000. He pays ₹ 4000 cash and agrees to pay the balance in annual installment of ₹ 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Ans. : Total cost of the scooter = ₹ 22000, Cash paid = ₹ 4000

$$\text{Balance to be paid} = 22000 - 4000 = ₹ 18000$$

$$\text{Annual installment} = ₹ 1000$$

$$\therefore \text{Number of installment} = \frac{18000}{1000} = 18$$

$$\text{Interest of 1}^{\text{st}} \text{ installment} = \frac{18000 \times 10 \times 1}{100} = ₹ 1800$$

$$\text{Amount of 1}^{\text{st}} \text{ installment} = 1000 + 1800 = ₹ 2800$$

$$\text{Interest of 2}^{\text{nd}} \text{ installment} = \frac{17000 \times 10 \times 1}{100} = ₹ 1700$$

$$\text{Amount of 2}^{\text{nd}} \text{ installment} = 1000 + 1700 = ₹ 2700$$

$$\text{Interest of 3}^{\text{rd}} \text{ installment} = \frac{16000 \times 10 \times 1}{100} = ₹ 1600$$

$$\text{Amount of 3}^{\text{rd}} \text{ installment} = 1000 + 1600 = ₹ 2600$$

\therefore Sequence of installments is 2800, 2700, 2600, ... in A.P

$$\text{Here, } a = 2800, d = 2700 - 2800 = -100 \text{ and } n = 18$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d] = \frac{18}{2} [2 \times 2800 + (18-1) \times (-100)]$$

$$= 9 [5600 - 1700] = ₹ 35100$$

Therefore, the total cost of tractor is $(35100 + 4000) = ₹ 39100$

85. Find the sum of the series up to n terms .6 + .66 + .666+...

Ans. : The given sum is not in GP but we can write it as follows: -

$$\text{Sum} = .6 + .66 + .666 + \dots \text{to } n \text{ terms}$$

$$= 6(0.1) + 6(0.11) + 6(0.111) + \dots \text{to } n \text{ terms}$$

taking 6 common

$$= 6[0.1 + 0.11 + 0.111 + \dots \text{to } n \text{ terms}]$$

divide & multiply by 9, we get

$$= \left(\frac{6}{9}\right)[9(0.1 + 0.11 + 0.111 + \dots \text{to } n \text{ terms})]$$

$$= \left(\frac{6}{9}\right)[0.9 + 0.99 + 0.999 + \dots \text{to } n \text{ terms}]$$

$$= \frac{6}{9} \left[\left(\frac{9}{10} \right) + \left(\frac{99}{100} \right) + \left(\frac{999}{1000} \right) + \dots \text{to } n \text{ terms} \right]$$

$$= \frac{6}{9} \left[\left(1 - \frac{1}{10} \right) + \left(1 - \frac{1}{100} \right) + \left(1 - \frac{1}{1000} \right) + \dots \text{to } n \text{ terms} \right]$$

$$= \frac{6}{9} \left[\{1 + 1 + 1 + \dots \text{to } n \text{ terms}\} - \left\{ \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{to } n \text{ terms} \right\} \right]$$

$$= \frac{6}{9} \left[n - \left\{ \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{to } n \text{ terms} \right\} \right]$$

Since $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{to } n \text{ terms}$ is in GP with

$$\text{first term}(a) = \frac{1}{10}$$

$$\text{common ratio}(r) = \frac{10^{-2}}{10^{-1}} = 10^{-1} = \frac{1}{10}$$

We know that

$$\text{Sum of } n \text{ terms} = \frac{a(1 - r^n)}{1 - r} \quad [\text{As } r < 1]$$

Substituting value of a & r

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{to } n \text{ terms} = \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{\frac{1}{10} \left(1 - \left(\frac{1}{10} \right)^n \right)}{\left(1 - \frac{1}{10} \right)}$$

$$= \frac{\frac{1}{10} \left(1 - \left(\frac{1}{10} \right)^n \right)}{\frac{9}{10}}$$

$$= \frac{1 \left(1 - 10^{-n} \right)}{9}$$

$$\text{Therefore, Sum} = \frac{6}{9} \left[n - \frac{1(1 - 10^{-n})}{9} \right]$$

86. Find the sum of the series up to n terms $5 + 55 + 555 + \dots$

Ans.: Now, to find sum = $5 + 55 + 555 + \dots$ n terms.

$$\begin{aligned} &= \frac{5}{9} [9 + 99 + 999 + \dots \text{ n terms}] \\ &= \frac{5}{9} [(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ n terms}] \\ &= \frac{5}{9} [10 + 100 + 1000 \dots - (1 + 1 + \dots 1)] \\ &= \frac{5}{9} [10(10^n - 1)/(10 - 1) + (1 + 1 + \dots \text{ n times})] \\ &= \frac{50}{81}(10^n - 1) - \frac{5n}{9} \end{aligned}$$

87. If a and b are the roots $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that $(q + p):(q - p) = 17:15$.

Ans.: Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$$\therefore \frac{b}{a} = k$$

$$\Rightarrow b = ak$$

$$\text{And } \frac{c}{b} = k$$

$$\Rightarrow c = bk = (ak)k = ak^2$$

$$\text{Also } \frac{d}{c} = k$$

$$\Rightarrow d = ck = (ak^2)k = ak^3$$

$\because a^a$ and b^b are the roots $x^2 - 3x + p = 0$

$$\therefore a + b = \frac{-(-3)}{1} = 3$$

$$\Rightarrow a + ak = 3$$

$$\Rightarrow a(1 + k) = 3 \dots (\text{i})$$

$$\text{And } ab = \frac{p}{1}$$

$$\Rightarrow a(ak) = p$$

$$\Rightarrow a^2k = p \dots (\text{ii})$$

Also c, d are roots of $x^2 - 12x + q = 0$

$$\therefore c + d = \frac{-(-12)}{1} = 12$$

$$\Rightarrow ak^2 + ak^3 = 12$$

$$\Rightarrow ak^2(1 + k) = 12 \dots (\text{iii})$$

$$\text{And } cd = \frac{q}{1}$$

$$\Rightarrow ak^2(ak^3) = q$$

$$\Rightarrow a^2 k^5 = q \dots \text{(iv)}$$

$$\text{Dividing eq. (iii) by eq. (i), } \frac{ak^2(1+k)}{a(1+k)} = \frac{12}{3}$$

$$\Rightarrow k^2 = 4$$

$$\Rightarrow k = \pm 2$$

$$\text{Now } \frac{q+p}{q-p} = \frac{a^2 k^5 + a^2 k}{a^2 k^5 - a^2 k} = \frac{a^2 k (k^4 + 1)}{a^2 k (k^4 - 1)}$$

$$= \frac{(\pm 2)^4 + 1}{(\pm 2)^4 - 1} = \frac{16 + 1}{16 - 1} = \frac{17}{15}$$

$$\text{Therefore, } (q+p):(q-p) = 17:15$$

88. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2 R^n = S^n$.

Ans. : Let the G.P be $a, ar, ar^2, ar^3, \dots, ar^{n-1}$

$$\text{Here } S = \frac{a(r^n - 1)}{r - 1}$$

$$P = a \cdot ar \cdot ar^2 \dots ar^{n-1} = a^n \cdot r^{1+2+3+\dots+(n-1)} = a^n \cdot r^{\frac{n(n-1)}{2}}$$

$$\text{and } R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}} = \frac{r^{n-1} + r^{n-2} + r^{n-3} + \dots + 1}{ar^{n-1}}$$

$$= \frac{1(r^n - 1)}{r - 1} \cdot \frac{1}{ar^{n-1}} = \frac{r^n - 1}{ar^{n-1}(r - 1)}$$

$$\text{Now } P^2 R^n = \frac{a^{2n} \cdot r^{n(n-1)} (r^{n-1})^n}{a^n r^{n(n-1)} (r - 1)^n} = \frac{a^n (r^n - 1)^n}{(r - 1)^n} = a^n \left(\frac{r^n - 1}{r - 1} \right)^n = S^n$$

Hence proved.

89. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

Ans. : Given: $a = 11$ and $S_4 = 56$

$$\Rightarrow S_4 = \frac{4}{2}[2 \times 11 + (4-1)d]$$

$$\Rightarrow 2(22 + 3d) = 56$$

$$\Rightarrow 22 + 3d = 28$$

$$\Rightarrow 3d = 6$$

$$\Rightarrow d = 2$$

$$\text{Also, } l + (l - d) + (l - 2d) + (l - 3d) = 112$$

$$\Rightarrow 4l - 6d = 112$$

$$\Rightarrow 4l = 112 + 6 \times 2$$

$$\Rightarrow 4l = 112 + 12$$

$$\Rightarrow 4l = 124$$

$$\Rightarrow l = 31$$

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow 31 = 11 + (n - 1) \times 2$$

$$\Rightarrow 2(n - 1) = 20$$

$$\Rightarrow n - 1 = 10$$

$$\Rightarrow n = 11$$

90. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Ans. : Let the number of terms be $2n$ then we have the number of odd terms is n
Let the G.P be $a, ar, ar^2, \dots, ar^{2n-1}$

Then the odd terms $a, ar^2, ar^4, ar^6, \dots$ form a G.P

$$\therefore S_{2n} = \frac{a(r^{2n} - 1)}{r - 1} \text{ and } S_n = a \left[\frac{(r^2)^n - 1}{r^2 - 1} \right]$$

According to question, $S_{2n} = 5S_n$

$$\Rightarrow a \left[\frac{r^{2n} - 1}{r - 1} \right] = 5a \left[\frac{(r^2)^n - 1}{r^2 - 1} \right]$$

$$\Rightarrow \frac{1}{r-1} = \frac{5}{r^2-1}$$

$$\Rightarrow r + 1 = 5$$

$$\Rightarrow r = 4$$

91. If A and G be A.M. and G.M. respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A + G)(A - G)}$.

Ans. : Let the two positive numbers be a and b

Therefore $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$

$$\text{Now, } A \pm \sqrt{(A + G)(A - G)} = A \pm \sqrt{A^2 - G^2}$$

$$= \frac{a+b}{2} \pm \sqrt{\left(\frac{a+b}{2}\right)^2 - (\sqrt{ab})^2}$$

$$= \frac{a+b}{2} \pm \sqrt{\frac{a^2 + b^2 + 2ab}{4} - ab}$$

$$= \frac{a+b}{2} \pm \sqrt{\frac{a^2 + b^2 + 2ab - 4ab}{4}}$$

$$= \frac{a+b}{2} \pm \sqrt{\frac{(a-b)^2}{4}} = \frac{a+b}{2} \pm \frac{a-b}{2}$$

$$\begin{aligned}
 &= \frac{a+b}{2} + \frac{a-b}{2} \text{ and } \frac{a+b}{2} - \frac{a-b}{2} \\
 &= \frac{a+b+a-b}{2} \text{ and } \frac{a+b-a+b}{2} \\
 &= \frac{2a}{2} = a \text{ and } \frac{2b}{2} = b
 \end{aligned}$$

92. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3 + 2\sqrt{2}):(3 - 2\sqrt{2})$.

Ans.: Let the numbers be a and b

$$\text{Given: } a+b = 6\sqrt{ab} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Applying componendo and dividendo, we get

$$\begin{aligned}
 \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} &= \frac{3+1}{3-1} \\
 \Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} &= \frac{4}{2} \\
 \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} &= \frac{\sqrt{2}}{1}
 \end{aligned}$$

Again applying componendo and dividendo, we get

$$\begin{aligned}
 \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} &= \frac{\sqrt{2}+1}{\sqrt{2}-1} \\
 \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} &= \frac{\sqrt{2}+1}{\sqrt{2}-1}
 \end{aligned}$$

$$\text{Squaring both sides, } \frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Therefore, the numbers are in the ratio $(3 + 2\sqrt{2}):(3 - 2\sqrt{2})$

93. Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b .

$$\text{Ans. : } \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a^{\frac{1}{2}} b^{\frac{1}{2}}}{1}$$

$$a^{n+1} + b^{n+1} = a^{\frac{1}{2}} b^{\frac{1}{2}} (a^n + b^n)$$

$$a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n+\frac{1}{2}}$$

$$a^{n+1} - a^{n+\frac{1}{2}} b^{\frac{1}{2}} = a^{\frac{1}{2}} b^{n+\frac{1}{2}} - b^{n+1}$$

$$a^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{\frac{1}{2}} \right) = b^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{\frac{1}{2}} \right)$$

$$\left(\frac{a}{b} \right)^{n+\frac{1}{2}} = 1$$

$$\left(\frac{a}{b} \right)^{n+\frac{1}{2}} = \left(\frac{a}{b} \right)^0$$

$$n + \frac{1}{2} = 0$$

$$n = \frac{-1}{2}$$

94. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms

$$\text{from } (n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term is } \frac{1}{r^n}.$$

Ans.: Let a be the first term and r be the common ratio of given G.P.

$$\begin{aligned} \text{Then } & \frac{\text{Sum of first } n \text{ terms}}{\text{Sum of terms from } (n+1)^{\text{th}} \text{ to } (2n)^{\text{th}}} \\ &= \frac{a + ar + ar^2 + \dots + ar^{n-1}}{ar^n + ar^{n+1} + \dots + ar^{2n-1}} \\ &= \frac{a + ar + ar^2 + \dots + ar^{n-1}}{r^n [a + ar + ar^2 + \dots + ar^{n-1}]} = \frac{1}{r^n} \end{aligned}$$

95. Find the sum to n terms of the sequences 8, 88, 888, 8888,

Ans.: Here $S_n = 8 + 88 + 888 + 8888 + \dots$ up to n terms

$$\Rightarrow S_n = 8(1 + 11 + 111 + 1111 + \dots \text{ up to } n \text{ terms})$$

$$\Rightarrow S_n = \frac{8}{9}(9 + 99 + 999 + 9999 + \dots \text{ up to } n \text{ terms}) \Rightarrow$$

$$\Rightarrow S_n = \frac{8}{9} \left[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots \text{ up to } n \text{ terms} \right]$$

$$\Rightarrow S_n = \frac{8}{9} [(10 + 10^2 + 10^3 + \dots \text{ up to } n \text{ terms}) - (1 + 1 + 1 + \dots \text{ up to } n \text{ terms})]$$

$$\Rightarrow S_n = \frac{8}{9} \left[\frac{10 \times (10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{8}{9} \left[\frac{10}{9} (10^n - 1) - n \right]$$

$$= \frac{80}{81} (10^n - 1) - \frac{8}{9} n$$

96. How many terms of G.P. $3, 3^2, 3^3, \dots$ are needed to give the sum 120?

Ans.: Here, $a = 3$ and $r = \frac{3^2}{3} = 3$

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1} \text{ when } r > 1$$

$$\Rightarrow 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$\Rightarrow 120 = \frac{3}{2}(3^n - 1)$$

$$\Rightarrow 120 \times \frac{2}{3} = 3^n - 1$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = (3)^4$$

$$\Rightarrow n = 4$$

Therefore, the sum of 4 terms of the given G.P. is 120.

97. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Ans.: Let $\frac{a}{r}, a, r$ be first three terms of the given G.P.

According to question, $\frac{a}{r} + a + ar = \frac{39}{10}$ (i)

And $\frac{a}{r} \times a \times r = 1$

$$\Rightarrow a^3 = 1$$

$$\Rightarrow a = 1$$

Putting value of a in eq. (i),

$$\Rightarrow 10 + 10r + 10r^2 = 39r$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow r = \frac{-(-29) \pm \sqrt{(-29)^2 - 4 \times 10 \times 10}}{2 \times 10}$$

$$\Rightarrow r = \frac{29 \pm \sqrt{841 - 400}}{20}$$

$$\Rightarrow r = \frac{29 \pm 21}{20}$$

Taking $r = \frac{29 + 21}{20} = \frac{50}{120} = \frac{5}{2}$ and

then the first three terms are $\frac{1}{5/2}, 1, 1 \times \frac{5}{2}$

$$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}$$

Taking $r = \frac{29 - 21}{20} = \frac{8}{20} = \frac{2}{5}$

then first three terms are $\frac{1}{2/5}, 1, 1 \times \frac{2}{5}$

$$\Rightarrow \frac{5}{2}, 1, \frac{2}{5}$$

- 98.

Evaluate: $\sum_{k=1}^{11} (2 + 3^k)$

Ans.: Given: $\sum_{k=1}^{11} (2 + 3^k)$

$$= (2 + 3^1) + (2 + 3^2) + (2 + 3^3) + (2 + 3^{11})$$

$$= (2 + 2 + 2 + \dots \text{11 times}) + (3 + 3^2 + 3^3 + \dots + 3^{11})$$

$$= 22 + (3 + 3^2 + 3^3 + \dots + 3^{11}) \dots \text{(i)}$$

Here $3, 3^2, 3^3, \dots, 3^{11}$ is in G.P.

$$\therefore a = 3 \text{ and } r = \frac{3^2}{3} = 3$$

$$S_n = \frac{3(3^{11} - 1)}{3 - 1} = \frac{3}{2}(3^{11} - 1)$$

$$\text{Putting the value of } S_n \text{ in eq. (i), we get } \sum_{k=1}^{11} (2 + 3^k) = 22 + \frac{3}{2}(3^{11} - 1)$$

99. Find the sum to indicated number of terms of the geometric progression x^3, x^5, x^7, \dots, n terms (if $x \neq \pm 1$).

Ans.: Here, $a = x^3$ and $r = \frac{x^5}{x^3} = x^2$

$$S_n = \frac{a(1 - r^n)}{1 - r} \text{ when } r < 1$$

$$\Rightarrow S_n = \frac{x^3 \left[1 - (x^2)^n \right]}{1 - x^2}$$

$$\Rightarrow S_n = \frac{x^3}{1 - x^2} \left[1 - x^{2n} \right]$$

100. The number of terms of an A.P. is even; the sum of odd terms is 24, of the even terms is 30, and the last term exceeds the first by $10\frac{1}{2}$, find the number of terms and the series.

Ans.: Let no. of term be $2n$

$$\text{Odd terms sum} = 24 = T_1 + T_3 + \dots + T_{2n-1}$$

$$\text{Even terms sum} = 30 = T_2 + T_4 + \dots + T_{2n}$$

Subtract above two equations

$$nd = 6$$

$$T_{2n} = T_1 + \frac{21}{2}$$

$$T_{2n} - a = \frac{21}{2}$$

$$(2n - 1)d = \frac{21}{2}$$

$$12 - \frac{21}{2} = d = \frac{3}{2}$$

$$\Rightarrow n = 6 \times \frac{2}{3} = 4$$

$$\text{Total terms} = 2n = 8$$

Subtitute above values in equation of
Sum of even terms or add terms, we get

$$a = \frac{3}{2}$$

So series is $\frac{3}{2}, 3\frac{9}{2}, \dots$

101. If a^2, b^2, c^2 are in A.P., prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

Ans. : $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

$$\text{LHS} = \frac{b}{a+c} - \frac{a}{b+c}$$

$$\Rightarrow \frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)}$$

$$\Rightarrow \frac{(b-a)(a+b+c)}{(a+c)(b+c)} \dots \dots (1)$$

$$\text{RHS} = \frac{a}{a+b} = \frac{b}{a+c}$$

$$\Rightarrow \frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)}$$

$$\Rightarrow \frac{(c-d)(a+b+c)}{(a+b)(b+c)} \dots \dots (2)$$

and

a^2, b^2, c^2 are in A.P

$$\therefore b^2 - a^2 = c^2 - b^2 \dots \dots (3)$$

Substituting $b^2 - a^2$ with $c^2 - b^2$

$$(1) = (2)$$

$$\therefore \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b} \text{ are in A.P}$$

102. There are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

Ans. : There are 25 trees at equal distance of 5 m in line with a well (w), and the distance of the well from the nearest tree = 10 m.

Thus,

The total distance travelled by gardener to tree 1 and back is $2 \times 10\text{m} = 20\text{m}$

Similarly for all the 25 trees.

The distance covered by gardener is

$$= 2[10 + (10 + 5) + (10 + 2 \times 5) + (10 + 3 \times 5) + \dots + (10 + 23 \times 5)]$$

This forms a series of 1st term $a = 10$, common difference $d = 5$ and $n = 25$

$$\therefore 10 + (10 + 5) + (10 + 2 \times 5) + \dots + (10 + 23 \times 5)$$

$$\Rightarrow S_{25} = \frac{25}{2}[2 \times 10 + (24)5] = 25[10 + 60] = 1750\text{m}$$

From(1) and (2)

$$\text{Total distance} = 2 \times 1750\text{m} = 3500\text{m}$$

103. A man is employed to count ₹ 10710. he count at the rate od ₹ 180 per minute for half an hour. after this he counts at the rate of ₹ 3 less every minute than the preceding minute. find the time takan by him to count the entire amount.

Ans. : The man of counts at the rate of ₹ 180 per minute for half an hour. After this he counts at the rate of ₹ 3 less every minute than preceding minute.

Then, the amount counted in first 30 mitnute

$$= ₹ 180 \times 30 = ₹ 5400$$

The amount left to be counted aftar 30 minute

$$= ₹ 10710 - 5400 = ₹ 5310$$

ATQ

$$\text{A.p formed is } (180 - 3) + (180 - 2 \times 3) + \dots = 5310$$

Let time takan to count 5310 be t

Then,

$$S_t = \frac{t}{2}[200 - 3t]$$

$$5310 = \frac{t}{2}[200 - 3t]$$

$$\text{or } t = 59 \text{ minute}$$

Thus, the total time takan by the man to count ₹ 10710 is $(59 - 30) = 89$ minutes.

104. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that 25th term of the A.P. is zero.

Ans. : Given:

$$10a_{10} = 15a_{15}$$

$$10(a + (10 - 1)d) = 15(a + (15 - 1)d)$$

$$10a + 90d = 15a + 210d$$

$$5a + 120d = 0$$

$$a + 24d = 0 \dots \dots (1)$$

$$a_{25} = a + (25 - 1)d$$

$$= a + 24d$$

$$= 0 [\because \text{from(1)} a + 24d = 0]$$

Hence proved.

105. How many terms of G.P. $3, \frac{3}{2}, \frac{3}{4}, \dots$ are needed to give the sum $\frac{3069}{512}$?

$$\text{Ans. : Sum} = \frac{3069}{512} = \frac{3\left(1 - \frac{1}{2^n}\right)}{\frac{1}{2}}$$

$$1 - \frac{1}{2^n} = \frac{3069}{512 \times 6} = \frac{1023}{512 \times 2}$$

$$1 - \frac{1023}{1024} = \frac{1}{2^n}$$

$$\frac{1}{2^n} = \frac{1}{1024}$$

$$n = 10$$

106. Find the sum: $\sum_{n=1}^{10} \left\{ \left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{5}\right)^{n+1} \right\}.$

$$\begin{aligned}\text{Ans. : } & \sum_{n=1}^{10} \left\{ \left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{5}\right)^{n+1} \right\} \\ &= \sum_{n=1}^{10} \left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{10} \left(\frac{1}{5}\right)^{n+1} \\ &= 1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \dots \\ &= \frac{\left(1 - \frac{1}{2^{10}}\right)}{1 - \frac{1}{2}} + \frac{\frac{1}{5} \left(1 - \frac{1}{5^{10}}\right)}{1 - \frac{1}{5}} \\ &= \frac{2^{10} - 1}{2^9} + \frac{5^{10} - 1}{5^{11}}\end{aligned}$$

107. Show that the ratio of the sum of the first n terms of a G.P. to the sum of terms from $(n+1)^{\text{th}}$ to $(2n)^{\text{th}}$ terms is $\frac{1}{r^n}$.

Ans. : Sum of first n term of G.P.

$$\begin{aligned}&= a + a_2 + a_3 + \dots + a_n \\ &= a + ar + ar^2 + \dots + ar^{n-1} [\because t_n = ar^{n-1}] \dots (i)\end{aligned}$$

Also sum of term from

$(n+1)^{\text{th}}$ to $(2n)^{\text{th}}$ term is

$$\begin{aligned}&= a_{n+1} + a_{n+2} + \dots + a_{2n} \\ &= ar^n + ar^{n+1} + \dots + ar^{2n-1} \dots (ii)\end{aligned}$$

Ratio of (i) and (ii) is:

$$\begin{aligned}&= \frac{a + ar + ar^2 + \dots + ar^{n-1}}{ar^n + ar^{n+1} + \dots + ar^{2n-1}} \left[\because S_n = \frac{a(1-r^n)}{1-r} \right] \\ &= \frac{\frac{a(1-r^n)}{1-r}}{\frac{ar^n(1-r^n)}{1-r}} \\ &= \frac{1}{r^n}\end{aligned}$$

108. The ratio of the sum of first three term is to that of first 6 terms of a G.P. is 125 : 152. Find the common ratio.

Ans. : Let Sum of first three terms = $a + ar + ar^2$

$$\begin{aligned}\text{The ratio } &= \frac{a + ar + ar^2}{a + ar + ar^2 + ar^3 + ar^4 + ar^5} \\ &= \frac{1 + r + r^2}{1 + r + r^2 + r^3(1 + r + r^2)} \dots (1)\end{aligned}$$

$$\text{Let } A = 1 + r + r^2 \dots (2)$$

$$\text{Ratio} = \frac{A}{A+r^3A} = \frac{125}{152}$$

$$\frac{1}{1+r^3} = \frac{125}{152}$$

$$152 + 125 + 125r^3$$

$$r^3 = \frac{27}{125}$$

$$r = \frac{3}{5}$$

109. Find the two numbers whose A.M. is 25 and G.M. is 20.

Ans.: Given,

$$\text{A.M.} = 25$$

$$\text{G.M.} = 20$$

Now,

$$\text{A.M.} = \frac{a+b}{2} = 25$$

$$\text{And, G.M.} = \sqrt{ab} = 20$$

$$a+b = 50, ab = 400$$

$$\begin{aligned} (a-b) &= \sqrt{(a+b)^2 - 4ab} \\ &= \sqrt{(50)^2 - 16000} \\ &= \sqrt{2500 - 1600} \\ &= \pm 30 \end{aligned}$$

$$a-b = \pm 30, a+b = 50$$

$$2a = 80$$

$$a = 40$$

$$\text{Also, } -2b = -20$$

$$b = 10$$

∴ The numbers are 40, 10.

110. If S_p denotes the sum of the series $1 + r^p + r^{2p} + \dots$ to ∞ and S_p the sum of the series $1 - r^p + r^{2p} - \dots$ to ∞ , prove that $S_p + S_p = 2S_{2p}$.

Ans.: $S_p = 1 + r^p + r^{2p} + \dots + \infty$

$$S_p = \frac{1}{1-r^p}$$

$$S_p = 1 - r^p + r^{2p} - \dots + \infty$$

$$S_p = \frac{1}{1+r^p}$$

Now,

$$S_p + S_p = \frac{1}{1-r^p} + \frac{1}{1+r^p}$$

$$= \frac{2}{1-r^{2p}}$$

$$S_p + S_p = 2 \times S_{2p}$$

111. Find k such that $k+9$, $k-6$ and 4 from three consecutive terms of a G.P.

Ans. : $k+9$, $k-6$, 4 are in G.P.

$$(k-6)^2 = (k+9)4$$

$$k^2 + 36 - 12k = 4k + 36$$

$$k^2 - 16k = 0$$

$$k(k-16) = 0$$

$$k = 0, k = 16$$

112. $\frac{1}{9}, \frac{1}{9}, \frac{1}{27}, \dots, \infty$

Prove that: $(9 \frac{1}{3} \cdot 9 \frac{1}{9} \cdot 9 \frac{1}{27} \dots, \infty) = 3$.

$$\text{Ans.} : 9 \frac{1}{3} \times 9 \frac{1}{9} \times 9 \frac{1}{27} \dots, \infty$$

$$= 9 \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \right)$$

$$= 9 \left(\frac{\frac{1}{3}}{1 - \frac{1}{3}} \right) \left[\text{Using } S_{\infty} = \frac{a}{1-r} \right]$$

$$= 9 \left(\frac{1}{3} \times \frac{3}{2} \right)$$

$$= 9 \frac{1}{2}$$

$$= 3$$

So,

$$9 \frac{1}{3} \times 9 \frac{1}{9} \times 9 \frac{1}{27} \dots, \infty = 3$$

113. Find the sum of the following series to infinit:

$$\frac{1}{3} + \frac{1}{5^2} + \frac{1}{3^3} + \frac{1}{5^4} + \frac{1}{3^5} + \frac{1}{5^6} + \dots, \infty$$

Ans. : The G.P can be written as follows:

$$\begin{aligned} & \frac{1}{3} + \frac{1}{5^2} + \frac{1}{3^3} + \frac{1}{5^4} + \frac{1}{3^5} + \frac{1}{5^6} + \dots, \infty \\ &= \left(\frac{1}{3} + \frac{1}{3^3} + \frac{1}{3^5} + \dots, \infty \right) + \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots, \infty \right) \\ &= \frac{\frac{1}{3}}{1 - \frac{1}{3^2}} + \frac{\frac{1}{5^2}}{1 - \frac{1}{5^2}} \\ &= \frac{3}{8} + \frac{1}{24} \\ &= \frac{10}{24} \\ &= \frac{5}{12} \end{aligned}$$

114. If a, b, c, d and p are different real numbers such that:

$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + a^2) \leq 0, \text{ then show that } a, b, c \text{ and } d \text{ are in G.P.}$$

$$\begin{aligned}
\text{Ans. : } & (a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + a^2) \leq 0, \\
\Rightarrow & (a^2p^2 + b^2p^2 + c^2p^2) - 2(abp + bcp + cdp) + (b^2 + c^2 + d^2) \leq 0 \\
\Rightarrow & (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \leq 0 \\
\Rightarrow & (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0 \\
\Rightarrow & (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0 \\
\Rightarrow & (ap - b)^2 = 0 \\
\Rightarrow & p = \frac{b}{a}
\end{aligned}$$

$$\text{Also, } (bp - c)^2 = 0$$

$$\Rightarrow p = \frac{c}{b}$$

$$\text{Similarly, } \Rightarrow (cp - d)^2 = 0$$

$$\Rightarrow p = \frac{d}{c}$$

$$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Thus, a, b, c and d are in G.P.

115. Find the sum of the following geometric series:

$$\frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \dots + \frac{a}{(1+i)^n}.$$

$$\text{Ans. : } \frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \dots + \frac{a}{(1+i)^n}.$$

$$a = \frac{a}{1+i}, r = \frac{\frac{a}{(1+i)^2}}{\frac{a}{1+i}} = \frac{1}{1+i}$$

$$\begin{aligned}
S_n &= a \frac{(1-r^n)}{1-r} \\
&= \frac{a}{1+i} \times \frac{\left(1 - \left(\frac{1}{1+i}\right)^n\right)}{1 - \frac{1}{1+i}} \\
&= \frac{a}{1+i} \times \frac{1+i}{(-i)} (1 - (1+i)^n) \\
&= -ai(1 - (1+i)^{-n})
\end{aligned}$$

116. Find the sum of the following series:

0.6 + 0.66 + 0.666 + ... to n terms.

$$\begin{aligned}
\text{Ans. : } & 0.6 + 0.66 + 0.666 + \dots \text{ to n terms} \\
& = 6 \times 0.1 + 6 \times 0.11 + 6 \times 0.111 + \dots \\
& = \frac{6}{9} \left\{ \frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots + - \right\} \\
& = \frac{6}{9} \left\{ \left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \dots + \right\} \\
& = \frac{6}{9} \left\{ n - \left(\frac{1}{10} + \frac{1}{10^2} + \dots + \frac{1}{10^n}\right) \right\}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{6}{9} \left[n - \frac{1}{10} \frac{\{1 - (\frac{1}{10})^n\}}{(1 - \frac{1}{10})} \right] \\
 &= \frac{6}{9} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n}\right) \right]
 \end{aligned}$$

117. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that:

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

Ans.: Let the numbers be a and b.

Then, $A = \frac{a+b}{2}$ or $2A = a + b \dots (1)$

Also, G_1 and G_2 are geometric means between a and b, the a, G_1, G_2, b are in G.P.

Let r be the common ratio.

$$\text{Then, } b = ar^4 - 1 = ar^3 \Rightarrow \frac{b}{a} = r^3 \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$\therefore G_1 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{1}{3}} = a^{\frac{2}{3}}b^{\frac{1}{3}}$$

$$\text{and } G_2 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}}b^{\frac{2}{3}}$$

$$\therefore \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{G_1^3 + G_2^3}{G_1 G_2} = \frac{a^2 b + a b^2}{a b} = a + b = 2A$$

118. In a cricket tournament 16 school teams participated. A sum of Rs. 8000 is to be awarded among themselves as prize money. If the last placed team is awarded Rs. 275 in prize money and the award increases by the same amount for successive finishing places, how much amount will the first place team receive?

Ans.: Let the first place team get Rs. a as the prize money.

Since award money increases by the same amount for successive finishing places, we get an A.P.

Let the constant amount be d.

Here, $t_{16} = 275$, $n = 16$ and $S_{16} = 8000$

$$\therefore t_{16} = a + (16 - 1)(-d)$$

$$\Rightarrow 275 = a - 15d \dots (1)$$

$$\text{Also, } S_{16} = \frac{16}{2} [2a + (n - 1)(-d)]$$

$$\Rightarrow 8000 = 8[2a + (16 - 1)(-d)]$$

$$\Rightarrow 1000 = 2a - 15d \dots (2)$$

Solving Eqs. (1) and (2), we get $a = 725$

Hence, first place team receives Rs. 725.

119. If $a_1, a_2, a_3, \dots, a_n$ are in A.P., where $a_i > 0$ for all i, show that:

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Ans.: Given that, a_1, a_2, \dots, a_n are in A.P., $\forall a_i > 0$

$\therefore a_1 - a_2 = a_2 - a_3 = \dots = a_{n-1} - a_n = -d$ (constant)

Now,

$$\begin{aligned} & \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \\ &= \frac{\sqrt{a_1} - \sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2} - \sqrt{a_3}}{a_2 - a_3} + \dots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{a_{n-1} - a_n} \quad (\text{rationalizing}) \\ &= \frac{\sqrt{a_1} - \sqrt{a_2}}{-d} + \frac{\sqrt{a_2} - \sqrt{a_3}}{-d} + \dots + \frac{\sqrt{a_{n-1}} - \sqrt{a_n}}{-d} \\ &= \frac{1}{-d} [\sqrt{a_1} - \sqrt{a_n}] \\ &= \frac{a_1 - a_n}{-d(\sqrt{a_1} + \sqrt{a_n})} \quad (\text{rationalizing}) \\ &= \frac{-(n-1)d}{-d(\sqrt{a_1} + \sqrt{a_n})} \quad (\text{as } a_n = a_1 + (n-1)d) \\ &= \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}} \end{aligned}$$

120. Find the r^{th} term of an A.P. sum of whose first n terms is $2n + 3n^2$.

[Hint: $a_n = S_n - S_{n-1}$]

Ans.: Given that $S_n = 2n + 3n^2$

$$\Rightarrow S_1 = 2n + 3n^2$$

$$\Rightarrow S_2 = 2 \times 1 + 3(1)^2 = 5$$

$$\Rightarrow S_3 = 2 \times 3 \times 9 = 33$$

$$\therefore S_1 = a_1 = 5$$

$$S_2 - S_1 = a_2 = 16 - 5 = 11$$

$$\therefore d = a_2 - a_1 = 11 - 5$$

$$\text{Now } T_r = a_1 + (r-1)d$$

$$= 5 + (r-1)6 = 5 + 6r - 6 = 6r - 1$$

Hence, the required r^{th} term is $6r - 1$.

121. A man accepts a position with an initial salary of Rs. 5200 per month. It is understood that he will receive an automatic increase of Rs. 320 in the very next month and each month thereafter.

a. Find his salary for the tenth month.

b. What is his total earnings during the first year?

Ans.: The man gets a fixed increment of Rs. 320 each month. Therefore, this forms an A.P. whose,

First term, $a = 5200$ and Common difference, $d = 320$

a. Salary for 10th month will be given by a_n , where $n = 10$.

$$\therefore \text{Total earning} = a_{10}$$

$$= a + (n - 1)d$$

$$= 5200 + (10 - 1) \times 320$$

$$= 5200 + 9 \times 320$$

$$= 5200 + 2880 = \text{Rs. 8080}$$

b. Total earnings during the first year is equal to the sum of 12 terms of the A.P.

$$\therefore \text{Total earnings} = S_{12}$$

$$= \frac{12}{2} [2 \times 5200 + (12 - 1)320]$$

$$= 6[10400 + 11 \times 320]$$

$$= 6[10400 + 3520]$$

$$= 6 \times 13920 = \text{Rs. 83520}$$

122. Match the questions given under Column I with their appropriate answers given under the Column II.

	Column I		Column II
(a)	$4, 1, \frac{1}{4}, \frac{1}{16}$	(i)	A.P.
(b)	$2, 3, 5, 7$	(ii)	Squence
(c)	$13, 8, 3, -2, -7$	(iii)	G.P.

Ans. :

	Column I		Column II
(a)	$4, 1, \frac{1}{4}, \frac{1}{16}$	(i)	G.P.
(b)	$2, 3, 5, 7$	(ii)	Squence
(c)	$13, 8, 3, -2, -7$	(iii)	A.P.

Solution:

1. $4, 1, \frac{1}{4}, \frac{1}{16}$

Here, $\frac{a_2}{a_1} = \frac{1}{4}$, $\frac{a_3}{a_2} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$ and $\frac{a_4}{a_3} = \frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ Hence, it is G.P.

2. $2, 3, 5, 7$

Here, $a_2 - a_1 = 3 - 2 = 1$ $a_3 - a_2 = 5 - 3 = 2$ $\therefore a_2 - a_1 \neq a_3 - a_2$ Hence, it is not A.P.

$\frac{a_2}{a_1} = \frac{3}{2}$, $\frac{a_3}{a_2} = \frac{5}{3}$ So, $\frac{3}{2} \neq \frac{5}{3}$ So, it is not G.P. Hence, it is squence.

3. $13, 8, 3, -2, -7$

Here, $a_3 - a_1 = 8 - 13 = -5$ $a_3 - a_2 = 3 - 8 = -5$ So, $a_2 - a_1 = a_3 - a_2 = -5$ So, it is an A.P.

123. If S_1, S_2, S_3 are respectively the sum of $n, 2n$ and $3n$ terms of G.P. then prove that $S_1^2 + S_2^2 = S_1 (S_2 + S_3)$.

Ans. : Let first term of A.P. be a and common ratio be r .

Then,

$$S_1 = \frac{a(r^n - 1)}{r - 1}, S_2 = \frac{a(r^{2n} - 1)}{r - 1}$$

$$\text{and } S_3 = \frac{a(r^{3n} - 1)}{r - 1}$$

$$S_1 = \frac{a(r^n - 1)}{r - 1}, S_2 = \frac{a(r^{2n} - 1)}{r - 1}$$

Now,
and $S_3 = \frac{a(r^{3n} - 1)}{r - 1}$

$$\begin{aligned} S_1^2 + S_2^2 &= \frac{a^2 (r^n - 1)^2}{(r - 1)^2} + \frac{a^2 (r^{2n} - 1)^2}{(r - 1)^2} \\ &= \frac{a^2}{(r - 1)^2} \left[(r^n - 1)^2 + (r^{2n} - 1)^2 \right] \\ &= \frac{a^2}{(r - 1)^2} \left\{ 1 + (r^n + 1)^2 \right\} \\ &\quad \left[\because r^{2n} - 1 = (r^n - 1)(r^n + 1) \right] \end{aligned}$$

$$\begin{aligned} S_1^2 + S_2^2 &= \frac{a^2 (r^n - 1)^2}{(r - 1)^2} + \frac{a^2 (r^{2n} - 1) a^2 (r^n - 1)^2 (r^{2n} + 2r^n + 2)}{(r - 1)^2 (r - 1)^2} \\ &= \frac{a^2}{(r - 1)^2} \left[(r^n - 1)^2 + (r^{2n} - 1)^2 \right] \\ &= \frac{a^2 (r^n - 1)^2}{(r - 1)^2} \left\{ 1 + (r^n + 1)^2 \right\} \\ \text{and } S_1 (S_2 + S_3) &= \frac{a (r^n - 1) \left[\frac{a (r^{2n} - 1)}{r - 1} + \frac{a (r^{3n} - 1)}{r - 1} \right]}{(r - 1)^2} \\ &= \frac{a^2 (r^n - 1)^2 (r^{2n} + 2r^n + 2)}{(r - 1)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{a^2}{(r-1)^2} (r^n - 1) \cdot [(r^{2n} - 1) + \\
&\quad (r^{3n} - 1)] \\
&= \frac{a^2 (r^n - 1)}{(r-1)^2} [(r^n - 1)(r^n + 1) + \\
&\quad [\because a^3 - b^3 = (r^n - 1)(r^{2n} + r^n + 1)] \\
&= \frac{a^2 (r^n - 1)^2}{(r-1)^2} (r^{2n} + 2b + r^2) \\
&= \frac{a^2 (r^n - 1)}{(r-1)^2} [(r^{2n} - 1)(r^n + 1) + \\
&\quad (r^{3n} - 1)] \\
&\text{From equations (i) and (ii)} \\
&[\because a^3 - b^3 = (r^n - 1)(r^{2n} + r^n + 1)] \\
&= \frac{a^2 (r^n - 1)^2}{(r-1)^2} (r^{2n} + 2b + r^2) + S_2^2 = S_1 (S_2 + S_3) \quad \text{Hence proved.}
\end{aligned}$$

124. $S_1^2 + S_2^2 = S_1 (S_2 + S_3)$ Hence proved.
 For a G.P., if $(m+n)$ th term is P and $(m-n)$ th term is q , then prove that m th and q th term are \sqrt{pq} and $p(\frac{q}{p})^{m/2n}$ respectively.

Ans. : Let first term of sequence be a and common ratio be r , then

$$\begin{aligned}
&T_{m+n} = p \\
\Rightarrow &\text{ and } T_{m-n} = q \\
\Rightarrow &\frac{a \cdot r^{m+n-1}}{a \cdot r^{m-n-1}} = p \\
&\text{ and } a \cdot \frac{p}{q}
\end{aligned}$$

$$\begin{aligned}
& T_{m+n} = p \\
\Rightarrow & \text{ and } T_{m-n} = q \\
\Rightarrow \Rightarrow & \frac{a \cdot r^{m+n-1}}{r^{2n}} = \frac{p}{q} \\
a \cdot r^{m-n-1} & \text{ and } a \cdot \frac{p}{q} \\
\Rightarrow & r = \left(\frac{p}{q} \right)^{1/2n} \\
\Rightarrow & \frac{1}{r} = \left(\frac{q}{p} \right)^{\frac{1}{2n}}
\end{aligned}$$

$$\text{Now, } T_m = a \cdot r^{m-1}$$

$$\begin{aligned}
& a \cdot r^{m+n-1} \left(\frac{1}{r} \right)^n \\
& T_{m+n} \left(\frac{1}{r} \right)^n \\
& = p \cdot \left(\frac{q}{p} \right)^{\frac{n}{2n}} \\
& \left[\because T_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p} \right)^{\frac{1}{2n}} \right] \\
\Rightarrow & T_m = p \cdot \left(\frac{q}{p} \right)^{1/2} = \sqrt{pq} \text{ Hence proved.}
\end{aligned}$$

$$\begin{aligned}
& \text{and } T_n = a \cdot r^{n-1} \\
& = a \cdot r^{m+n-1} \left(\frac{1}{r} \right)^m = T_{m+n} \left(\frac{1}{r} \right)^m
\end{aligned}$$

$$\begin{aligned}
& = p \cdot \left(\frac{q}{p} \right)^{\frac{n}{2n}} \quad = p \cdot \left(\frac{q}{r} \right)^{\frac{m}{2n}} \\
& \left[\because T_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p} \right)^{\frac{1}{2n}} \right] \text{ proved.}
\end{aligned}$$

$$\Rightarrow T_m = p \cdot \left(\frac{q}{p} \right)^{1/2} = \sqrt{pq} \text{ Hence proved.}$$

$$\text{and } T_n = a \cdot r^{n-1}$$

* Given section consists of $\binom{n}{r}$ questions of 5 marks each.

[115]

$$\begin{aligned}
125. & = p \cdot \left(\frac{q}{r} \right)^{\frac{m}{2n}} \quad \frac{1 \times 2^2 + 2 \times 3^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + n^2(n+1)} = \frac{3n+5}{3n+1} \\
& \text{Show that} \\
& \text{Hence proved.}
\end{aligned}$$

$$\text{Ans. : Given: } \frac{1 \times 2^2 + 2 \times 3^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + n^2(n+1)} = \frac{3n+5}{3n+1}$$

$$\begin{aligned}
&= \frac{\sum n(n+1)^2}{\sum n^2(n+1)} = \frac{\sum n(n^2+2n+1)}{\sum (n^3+n^2)} \\
&= \frac{\sum n^3 + 2\sum n^2 + \sum n}{\frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}} \\
&= \frac{\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1 \right]} \\
&= \frac{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{(2n+1)}{3} \right]}{\frac{3n^2+11n+10}{3n^2+7n+2}} = \frac{(n+2)(3n+5)}{(n+2)(3n+1)} = \frac{3n+5}{3n+1}
\end{aligned}$$

126. Find the sum of the following series up to n terms:

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots$$

Ans.: Given: $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots$ up to n terms

$$\begin{aligned}
\therefore a_n &= \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{1+3+5+\dots+(2n-1)} \\
&= \frac{\sum n^3}{\frac{n}{2}[2+(n-1)2]} = \frac{\sum n^3}{\frac{n}{2}(2n)} = \frac{\sum n^3}{n^2} = \frac{n^2(n+1)^2}{4n^2} \\
&= \frac{1}{4}(n^2 + 2n + 1)
\end{aligned}$$

$$\begin{aligned}
\therefore S_n &= \sum_{k=1}^n a_k = \sum_{k=1}^n \frac{k^2 + 2k + 1}{4} \\
&= \frac{1}{4}[(1^2 + 2 \cdot 1 + 1) + (2^2 + 2 \cdot 2 + 1) + (3^2 + 2 \cdot 3 + 1) + \dots + (n^2 + 2n + 1)] \\
&= \frac{1}{4} \left[\sum n^2 + 2 \sum n + n \right] \\
&= \frac{1}{4} \left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n \right] \\
&= \frac{n}{4} \left[\frac{2n^2 + 3n + 1 + 6n + 6 + 6}{6} \right] \\
&= \frac{n}{24} (2n^2 + 9n + 13)
\end{aligned}$$

127. If S_1, S_2, S_3 are the sum of first n natural no. their squares and their cubes respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.

$$\text{Ans. : } S_1 = \frac{n(n+1)}{2}$$

$$S_2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_3 = \left(\frac{n(n+1)}{2} \right)^2$$

$$\text{R.H.S.} = S_3(1 + 8S_1)$$

$$= \frac{(n(n+1))^2}{2} \left[1 + 8 \frac{n(n+1)}{2} \right]$$

$$= 9 \left(\frac{n(n+1)(2n+1)}{6} \right)^2$$

$$= 9S_2^2$$

128. If a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P., prove that a, c, e are in G.P.

Ans. : Since, a, b, c are in A.P.

$$\therefore b - a = c - b$$

$$\Rightarrow 2b = a + c$$

$$\Rightarrow b = \frac{a+c}{2}$$

Since, b, c, d are in G.P.

$$\therefore \frac{c}{b} = \frac{d}{c}$$

$$\Rightarrow c^2 = bd \dots \dots \dots \text{(i)}$$

Also $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.

$$\therefore \frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d}$$

$$\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$$

$$\Rightarrow \frac{2}{d} = \frac{c+e}{ce}$$

$$\Rightarrow d = \frac{2ce}{c+e}$$

Putting values of b and d in eq. (i), $c^2 = \left(\frac{c+a}{2} \right) \left(\frac{2ce}{c+e} \right)$

$$\Rightarrow c^2 = \frac{ce(c+a)}{c+e}$$

$$\Rightarrow c^2(c+e) = ec(c+a)$$

$$\Rightarrow c^2 + ce = ce + ae$$

$$\Rightarrow c^2 = ae \text{ which shows that } a, c, e \text{ are in G.P.}$$

129. The ratio of the A.M. and G.M. of two positive numbers a and b is $m:n$. Show

$$\text{that } a:b = (m + \sqrt{m^2 - n^2}):(m - \sqrt{m^2 - n^2})$$

Ans.: Given: $\frac{a+b}{2} : \sqrt{ab} = m : n$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

By componendo and dividendo,

$$\begin{aligned} \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} &= \frac{m+n}{m-n} \\ \Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} &= \frac{m+n}{m-n} \\ \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} &= \frac{\sqrt{m+n}}{\sqrt{m-n}} \end{aligned}$$

Again by componendo and dividendo,

$$\begin{aligned} \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} &= \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}} \\ \Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} &= \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}} \\ \Rightarrow \frac{a}{b} &= \frac{(\sqrt{m+n}+\sqrt{m-n})^2}{(\sqrt{m+n}-\sqrt{m-n})^2} \\ \Rightarrow \frac{a}{b} &= \frac{m+n+m-n+2\sqrt{(m+n)(m-n)}}{m+n+m-n-2\sqrt{(m+n)(m-n)}} \\ \Rightarrow \frac{a}{b} &= \frac{2m+2\sqrt{(m+n)(m-n)}}{2m-2\sqrt{(m+n)(m-n)}} \\ \Rightarrow \frac{a}{b} &= \frac{m+\sqrt{(m+n)(m-n)}}{m-\sqrt{(m+n)(m-n)}} \end{aligned}$$

Therefore, $a:b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$

130. If a, b, c are in A.P., prove that:

$$a^3 + c^3 + 6abc = 8b^3$$

Ans.: If $a^3 + c^3 + 6abc = 8c^3$

$$\text{or } a^3 + c^3 - (2b)^3 + 6abc = 0$$

$$\text{or } a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$$

$$\therefore (a - 2b + c) = 0 \left[\begin{array}{l} \therefore x^3 + y^3 + z^3 + 3xyz = 0 \\ \text{or if } x+y+z = 0 \end{array} \right]$$

$$\text{or } a+c = 2b$$

$$a-b = c-b$$

and since, a, b, c are in A.P

$$\text{Thus, } a-b = c-d$$

Hence proved. $a^3 + c^3 + 6abc = 8b^3$

131. If a, b, c are in A.P., prove that:

$$(a - c)^2 = 4(a - b)(b - c)$$

Ans.: If $(a - c)^2 = 4(a - b)(b - c)$

Then,

$$a^2 + c^2 - 2ac = 4(ab) - b^2 - ac + bc$$

$$\Rightarrow a^2 + c^2 - 4b^2 + 2ac - 4ac - 4bc = 0$$

$$\Rightarrow (a + c - 2b)^2 = 0 \quad [\text{Using } (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc]$$

$$\therefore a + c - 2b = 0$$

$$\text{or } a + c = 2b$$

and since,

a, b, c are in A.P [Given]

$$a + c = 2b$$

Hence proved

$$(a - b)^2 = 4(a - b)(b - c)$$

132. If a, b, c are in A.P., prove that:

$$a^2 + c^2 + 4ac = 2(ab + bc + ca)$$

Ans.: If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$$a^2 + c^2 + 2ac - 2ab = 2(ab + bc + ca)$$

or

$$(a + b + -c)^2 - b^2 = 0$$

$$[\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc]$$

$$\text{or } b = a + c - b$$

$$\text{or } 2b = a + c$$

$$b = \frac{a + c}{2}$$

and since,

a, b, c are in A.P

$$b = \frac{a + c}{2}$$

$$\text{Thus, } a^2 + c^2 + 4ac = 2(ab + bc + ca)$$

Hence proved.

133. Show that $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., if x, y and z are in A.P.

Ans.: x, y and z are in A.P.

Let d be the common difference then,

$$y = x + d \text{ and } x = x + 2d$$

To show $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., it is enough to show that,

$$(z^2 + zx + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + zx + x^2)$$

$$\begin{aligned}
 \text{LHS} &= (z^2 + zx + x^2) - (x^2 + xy + y^2) \\
 &= (z^2 + zx - zy - y^2) \\
 &= (x2d)^2 + (x + 2d)x - x(x + d) - (x + d)^2 \\
 &= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2 \\
 &= 3xd + d^3
 \end{aligned}$$

$$\begin{aligned}
 \text{RHS} &= (y^2 + yz + z^2) - (z^2 + zx + x^2) \\
 &= (y^2 + yz + z^2) - (z^2 + zx + x^2) \\
 &= (x + d)^2 + (x + d)(x + 2d) - (x + 2d)x = x^2 \\
 &= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2 \\
 &= 3xd + 3d^2
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore x^2 + xy + y^2, z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

134. Show that $x^2 + xy + y^2, z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., if x, y and z are in A.P.

Ans. : x, y and z are in A.P.

Let d be the common difference then,

$$y = x + d \text{ and } x = x + 2d$$

To show $x^2 + xy + y^2, z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., it is enough to show that,

$$(z^2 + zx + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + zx + x^2)$$

$$\text{LHS} = (z^2 + zx + x^2) - (x^2 + xy + y^2)$$

$$\begin{aligned}
 &(z^2 + zx - zy - y^2) \\
 &= (x2d)^2 + (x + 2d)x - x(x + d) - (x + d)^2 \\
 &= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2 \\
 &= 3xd + d^3
 \end{aligned}$$

$$\text{RHS} = (y^2 + yz + z^2) - (z^2 + zx + x^2)$$

$$= (y^2 + yz + z^2) - (z^2 + zx + x^2)$$

$$= (x + d)^2 + (x + d)(x + 2d) - (x + 2d)x = x^2$$

$$= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2$$

$$= 3xd + 3d^2$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore x^2 + xy + y^2, z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

135. If a, b, c are in A.P., prove that:

$$(a - c)^2 = 4(a - b)(b - c)$$

Ans. : If $(a - c)^2 = 4(a - b)(b - c)$

Then,

$$\begin{aligned}
 a^2 + c^2 - 2ac &= 4(ab) - b^2 - ac + bc \\
 \Rightarrow a^2 + c^2 - 4b^2 + 2ac - 4ac - 4bc &= 0 \\
 \Rightarrow (a + c - 2b)^2 &= 0 \quad [\text{Using } (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc] \\
 \therefore a + c - 2b &= 0
 \end{aligned}$$

or $a + c = 2b$

and since,

a, b, c are in A.P [Given]

$$a + c = 2b$$

Hence proved

$$(a - b)^2 = 4(a - b)(b - c)$$

136. Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.

Ans. : sum of all two digit numbers which when divided by 4, yields 1 as remainder,

\Rightarrow all $4n + 1$ terms with $n \geq 3$

$$n = 22, a = 13, d = 4$$

$$\text{Sum of terms} = \frac{22}{2}[26 + 21 \times 4] = 11 \times 110 = 1210$$

137. A man accepts a position with an initial salary of ₹ 5200 per month. It is understood that he will receive an automatic increase of ₹ 320 in the very next month and each month thereafter.

i. Find his salary for the tenth month.

ii. What is his total earnings during the first year?

Ans. : A man accepts a position with an initial salary of ₹ 5200 per month.

$$a = 5200$$

Man will receive an automatic increase of ₹ 320.

$$d = 320$$

Man's salary for the n^{th} month is given by,

$$a_n = a_1 + (n - 1)d$$

Total earning of the man for the first year

$$\begin{aligned}
 &= \frac{12}{2}[a_1 + a_{12}] \\
 &= 6[5200 + 5200 + (12 - 1)320] \\
 &= 83520
 \end{aligned}$$

Total earning of the man for the first year is ₹ 83,520.

138. If a, b, c, d are in G.P., prove that:

$$(b + c)(b + d) = (c + a)(c + d)$$

Ans. : a, b and c are in G.P.

$$\therefore b^2 = ac \dots (1)$$

$$\text{L.H.S} = (b + a)(b + d)$$

$$\begin{aligned}
 &= b^2 + bd + bc + cd \\
 &= ac + c^2 + ad + cd \quad [\text{Using (1)}] \\
 &= c(a + c) + d(a + c) \\
 &= (c + a)(c + d) \\
 &= \text{R.H.S} \\
 \therefore \text{R.H.S} &= \text{L.H.S}
 \end{aligned}$$

139. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d from a G.P. Prove that $(q + p) : (q - p) = 17 : 15$.

Ans. : Given,

a, b are roots of the equation $x^2 - 3x + p = 0$

$$\Rightarrow a + b = 3, ab = p$$

And c, d are roots of the equation $x^2 - 12x + q = 0$

$$\Rightarrow c + d = 12, cd = q$$

Let $b = ar, c = ar^2$ and $d = ar^3$, then $a + b = 3$ and $c + d = 12$

$$a(1 + r) = 3 \text{ and } ar^2(1 + r) = 12$$

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$

$$\Rightarrow r = 2$$

$$\text{And } a(r + 2) = 3$$

$$\Rightarrow a = 1$$

$$p = ab$$

$$p = a \times ar$$

$$p = 2$$

$$q = cd$$

$$= ar^2 \times ar^3$$

$$a = 32$$

$$\begin{aligned}
 \frac{q+p}{q-p} &= \frac{32+2}{32-2} \\
 &= \frac{34}{30}
 \end{aligned}$$

$$(q + p) : (q - p) = 17 : 15$$

140. The product of three numbers in G.P. is 125 and the sum of their products taken in pairs is $87\frac{1}{2}$. Find them.

Ans. : Let the three numbers in G.P. be $\frac{a}{r}, a, ar$ then product of these numbers

$$\left(\frac{a}{r}\right)(a)(ar)$$

$$\Rightarrow a^3 = 125 = 5^3$$

$$a = 5$$

Also, sum of these product in pair

$$\left(\frac{a}{r}\right)(a) + (a)(ar) + \left(\frac{a}{r}\right)(ar)$$

$$= 87\frac{1}{2} = \frac{195}{2}$$

$$= (5)^2 \left(\frac{1+r^2+r}{r}\right) = \frac{195}{2}$$

$$1 + r^2 + r = \left(\frac{195}{2 \times 25}\right)^r$$

$$2(1 + r^2 + r) = \frac{39}{5}r$$

$$10 + 10r^2 + 10r = 39r$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r - 5) - 2(2r - 5) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

∴ G.P. is $\frac{a}{r}, a, ar$

$$10, 5, \frac{5}{2}, \dots \text{ or } \frac{5}{2}, 5, 10 \dots$$

141. If a, b, c are in G.P., prove that:

$$(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$$

Ans. : a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\text{L.H.S} = (a+2b+2c)(a-2ar+2c)$$

$$= (a+2ar+2ar^2)(1-2ar+2ar^2)$$

$$= a^2(1+2ar+2ar^2)(1-2r+2r^2)$$

$$= a^2[(1+2r^2)^2 - (2r)^2]$$

$$= a^2[1+4r^4+4r^2-4r^2]$$

$$= a^2[1+4r^4]$$

$$= a^2 + 4(ar^2)^2$$

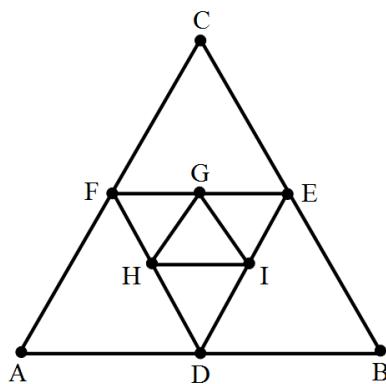
$$= a^2 + 4c^2$$

$$= \text{R.H.S}$$

$$\therefore \text{R.H.S} = \text{L.H.S}$$

142. One side of equilateral triangle is 18 cm. The mid-points of its sides are joined to from another triangle whose mid-points, in turn, are joined to from still another triangle. the process is continued indefinitely. Find the sum of the (i) Perimeters of all the triangles. (ii) Areas of all triangles.

Ans. :



Side of triangle = 18cm.

$$AD = BD = 9\text{cm.}$$

$$DE = BD = 9\text{cm.}$$

$$GI = IF = \frac{9}{2}\text{cm.}$$

Sides of the triangles are 18, 9, $\frac{9}{2}$...

$$\begin{aligned} \text{(i) Sum of perimeters of the equilateral triangle} &= \left(54 + 27 + \frac{27}{2} + \dots \right) \\ &= \frac{54}{1 - \frac{1}{2}} \\ &= 54 \times 2 \end{aligned}$$

Perimeter = 108cm.

(ii) Sum of area of equilateral triangle

$$\begin{aligned} &= \left[\frac{\sqrt{3}}{4}(18)^2 + \frac{\sqrt{3}}{4}(9)^2 + \frac{\sqrt{3}}{4}\left(\frac{9}{2}\right)^2 + \dots \right] \\ &= \frac{\sqrt{3}}{4} \left[324 + 81 + \frac{81}{4} + \dots \right] \\ &= \frac{\sqrt{3}}{4} \left[\frac{324}{1 - \frac{1}{4}} \right] \\ &= \frac{\sqrt{3}}{4} \left[\frac{324 \times 4}{3} \right] \\ &= \sqrt{3}(108) \end{aligned}$$

143. If a, b, c , are in G.P., prove that:

$(a^2 + b^2), (b^2 + c^2), (c^2 + d^2)$ are in G.P.

Ans. : a, b, c, d are in G.P.

$$\therefore b^2 = ac$$

$$ab = bc$$

$$c^2 = bd \dots (1)$$

Now,

$$\begin{aligned} (b^2 + c^2)^2 &= (b^2)^2 2b^2c^2 + (c^2)^2 \\ &\Rightarrow (b^2 + c^2)^2 = (ac)^2 + b^2c^2 + b^2c^2 + (bd)^2 \quad [\text{Using (1)}] \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow (b^2 + c^2)^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 \text{ [Using (1)]} \\
 & \Rightarrow (b^2 + c^2)^2 = a^2(c^2 + d^2) + b^2(c^2 + d^2)^2 \\
 & \Rightarrow (b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2) \\
 \therefore (a^2 + b^2), (c^2 + d^2) \text{ and } (b^2 + c^2) \text{ are also in G.P.}
 \end{aligned}$$

144. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively. Prove that x, y, z are in G.P.

Ans. : $a_4 = x$

$$\Rightarrow ar^3 = x$$

Also, $a_{16} = y$

$$\Rightarrow ar^9 = y$$

And, $a_{16} = z$

$$\Rightarrow ar^{15} = z$$

$$\therefore \frac{z}{x} = \frac{ar^9}{ar^3} = r^6$$

$$\text{And, } \frac{z}{y} = \frac{ar^{15}}{ar^9} = r^6$$

$$\therefore \frac{y}{x} = \frac{z}{y}$$

$\therefore x, y$ and z are in G.P.

145. Find the 4th term from the end of the G.P. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \dots, \frac{1}{4374}$.

Ans. : $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \dots, \frac{1}{4374}$.

$$a = \frac{1}{2}, l = \frac{1}{4374}, r = \frac{t_{n-1}}{t_n} = \frac{t_2}{t_1} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

Term from the end is:

$$a_n = l \left(\frac{1}{r} \right)^{n-1}$$

$$t_4 = \left(\frac{1}{4374} \right) (3)^{n-1}$$

$$= \frac{1}{4374} \times 3^3$$

$$= \frac{1}{162}$$

$$\therefore 4^{\text{th}} \text{ term from the end is } = \frac{1}{162}$$

146. If the pth and qth terms of a G.P. are q and p respectively, show that its (p + q)th term is $\left(\frac{q^p}{p^q} \right) \frac{1}{p - q}$.

Ans. : Let the first term and common ratio of G.P. are a and r, respectively.

Given that, pth term = q $\Rightarrow ar^{p-1} = q \dots (1)$

and q^{th} term $= p \Rightarrow ar^{q-1} = p \dots (2)$

On dividing Eq. (1) by (2), we get,

$$\begin{aligned}\frac{ar^{p-1}}{ar^{q-1}} &= \frac{q}{p} \\ \Rightarrow r^{p-q} &= \frac{q}{p} \\ \Rightarrow r &= \left(\frac{q}{p}\right)^{\frac{1}{p-q}}\end{aligned}$$

On substituting the value of r in Eq. (1), we get

$$\begin{aligned}a \left(\frac{q}{p}\right)^{\frac{p-1}{p-q}} &= q \\ \Rightarrow a &= q \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}} \\ \therefore (p+q)^{\text{th}} \text{ term, } T_{p+q} &= a \cdot r^{p+q-1} \\ &= q \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}} \left(\frac{a}{p}\right)^{\frac{p+q-1}{p-q}} \\ &= q \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}} - \frac{p+q-1}{p-q} \\ &= q \left(\frac{q}{p}\right)^{\frac{q}{p-q}} \\ &= \frac{q^{\frac{q}{p-q}} + 1}{p^{\frac{q}{p-q}}} \\ &= \frac{q^{\frac{q}{p-q}}}{p^{\frac{q}{p-q}}} \\ &= \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}\end{aligned}$$

147. Match the questions given under Column I with their appropriate answers given under the Column II.

	Column I		Column II
(a)	$1^2 + 2^2 + 3^2 + \dots + n^2$	(i)	$\left[\frac{n(n+1)}{2}\right]^2$
(b)	$1^3 + 2^3 + 3^3 + \dots + n^3$	(ii)	$n(n+1)$
(c)	$2 + 4 + 6 + \dots + 2n$	(iii)	$\frac{n(n+1)(2n+1)}{6}$
(d)	$1 + 2 + 3 + \dots + n$	(iv)	$\frac{n(n+1)}{2}$

Ans. :

	Column I		Column II
(a)	$1^2 + 2^2 + 3^2 + \dots + n^2$	(iii)	$\frac{n(n+1)(2n+1)}{6}$
(b)	$1^3 + 2^3 + 3^3 + \dots + n^3$	(i)	$\left[\frac{n(n+1)}{2}\right]^2$
(c)	$2 + 4 + 6 + \dots + 2n$	(ii)	$n(n+1)$

(d)	$1 + 2 + 3 + \dots + n$	(iv)	$\frac{n(n+1)}{2}$
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Solution:

$$1. \text{ Let } S = 1^2 + 2^2 + 3^2 + \dots + n^2$$

We have, $n^3 - (n-1)^3 = 3n^2 - 3n + 1$; And by changing n into $n-1$,

$$(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1;$$

$$(n-2)^2 - (n-3)^3 = 3(n-2)^2 - 3(n-2) + 1; \dots \dots \dots$$

$$3^3 - 2^3 = 3 \cdot 3^2 - 3 \cdot 3 + 1; 2^3 - 1^2 = 3 \cdot 2^2 - 3 \cdot 2 + 1; 1^2 - 0^2 = 3 \cdot 1^2 - 3 \cdot 1 + 1$$

Hence, by addition,

$$n^3 = 3(1^2 + 2^2 + 3^2 + \dots + n^2) - 3(1 + 2 + 3 + \dots + n) + n$$

$$= 3S - \frac{3n(n+1)}{2} + n \Rightarrow 3S = n^3 - n + \frac{3n(n+1)}{2} = n(n+1)\left(n-1 + \frac{3}{2}\right)$$

$$\Rightarrow S = \frac{n(n+1)(2n+1)}{6}$$

$$2. \text{ Let } S = 1^3 + 2^3 + 3^3 + \dots + n^3$$

$$\text{We have, } n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1;$$

$$(n-1)^4 - (n-2)^4 = 4(n-1)^3 - 6(n-1)^2 + 4(n-1) - 1;$$

$$(n-2)^4 - (n-3)^4 = 4(n-2)^3 - 6(n-2)^2 + 4(n-2) - 1;$$

$$\dots \dots \dots \quad 3^4 - 2^4 = 4 \cdot 3^3 - 6 \cdot 3^2 + 4 \cdot 3 - 1;$$

$$2^4 - 1^4 = 4 \cdot 2^3 - 6 \cdot 2^2 + 4 \cdot 2 - 1; 1^4 - 0^4 = 4 \cdot 1^3 - 6 \cdot 1^2 + 4 \cdot 1 - 1. \text{ Hence, by}$$

$$\text{addition, } n^4 = 4S - 6(1^2 + 2^2 + \dots + n^2) + 4(1 + 2 + \dots + n) - n;$$

$$\therefore 4S = n^4 + n + 6(1^2 + 2^2 + \dots + n^2) - 4(1 + 2 + \dots + n)$$

$$= n^4 + n + n(n+1)(2n+1) - 2n(n+1) = n(n+1)(n^2 - n + 1 + 2n + 1 - 2)$$

$$= n(n+1)(n^2 + n) \therefore S = \frac{n^2(n+1)^2}{4} = \left\{ \frac{n(n+1)}{2} \right\}^2$$

$$3. \quad 1 + 2 + 3 + \dots + n = \frac{n(n+2)}{2}$$

$$2 + 4 + 6 + \dots + 2n = 2(1 + 2 + 3 + \dots + n) = 2 \times \frac{n}{2}(1 + n) = n(n + 1)$$

4. $1 + 2 + 3 + \dots + n$ = Sum of n terms of A.P. with first term '1' and common

$$\text{difference '1'} = \frac{n}{2}(1 + n)$$

*** Case study based questions**

[8]

148. A company produces 500 computers in the third year and 600 computers in the seventh year. Assuming that the production increases uniformly by a constant number every year.



Based on the above information, answer the following questions.

- (i) The value of the fixed number by which production is increasing every year is
(a) 25 (b) 20 (c) 10 (d) 30
- (ii) The production in first year is
(a) 400 (b) 250 (c) 450 (d) 300
- (iii) The total production in 10 years is
(a) 5625 (b) 5265 (c) 2655 (d) 6525
- (iv) The number of computers produced in 21 st year is
(a) 650 (b) 700 (c) 850 (d) 950
- (v) The difference in number of computers produced in 10th year and 8th year is
(a) 25 (b) 50 (c) 100 (d) 75

Ans. : (i) Since, it is given that, production increases uniformly by a constant number, hence number of productions every year forms an AP.

$$\therefore a_3 = 500 \Rightarrow a + 2d = 500$$

$$a_7 = 600 \Rightarrow a + 6d = 600$$

Now, subtracting Eq. (i) from Eq. (ii), we get

$$4d = 100 \Rightarrow d = 25$$

(ii) Put $d = 25$ in Eq. (i), we get

$$a + 50 = 500 \Rightarrow a = 450$$

(iii) The total production in 10 years = S_{10}

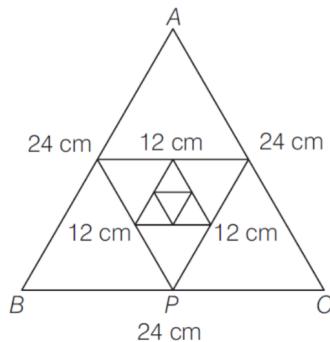
$$\begin{aligned}\therefore S_{10} &= \frac{10}{2}[2 \times 450 + 9 \times 25] \\ &= 5[900 + 225] = 5625\end{aligned}$$

(iv) The number of computers produced in 21st year = a_{21}

$$\therefore a_{21} = 450 + 20 \times 25 = 450 + 500 = 950$$

$$(v) a_{10} - a_8 = (a + 9d) - (a + 7d) = 2d = 2 \times 25 = 50$$

149. Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



Based on above information, answer the following questions.

(i) The side of the 5th triangle is (in cm)

- (a) 3 (b) 6 (c) 1.5 (d) 0.75

(ii) The sum of perimeter of first 6 triangle is (in cm)

- (a) $\frac{569}{4}$ (b) $\frac{567}{4}$ (c) 120 (d) 144

(iii) The area of all the triangle is (in sq cm)

- (a) 576 (b) $192\sqrt{3}$ (c) $144\sqrt{3}$ (d) $169\sqrt{3}$

(iv) The sum of perimeter of all triangle is (in cm)

- (a) 144 (b) 169 (c) 400 (d) 625

(v) The perimeter of 7 th triangle is (in cm)

- (a) $\frac{7}{8}$ (b) $\frac{9}{8}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

Ans. : (i) Side of first triangle is 24 .

Side of second triangle is $\frac{24}{2} = 12$

Similarly, side of second triangle is $\frac{12}{2} = 6$

$$\therefore a = 24, r = \frac{12}{24} = \frac{1}{2}$$

\therefore Side of the fifth triangle,

$$a_5 = ar^4 = 24 \times \left(\frac{1}{2}\right)^4$$

$$= \frac{24}{16} = \frac{3}{2} = 1.5 \text{ cm}$$

$$\begin{aligned}
 \text{(ii)} \quad &= \frac{72 \left(1 - \left(\frac{1}{2} \right)^6 \right)}{1 - \frac{1}{2}} = \frac{72 \times 63 \times 2}{2^6} \\
 &= \frac{567}{4} \text{ cm}
 \end{aligned}$$

(iii) Area of first triangle is $\frac{\sqrt{3}}{4}(24)^2$

$$\text{Area of second triangle} = \frac{\sqrt{3}}{4} \left(\frac{24}{2} \right)^2 = \frac{\sqrt{3}}{4}(24)^2 \times \frac{1}{4}$$

$$\therefore a = \frac{\sqrt{3}}{4}(24)^2, r = \frac{1}{4}$$

\therefore Sum of area of all triangles

$$\begin{aligned}
 &= \frac{a}{1-r} = \frac{\sqrt{3}}{4} \frac{(24)^2}{1-\frac{1}{4}} \\
 &= \frac{\sqrt{3} \times (24)^2}{3} = 192\sqrt{3} \text{ cm}^2
 \end{aligned}$$

(iv) The sum of perimeter of all triangle $3(24 + 12 + 6 + \dots)$ is

$$3 \left(\frac{24}{1 - \frac{1}{2}} \right) = 144 \text{ cm} \left[\because a = 24, r = \frac{1}{2} \right]$$

(v) Here, $a = 72, r = \frac{1}{2}$

$$a_7 = (72) \left(\frac{1}{2} \right)^6$$

$$= \frac{72}{64} = \frac{9}{8} \text{ cm}$$

----- "Opportunities don't happen, you create them. -----