

* Match the following.

[5]

1.	Part (a)	Part (b)
1.	Value of $(1 + i)(1 + i^2)(1 + i^3)(1 + i^4)$	(a) 0
2.	$a = 1 + i$ then value of a^2	(b) $-i$
3.	Square root of $-i$	(c) $2i$
4.	i^{135}	(d) $\pm \frac{1}{\sqrt{2}}(1 - i)$
5.	i^{-999}	(e) i

* Choose the right answer from the given options. [1 Marks Each]

[52]

- If $i^2 = -1$, then the sum $i + i^2 + i^3 + \dots$ upto 1000 terms is equal to:
 (A) 1 (B) -1 (C) i (D) 0
- If, $x^4 + 4x^3 + 6ax^2 + 6bx + c$ is divisible by $x^3 + 3x^2 + 9x + 3$. Then, what is the value of $a + b + c$?
 (A) 4 (B) 6 (C) 7 (D) 10
- If p and q are the roots of the equation $x^2 + px + q = 0$ then, what are the values of p and q ?
 (A) $p = 1, q = -2$ (B) $p = 0, q = 1$ (C) $p = -2, q = 0$ (D) $p = -2, q = 1$
- A real value of x satisfies the equation $\frac{3-4ix}{3+4ix} = a - ib$ ($a, b \in \mathbb{R}$), if $a^2 + b^2 =$
 (A) 1 (B) -1 (C) 2 (D) -2
- If $x^2 + px + 1 = 0$ and $(a - b)x^2 + (b - c)x + (c - a) = 0$ have both roots common, then what is the form of a, b, c ?
 (A) a, b, c are in A.P (B) b, a, c are in A.P (C) b, a, c are in G.P (D) b, a, c are in H.P
- If $i^2 = -1$, then the sum $i + i^2 + i^3 + \dots$ upto 1000 terms is equal to:
 (A) 1 (B) -1 (C) i (D) 0
- According to De Moivre's theorem what is the value of $z^{\frac{1}{n}}$
 (A) $r^{\frac{1}{n}} [\cos 2kn + \theta] + i \sin(2kn + \theta)$
 (B) $r^{\frac{1}{n}} \left[\frac{\cos 2kn + \theta}{n} - \frac{i \sin(2kn + \theta)}{n} \right]$
 (C) $r^{\frac{1}{n}} \left[\frac{\cos 2kn + \theta}{n} + \frac{i \sin(2kn + \theta)}{n} \right]$
 (D) $r^{\frac{1}{n}} [\cos 2kn + \theta) - i \sin(2kn + \theta)]$

9. If $\frac{1-ix}{1+ix} = a + ib$, then $a^2 + b^2 =$
 (A) 1 (B) -1 (C) 0 (D) none of these
10. Find mirror image of point representing $x + iy$ on real axis:
 (A) (x, y) (B) $(-x, -y)$ (C) $(-x, y)$ (D) $(x, -y)$
11. If $(1+i)(1+2i)(1+3i)\dots(1+ni) = a + ib$, then $2.5.10.17\dots(1+n^2) =$
 (A) $a - ib$ (B) $a^2 - b^2$ (C) $a^2 + b^2$ (D) none of these
12. The value of $(1+i)(1+i^2)(1+i^3)(1+i^4)$ is
 (A) 2 (B) 0 (C) 1 (D) i
13. The square root of $(-15 - 8i)$ is:
 (A) $\pm(1 - 4i)$ (B) $\pm(1 + 4i)$ (C) $\pm(-2 + 4i)$ (D) None of these
14. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and $0 < \theta < 2\pi$, then $\theta =$
 (A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$
15. If, α and β are the roots of the equation $2x^2 - 3x - 6 = 0$, then what is the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$
 (A) $4x^2 + 49x + 118 = 0$ (B) $4x^2 - 49x + 118 = 0$ (C) $4x^2 - 49x - 118 = 0$ (D) $x^2 - 49x + 118 = 0$
16. If $x + iy = (1+i)(1+2i)(1+3i)$, then $x^2 + y^2 =$
 (A) 0 (B) 1 (C) 100 (D) none of these
17. If, $(a+1)x^2 + 2(a+1)x + (a-2) = 0$ then, for what parameter of 'a' the given equation have real and distinct roots?
 (A) $(-\infty, \infty)$ (B) $(-1, \infty)$ (C) $[-1, \infty)$ (D) $(-1, 1)$
18. If $z = 1 - \cos\theta + i\sin\theta$, then $|z| =$
 (A) $2\sin\frac{\theta}{2}$ (B) $2\cos\frac{\theta}{2}$ (C) $2\left|\sin\frac{\theta}{2}\right|$ (D) $2\left|\cos\frac{\theta}{2}\right|$
19. If one root of the equation $x^2 + px + 12 = 0$, is 4, while the equation $x^2 + px + q = 0$ has equal roots, the value of q is:
 (A) $\frac{49}{4}$ (B) $\frac{4}{49}$ (C) 4 (D) None of these.
20. If $z = 2 - 3i$ then $z^2 - 4z + 13 =$
 (A) 0 (B) 1 (C) 2 (D) 3
21. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on:
 (A) Circle $x^2 + y^2 = 1$ (B) The x-axis (C) The y-axis (D) The line $x + y = 1$
22. If $a = \cos\theta + i\sin\theta$, then $\frac{1+a}{1-a} =$
 (A) $\cot\frac{\theta}{2}$ (B) $\cot\theta$ (C) $i\cot\frac{\theta}{2}$ (D) $i\tan\frac{\theta}{2}$

23. The argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is:
 (A) 60° (B) 120° (C) 210° (D) 240°
24. If, $(a+1)x^2 + 2(a+1)x + a - 2 = 0$ then, for what parameter of 'a' the given equation have equal roots?
 (A) $(-\infty, -1)$ (B) $[-1, \infty)$ (C) $(0, 1)$ (D) Not possible
25. Choose the correct answer.
 $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for:
 (A) $x = n\pi$ (B) $x = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$ (C) $x = 0$ (D) no value of x
26. Convert $-1 + i$ into polar form:
 (A) $\sqrt{2}, \frac{5\pi}{4}$ (B) $\sqrt{2}, \frac{3\pi}{4}$ (C) $-\sqrt{2}, \frac{\pi}{4}$ (D) $\sqrt{2}, \frac{\pi}{4}$
27. Choose the correct answer.
 The value of $(z+3)(\bar{z}+3)$ is equivalent to:
 (A) $|z+3|^2$ (B) $|z-3|$ (C) z^2+3 (D) None of these.
28. A quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots, if
 (A) $a = 0$ (B) $b^2 - 4ac = 0$ (C) $b^2 - 4ac < 0$ (D) $b^2 - 4ac > 0$
29. The number of real roots of the equation $(x^2 + 2x)^2 - (x+1)^2 - 55 = 0$:
 (A) 2 (B) 1 (C) 4 (D) None of these.
30. The polar form of $(i^{25})^3$ is:
 (A) $\cos \frac{\pi}{2} + i\sin \frac{\pi}{2}$ (B) $\cos \pi + i\sin \pi$ (C) $\cos \pi - i\sin \pi$ (D) $\cos \frac{\pi}{2} - i\sin \frac{\pi}{2}$
31. What will be the sum of $b + c$ if the equations $x^2 + bx + c = 0$ and $x^2 + 3x + 3 = 0$ have one common root:
 (A) 2 (B) 4 (C) 6 (D) 8
32. Choose the correct answer.
 The real value of α for which the expression $\frac{1-i\sin \alpha}{1+2i\sin \alpha}$ is purely real is:
 (A) $(n+1)\frac{\pi}{2}$ (B) $(2n+1)\frac{\pi}{2}$
 (C) $n\pi$ (D) None of these, where $n \in \mathbb{N}$
33. If $z_1 = 2 + 3i$ and $z_2 = 5 + 2i$, then find sum of two complex numbers:
 (A) $4 + 8i$ (B) $3 - i$ (C) $7 + 5i$ (D) $7 - 5i$
34. Choose the correct answer.
 The real value of θ for which the expression $\frac{1+i\cos \theta}{1-2i\cos \theta}$ is a real number is:
 (A) $n\pi + \frac{\pi}{4}$ (B) $n\pi + (-1)^n \frac{\pi}{4}$ (C) $2n\pi \pm \frac{\pi}{2}$ (D) None of these.

35. If $z = \left(\frac{1+i}{1-i}\right)$, then z^4 equals:
 (A) 1 (B) -1 (C) 0 (D) none of these.
36. What is the number of solution(s) of the equation $|\sqrt{x-2}| + \sqrt{x(\sqrt{x-4})} + 2 = 0$
 (A) 2 (B) 4
 (C) No solution (D) Infinitely many solutions
37. $\frac{1+2i+3i^2}{1-2i+3i^2}$ equals:
 (A) i (B) -1 (C) -i (D) 4
38. The value of $(1+i)^4 + (1-i)^4$ is:
 (A) 8 (B) 4 (C) -8 (D) -4
39. Choose the correct answer.
 A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta (\alpha, \beta \in \mathbb{R})$ if $\alpha^2 + \beta^2 =$
 (A) 1 (B) -1 (C) 2 (D) -2
40. Solve $\sqrt{3x^2} + x + \sqrt{3} = 0$
 (A) $\frac{-1 \pm i\sqrt{11}}{6\sqrt{3}}$ (B) $\frac{1 \pm i\sqrt{11}}{6\sqrt{3}}$ (C) $\frac{1 \pm \sqrt{11}}{6\sqrt{3}}$ (D) $\frac{-1 \pm \sqrt{11}}{6\sqrt{3}}$
41. If $\frac{1+7i}{(2-i)^2}$, then:
 (A) $|z| = 2$ (B) $|z| = \frac{1}{2}$ (C) $\text{amp}(z) = \frac{\pi}{4}$ (D) $\text{amp}(z) = \frac{3\pi}{4}$
42. If α, β are the roots of the equation $x^2 - p(x+1) - c = 0$, then $(\alpha+1)(\beta+1) =$
 (A) c (B) c - 1 (C) 1 - c (D) None of these.
43. If $(x+iy)^{\frac{1}{3}} = a+ib$, then $\frac{x}{a} + \frac{y}{b} =$
 (A) 0 (B) 1 (C) -1 (D) none of these
44. If $\alpha\beta$ are the roots of the equation $x^2 + px + q = 0$ then $-\frac{1}{\alpha} + \frac{1}{\beta}$ are the roots of the equation:
 (A) $x^2 - px + q = 0$ (B) $x^2 + px + q = 0$ (C) $qx^2 + px + 1 = 0$ (D) $qx^2 - px + 1 = 0$
45. The number of roots of the equation $\frac{(x+2)(x-5)}{(x+3)(x+6)} = \frac{x-2}{x+4}$ is:
 (A) 0 (B) 1 (C) 2 (D) 3
46. If α, β are the roots of the equation $x^2 + px + 1 = 0$; γ, δ the roots of the equation $x^2 + qx + 1 = 0$, then $(\alpha - \gamma)(\alpha + \delta)(\beta - \delta)(\beta + \delta) =$
 (A) $q^2 - p^2$ (B) $p^2 - q^2$ (C) $p^2 + q^2$ (D) None of these.
47. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer, is:

(A) 16

(B) 8

(C) 4

(D) 2

48. Solve $2x^2 + \sqrt{2x+2} = 0$

(A) $\frac{-1 \pm i\sqrt{7}}{2\sqrt{2}}$

(B) $\frac{1 \pm i\sqrt{7}}{2\sqrt{2}}$

(C) $\frac{1 \pm \sqrt{7}}{2\sqrt{2}}$

(D) $\frac{-1 \pm \sqrt{7}}{2\sqrt{2}}$

49. The value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$ is

(A) -1

(B) -2

(C) -3

(D) -4

50. The value of $\frac{(i^5 + i^6 + i^7 + i^8 + i^9)}{(1+i)}$ is:

(A) $\frac{1}{2}(1+i)$

(B) $\frac{1}{2}(1-i)$

(C) 1

(D) $\frac{1}{2}$

51. If $z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{6}$, then

(A) $|z| = 1, \arg(z) = \frac{\pi}{4}$

(B) $|z| = 1, \arg(z) = \frac{\pi}{6}$

(C) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \frac{5\pi}{24}$

(D) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \tan^{-1} \frac{1}{\sqrt{2}}$

52. The number of real solutions of $|2x - x^2 - 3| = 1$ is:

(A) 0

(B) 2

(C) 3

(D) 4

53. If α and β are imaginary cube roots of unity, then the value of $\alpha^4 + \beta^{28} + \frac{1}{\alpha\beta}$ is:

(A) 1

(B) -1

(C) 0

(D) None of these

* Given section consists of questions of 2 marks each.

[54]

54. Express the complex number $(-2 - \frac{1}{3}i)^3$ in the form of $a + ib$.

55. Find the multiplicative inverse of the complex numbers $= \sqrt{5} + 3i$

56. Express the following in the form of $a + ib$.

$$\frac{(3 + \sqrt{5}i)(3 - \sqrt{5}i)}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - \sqrt{2}i)}$$

57. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

58. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$.

59. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$ then show that

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

60. If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of m .

61. Express $(5 - 3i)^3$ in the form $a + ib$.

62. Represent the complex number $z = 1 + i\sqrt{3}$ in the polar form.

63. Convert the complex number $\frac{-16}{1 + \sqrt{3}i}$ into polar form.

64. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

65. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

66. Evaluate the following:

$$i^{528}$$

67. Find the square root of the following complex numbers:

$$4i$$

68. Evaluate the following:

$$i^{30} + i^{40} + i^{60}$$

69. Evaluate the following:

$$i^{49} + i^{68} + i^{89} + i^{110}$$

70. Express the following complex numbers in the standard form $a + ib$:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right)$$

71. If z_1, z_2, z_3 are complex numbers such that

$$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1, \text{ then find the value of } |z_1 + z_2 + z_3|.$$

72. Express the following complex numbers in the standard form $a + ib$:

$$\frac{(1-i)^3}{1-i^3}$$

73. Find the number of solutions of $z^2 + |z|^2 = 0$.

74. Show the following quadratic equation by factorization method:

$$27x^2 - 10x + 1 = 0$$

75. Show the following quadratic equation:

$$x^2 - (2 + i)x - (1 - 7i) = 0$$

76. Show the following quadratic equation:

$$x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$$

77. Show the following quadratic equation:

$$(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$$

78. Show the following quadratic equation by factorization method:

$$x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$$

79. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find (x, y) .

80. Find the value of the following :

$$(1 + i)^8 + (1 - i)^8$$

*** Given section consists of questions of 3 marks each.**

[48]

81. Evaluate $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

82. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right)$ to the standard form.
83. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$
84. Convert in the polar form: $\frac{1+7i}{(2-i)^2}$
85. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.
86. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$
87. If α and β are different complex numbers with $|\beta| = 1$ then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$
88. Express $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ in the form of $a + ib$.
89. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$
90. Write the complex number $z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ in the polar form.
91. $(1+i)^6 + (1-i)^3$
92. Find the values of the following expressions:
 $i^{30} + i^{80} + i^{120}$
93. Show the following quadratic equation by factorization method:
 $\sqrt{2}x^2 + x + \sqrt{2} = 0$
94. Show the following quadratic equation:
 $ix^2 - x + 12i = 0$
95. If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of $x + y$.
96. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, what is the value of $x^2 + y^2$?

* Given section consists of questions of 5 marks each.

[55]

97. Express the following complex numbers in the form $r(\cos \theta + i \sin \theta)$:
 $1 - \sin \alpha + i \cos \alpha$
98. Find the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is real.
99. Evaluate the following:
 $2x^3 + 2x^2 - 7x + 72$, when $x = \frac{3-5i}{2}$
100. $x^4 + 4x^3 + 6x^2 + 4x + 9$, when $x = -1 + i\sqrt{2}$
101. Express the following complex numbers in the form $r(\cos \theta + i \sin \theta)$:
 $\tan \alpha - i$
102. $2x^4 + 5x^3 + 7x^2 - x + 41$, when $x = -2 - \sqrt{3}i$

103. For a positive integer n , find the value of $(1-i)^n \left(1 - \frac{1}{i}\right)^i$.
104. Express the following complex numbers in the form $r(\cos \theta + i \sin \theta)$:
 $1 + i \tan \alpha$
105. Evaluate the following:
 $x^4 - 4x^3 + 4x^2 + 8x + 44$, when $x = 3 + 2i$
106. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, find (a, b) .
107. If $a = \cos \theta + i \sin \theta$, find the value of $\left(\frac{1+a}{1-a}\right)$

*** Case study based questions**

[8]

108. A complex number z is pure real if and only if $\bar{z} = z$ and is pure imaginary if and only if $\bar{z} = -z$.

Based on the above information, answer the following questions.

(i) If $(1+i)z = (1-i)\bar{z}$, then $-i\bar{z}$ is

- (a) $-\bar{z}$ (b) z (c) \bar{z} (d) z^{-1}

(ii) $\overline{Z_1 Z_2}$ is

- (a) $\bar{z}_1 \bar{z}_2$ (b) $\bar{z}_1 + \bar{z}_2$ (c) $\frac{z_1}{z_2}$ (d) $\frac{1}{z_1 z_2}$

(iii) If x and y are real numbers and the complex number $\frac{(2+i)x-i}{4+i} + \frac{(1-i)y+2i}{4i}$ is pure real, the relation between x and y is

- (a) $8x - 17y = 16$ (b) $8x + 17y = 16$
(c) $17x - 8y = 16$ (d) $17x - 8y = -16$

(iv) If $z = \frac{3+2i \sin \theta}{1-2i \sin \theta}$ ($0 < \theta \leq \frac{\pi}{2}$) is pure imaginary, then θ is equal to

- (a) $\frac{\pi}{4}$ (b) $\frac{4}{6}$ (c) $\frac{6}{3}$ (d) $\frac{\pi}{12}$

(v) If z_1 and z_2 are complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$

- (a) $\frac{z_1}{z_2}$ is pure real (b) $\frac{z_1}{z_2}$ is pure imaginary
(c) z_1 is pure real (d) z_1 and z_2 are pure imaginary

109. We have, $i = \sqrt{-1}$. So, we can write the higher powers of i as follows

- (i) $i^2 = -1$
(ii) $i^3 = i^2 \cdot i = (-1) \cdot i = -i$
(iii) $i^4 = (i^2)^2 = (-1)^2 = 1$
(iv) $i^5 = i^{4+1} = i^4 \cdot i = 1 \cdot i = i$
(v) $i^6 = i^{4+2} = i^4 \cdot i^2 = 1 \cdot i^2 = -1$

In order to compute i^n for $n > 4$, write $i^n = i^{4q+r}$ for some $q, r \in \mathbb{N}$ and $0 \leq r \leq 3$.

Then, $i^n = i^{4q} \cdot i^r = (i^4)^q \cdot i^r = (1)^q \cdot i^r = i^r$.

In general, for any integer k , $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$ and $i^{4k+3} = -i$.

On the basis of above information, answer the following questions.

(i) The value of i^{37} is equal to

- (a) i (b) $-i$ (c) 1 (d) -1

(ii) The value of i^{-30} is equal to

- (a) i (b) 1 (c) -1 (d) $-i$

(iii) If $z = i^9 + i^{19}$, then z is equal to

- (a) $0 + 0i$ (b) $1 + 0i$ (c) $0 + i$ (d) $1 + 2i$

(iv) The value of $\left[i^{19} + \left(\frac{1}{i} \right)^{25} \right]^2$ is equal to

- (a) -4 (b) 4 (c) i (d) 1

(v) If $z = i^{-39}$, then simplest form of z is equal to

- (a) $1 + 0i$ (b) $0 + i$ (c) $0 + 0i$ (d) $1 + i$

----- "The only one who can tell you 'you can't win' is you, and you don't have to listen." -----