

* Choose the right answer from the given options. [1 Marks Each]

[85]

1. If $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$, then $\sin 3x + \sin 3y =$
 (A) $2 \sin 3x$ (B) 0 (C) 1 (D) None of these
2. The value of $\frac{2(\sin 2x + 2 \cos^2 x - 1)}{\cos x - \sin x - \cos 3x + \sin 3x}$ is:
 (A) $\cos x$ (B) $\sec x$ (C) $\operatorname{cosec} x$ (D) $\sin x$
3. Choose the correct answer.
 If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, then the value of $\theta + \phi$ is:
 (A) $\frac{\pi}{6}$ (B) π (C) 0 (D) $\frac{\pi}{4}$
4. If $\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ$ is equal to:
 (A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) 1
5. The value of $\sin^2 \frac{5\pi}{12} - \sin^2 \frac{\pi}{12}$ is:
 (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) 0
6. If $n = 1, 2, 3, \dots$, then $\cos \alpha \cos 4\alpha \dots \cos 2^{n-1}\alpha$ is equal to:
 (A) $\frac{\sin 2n\alpha}{2^n \sin \alpha}$ (B) $\frac{\sin 2^n \alpha}{2^n \sin 2^{n-1} \alpha}$ (C) $\frac{\sin 4^{n-1} \alpha}{4^{n-1} \sin \alpha}$ (D) $\frac{\sin 4^{n-1} \alpha}{4^{n-1} \sin \alpha}$
7. Choose the correct answer.
 The value of $\sin \frac{\pi}{10} \sin \frac{13\pi}{10}$ is:
 (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) 1
8. $2(1 - 2 \sin^2 7x) \sin 3x$ is equal to:
 (A) $\sin 17x - \sin 11x$
 (B) $\sin 11x - \sin 17x$
 (C) $\cos 17x - \cos 11x$
 (D) $\cos 17x + \cos 11x$
9. If α and β are acute angles satisfying $\cos 2\alpha = \frac{3 \cos 2\beta - 1}{3 - \cos 2\beta}$, then $\tan \alpha =$
 (A) $\sqrt{2} \tan \beta$ (B) $\frac{1}{\sqrt{2}} \tan \beta$ (C) $\sqrt{2} \cot \beta$ (D) $\frac{1}{\sqrt{2}} \cot \beta$
10. If D, G and R denote respectively the number of degrees, grades and radians in an angle, then:
 (A) $\frac{D}{100} = \frac{G}{90} = \frac{2R}{\pi}$ (B) $\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi}$
 (C) $\frac{D}{100} = \frac{G}{100} = \frac{2R}{\pi}$ (D) $\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi}$

11. If $x = r \sin \theta \cos \theta$, $y = r \sin \theta$ and $z = r \cos \theta$, then $x^2 + y^2 + z^2$ is independent of
 (A) θ, ϕ (B) r, θ (C) r, ϕ (D) r .
12. If $5 \sin \alpha = 3 \sin(\alpha + 2\beta) \neq 0$, then $\tan(\alpha + \beta)$ is equal to:
 (A) $2 \tan \beta$ (B) $3 \tan \beta$ (C) $4 \tan \beta$ (D) $6 \tan \beta$
13. If $\cot(\alpha + \beta) = 0$, then $\sin(\alpha + 2\beta)$ is equal to:
 (A) $\sin \alpha$ (B) $\cos 2\beta$ (C) $\cos \alpha$ (D) $\sin 2\alpha$
14. If $\sec x + \tan x = k$, $\cos x =$
 (A) $\frac{x^2+1}{2k}$ (B) $\frac{2k}{x^2+1}$ (C) $\frac{k}{x^2+1}$ (D) $\frac{k}{x^2-1}$
15. The value of $\cos^4 x + \sin^4 x - 6 \cos^2 x \sin^2 x$ is:
 (A) $\cos 2x$ (B) $\sin 2x$ (C) $\cos 4x$ (D) None of these
16. The radius of the circle whose arc of length 15π makes an angle of $\frac{3\pi}{4}$ radian at the centre is:
 (A) 10cm (B) 20cm (C) $11\frac{1}{4}$ cm (D) $22\frac{1}{2}$ cm
17. In the sides of a triangle are in the ratio $1 : \sqrt{3} : 2$, then the measure of its greatest angle is:
 (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$
18. Choose the correct answer.
 The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is:
 (A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2
19. If $\tan \alpha = \frac{1 - \cos \beta}{\sin \beta}$, then:
 (A) $\tan 3\alpha = \tan 2\beta$ (B) $\tan 2\alpha = \tan \beta$
 (C) $\tan 2\beta = \tan \alpha$ (D) None of these
20. If $\cos x + \sqrt{3} \sin x = 2$, then $x =$
 (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$
21. If $\alpha + \beta + \gamma = 2\pi$, then:
 (A) $\frac{\tan \alpha}{2} + \frac{\tan \beta}{2} + \frac{\tan \gamma}{2} = \frac{\tan \alpha}{2} \cdot \frac{\tan \beta}{2} \cdot \frac{\tan \gamma}{2}$
 (B) $\frac{\tan \alpha}{2} \cdot \frac{\tan \beta}{2} + \frac{\tan \beta}{2} \cdot \frac{\tan \gamma}{2} = \frac{\tan \gamma}{2} \cdot \frac{\tan \alpha}{2} = 1$
 (C) $\frac{\tan \alpha}{2} + \frac{\tan \beta}{2} + \frac{\tan \gamma}{2} = \frac{\tan \alpha}{2} \cdot \frac{\tan \beta}{2} \cdot \frac{\tan \gamma}{2}$
 (D) None of these
22. If the angles of a triangle are in A.P. then the measures of one of the angles in radians is:
 (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

23. If $\tan \frac{x}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{\alpha}{2}$ then $\cos \alpha =$
 (A) $1 - e \cos(\cos x + e)$ (B) $\frac{1+e \cos x}{\cos x - e}$
 (C) $\frac{1-e \cos x}{\cos x - e}$ (D) $\frac{\cos x - e}{1 - e \cos x}$
24. If $\tan \alpha = \frac{1}{7}, \tan \beta = -\frac{1}{3}$, then $\cos 2\alpha$ is equal to:
 (A) $\sin 2\beta$ (B) $\sin 4\beta$ (C) $\sin 3\beta$ (D) $\cos^2 \beta$
25. If $2 \tan \alpha = 3 \tan \beta$ then $\tan(\alpha - \beta) =$
 (A) $\frac{\sin 2\beta}{5 - \cos 2\beta}$ (B) $\frac{\cos 2\beta}{5 - \cos 2\beta}$ (C) $\frac{\sin 2\beta}{5 + \cos 2\beta}$ (D) None of these
26. If $\frac{\pi}{2} < x < \pi$, and if $\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}}$, is equal to:
 (A) $2 \sec x$ (B) $-2 \sec x$ (C) $\sec x$ (D) $-\sec x$
27. The angle between the minute and hour hands of a clock at 8 : 30 is:
 (A) 80° (B) 75° (C) 60° (D) 105°
28. If $\tan \alpha = \frac{x}{x+1}$ and $\tan \beta = \frac{1}{2x+1}$, then $\alpha + \beta$ is equal to
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$
29. Choose the correct answer.
 The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ is:
 (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) Not defined
30. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$ is:
 (A) $\frac{1}{\sqrt{2}}$ (B) 0 (C) 1 (D) -1
31. The value of $\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right)$ is:
 (A) 1 (B) 2 (C) 3 (D) None of these
32. The value of $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ$ is:
 (A) 7 (B) 8 (C) 9.5 (D) 10
33. If $\tan X = \frac{a}{b}$, then $b \cos 2x + a \sin 2x$ is equal to:
 (A) a (B) b (C) $\frac{a}{b}$ (D) $\frac{b}{a}$
34. The value of $\sin 78^\circ - \sin 66^\circ - \sin 42^\circ + \sin 6^\circ$ is
 (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) None of these
35. The value of $\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$ is
 (A) 0 (B) 1 (C) 2 (D) $\frac{3}{2}$
36. The value of $\left(\cot \frac{x}{2} - \tan \frac{x}{2}\right)^2 (1 - 2 \tan x \cot 2x)$ is:
 (A) 1 (B) 2 (C) 3 (D) 4

37. Choose the correct answer.

The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is equal to:

- (A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 2

38. If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then $\tan x =$

- (A) $\frac{21}{22}$ (B) $\frac{15}{16}$ (C) $\frac{44}{117}$ (D) $\frac{117}{43}$

39. If $\tan \theta_1 \tan \theta_2 = k$, then $\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)} =$

- (A) $\frac{1+k}{1-k}$ (B) $\frac{1-k}{1+k}$ (C) $\frac{k+1}{k-1}$ (D) $\frac{k-1}{k+1}$

40. If A lies in second quadrant $3 \tan A + 4 = 0$, then the value of $2 \cot A - 5 \cot A + \sin A$ is:

- (A) $-\frac{53}{10}$ (B) $\frac{23}{10}$ (C) $\frac{37}{10}$ (D) $\frac{7}{10}$

41. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha - \cos \beta = b$, then $\tan \frac{\alpha - \beta}{2} =$

- (A) $-\frac{a}{b}$ (B) $-\frac{b}{a}$ (C) $\sqrt{a^2 + b^2}$ (D) None of these

42. The value of $\frac{\sin 5\alpha - \sin \beta}{\cos 5\alpha + 2 \cos 4\alpha + \cos 3\alpha}$ is:

- (A) $\cot \frac{\alpha}{2}$ (B) $\cot \alpha$ (C) $\tan \frac{\alpha}{2}$ (D) None of these

43. The smallest value of x satisfying the equation $\sqrt{3}(\cot x + \tan x) = 4$ is:

- (A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

44. If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then $\tan x$ is equal to:

- (A) $\frac{21}{22}$ (B) $\frac{15}{16}$ (C) $\frac{44}{117}$ (D) $\frac{117}{44}$

45. What is the value of $\cot (-870^\circ)$?

- (A) 3 (B) $\frac{1}{\sqrt{3}}$ (C) $-\sqrt{3}$ (D) $-\frac{1}{\sqrt{3}}$

46. If $\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = a$, then $\tan^2\left(\frac{\pi}{4} + x\right) + \tan^2\left(\frac{\pi}{4} - x\right) =$

- (A) $a^2 + 1$ (B) $a^2 + 2$ (C) $a^2 - 2$ (D) none of these

47. If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A$ is equal to:

- (A) 110 (B) 191 (C) 80 (D) 194

48. The value of $\cos(36^\circ - A)\cos(36^\circ + A) + \cos(54^\circ + A)\cos(54^\circ - A)$ is:

- (A) $\sin 2A$ (B) $\cos 2A$ (C) $\cos 3A$ (D) $\sin 3A$

49. $8 \sin \frac{x}{8} \cos \frac{x}{2} \cos \frac{x}{4} \cos \frac{x}{8}$ is equal to:

- (A) $8 \cos x$ (B) $\cos x$ (C) $8 \sin x$ (D) $\sin x$

50. If x is an acute angle and $x = \frac{1}{\sqrt{7}}$, then the value of $\frac{\operatorname{cosec}^2 x - \sec^2 x}{\operatorname{cosec}^2 x + \sec^2 x}$ is:

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) $\frac{5}{4}$

51. $\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ =$

- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

52. A circular wire of radius 7cm is cut and bent again into an arc of a circle of radius 12cm. The angle subtended by the arc at the centre is:

- (A) 50° (B) 210° (C) 100° (D) 60°

53. The solution of the equation $\cos^2 q + \sin q + 1 = 0$, lies in the interval:

- (A) $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (C) $\left(-\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (D) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

54. $\frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ}$ is equal to:

- (A) $\tan 55^\circ$ (B) $\cot 55^\circ$ (C) $-\tan 35^\circ$ (D) $-\cot 35^\circ$

55. The value of $\cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right)$ is:

- (A) $\frac{1}{2}\cos^2 x$ (B) 0 (C) $-\frac{1}{2}\cos 2x$ (D) $\frac{1}{2}$

56. The value of $\frac{\cos 3x}{2\cos 2x - 1}$ is equal to:

- (A) $\cos x$ (B) $\sin x$ (C) $\tan x$ (D) None of these

57. Choose the correct answer.

If $\sin \theta + \cos \theta = 1$, then the value of $\sin 2\theta$ is equal to:

- (A) 1 (B) $\frac{1}{2}$ (C) 0 (D) -1

58. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha - \cos \beta = b$ then $\tan \frac{\alpha - \beta}{2} =$

- (A) $-\frac{a}{b}$ (B) $-\frac{b}{a}$ (C) $\sqrt{a^2 + b^2}$ (D) None of these

59. The value of $\cos(36^\circ - A)\cos(36^\circ + A) + \cos(54^\circ - A)\cos(54^\circ + A)$ is:

- (A) $\cos 2A$ (B) $\sin 2A$ (C) $\cos A$ (D) 0

60. If $A = 2\sin^2 x - \cos 2x$, then A lies in the interval:

- (A) $[-1, 3]$ (B) $[1, 2]$ (C) $[-2, 4]$ (D) None of these

61. If $(2^n + 1)x = \pi$, then $2^n \cos x \cos 2x^2 x \cos^{n-1} x =$

- (A) -1 (B) 1 (C) $\frac{1}{2}$ (D) None of these

62. Let a, b be such that $\pi < \alpha - \beta < 3\pi$ D

If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and, $\cos \alpha + \cos \beta = -\frac{27}{65}$. then the value is $\alpha - \frac{\beta}{2}$

- (A) $\frac{-6}{65}$ (B) $\frac{3}{\sqrt{130}}$ (C) $\frac{6}{65}$ (D) $\frac{-3}{130}$

63. Choose the correct answer.

If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan \alpha)(1 + \tan \beta)$ is:

- (A) 1 (B) 2 (C) -2 (D) Not defined

64. Choose the correct answer.

If $\tan \alpha = \frac{m}{m+1}$, $\tan \beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to:

- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

65. If $\cos A = m \cos B$, then $\cot \frac{A+B}{2} \cot \frac{B-A}{2} =$
 (A) $\frac{m-1}{m+1}$ (B) $\frac{m+2}{m-2}$ (C) $\frac{m+1}{m-1}$ (D) None of these
66. If $\tan \alpha = \frac{x}{x+1}$ and $\tan \beta = \frac{1}{2x+1}$, then $\alpha + \beta$ is equal to:
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$
67. At 3 : 40, the hour and minute hands of a clock are inclined at:
 (A) $\frac{2\pi^\circ}{3}$ (B) $\frac{7\pi^\circ}{12}$ (C) $\frac{13\pi^\circ}{18}$ (D) $\frac{13\pi^\circ}{4}$
68. If $\sin 2\theta + \sin 2\phi = \frac{1}{2}$ and $\cos 2\theta + \cos 2\phi = \frac{3}{2}$, then $\cos^2(\theta - \phi) =$
 (A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$
69. If $\sec x = x + \frac{1}{4x}$, then $\sec x + \tan x =$
 (A) $x, \frac{1}{x}$ (B) $2x, \frac{1}{2x}$ (C) $-2x, \frac{1}{2x}$ (D) $-\frac{1}{x}, x$
70. For all real values of x , $\cot x - 2 \cot$ is equal to:
 (A) $\tan 2x$ (B) $\tan x$ (C) $-\cot 3x$ (D) None of these
71. If $\tan x = t$ then $\tan 2x + \sec 2x$ is equal to:
 (A) $\frac{1+t}{1-t}$ (B) $\frac{1-t}{1+t}$ (C) $\frac{2t}{1-t}$ (D) $\frac{2t}{1+t}$
72. Choose the correct answer.
 The minimum value of $3 \cos x + 4 \sin x + 8$ is:
 (A) 5 (B) 9 (C) 7 (D) 3
73. A solution of the equation $\cos^2 x + \sin x + 1 = 0$, lies in the interval:
 (A) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (C) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (D) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
74. The value of $\cos^2 48^\circ - \sin^2 48^\circ$ is:
 (A) $\sqrt{5} + \frac{1}{8}$ (B) $\sqrt{5} - \frac{1}{8}$ (C) $\sqrt{5} + \frac{1}{5}$ (D) $\sqrt{5} + \frac{1}{2\sqrt{2}}$
75. If $\tan px - \tan qx = 0$, then the values of θ form a series in:
 (A) AP (B) GP (C) HP (D) None of these
76. If $\tan \alpha = \frac{1}{7}, \tan \beta = \frac{1}{3}$, then $\cos 2\alpha$ is equal to:
 a. $\sin 2\beta$
 b. $\sin 4\beta$
 c. $\sin 3\beta$
 d. $\cos 2\beta$
77. If $f(x) = \cos^2 x + \sec^2 x$, then:
 a. $f(x) < 1$
 b. $f(x) = 1$
 c. $2 < f(x) < 1$

d. $f(x) \geq 2$

[Hint: A.M \geq G.M.]

78. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$ is:

- a. $\frac{1}{\sqrt{2}}$
- b. 0
- c. 1
- d. -1

79. If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan \alpha)(1 + \tan \beta)$ is:

- a. 1
- b. 2
- c. -2
- d. Not defined

80. If $\tan \alpha = \frac{m}{m+1}$, $\tan \beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to:

- a. $\frac{\pi}{2}$
- b. $\frac{\pi}{3}$
- c. $\frac{\pi}{6}$
- d. $\frac{\pi}{4}$

81. The value of $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$ is:

- a. $\frac{1}{2}$
- b. 1
- c. $-\frac{1}{2}$
- d. $\frac{1}{8}$

82. If $\tan \theta = \frac{a}{b}$, then $b \cos 2\theta + a \sin 2\theta$ is equal to:

- a. a
- b. b
- c. $\frac{a}{b}$
- d. None

83. The value of $\cot\left(\frac{\pi}{4} + \theta\right) \cot\left(\frac{\pi}{4} - \theta\right)$ is:

- a. -1
- b. 0
- c. 1
- d. Not defined

84. The value of $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is:

- a. $2 \cos \theta$
- b. $2 \sin \theta$
- c. 1
- d. 0

85. The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is equal to:

- a. 1
- b. 0
- c. $\frac{1}{2}$
- d. 2

*** Given section consists of questions of 2 marks each.**

[4]

86. Find the value of $\tan 22^\circ 30'$.

87. If $\tan(A + B) = p, \tan(A - B) = q$, then show that $\tan 2A = \frac{p+q}{1-pq}$

[Hint: Use $2A = (A + B) + (A - B)$]

*** Given section consists of questions of 3 marks each.**

[36]

88. Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

89. Find $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ in the $\sin x = \frac{1}{4}$, x in quadrant II.

90. Solve: $2 \cos^2 x + 3 \sin x = 0$

91. Prove that: $\cos^2 x + \cos^2 \left(x + \frac{\pi}{3}\right) + \cos^2 \left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

92. Show that:

$$(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$$

93. Prove that:

$$\sin^2 42^\circ - \cos^2 78^\circ = \frac{\sqrt{15}+1}{8}$$

94. If $\cos \alpha + \cos \beta = \frac{1}{3}$ and $\sin \alpha + \sin \beta = \frac{1}{4}$, prove that $\cos \frac{\alpha-\beta}{2} = \pm \frac{5}{24}$

95. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha + \cos \beta = b$ prove that

$$\cos(\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$$

96. Prove that:

$$\cos 36^\circ \cos 42^\circ \cos 60^\circ \cos 78^\circ = \frac{1}{16}$$

97. If $a \cos 2x + b \sin 2x = c$ has α and β as its roots, then prove that,

$$\tan \alpha \tan \beta = \frac{c-a}{c+a}$$

98. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$

99. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

[Hint: Use Componendo and Dividendo]

*** Given section consists of questions of 5 marks each.**

[505]

100. In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor arc of the chord.

101. If $\alpha + \beta = \frac{\pi}{2}$, show that the maximum value of $\cos \alpha \cos \beta$ is $\frac{1}{2}$.

102. If θ lies in the first quadrant and $\cos \theta = \frac{8}{17}$, then find the value of $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.
103. Prove that $(2\sqrt{3} + 3)\sin x + 2\sqrt{3}\cos x$ lies between $-(2\sqrt{3} + \sqrt{15})$ and $(2\sqrt{3} + \sqrt{15})$.
104. Prove that:
- $$\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \cos \frac{1}{128}$$
105. Prove that:
- $$\begin{aligned} & \sin \alpha + \sin \beta + \sin \gamma - \sin(\alpha + \beta + \gamma) \\ &= 4 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\beta + \gamma}{2} \right) \sin \left(\frac{\gamma + \alpha}{2} \right) \end{aligned}$$
106. If $\cos(\alpha + \beta) \sin(\gamma + \delta) = \cos(\alpha - \beta) \sin(\gamma - \delta)$,
prove that $\cot \alpha \cot \beta \cot \gamma = \cot \delta$
107. If $T_n = \sin^n x + \cos^n x$, Prove that
- $$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$
108. $4 \left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} \right) = (a + b + c)^2$
109. If are two different values of X lying between 0 and which satisfy the equation $6 \cos x + 8 \sin x = 9$ find the value of $\sin(\alpha + \beta)$.
110. A person observes the angle of elevation of the peak of a hill from a station to be α . He walks c metres along a slope inclined at an angle β and finds the angle of elevation of the peak of the hill to be γ . Show that the height of the peak above the ground is $\frac{c \sin \alpha \sin(\gamma - \beta)}{(\sin \gamma - \alpha)}$.
111. If the sides a, b, c of a $\triangle ABC$ are in H.P., prove that $\sin^2 \frac{A}{2}, \sin^2 \frac{B}{2}, \sin^2 \frac{C}{2}$ are in H.P.
112. Prove that:
- $$\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ - \cos 140^\circ \cos 200^\circ = -\frac{3}{4}$$
113. Prove that: $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A - B) = \sin^2(A + B)$
114. If $\cos x = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$ prove that $\tan \frac{x}{2} = \pm \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$
115. Reduce each of the following expressions to the sine and cosine of a single expression:
- $$24 \cos x + 7 \sin x$$
116. The angles of a triangle are in A.P. such that the greatest is 5 times the least. Find the angles in radians.
117. Prove the following identities:
- $$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

118. The number of sides of two regular polygons are as 5 : 4 and the difference between their angles is 9° . Find the number of sides of the polygons.

119. Prove that:

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$$

120. Solve the following equations:

$$\tan x + \tan 2x + \tan 3x = 0$$

121. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that

$$\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$$

[Hint: Express $\cos(\alpha - \beta) = \cos((\theta + \alpha) - (\theta + \beta))$]

122. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$

[Hint: Express $\tan 2\alpha$ as $\tan(\alpha + \beta + \alpha - \beta)$]

123. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha + \cos \beta = b$, show that

$$\cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

124. If $\sin \alpha = \frac{4}{5}$ and $\cos \beta = \frac{5}{13}$, prove that $\cos \frac{\alpha - \beta}{2} = \frac{8}{\sqrt{65}}$

125. Show that:

$$3(\sin x - \cos x) + 6(\sin x + \cos)^2 + 4(\sin^6 x + \cos^6 x) = 13$$

126. If $\cos A + \sin B = m$ and $\sin A + \cos B = n$, prove that $2 \sin(A + B) = m^2 + n^2 - 2$.

127. The angle of a quadrilateral are in A.P. and the greatest angle is 120° . Express the angles in radians.

128. If $\sin x + \sin \cos x = m$, then prove that $\sin^6 x + \cos^6 x = \frac{4 - 3(m^2 - 1)^2}{4}$, where $m^2 \leq 2$

129. Prove that:

$$\cos^3 2x + 3 \cos 2x = 4(\cos^6 x - \sin^6 x)$$

130. Prove that:

$$\cos 3A + \cos 5A + \cos 7A + \cos 15A = 4 \cos 4A \cos 5A \cos 6A$$

131. Prove the following identities:

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x} = \frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

132. If $\sin 2A = \lambda \sin 2B$, prove that:

$$\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$

133. If $2 \tan \alpha = 3 \tan \beta$, prove that $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$

134. Solve the following equation:

$$\sin^2 x - \cos x = \frac{1}{4}$$

135. If $m \sin \theta = n \sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$

[Hint: Express $\frac{\sin(\theta+2\alpha)}{\sin \theta} = \frac{m}{n}$ and apply componendo and dividendo]

136. If $2 \tan \frac{\alpha}{2} = \tan \frac{\beta}{2}$, prove that $\cos \alpha = \frac{3+5 \cos \beta}{5+3 \cos \beta}$

137. $\sin^3 x + \sin^3 \left(\frac{2\pi}{3} + x \right) + \sin^3 \left(\frac{4\pi}{3} + x \right) = -\frac{3}{4} \sin 3x$

138. If $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$, then prove that $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$ is also equal to y .

[Hint: Express $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} \cdot \frac{1 + \cos \alpha + \sin \alpha}{1 + \cos \alpha + \sin \alpha}$]

139. Solve the following equations:

$$\sqrt{3} \cos x + \sin x = 1$$

140. Prove the following identities:

$$(1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 = \sec^2 \alpha \sec^2 \beta$$

141. Find the value of the expression

$$3[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha)] - 2[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi - \alpha)]$$

142. If $\cos \theta + \tan \theta = 2 \operatorname{cosec} \theta$, then find the general value of θ .

143. If $y \sin \phi = x \sin (2\theta + \phi)$, prove that $(x+y) \cot (\theta + \phi) = (y-x) \cot \theta$

144. If $\cos (\theta + \phi) = m \cos (\theta - \phi)$, then prove that $\tan \theta = \frac{1-m}{1+m} \cot \phi$.

[Hint: Express $\frac{\cos (\theta + \phi)}{\cos (\theta - \phi)} = \frac{m}{1}$ and apply Componendo and Dividendo]

145. Prove that:

$$\sin 3A + \sin 2A - \sin A = 4 \sin A \cos \frac{A}{2} \cos \frac{3A}{2}$$

146. If $\sec (x + \alpha) + \sec (x - \alpha) = 2 \sec x$, prove that $\cos x = \pm \sqrt{2} \cos \frac{\alpha}{2}$

147. $\frac{\sqrt{\sin A} - \sqrt{\sin B}}{\sqrt{\sin A} + \sqrt{\sin B}} = \frac{a + b - 2\sqrt{ab}}{a - b}$

148. Prove that:

$$\frac{1}{\sin (x-b) \sin (x-b)} = \frac{\cot (x-b) - \cot (x-b)}{\sin (a-b)}$$

149. Solve the following equations:

$$\cos x \cos 2x \cos 3x = \frac{1}{4}$$

150. If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then show that $\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$.

[Hint: Express $\tan \theta = \tan \left(\alpha - \frac{\pi}{4} \right) \theta = \alpha - \frac{\pi}{4}$]

151. Prove that:

$$\cos 40^\circ \cos 80^\circ \cos 160^\circ = -\frac{1}{8}$$

152. Prove that:

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$$

153. Solve the following equations:

$$\sin x + \cos x = 1$$

154. If $\tan x = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then show that $\sin \alpha + \cos \alpha = \sqrt{2} \cos x$.
155. In any $\triangle ABC$, if a^2, b^2, c^2 are in A.P., prove that $\cot A, \cot B$ and $\cot C$ are also in A.P.
156. If $a \cos 2\theta + b \sin 2\theta = c$ has α and β as its roots, then prove that $\tan \alpha + \tan \beta = \frac{2b}{a+b}$.
- [Hint: Use the identities $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$ and $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$].
157. Prove the following identities:
- $$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x \cos^3 x} = \sin x$$
158. A railway train is travelling on a circular curve of 1500 metres radius at the rate of 66km/ hr. Through what angle has it turned in 10 seconds?
159. If $m \sin \theta = n \sin(\theta + 2\alpha)$, prove that $\tan(\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$.
160. Prove that:
- $$\cot \frac{\pi}{8} = \sqrt{2} + 1$$
161. Prove that:
- $$\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ = 1$$
162. Prove that:
- $$\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = \frac{1}{16}$$
163. Prove that:
- $$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$$
164. Show that:
- $$\sin 25^\circ \cos 115^\circ = \frac{1}{2} (\sin 140^\circ - 1)$$
165. If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$.
- [Hint: $(\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$]
166. Prove that
- $$\cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} = \frac{1}{64}$$
167. Prove that
- $$\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = \frac{1}{16}$$
168. If $\cos x - \sin x = a^3, \sec x - \cos x = b^3$, then proved that $a^2 b^2 (a^2 + b^2) = 1$.
169. Prove that:
- $$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$$
170. Prove that:
- $$\frac{1}{\cos(x-b) \cos(x-b)} = \frac{\tan(x-a) - \tan(x-b)}{\sin(a-b)}$$
171. If $x = \sec \phi - \tan \phi$ and $y = \operatorname{cosec} \phi + \cot \phi$ then show that $xy + x - y + 1 = 0$

[Hint: Find $xy + 1$ and then show that $x - y = -(xy + 1)$]

172. Prove that:

$$\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$$

173. Find the distance from the eye at which a coin of 2cm diameter should be held so as to conceal the full moon whose angular diameter is $31'$.

174. Prove that:

$$\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$$

175. Prove that:

$$\sin A + \sin 2A + \sin 4A + \sin 5A = 4 \cos \frac{A}{2} \cos \frac{2A}{2} \cos 4A$$

176. Prove that:

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \frac{x-y}{2}$$

177. If $\sin x + \sin \cos x = m$, then prove that $\sin^6 x + \cos^6 x = \frac{4-3(m^2-1)^2}{4}$, where $m^2 \leq 2$

178. Prove the following identities:

$$(1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 = \sec^2 \alpha \sec^2 \beta$$

179. If $a = \sec x - \tan x$ and $b = \operatorname{cosec} x + \cot x$, then show that $ab + a - b + 1 = 0$.

180. If $\sin \alpha = \frac{4}{5}$ and $\cos \beta = \frac{5}{13}$, prove that $\cos \frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$

181. Show that:

$$3(\sin x - \cos x) + 6(\sin x + \cos)^2 + 4(\sin^6 x + \cos^6 x) = 13$$

182. If $\sin \alpha + \sin \beta = a$ and $\cos \alpha + \cos \beta = b$ prove that

$$\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

183. If $2 \tan \alpha = 3 \tan \beta$, prove that $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$

184. Prove that:

$$\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{16}$$

185. If $\sec(x + \alpha) + \sec(x - \alpha) = 2 \sec x$, prove that $\cos x = \pm \sqrt{2} \cos \frac{\alpha}{2}$

186. Prove that:

$$\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$$

187. Prove that:

$$\tan 82\frac{1}{2}^\circ = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1) = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$$

188. Prove that:

$$\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = \frac{1}{16}$$

189. Prove that

$$\cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} = \frac{1}{64}$$

190. Prove that:

$$\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$$

191. If θ lies in the first quadrant and $\cos \theta = \frac{8}{17}$, then find the value of $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.

192. If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, then prove that $m^2 - n^2 = 4 \sin \theta \tan \theta$

193. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$

[Hint: Express $\cos(\alpha - \beta) = \cos((\theta + \alpha) - (\theta + \beta))$]

194. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$

[Hint: Express $\tan 2\alpha$ as $\tan(\alpha + \beta + \alpha - \beta)$]

195. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$

196. If $m \sin \theta = n \sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$

[Hint: Express $\frac{\sin(\theta+2\alpha)}{\sin \theta} = \frac{m}{n}$ and apply componendo and dividendo]

197. If $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$, then prove that $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$ is also equal to y .

[Hint: Express $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} \cdot \frac{1 + \cos \alpha + \sin \alpha}{1 + \cos \alpha + \sin \alpha}$]

198. Find the value of the expression

$$3[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)] - 2[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)]$$

199. If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then prove that $\tan \theta = \frac{1-m}{1+m} \cot \phi$.

[Hint: Express $\frac{\cos(\theta+\phi)}{\cos(\theta-\phi)} = \frac{m}{1}$ and apply Componendo and Dividendo]

200. If $a \cos 2\theta + b \sin 2\theta = c$ has α and β as its roots, then prove that $\tan \alpha + \tan \beta = \frac{2b}{a+b}$.

[Hint: Use the identities $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$ and $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$].

----- the journey of thousands miles begins with a single step -----