KD EDUCATION ACADEMY [9582701166]

Time: 5 Hour

(A) $\frac{5}{3}$

(B) $\frac{5}{2}$

STD 11 Science Physics

Total Marks: 280

kd 90+ ch-12 kinetic theory of gases

*	Choose The Right Ar	swer From The	e Given Options.[1 Marks	Each]	[59
1.	The internal energy of	2 moles of a mo	no atomic gas is:		
	(A) $\frac{3}{2}$ RT	(B) 3RT	(C) 2RT	(D) 5RT	
	Ans.: b. $3RT$ Explanation:				
	Internal energy, ¹	$U = \left(\frac{3}{2}k_{\mathrm{B}}T\right)2$	N _A)	
	$=3(\mathrm{k_{B} imes N_{A}})\mathrm{T}$				
2.	, =,	ygen molecules	at 27°C is 318m/s. the r.m	s velocity of hydrogen	
	(A) 1470m/s	(B) 1603m/s	(C) 1869m/ s	(D) 2240m/ s	
	Ans.: a. 1470m/ s				
3.	average K.E. at 227°C	?	ecules of a gas at 27°C is 9		
	(A) 5 10-20J	(B) 10 10-20J	(C) 15 10-20J	(D) 20 10-20J	
	Ans.: c. 15 10-20J		V		
4.	Cooking gas containers of the gas molecules i (A) Remains the same		rry moving with uniform sp	eed. The temperature	
	(B) Decrease for some	e and increase fo	or others		
	(C) Decrease				
	(D) Increase				
	Ans.: a. Remains the s	ame			
5.	In equilibrium, the tota an energy equal to 1/		ly distributed in all possible led as:	e energy modes having	
	(A) Boyle's law		(B) Charle's law		
	(C) Law of equipartitio	n of energy	(D) None		
	Ans.:				
_		rtition of energy			
6.	What is the ratio of spe	ecific heats for a	monatomic gas?		

(C) $\frac{7}{5}$

(D) $\frac{9}{5}$

Ans.:

a.
$$\frac{5}{3}$$

Explanation:

The value of Cv for a monatomic gas is $\frac{3}{2}$ R and C_P is $\frac{5}{2}$ R.

Thus the value of γ is:

$$rac{\mathrm{C_p}}{\mathrm{C_V}} = rac{5}{3}$$

7. Latent heat of ice is:

- (A) Less than external latent heat of fusion.
- (C) Twice the external latent heat of fusion.
- (B) More than external latent heat of fusion.
- (D) Equal to external latent heat of fusion.

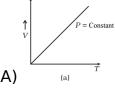
Ans.:

Equal to external latent heat of fusion.

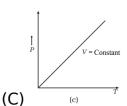
Explanation:

Latent heat of ice is more than external latent heat of fusion.

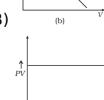
Which of the following diagrams (figure) depicts ideal gas behaviour? 8.





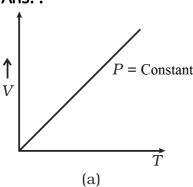


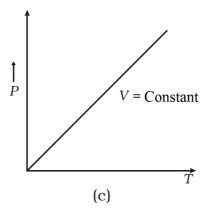




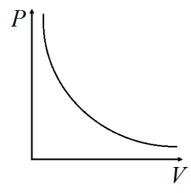








Explanation:



For ideal gas behaviour,

PV = nRT

1. When pressure, P = constant.

From (i) Volume V \propto Temperature T

Graph of V versus T will be straight line.

2. When T = constant.

So, graph of P versus V will be a rectangular hyperbola.

Hence this graph is wrong.

The correct graph is shown below:

3. When V = constant.

From (i)
$$P \propto T$$

So, graph is a straight line passing throught the origin.

4. From (i) $PV \propto T$

$$\Rightarrow \frac{PV}{T} = constant$$

So, graph of PV versus T will be a straight line parallel to the temperature axis (x-axis).

i.e., slope of this graph will be zero.

So, (d) is not correct.

- 9. Temperature remaining constant, the pressure of gas is decreased by 20%. The percentage change in volume:
 - (A) Increases by 20%

(B) Decreases by 20%

(C) Increases by 25%

(D) Decreases by 25%

Ans.:

	c. Increase	s by 25%					
10.	What is the num	What is the number of molecules in 2.24L of SO ₂ at STP?					
	(A) 6.023×10^{23}		(B) 6.023 ×				
	(C) 6.023×10^{20})	(D) 6.023 ×	10 ²¹			
	Ans.:	22					
	b. 6.023 ×						
	Explanation						
			2.4L of any gas at STP is 6	5.023×10^{23} .			
	So, in 2.24L = 6.023×1	there will be $\frac{6.023 imes}{10}$.	10-0				
11.	What is meant b	y mean free path?					
			cule travels without collid	ding.			
	(B) Average dista	ance between 2 mol	ecules.	V			
	(C) Average dista	ance travelled by a r	molecule before colliding	with a wall of the container.			
	(D) Sum of distar	nce travelled by all r	molecules.				
	Ans.:						
	a. It is the a	iverage distance a n	nolecule travels without	colliding.			
	Explanatio						
		oath of a molecule is pefore colliding.	s defined as the average	distance travelled by			
12.	A man is climbir	ng up a spiral type of	f staircase. His degrees c	of freedom are:			
	(A) 1	(B) 2	(C) 3	(D) More than 3			
	Ans.:						
	c. 3						
13.	The value of C_V		(0)	2 _			
	(A) 3R	(B) 2	(C) 4R	(D) $\frac{3}{2}$ R			
	Ans.:						
	a. 3R						
14.		ar specific heats of	a solid and R is universal				
	(A) $C_P - C_V = R$	221	(B) C _P - C _V , =				
	(C) C_P - C_V is neg	jative	(D) (C _P - C _V)	<< R			
	Ans.:						
	d. (C _P - C _V)						
	Explanation of second s	on: olides, C _P = C _V					
	∴ C _P - C _V <	•					
15.			as at temperature 120%;	s v. At what temperature			
٠,	will the velocity I	_	as at temperature 120K i	5 v. At what temperature			
	(A) 120K	(B) 240K	(C) 480K	(D) 1120K			

	Ans.:	480K					
16.	Law of (A) Pre	equipartition	of energy is used ic heats of gases.			ct the specifi er (a) nor (b	ic heats of solids.
		.ii (a) aiiu (b).			(D) Neith	er (a) nor (b)).
	Ans.:	Both (a) and	(h)				
		xplanation:	(2).				
		_	ition of energy is	used to p	redict the	specific heat	t of gases and solids.
17.	We too	ok two separat	e gases with the s	same num	nber densi	ties for both	. If the ratio of the
		ters of their m	olecules is 4:1, t			ean free pat	
((A) 1 : 4		(B) 4:1		(C) 2 : 1		(D) 1:16
	Ans.:	1 10					
10	d.	1:16			2	9	
18.	•	s have: ed shape and v	/olume		(B) Varia) ble shape ar	nd volume
		-	ut fixed volume.				ariable volume.
	Ans.:	•				•	
	C.	Variable shap	oe but fixed volur	ne. /			
19.	The de	eviation of gas	es from the beha	viour of ic	leal gas is	due to:	
	(A) Attı	raction of mole	ecules		(B) Absol	lute scale of	temp
	(C) Cov	valent bonding	of molecules		(D) Colou	ırless molec	ules
	Ans.:		~				
	a.	Attraction of					
20.				,			nergy is 50J. If the
	(A) 4.2		s 35J, the mechar	iicai equi	(B) 1.26J		
	(C) 4.9				(D) 2.1J/		
	Ans.:						
	a.	4.25J/ cal					
	E	xplanation:	441				
		16	st law of thermod	ynamics .	$\mathbf{J}\triangle\mathbf{Q}=\triangle$	$\Delta \mathbf{W} + \Delta \mathbf{U},$	where J is the
		\times 20 = 50 + 3	ivalent of heat.				
	_	× 20 = 30 + 3. = 4.25J/ cal	,				
21.		-	22.4L of CO ₂ at S	TP?			
	(A) 1g	5 GIC 111033 01 /	(B) 44g		(C) 44kg		(D) 1kg
	Ans.:		-		_		-
	b.	44g					
	E	xplanation:					

22.4L of a gas at STP has a weight equal to its molar mass. So, the weight of CO_2 will be 12 + 16 + 16 = 44g.

- 22. How many degrees of freedom are there in a monatomic gas?
 - (A) 1

(B) 2

(C) 3

(D) 0

Ans.:

c. 3

Explanation:

A monatomic gas has 3 translational degrees of freedom.

- 23. The molar specific heat at constant pressure of an ideal gas is $\left(\frac{7}{2}R\right)$. The ratio of specific heat at constant pressure to that at constant volume is:
 - (A) $\frac{9}{7}$

(B) $\frac{7}{5}$

(C) $\frac{5}{7}$

(D) $\frac{8}{7}$

Ans.:

b. $\frac{7}{5}$

Explanation:

$$C_P = \frac{7}{2}R$$

$$C_{V} = C_{P} - R = \frac{7}{2}R - R = \frac{5}{2}R$$

$$\gamma = rac{\mathrm{C_P}}{\mathrm{C_V}} = rac{rac{7}{2}\mathrm{P}}{rac{5}{2}\mathrm{R}} = rac{7}{5}$$

- 24. According to the kinetic theory of gases, the temperature of a gas is a measure of average:
 - (A) Velocities of its molecules.

- (B) Linear momenta of its molecules.
- (C) Kinetic energies of its molecules.
- (D) Angular momenta of its molecules.

Ans.:

- c. Kinetic energies of its molecules.
- 25. The rms speed of oxygen at room temperature is about 500m/ s. The rms speed of hydrogen at the same temperature is about:
 - (A) 125ms⁻¹
- (B) 2000ms⁻¹
- (C) 8000ms⁻¹
- (D) 31ms⁻¹

Ans.:

b. 2000ms⁻¹

Explanation:

Given,

Molecular mass of hydrogen, $M_H = 2$

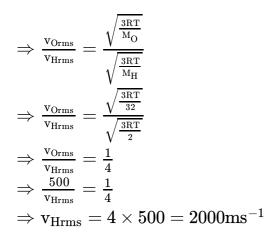
Molecular mass of oxygen, $M_0 = 32$

RMS speed is given by,

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \sqrt{rac{3 ext{RT}}{ ext{M}_{ ext{O}}}} = 500$$

Now,



- 26. One mole of ideal gas required 207J heat to rise the temperature by 10° K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10° K the heat required is (R = 8/ 3J/ mole $^{\circ}$ K)
 - (A) 1987I
- (B) 29I

- (C) 215.3J
- (D) 124J

Ans.:

d. 124J

- 27. K.E. of gas molecules is zero at:
 - (A) 0°C

(B) 273°t

(C) -273°C

(D) None of the above

Ans.:

c. -273°C

- 28. A rigid container of negligible heat capacity contains one mole of an ideal gas. The temperature of the gas increases by 1°C if 3.0cal of heat is added to it. The gas may be:
 - (A) Helium.

(B) Argon.

(C) Oxygen.

(D) Carbon dioxide.

Ans.:

c. Helium.

d. Argon.

Explanation:

The temperature of one mole of a gas kept in a container of fixed volume is increased by 1 degree Celsius if 3 calories, i.e. 12.54J of heat is added to it. So, its molar heat capacity, $\text{C}_{\text{V}} = 12.54\text{J-JK}^{-1}\text{mol}^{-1}$, as molar heat capacity at fixed volume is the heat supplied to a mole of gas to increase its temperature by a degree. For a monatomic gas, $C_{\text{v}} \simeq \frac{3}{2}R = 1.5 \times 8.314 = 12.54\text{JK}^{-1}\text{mol}^{-1}$.

Among the given gases, only helium and argon are inert and hence, monoatomic. Therefore, the gas may be helium or argon.

- 29. The total energy of water molecule is given by:
 - (A) 8RT

(B) 3RT

(C) 9RT

(D) RT

Ans.:

c. 9RT

- 30. Moon has no atmosphere because:
 - (A) It is far away form the surface of the earth.

(B) Its surfa	ace temperature is 10°C.		
(C) The r.m surface.	n.s. velocity of all the gas mole	cules is more then the ϵ	escape velocity of the moons
(D) The es	cape velocity of the moons sur	face is more than the r.	m.s velocity of all molecules.
	e r.m.s. velocity of all the gas noons surface.	nolecules is more then	the escape velocity of the
31. During an	adiabatic process, the pressur	e of a gas is found to be	e proportional to the
cube of its	temperature. The ratio of $\frac{C_P}{C_V}$	for the gas is:	
(A) $\frac{4}{3}$	(B) 2	(C) $\frac{5}{3}$	(D) $\frac{3}{2}$
	erature of the mixture of one m from 0°C to 100°C at constant		
be:	ITOTILO C to 100 C at constant	pressure. The amount	or near delivered will
(A) 600 cal	(B) 1200 cal	(C) 1800 cal	(D) 3600 cal
Ans. : b. 12	00 cal		
33. The Brown	nian Motion was discovered by	the scientist:	
(A) Albert		(B) John Brown	
(C) Robert	Brown	(D) Isaac Browr	1
Ans. : c. Ro	bert Brown	7	
	3 non-interacting ideal gases i		
in the ration (A) 6Pa	o 1:3:5. If the total pressure is 5 (B) 12Pa	(C) 18Pa	artial pressure of gas 1. (D) 28Pa
Ans.: a. 6Pa		(C) Iora	(D) Zora
_	anation: atio of partial pressures will be	in the same ratio as th	at of moles i.e. 1:3:5
	e partial pressure of gas 1 be '		at of moles, i.e. 1.3.3.
	x + 3x + 5x = 54.		
35. A diatomi	c molecule has how many degr	ees of freedom:	
(A) 3	(B) 4	(C) 5	(D) 6
Ans.: c. 5	anation:		
_	per of degree of freedom.		
Name			

d = 3N - 1

where N is the number of atoms in a molecules

In diatomic molecules, N = 2

$$\implies$$
 d = 3(2) - 1

= 5

Hence diatomic molecule has 5 degrees of freedom (3 translational and 2 rotational).

- 36. Calculate the RMS velocity of molecules of a gas of which the ratio of two specific heats is 1.42 and velocity of sound in the gas is 500m/s:
 - (A) 727m/s

(B) 527m/s

(C) 927m/s

(D) 750m/s

Ans.:

a. 727m/s

37. At constant temperature, pressure is inversely proportional to volume is called as:

(A) Charle's law

(B) Boyle's law

(C) Zeroth law of thermodynamics

(D) First law of thermodynamics

Ans.:

b. Boyle's law

- 38. Boyle's law is applicable for an:
 - (A) Diabatic process.

(B) Isothermal process.

(C) Isobaric process.

(D) Isochoric process.

Ans.:

b. Isothermal process.

Explanation:

Boyle's law is applicable at constant temperature, and temperature remains constant in isothermal process,

PV = nRT (n, R and T are constant)

∴ PV = constant

 $P\alpha \frac{1}{V}$ (where constant = nRT)

- 39. Which of the following gases has maximum rms speed at a given temperature?
 - (A) Hydrogen.

(B) Nitrogen.

(C) Oxygen.

(D) Carbon dioxide.

Ans.:

a. Hydrogen.

Explanation:

The rms speed of a gas is given by $\sqrt{\frac{3RT}{M_O}}$.

Since hydrogen has the lowest $M_{\rm O}$ compared to other molecules, it will have the highest rms speed.

		speed of molecules of	this gas is C. If $\gamma=rac{\mathrm{C_p}}{\mathrm{C_v}},$
then the ratio of v and $rac{3}{\gamma}$	(B) 0.33γ	(C) $\sqrt{\frac{3}{\gamma}}$	(D) $\sqrt{\frac{\gamma}{3}}$

d.
$$\sqrt{\frac{\gamma}{3}}$$

41. A gas is taken in a sealed container at 300K. it is heated at constant volume to a temperature 600K. the mean K.E. of its molecules is:

Ans.:

b. Doubled

42. What is the ratio of densities of 2 gases, $O_2 \& N_2$, having partial pressures in the ratio 2:3?

(A)
$$\frac{16}{21}$$

(B)
$$\frac{12}{7}$$

(C)
$$\frac{21}{16}$$

(D)
$$\frac{7}{12}$$

Ans.:

a.
$$\frac{16}{21}$$

Explanation:

The ratio of moles is the same as the ratio of partial pressures.

The ratio of densities:

$$egin{aligned} rac{\mathrm{d}_1}{\mathrm{d}_2} &= rac{\left(rac{\mathrm{m}_1}{\mathrm{V}}
ight)}{\left(rac{\mathrm{m}_2}{\mathrm{V}}
ight)} \ &= rac{\mathrm{m}_1}{\mathrm{m}_2} &= rac{\left(rac{\mathrm{n}_1}{\mathrm{M}_1}
ight)}{\left(rac{\mathrm{n}_1}{\mathrm{M}_1}
ight)} \end{aligned}$$

where n is the number of moles and M is the molecular mass.

$$\frac{d_1}{d_2} = \left(\frac{2}{3}\right) \times \left(\frac{16}{14}\right)$$
$$= \frac{16}{21}$$

43. The total internal energy of the monoatomic gas molecule is given by:

(A)
$$\frac{1}{2}$$
RT

(B)
$$\frac{5}{2}\mathrm{RT}$$

(C)
$$\frac{3}{2}$$
RT

(D)
$$m RT$$

Ans.:

c.
$$\frac{3}{2}$$
RT

44. The total internal energy of a mole of diatomic gas is given by:

(A)
$$\frac{5}{3}$$
RT

(B)
$$\frac{5}{2}\mathrm{R}$$

(C)
$$\frac{5}{2}$$
RT

(D)
$$\frac{3}{2}$$
RT

Ans.:

c.
$$\frac{5}{2}$$
RT

45.	A container has 3 gases whose mass ratio is 1:3:5. What is the ratio of mean square speed of the molecules of two gases? Their atomic masses are 20u, 30u & 40u corresponding to the order in which the ratios are given.			
	(A) $2:3:4$		(B) $4:3:2$	
	(C) $2:\sqrt{3}:\sqrt{2}$		(D) $\sqrt{2}:\sqrt{3}:2$	
	Ans.: b. 4:3:2.			
	Explanation:			
	Their average ki	netic energies will be the	ϵ same. Thus, $rac{1}{2}$ mv 2 will b	e the same.
	$v_1^2 : v_2^2 : v_3^2$ = m_3 : m_2 : m_1 = 40:30:20 = 4:3:2.			
46.	A room temperature	the r.m.s. velocity of the	molecules of a certain di	atomic gas is
	found to be 1930m/s		40	-
(,	A) H ²	(B) F ²	(C) O ²	(D) Cl ²
	Ans. : a. H ²			
47.	=	quipartition of energy, in ible energy modes having (B) $rac{1}{2} ext{KBT}$	equilibrium the tot energe an energy equal to: (C) $\overline{\mathrm{KBT}}$	gy is equally $ \text{(D) } \frac{5}{2} \text{KBT} $
(4	2		(0) 112 1	$(D)_{2}$ \mathbb{R} \mathbb{D} \mathbb{I}
	Ans.: b. $\frac{1}{2}$ KBT	T		
	·	$-C_v = b$. The relation be		
(,	A) a = 16b	(B) b = 16	(C) $a = b$	(D) $a = 4b$
	Ans.:			
	c. a = b			
	Explanation: For any gas C _P -	C – P		
	∴ a = b	CV -K		
49.	The two gases with th	ne ratio 3 : 2 of their mass c energies of the molecu	ses in a container are at a	a temperature T.
(,	A) 3 : 2	(B) 9 : 4	(C) 1:1	(D) 4 : 9
	Ans.: c. 1:1			
50.	What is the ratio of sp	pecific heats for a diatom	ic gas?	
(,	A) $\frac{7}{5}$	(B) $\frac{5}{3}$	(C) $\frac{9}{7}$	(D) $\frac{7}{2}$
	Ans.:			
	a. $\frac{7}{5}$			

Explanation:

The value of CV for a diatomic gas is $\frac{7}{2}$ R and

CP is
$$\frac{9}{2}$$
 R.

Thus the value of γ is:

$$\frac{C_P}{C_V} = \frac{7}{5}$$

51. The pressure P of a gas and its mean K.E. per unit volume are related as:

(A)
$$P = \frac{1}{2}E$$

(B)
$$\mathrm{P}=\mathrm{E}$$

(C)
$$P = \frac{3}{2}E$$

(D)
$$P = \frac{2}{3}E$$

Ans.:

d.
$$P = \frac{2}{3}E$$

Explanation:

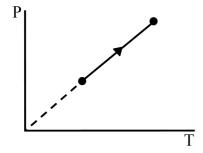
$$\mathrm{P}=rac{1}{3}
ho\mathrm{C}^2$$

Mean K.E./ volume $= E = {1 \over 2}
ho C^2$

$$\therefore P = \frac{1}{3}\rho C^2 = \frac{2}{3} \left(\frac{1}{2}\rho C^2\right) = \frac{2}{3}E$$

52. The process on an ideal gas, shown in figure. is:

- a. Isothermal.
- b. Isobaric.
- c. Isochoric.
- d. None of these.



Ans.:

c. Isochoric.

Explanation:

According to the graph, P is directly proportional to T.

Applying the equation of state, we get,

$$PV = nRT$$

$$= \frac{nR}{V}T$$

Given:
$$P \propto T$$

This means $\frac{nR}{V}$ is a constant. So, V is also a constant.

Constant V implies the process is isochoric.

- 53. The rms speed of oxygen at room temperature is about 500m/ s. The rms speed of hydrogen at the same temperature is about:
 - a. 125ms⁻¹
 - b. 2000ms⁻¹

- c. 8000ms⁻¹
- d. 31ms⁻¹

Ans.:

b. 2000ms⁻¹

Explanation:

Given,

Molecular mass of hydrogen, $M_H = 2$

Molecular mass of oxygen, $M_O = 32$

RMS speed is given by,

$$egin{aligned} v_{rms} &= \sqrt{rac{3RT}{M}} \ \Rightarrow \sqrt{rac{3RT}{M_O}} &= 500 \end{aligned}$$

Now,

$$\begin{split} &\Rightarrow \frac{v_{\mathrm{Orms}}}{v_{\mathrm{Hrms}}} = \frac{\sqrt{\frac{3\mathrm{RT}}{\mathrm{M}_{\mathrm{O}}}}}{\sqrt{\frac{3\mathrm{RT}}{\mathrm{M}_{\mathrm{H}}}}} \\ &\Rightarrow \frac{v_{\mathrm{Orms}}}{v_{\mathrm{Hrms}}} = \frac{\sqrt{\frac{3\mathrm{RT}}{\mathrm{M}_{\mathrm{H}}}}}{\sqrt{\frac{3\mathrm{RT}}{32}}} \\ &\Rightarrow \frac{v_{\mathrm{Orms}}}{v_{\mathrm{Hrms}}} = \frac{1}{4} \\ &\Rightarrow \frac{500}{v_{\mathrm{Hrms}}} = \frac{1}{4} \\ &\Rightarrow v_{\mathrm{Hrms}} = 4 \times 500 = 2000 \mathrm{ms}^{-1} \end{split}$$

- 54. Which of the following gases has maximum rms speed at a given temperature?
 - a. Hydrogen.
 - b. Nitrogen.
 - c. Oxygen.
 - d. Carbon dioxide.

Ans.:

a. Hydrogen.

Explanation:

The rms speed of a gas is given by $\sqrt{rac{3RT}{M_O}}$.

Since hydrogen has the lowest $\ensuremath{\text{M}}_{\ensuremath{\text{O}}}$ compared to other molecules, it will have the highest rms speed.

- 55. The quantity $\frac{PV}{kT}$ represents:
 - a. Mass of the gas.
 - b. Kinetic energy of the gas.
 - c. Number of moles of the gas.
 - d. Number of molecules in the gas.

Ans.:

d. Number of molecules in the gas.

Explanation:

Here,

$$PV = nRT ...(1)$$

Also,

$$egin{aligned} k &= rac{R}{N} \ \Rightarrow R &= kN \dots (2) \end{aligned}$$

Now.

PV = nkNT [From eq. (1) and eq. (2)]

$$\Rightarrow nN = \frac{PV}{kT}$$

nN = Number of molecules

 $\frac{PV}{kT}$ = Number of molecules.

- 56. The mean square speed of the molecules of a gas at absolute temperature T is proportional to:
 - a. $\frac{1}{7}$
 - b. \sqrt{T}
 - c. T
 - d. T^2

Ans.:

c. T

Explanation:

Root mean squared velocity is given by

$$egin{aligned} \mathbf{v}_{
m rms} &= \sqrt{rac{3
m RT}{
m M}} \ \Rightarrow (\mathbf{v}_{
m rms})^2 &= rac{3
m RT}{
m M} \ \Rightarrow (\mathbf{v}_{
m rms})^2 lpha \mathrm{T} \end{aligned}$$

57. Vapour is injected at a uniform rate in a closed veseel which was initially evacuated.

The pressure in the vessel:

- a. Increaaes continuously.
- b. Decreases continuously.
- c. First increases and then decreases.
- d. First increases and then becomes constant.

Ans.:

d. First increases and then becomes constant.

Explanation:

As the vapour is injected, the pressure of the chamber increases. But when the pressure becomes equal to the saturated vapour pressure, it condenses. So, if more vapour is injected beyond the saturated vapour pressure, the vapour will condense and thus the vapour pressure will be constant.

- 58. The average momentum of a molecule in a sample of an ideal gas depends on:
 - a. Temperature.
 - b. Number of moles.

- c. Volume.
- d. None of these.

Ans.:

d. None of these.

Explanation:

Average momentum of a gas sample is zero, so it does not depend upon any of these parameters.

- 59. There is some liquid in a closed bottle. The amount of liquid is continuously decreasing. The vapour in the remaining part:
 - a. Must be saturated.
 - b. Must be unsaturated.
 - c. May be saturated.
 - d. There will be no vapour.

Ans.:

b. Must be unsaturated.

Explanation:

As the liquid is decreasing, the liquid is vapourised. We know that vapourisation cannot occur in saturated air and there cannot be any liquid with no vapour at all. So, the vapour in the remaining part is unsaturated.

* Answer The Following Questions In One Sentence.[1 Marks Each]

[5]

60. A ball is dropped on a floor from a height of 2.0m. After the collision it rises up to a height of 1.5m. Assume that 40% of the mechanical energy lost goes as thermal energy into the ball. Calculate the rise in the temperature of the ball in the collision. Heat capacity of the ball is $800JK^{-1}$.

Ans.: Height of the floor from which ball is dropped, $h_1 = 2.0$ m

Height to which the ball rises after collision, $h_2 = 1.5m$

Let the mass of ball be m kg.

Let the speed of the ball when it falls from h_1 and h_2 be v_1 and v_2 , respectively.

$$\begin{aligned} v_1 &= \sqrt{2gh_1} = \sqrt{2\times10\times2} = \sqrt{40}\,\text{m/s} \\ v_2 &= \sqrt{2gh_2} = \sqrt{2\times10\times1.5} = \sqrt{30}\,\text{m/s} \end{aligned}$$

Change in kinetic energy is given by,

$$\triangle \mathbf{K} = \frac{1}{2} \times \mathbf{m} \times 40 - \left(\frac{1}{2}\mathbf{m}\right) \times 30 = \left(\frac{10}{2}\right)\mathbf{m}$$

$$\Rightarrow \triangle \mathbf{K} = 5\mathbf{m}$$

If the position of the ball is considered just before hitting the ground and after its first collision, then 40% of the change in its KE will give the change in thermal energy of the ball. At these positions, the PE of the ball is same. Thus,

Loss in PE = 0

The change in kinetic energy is utilised in increasing the temperature of the ball. Let the change in temperature be $\triangle T$. Then,

$$\left(\frac{40}{100}\right) imes \triangle ext{K} = ext{m} imes 800 imes \triangle ext{T}$$

$$\begin{split} \left(\frac{40}{100}\right) \times \frac{10}{2} m &= m \times 800 \times \triangle T \\ \Rightarrow \triangle T &= \frac{1}{400} = 0.0025 \\ \Rightarrow 2.5 \times 10^{-3} ^{\circ} C \end{split}$$

61. A 50kg man is running at a speed of 18kmh⁻¹. If all the kinetic energy of the man can be used to increase the temperature of water from 20°C to 30°C, how much water can be heated with this energy?

Ans.: Given.

Mass of the man, m = 50kg

Speed of the man, $v=18 \mathrm{km/h}$

$$=18 imes rac{5}{18} = 5 ext{m/s}$$

Kinetic energy of the man is given by,

$$K=\tfrac{1}{2}mV^2$$

$$K = \left(\frac{1}{2}\right) 50 \times 5^2$$

$$K = 25 \times 25 = 625J$$

Specific heat of the water, s = 4200J/Kg-K

Let the mass of the water heated be M.

The amount of heat required to raise the temperature of water from 20°C to 30°C is given by,

$$\mathrm{Q} = \mathrm{ms} \triangle \mathrm{T} = \mathrm{M} imes 4200 imes (30-20)$$

$$Q = 42000M$$

According to the question,

$$Q = K$$

$$\Rightarrow 42000M = 625$$

$$\Rightarrow M = \frac{625}{42} \times 10^{-3}$$

$$\Rightarrow M = 14.88 \times 10^{-3}$$

$$\Rightarrow M = 15g$$

62. The ratio of vapour densities of two gases at the same temperature is 6 : 9. Compare the r.m.s. velocities of their molecules.

Ans.: The ratio of r.m.s velocities is given as

$$rac{\mathrm{C_1}}{\mathrm{C_2}} = \sqrt{rac{\mathrm{M_2}}{\mathrm{M_1}}} = \sqrt{rac{
ho_2}{
ho_1}};$$

$$rac{\mathrm{C_1}}{\mathrm{C_2}} = \sqrt{rac{9}{6}} = \sqrt{3}:\sqrt{2}$$

63. Consider a gas of neutrons. Do you expect it to behave much better as an ideal gas as compared to hydrogen gas at the same pressure and temperature?

Ans.: In case of neutron gas there would be no internal forces such as electrostatic forces between molecule between atoms of gases thus it can behave like ideal gas better than hydrogen gas.

64. Calculate the volume of 1 mole of an ideal gas at STP.

Ans.: Volume of 1 mole of gas,

$$PV = nRT$$

$$\Rightarrow V = \frac{RT}{P} = \frac{0.082 \times 273}{1}$$

$$\Rightarrow 22.38 \approx 22.4 L = 22.4 \times 10^{-3}$$

$$\Rightarrow 2.24 \times 10^{-2} \text{m}^3$$

* Given Section consists of questions of 2 marks each.

[6]

65. Find the average magnitude of linear momentum of a helium molecule in a sample of helium gas at 0°C. Mass of a helium molecule = 6.64×10^{-27} kg nd Boltzmann constant = 1.38×10^{-23} JK⁻¹.

Ans. :
$$M = 4 \times 10^{-3} \text{Kg}$$

$$m V_{avg} = \sqrt{rac{8RT}{\pi M}} = \sqrt{rac{8 imes 8.3 imes 273}{3.14 imes 4 imes 10^{-3}}} = 1201.35$$

$$\text{Momentum} = M \times V_{avg} = 6.64 \times 10^{-27} \times 1201.35$$

$$= 7.97 \times 10^{-24} \approx 8 \times 10^{-24} {
m Kg-m/s}.$$

66. The temperature and the dew point in an open room are 20°C and 10°C. If the room temperature drops to 15°C, what will be the new dew point?

The place is saturated at 10°C

Even if the temp drop dew point remains unaffected.

The air has V.P. which is the saturation VP at 10°C. It (SVP) does not change on temp.

67. The weather report reads, "Temperature 20°C : Relative humidity 100%". What is the dew point?

Ans.: Temp is 20° Relative humidity = 100%

So the air is saturated at 20°C

Dew point is the temperature at which SVP is equal to present vapour pressure, So 20°C is the dew point.

* Given Section consists of questions of 3 marks each.

[63]

- 68. A flask contains argon and chlorine in the ratio of 2:1 by mass. The temperature of the mixture is 27C. Obtain the ratio of (i) average kinetic energy per molecule, and (ii) root mean square speed $V_{\rm rms}$ of the molecules of the two gases. Atomic mass of argon =39.9u; Molecular mass of chlorine =70.9u.
 - **Ans.:** The important point to remember is that the average kinetic energy (per molecule) of any (ideal) gas (be it monatomic like argon, diatomic like chlorine or polyatomic) is always equal to $(3/2)k_BT$. It depends only on temperature, and is independent of the nature of the gas.
 - (i) Since argon and chlorine both have the same temperature in the flask, the ratio of average kinetic energy (per molecule) of the two gases is 1:1.
 - (ii) Now $1/2mv_{rms}^2=$ average kinetic energy per molecule $=(3/2))k_BT$ where

m is the mass of a molecule of the gas. Therefore,

$$rac{\left(v_{rms}^2
ight)_{Ar}}{\left(v_{rms}^2
ight)_{Cl}} = rac{(m)_{Cl}}{(m)_{Ar}} = rac{(M)_{Cl}}{(M)_{Ar}} = rac{70.9}{39.9} = 1.77$$

where M denotes the molecular mass of the gas. (For argon, a molecule is just an atom of argon.) Taking square root of both sides,

$$rac{(v_{rms})_{Ar}}{(v_{rms})_{Cl}} = 1.33$$

You should note that the composition of the mixture by mass is quite irrelevant to the above calculation. Any other proportion by mass of argon and chlorine would give the same answers to (i) and (ii), provided the temperature remains unaltered.

69. Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the masses of the molecules are m_1 and m_2 and the number of the molecules in the gases are n_1 and n_2 respectively.

Ans. : According to kinetic theory, the average kinetic energy per molecule of a gas $= \frac{3}{2} k_B T$

Before mixing, the two gases, the average K.E. of all the molecules of two gases

$$= \tfrac{3}{2} k_B n_1 T_1 + \tfrac{3}{2} k_B n_2 T_2$$

After mixing, the average K.E. of both the gases $=\frac{3}{2}k_B(n_1+n_2)T$ where T is the temperature of mixture. Since there is no loss of energy,

Hence,
$$rac{3}{2}k_{\mathrm{B}}(n_{1}+n_{2})T$$
 $=rac{3}{2}k_{\mathrm{B}}n_{1}T_{1}+rac{3}{2}k_{\mathrm{B}}n_{2}T_{2}$ $T=rac{n_{1}T_{1}+n_{2}T_{2}}{(n_{2}+n_{2})}$

70. The volume of air bubble increases 15 times when it rises from bottom to the top of a lake. Calculate the depth of the lake if density of lake water is 1.02×10^3 kg/ m³ and atmospheric pressure is 75cm of mercury.

Ans.: According to Boyle's law,

$$P_1V_1 = P_2V_2 ...(1)$$

Here $P_1 = 75$ cm of Hg = 75m of Hg = $0.75 \times 13.6 \times 10^3 \times 9.8$

$$= 99.96 \times 10^3 \text{Nm}^{-2}$$

Let volume of bubble at depth h = x

i.e.,
$$V_2 = x : V_1 = 16x$$

 $P_2 = 75$ cm of Hg + hp_{water} g = $99.96 \times 10^3 + h \times 10^3 \times 9.8$

Using eqn. (1), we get

$$99.96 \times 10^3 \times 16x = (99.96 \times 10^3 + h \times 10^3 \times 9.8)x$$

$$9.8h = 99.96 \times 16 - 99.96 = 99.96 \times 15$$

$$\therefore h = \frac{99.96 \times 15}{9.8} = 153 \text{m}$$

71. The condition of air in a closed room is described as follows. Temperature = 25° C, relative humidity = 60° , pressure = 104kPa. If all the water vapour is removed from the room without changing the temperature, what will be the new pressure? The saturation vapour pressure at 25° C = 3.2kPa.

Ans.:
$$T = 25^{\circ}CP = 104KPa$$

$$\mathrm{RH} = rac{\mathrm{VP}}{\mathrm{SVP}}$$
[SVP = 3.2KPa, RH = 0.6]

$$ext{VP} = 0.6 imes 3.2 imes 10^3 = 1.92 imes 10^3 pprox 2 imes 10^3$$

When vapours are removed VP reduces to zero

Net pressure inside the room now = 104×10^3 - 2×10^3 = 102×10^3 = 102 KPa.

72. Find the kinetic energy of 1g of nitrogen gas at 77°C. Given, R = 8.31J-mol⁻¹K⁻¹.

Ans.: For, nitrogen,
$$M = 28$$

$$T = 77 + 273 = 350K$$

$$R = 8.31$$
J-mol⁻¹K⁻¹

$$=\frac{3}{2}\frac{\text{RT}}{\text{M}}=\frac{3\times8.31\times350}{2\times50}=155.8\text{J}$$

73. Calculate (i) r.m.s. velocity and (ii) mean kinetic energy of one gram molecule of hydrogen at S.T.P. Given density of hydrogen at S.T.P. is 0.09kg-m⁻³.

Ans. : Here,
$$ho=0.09 {
m kg \cdot m}^{-3}$$

At S.T.P., pressure
$$P=1.01 imes10^5Pa$$

According to kinetic theory of gases.

$$P = \frac{1}{3}\rho C^2$$

$$C = \sqrt{\frac{3P}{\rho}}$$

$$=\sqrt{rac{3 imes 1.01 imes 10^5}{0.09}}=1837.5 ext{ms}^{-1}$$

Volume occupied by one mole of hydrogen at S.T.P. = 22.4 liters = $22.4 \times 10^{-3} \text{ m}^3$

 \therefore Mass of hydrogen, M = volume x density

$$=22.4 imes 10^{-3} imes 0.09$$

$$=2.016 imes10^{-3}\mathrm{kg}$$

Average K.E./ mole
$$=\frac{1}{2}\mathrm{MC}^2$$

$$=rac{1}{2} imes (2.016 imes 10^{-3})$$

$$\times (1837.5)^2 = 3403.4$$
J

74. A vessel is filled with a gas at a pressure of 76cm of Hg at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.

Ans.: According to kinetic theory of gases, the pressure exerted by a gas is

$$P = \frac{1}{3}\rho c^2 = \frac{1}{3}\frac{M}{V}c^2$$

As temperature T is kept constant, therefore, c^2 is constant.

Also, V is constant.

$$\therefore$$
 P \propto M or $\frac{P_2}{P_1} = \frac{M_2}{M_1}$

$$rac{ ext{P}_2}{76} = rac{\left(ext{M}_1 + rac{50}{100} ext{M}_1
ight)}{ ext{M}_1} = rac{3}{2}$$

$$P_2=rac{3}{2} imes 76=144 cm$$
 of mercury (Hg).

75. The velocities of ten particles in ms⁻¹ are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate (i) Average speed and (ii) r.m.s. speed.

Ans.:

i. Average speed:

$$v_{av} = \frac{0+2+3+4+4+4+5+5+6+9}{10} = \frac{42}{10} = 4.2$$

ii. R.M.S. speed:

$$\begin{split} v_{rms} &= \left[\frac{(0)^2 + (2)^2 + (3)^2 + (4)^2 + (4)^2 + (5)^2 + (5)^2 + (6)^2 + (9)^2}{10} \right]^{\frac{1}{2}} \\ &= \left[\frac{228}{10} \right]^{\frac{1}{2}} = 4.77 ms^{-1} \end{split}$$

76. Two moles of gas A at 27°C are mixed with 3 moles of gas B at 37°C. If both are monoatomic ideal gases, what will be the temperature of the mixture?

Ans.: As there is no loss of energy in the process, therefore, sum of KE of gases A and B = KE of mixture,

$$egin{aligned} &\mu_1\Big(rac{3}{2}\mathrm{RT}_1\Big) + \mu_2\Big(rac{3}{2}\mathrm{RT}_2\Big) \ &= (\mu_1 + \mu_2)rac{3}{2}\mathrm{RT} \end{aligned}$$

where T is temperature of the mixture.

$$\begin{array}{l} \therefore \mathbf{T} = \frac{\mu_1 \mathbf{T}_1 + \mu_2 \mathbf{T}_2}{\mu_1 + \mu_2} \\ = \frac{2(27 + 273) + 3(37 + 273)}{2 + 3} \\ = \frac{600 + 930}{5} = \frac{1530}{5} \\ = 306 - 273 = 3^{\circ}\mathbf{C} \end{array}$$

77. Two ideal monoatomic gases A and B at 27°C and 37°C are mixed. The number of moles in gas A are 2 and number of moles in gas B are 3. What will be the temperature of the mixture?

Ans.: Sum of K.E. of gases A and B = K.E. of the mixture

$$\mu_{1}\left(\frac{3}{2}RT_{1}\right) + \mu_{2}\left(\frac{3}{2}RT_{2}\right)$$

$$= (\mu_{1} + \mu_{2})\left(\frac{3}{2}RT\right)$$

$$\therefore T = \frac{\mu_{1}T_{1} + \mu_{2}T_{2}}{\mu_{1} + \mu_{2}}$$

$$= \frac{2 \times 300 + 3 \times 310}{2 + 3} = 306K$$

 \cdot . Temperature of mixture $=33^{\circ}\mathrm{C}$

78. Explain the pressure exerted by an ideal gas and also find the average kinetic energy per molecule of the gas.

Ans.: From kinetic theory of gases, the pressure P exerted by an ideal gas of density p and r.m.s. velocity of its gas molecules C is given by

$$P = \frac{1}{3}\rho C^2$$

Mass of unit volume of the gas = 1 imes
ho =
ho

Mean kinetic energy of translation per unit volume of the gas is

$$E = \frac{1}{2}\rho C^2,$$

$$\therefore \frac{\mathrm{P}}{\mathrm{E}} = \frac{\left(\frac{1}{3}\right)\rho\mathrm{C}^2}{\left(\frac{1}{2}\right)\rho\mathrm{C}^2} = \frac{2}{3}$$

$$P = \frac{2}{3}E$$

"The pressure exerted by an ideal gas is numerically equal to two third of the mean kinetic energy of translation per unit volume of the gas."

Average Kinetic Energy per Molecule of the Gas: Consider one gram mole of an ideal gas occupying a volume V at temperature T. Let m be the mass of each molecule of the gas. Then

$$M = m \times N_A$$

where N_A is Avogadro's number.

If C is the r.m.s. velocity of the gas molecules, then pressure P exerted by ideal gas is

$$P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}\rho C^2$$

$$PV = \frac{1}{3}MC^2$$

From perfect gas equation, PV = RT, where R is a universal gas constant for one gram mole of the gas.

$$\therefore \frac{1}{3}MC^2 = RT$$

$$\frac{1}{3}MC^2 = \frac{3}{2}RT$$

 \therefore Average kinetic energy of translation of one mole of the gas $= \frac{1}{2} M C^2 = \frac{3}{2} R T$

$$\frac{1}{2}mN_AC^2=\frac{3}{2}RT\ (\because M=mN_A)$$

$$=rac{1}{2}mC^2=rac{3}{2}\Big(rac{R}{N_A}\Big)T=rac{3}{2}k_BT$$
 ($\because rac{R}{N}=k_B\Big)$

where k_B is called Boltzmarm constant.

- . Average K.E. of trarulation per molecule of gas $= \frac{1}{2} m C^2 = \frac{3}{2} k_B T$.
- 79. If one mole of ideal monoatomic gas $\left(\gamma=\frac{7}{5}\right)$ is mixed with one mole of diatomic gas $\left(\gamma=\frac{7}{5}\right)$. What is the value of y for the mixtures? (Here, γ represents the ratio of specific heat at constant pressure to that at constant volume)

Ans.: For monoatomic gas, $C_V=\frac{3}{2}R$

For diatomic gas, $C_V'=rac{5}{2}R$

Let, μ and μ' be moles of mono and diatomic gases then. C_V (mixture) $= \frac{\mu C_V + \mu' C_V'}{\mu + \mu'}$

$${
m C_V} = rac{1 imes rac{3}{2} {
m R} + 1 imes rac{5}{2} {
m R}}{1 + 1} = 2 {
m R}$$

$$\gamma(ext{mixture}) = 1 + rac{ ext{R}}{ ext{C}_{ ext{V (mixture)}}}$$

$$=1+\frac{R}{2R}=1.5$$

80. Calculate the root-mean square speed of oxygen molecules at 1092K. Density of oxygen at STP = 1.424kg-m⁻³.

Ans.: We first calculate the root-mean square speed of oxygen at STP.

$$P_0=0.76m$$
 of $Hg=1.01\times 10^5 Nm^{-2}$

$$\rho_0=1.424 \mathrm{kg}\text{-m}^{-3}$$

The root-mean square speed at 0°C is given by

$$c_0 = \sqrt{\frac{3P_0}{\rho_0}} = \sqrt{\frac{3\times 1.01\times 10^5}{1.424}} ms^{-1}$$

$$=4.61 \times 10^{2} {
m ms}^{-1}$$

Now c_{rms} is also given by

$$c_{rms} = \sqrt{\tfrac{3kT}{m}}$$

$$\therefore rac{c_{rms}}{c_0} = \sqrt{rac{T}{T_0}}$$

Here $T_0=273 \mathrm{K}$ and $T=1092 \mathrm{K}$

$$c_{
m rms} = c_0 \sqrt{rac{T}{T_0}}$$

$$=4.61 imes 10^2 imes \sqrt{rac{1092}{273}}$$

$$=9.22 \times 10^2 {
m ms}^{-1}$$

81. Three moles of an ideal diatomic gas is taken at a temperature of 300K. Its volume is doubled keeping its pressure constant. Find the change in internal energy of gas.

Ans. : Here, $\mu=3, T_1=300 K$ and for an ideal monoatomic gas

$$C_v = \frac{5}{2}R$$

As volume of gas is doubled ($V_2 = 2V_1$) at corstant pressure, hence according to Charlds law

$$ext{T}_2 = rac{ ext{T}_1 ext{V}_2}{ ext{V}_1} = rac{300 imes2 ext{V}_1}{ ext{V}_1} = 600 ext{K}$$

 \therefore Gain in internal energy $\mathrm{u}_2-\mathrm{u}_1=\mu\mathrm{C}_\mathrm{v}(\mathrm{T}_2-\mathrm{T}_1)$

$$=3 imesrac{5}{2} ext{R} imes(600-300)$$

$$= 2250 \mathrm{R} = 2250 \times 8.31 \times 10^4 \mathrm{J}$$

82. An ideal gas has a specific heat at constant pressure $C_p=\frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083m^3 at a temperature of 300K and a pressure of $1.6\times10^6\text{Nm}^{-2}$. An amount of $2.49\times10^4\text{J}$ of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

Ans.:
$$P = 1.6 \times 10^6 \text{Nm}^{-2}$$
, $V = 0.0083 \text{m}^3$, $T = 300 \text{K}$

We know that PV = nRT, where R = $8.3JK^{-1}$ mol⁻¹

Therefore
$$n=\frac{\mathrm{PV}}{\mathrm{RT}}$$

$$=\frac{1.6\times10^{6}\times0.0083}{8.3\times300}=\frac{16}{3}$$
mole

Now
$$C_P-C_V=R,$$
 therefore $C_V=C_P-R=rac{5R}{2}-R=rac{3R}{2}$

When heat energy Q is supplied to the gas, the inoease ΔT in its temperatue is obtained from the relation

$$m Q=nC_V\Delta T$$
 or $m \Delta T=rac{Q}{nC_V}=rac{2.49 imes 10^4}{rac{16}{3} imesrac{3}{2} imes 8.3}=375K$

 \therefore Final temperature T' = 300 + 375 = 675K. Since the gas is kept in a closed vessel, its volume remains constant. Hence the final pressure P is obtained from the relation.

$$\begin{array}{l} \frac{P'}{T'} = \frac{P}{T} \text{ or } P' = P \times \frac{T'}{T} \\ = \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6 Nm^{-2} \end{array}$$

83. The temperature and relative humidity in a room are 300K and 20% respectively. The volume of the room is 50m³. The saturation vapour pressure at 300K is 3.3kPa. Calculate the mass of the water vapour present in the room.

Ans.:
$$T = 300K$$
, Rel. humidity = 20%, $V = 50m^3$

SVP at 300K = 3.3KPa, V.P. = Relative humidity
$$\times$$
 SVP = 0.2 \times 3.3 \times 10³

$$PV = \frac{m}{M}RT$$

$$\Rightarrow 0.2 \times 3.3 \times 10^3 \times 50 = \frac{\text{m}}{18} \times 8.3 \times 300$$

$$\Rightarrow {
m m} = rac{0.2 imes 3.3 imes 50 imes 18 imes 10^3}{8.3 imes 300} = 238.55 {
m g} pprox 238 {
m g}$$

Mass of water present in the room = 238g.

84. 2g of hydrogen is sealed in a vessel of volume 0.02m³ and is maintained at 300K. Calculate the pressure in the vessel.

Ans.:
$$m = 2g$$
, $V = 0.02m^3 = 0.02 \times 10^6 cc = 0.02 \times 10^3 L$, $T = 300K$, $P = ?$

$$M = 2g$$
,

$$PV = nRT \Rightarrow PV = \frac{m}{M}RT$$

$$\Rightarrow P \times 20 = \frac{2}{2} \times 0.082 \times 300$$

$$\Rightarrow$$
 P = $\frac{0.082 \times 300}{20}$

$$=1.23~\mathrm{atm}=1.23 imes10^5~\mathrm{pa}pprox1.23 imes10^5\mathrm{pa}$$

85. A gas cylinder has walls that can bear a maximum pressure of 1.0×10^6 Pa. It contains a gas at 8.0×10^5 Pa and 300K. The cylinder is steadily heated. Neglecting any change in the volume, calculate the temperature at which the cylinder will break.

Ans.:
$$P_1 = 8.0 \times 10^5 \text{ Pa}$$
, $P_2 = 1 \times 10^6 \text{Pa}$, $T_1 = 300 \text{K}$, $T_2 = ?$

Since,
$$V_1 = V_2 = V$$

$$rac{\mathrm{P_1V_1}}{\mathrm{T_1}} = rac{\mathrm{P_2V_2}}{\mathrm{T_2}}$$

$$\Rightarrow \frac{8 \times 10^5 \times V}{300} = \frac{1 \times 10^6 \times V}{T_2}$$

$$ightarrow \mathrm{T}_2 = rac{1 imes 10^6 imes 300}{8 imes 10^5} = 375^\circ \mathrm{K}$$

86. An electric bulb of volume 250cc was sealed during manufacturing at a pressure of 10^{-3} mm of mercury at 27°C. Compute the number of air molecules contained in the bulb. Avogadro constant = 6×10^{23} mol⁻¹, density of mercury = 13600kg/ m⁻³ and g = 10ms⁻².

Ans.:
$$V = 250cc = 250 \times 10^{-3}$$

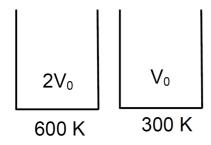
 $P = 10^{-3}mm = 10^{-3} \times 10^{-3}m$
 $= 10^{-6} \times 13600 \times 10 \text{ pascal}$
 $= 136 \times 10^{-3} \text{ pascal}$
 $T = 27^{\circ}C = 300K$
 $n = \frac{PV}{RT} = \frac{136 \times 10^{-3} \times 250}{8.3 \times 300} \times 10^{-3}$

$$n = \frac{PV}{RT} = \frac{136 \times 10^{-3} \times 250}{8.3 \times 300} \times 10^{-3}$$

$$= \frac{136 \times 250}{8.3 \times 300} \times 10^{-6}$$

No. of molecules
$$=rac{136 imes250}{8.3 imes300} imes10^{-6} imes6 imes10^{23}$$
 $=81 imes10^{17}pprox0.8 imes10^{15}$

- 87. Equal masses of air are sealed in two vessels, one of volume V₀ and the other of volume 2V₀. If the first vessel is maintained at a temperature 300K and the other at 600K, find the ratio of the pressures in the two vessels.
 - **Ans.:** Since mass is same



$$\begin{array}{l} n_1 = n_2 = n \\ P_1 = \frac{nR \times 300}{V_0}, P_2 = \frac{nR \times 600}{2V_0} \end{array}$$

$$rac{ ext{P}_1}{ ext{P}_2} = rac{ ext{nR} imes 300}{ ext{V}_0} imes rac{2 ext{V}_0}{ ext{nR} imes 600} = rac{1}{1} = 1:1$$

88. At what temperature the mean speed of the molecules of hydrogen gas equals the escape epeed from the earth?

Ans.: Mean speed of the molecule $= \sqrt{}$

Escape velocity
$$=\sqrt{2 {
m gr}}$$

$$\sqrt{rac{8 ext{RT}}{\pi ext{M}}} = \sqrt{2 ext{gr}} \Rightarrow rac{8 ext{RT}}{\pi ext{M}} = 2 ext{gr}$$

$$\Rightarrow T = \frac{2gr\pi M}{8R} = \frac{2 \times 9.8 \times 6400000 \times 3.14 \times 2 \times 10^{-3}}{8 \times 8.3} = 11863.9 \approx 11800 \text{m/s}.$$

Given Section consists of questions of 5 marks each.

89. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at- 20° C? (atomic mass of Ar = 39.9u, of He = 4.0u).

Ans.: Temperature of the helium atom, $T_{He} = -20$ °C = 253K

Atomic mass of argon, $M_{Ar} = 39.9u$

Atomic mass of helium, $M_{He} = 4.0u$

Let, $(v_{rms})_{Ar}$ be the rms speed of argon.

Let $(v_{rms})_{He}$ be the rms speed of helium.

[135]

The rms speed of argon is given by

$$(v_{rms})_{Ar} = \sqrt{\frac{3RT_{Ar}}{M_{Ar}}} \ldots (i)$$

where,

R is the universal gas constant

TAr is temperature of argon gas

The rms speed of helium is given by,

$$(\mathrm{v_{rms}})_{\mathrm{He}} = \sqrt{rac{3\mathrm{RT_{He}}}{\mathrm{M_{He}}}} \ldots (\mathrm{ii})$$

It is given that,

$$egin{aligned} (\mathrm{v_{rms}})_{\mathrm{Ar}} &= (\mathrm{v_{rms}})_{\mathrm{He}} \ \sqrt{rac{3\mathrm{RT_{Ar}}}{\mathrm{M_{Ar}}}} &= \sqrt{rac{3\mathrm{RT_{He}}}{\mathrm{M_{He}}}} \end{aligned}$$

$$rac{\mathrm{T_{Ar}}}{\mathrm{M_{Ar}}} = rac{\mathrm{T_{He}}}{\mathrm{M_{He}}}$$

$$T_{Ar} = rac{T_{He}}{M_{He}} imes M_{Ar}$$

$$= \frac{253}{4} \times 39.9$$

$$=2523.675=2.52\times103\mathrm{K}$$

Therefore, the temperature of the argon atom is 2.52×10^3 K.

90. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder (R = 8.31J mol⁻¹K⁻¹, molecular mass of O_2 = 32u).

Ans.: Volume of oxygen, $V_1 = 30$ litres = 30×10^{-3} m³

Gauge pressure, $P_1 = 15$ atm $= 15 \times 1.013 \times 10^5$ Pa

Temperature, $T_1 = 27$ °C = 300K

Universal gas constant, R = 8.314J mole⁻¹K⁻¹

Let the initial number of moles of oxygen gas in the cylinder be n_1 .

The gas equation is given as,

$$P_1V_1 = n_1RT_1$$

$$n_1 = \frac{P_1 V_1}{RT_1}$$

$$\begin{split} n_1 &= \frac{P_1 V_1}{R T_1} \\ &= \frac{(15.195 \times 10^5 \times 30 \times 10^{-3})}{(8.314 \times 300)} = 18.276 \end{split}$$

But
$$n_1 = \frac{m_1}{M}$$

Where,

 m_1 = Initial mass of oxygen

M = Molecular mass of oxygen = 32g

$$m_1 = n_1 M = 18.276 \times 32 = 584.84g$$

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduces.

Volume,
$$V_2 = 30$$
 litres = 30×10^{-3} m³

Gauge pressure, $P_2 = 11$ atm $= 11 \times 1.013 \times 10^5$ Pa

Temperature, $T_2 = 17$ °C = 290K

Let n_2 be the number of moles of oxygen left in the cylinder.

The gas equation is given as,

$$\mathsf{P}_2\mathsf{V}_2=\mathsf{n}_2\mathsf{RT}_2$$

$$\therefore n_2 = \frac{P_2 V_2}{RT_2}$$

$$=rac{11.143 imes10^5 imes30 imes10^{-3}}{8.314 imes290}=13.86$$

But
$$m n_2=rac{m_2}{M}$$

Where,

m2 is the mass of oxygen remaining in the cylinder

$$m_2 = n_2 M = 13.86 \times 32 = 453.1g$$

The mass of oxygen taken out of the cylinder is given by the relation,

Initial mass of oxygen in the cylinder - Final mass of oxygen in the cylinder

$$= m_1 - m_2$$

$$= 584.84 g - 453.1g$$

$$= 131.74g$$

$$= 0.131 kg$$

Therefore, 0.131kg of oxygen is taken out of the cylinder.

91. An air bubble of volume 1.0cm³ rises from the bottom of a lake 40m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35°C?

Ans. : Volume of the air bubble,
$$V_1 = 1.0 \text{cm}^3 = 1.0 \times 10^{-6} \text{m}^3$$

Bubble rises to height, d = 40m

Temperature at a depth of 40 m,
$$T_1 = 12$$
°C = 285K

Temperature at the surface of the lake,
$$T_2 = 35$$
°C = 308K

The pressure on the surface of the lake,

$$P_2 = 1 \text{ atm} = 1 \times 1.013 \times 10^5 \text{Pa}$$

The pressure at the depth of 40m

$$P_1 = 1 atm + dpg$$

Where, p is the density of water = 10^3 kg/m³

g is the acceleration due to gravity = 9.8m/s^2

$$\therefore P_1 = 1.013 \times 10^5 + 40 \times 10^3 \times 9.8 = 493300$$
Pa

We have,
$$rac{\mathrm{P_1V_1}}{\mathrm{T_1}} = rac{\mathrm{P_2V_2}}{\mathrm{T_2}}$$

Where, V_2 is the volume of the air bubble when it reaches the surface

$$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$$

$$=\frac{493300\times(1.0\times10^{-6})308}{285\times1.013\times10^{5}}$$

$$= 5.263 \times 10^{-6} \text{m}^3 \text{ or } 5.263 \text{cm}^3$$

Therefore, when the air bubble reaches the surface, its volume becomes 5.263cm³.

92. Consider a rectangular block of wood moving with a velocity v_0 in a gas at temperature T and mass density ρ . Assume the velocity is along x-axis and the area of cross-section

of the block perpendicular to v0 is A. Show that the drag force on the block is

$$4\rho~Av_0\sqrt{\frac{\mathrm{KT}}{\mathrm{m}}}$$
 where m is the mass of the gas molecule.

Ans.: Let $ho_{
m m}$ is the number of molecule per unit volume i.e. $ho_{
m m}$ is molecular density per unit volume.

 $v = v_{rms}$ is velocity of gas molecules

When box moves in gas the molecules of gas strike to front face in opposite direction and on back face in same direction as $v >> v_0$ (box) so relative velocity on back face = $(v - v_0)$

Change in momentum by a molecule on front face = $2m(v + v_0)$

Change in momentum by a molecule on back side = $2m(v - v_0)$

Number of molecule striking on front face in Δt $time = \frac{1}{2}$ volume x molecular density/vol.

To front face

$$=rac{1}{2}[\mathrm{A.}\,(\mathrm{v}+\mathrm{v}_0)\Delta\mathrm{t}]
ho_\mathrm{m}$$

Number of molecules striking to front face,

$$N_{\mathrm{F}}=rac{1}{2}(\mathrm{v}+\mathrm{v}_{0})A\;
ho_{\mathrm{m}}\Delta\mathrm{t}$$

Similarly as the speed of molecule and block are same so number of molecule striking on backend face,

$$m N_B = rac{1}{2}(v-v_0)A~
ho_m \Delta t$$

Total change in momentum due to striking the molecule on front face,

$$m P_F=2m(v+v_0)N_F=2m(v+v_0) imesrac{1}{2}(v+v_0)A
ho_m\Delta t$$

$$P_{\mathrm{F}} = -m(v+v_0)^2 \; A
ho_{\mathrm{m}} \Delta t$$
 (Backward direction)

So rate of change of momentum on front face is equal to the force,

$$F_{\mathrm{F}} = -\mathrm{m}(\mathrm{v} + \mathrm{v}_0)^2 \; \mathrm{A}
ho_{\mathrm{m}}$$
 in Backward direction.

Similarly force on back end $F_{
m B}=+{
m m}({
m v}-{
m v}_0)^2\;{
m A}
ho_{
m m}$

Net dragging force $=-\mathrm{m}(\mathrm{v}+\mathrm{v}_0)^2~\mathrm{A}
ho_\mathrm{m}+\mathrm{m}(\mathrm{v}-\mathrm{v}_0)^2~\mathrm{A}
ho_\mathrm{m}$

$$=-\mathrm{mA}
ho_{\mathrm{m}}[(\mathrm{v}+\mathrm{v}_{0})^{2}-(\mathrm{v}-\mathrm{v}_{0})^{2}]$$

$$=-mA\rho_m[v^2+v_0^2+2vv_0-(v^2+v_0^2-2v.\,v_0]$$

$$=-\mathrm{mA}
ho_\mathrm{m}\,4\mathrm{v.v_0}$$

So magnitude of dragging force due to gas molecule $= 4 m v v_0 A P_{\rm m}$

KE of gas molecule.

$$= \tfrac{1}{2} m v^2 = \tfrac{3}{2} K_B T$$

$$\therefore$$
 $m v = \sqrt{rac{K_B T}{m}}$ [using equ. (A) of Q.13.30]

... Dragging force becomes,

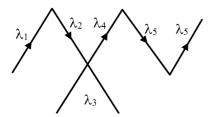
$$= 4m\; A P_m v_0 \sqrt{\frac{K_B T}{m}}$$

93.

- i. Define mean free path.
- ii. Derive an expression for mean free path of a gas molecule.

Ans.:

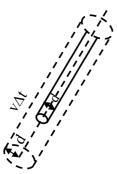
i. The mean free path of a gas molecule is defined as the average distance travelled by a molecule between two successive collisions.



According to figure, if a molecule covers free path $\lambda_1, \lambda_2, \lambda_3, \ldots$ after successive collisions, then its mean free path is given by

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 ...}{ ext{(total number of collisions)}}$$

ii. **Expression for mean free path:** Let d be the diameter of each molecule of the gas, then a particular molecule will suffer collision with any molecule that comes within a distance d between centers of two molecules.



Volume swept by a molecule in time Δ

If \bar{v} is average speed of molecule, then from figure, the volume swept by the molecule in small time Δt in which any molecule will collide with it

$$=\pi d^2\langle v\rangle \Delta t$$

If n is number of molecules per unit volume of the gas, then number of collision suffered by the molecule in time $\Delta t\,$

$$=\pi d^2\langle v
angle \Delta t imes n$$

So, number of collisions per second

$$= \tfrac{\pi d^2 \langle v \rangle \Delta t \times n}{\Delta t} = n \pi d^2 \langle v \rangle$$

... Average time between two successive collisions

i.e.
$$au = rac{1}{\mathrm{n}\pi\mathrm{d}^2\langle\mathrm{v}
angle}$$

... Mean free path = average distance between two successive collision

$$\Rightarrow \lambda = \tau \times \text{mean velocity}$$

$$=\frac{1}{n\pi d^2\langle v\rangle}\times \bar{\mathbf{v}}=\frac{1}{n\pi d^2}$$

Mean free path, $\lambda=rac{1}{\mathrm{n}\pi\mathrm{d}^2}$

where, d = diameter of each molecule

and n = number of molecules per unit volume

94. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. Obtain the ratio of (i) average kinetic energy per molecule, and (ii) root

mean square speed v_{rms} of the molecules of the two gases. Atomic mass of argon = 39.9u; Molecular mass of chlorine = 70.9u.

Ans.: The important point to remember is that the average kinetic energy (per molecule) of any (ideal) gas (be it monatomic like argon, diatomic like chlorine or polyatomic) is always equal to $\left(\frac{3}{2}\right)k_BT$. It depends only on temperature, and is independent of the nature of the gas.

- i. Since argon and chlorine both have the same temperature in the flask, the ratio of average kinetic energy (per molecule) of the two gases is 1:1.
- ii. Now $\frac{1}{2}mv_{rms}^2$ = average kinetic energy per molecule = $\left(\frac{3}{2}\right)k_BT$ where m is the mass of a molecule of the gas. Therefore,

$$egin{aligned} &rac{\left(V_{
m rms}^2
ight)_{
m Ar}}{\left(V_{
m rms}^2
ight)_{
m Cl}} = rac{\left(m
ight)_{
m Cl}}{\left(m
ight)_{
m Ar}} = rac{\left(M
ight)_{
m Cl}}{\left(M
ight)_{
m Ar}} \ = rac{70.9}{39.9} = 1.77 \end{aligned}$$

where M denotes the molecular mass of the gas. (For argon, a molecule is just an atom of argon) Taking square root of both sides.

$$rac{{
m (V_{rms})}_{
m Ar}}{{
m (V_{rms})}_{
m Cl}}=1.33$$

You should note that the composition of the mixture by mass is quite irrelevant to the above calculation. Any other proportion by mass of argon and chlorine would give the same answers to (i) and (ii), provided the temperature remains unaltered.

95. 0.040g of He is kept in a closed container initially at 100.0°C. The container is now heated. Neglecting the expansion of the container, calculate the temperature at which the internal energy is increased by 12J.

Ans.:
$$m = 0.040 g, T = 100^{\circ} C, M_{He} = 4 g$$
 $U = \frac{3}{2} n Rt = \frac{3}{2} \times \frac{m}{M} \times RT, \ T' = ?$ Given $\frac{3}{2} \times \frac{m}{M} \times RT + 12 = \frac{3}{2} \times \frac{m}{M} \times RT'$ $\Rightarrow 1.5 \times 0.01 \times 8.3 \times 373 + 12 = 1.5 \times 0.01 \times 8.3 \times T'$ $\Rightarrow T' = \frac{58.4385}{0.1245} = 469.3855 K = 196.3^{\circ} C \approx 196^{\circ} C$

96. The temperature and pressure at Simla are 15.0°C and 72.0cm of mercury and at Kalka these are 35.0°C and 76.0cm of mercury. Find the ratio of air density at Kalka to the air density at Simla.

$$\begin{array}{l} \textbf{Ans.:} \ \mathsf{T} \ \mathsf{at} \ \mathsf{Simla} = 15^{\circ}\mathsf{C} = 15 + 273 = 288\mathsf{K} \\ \mathsf{P} \ \mathsf{at} \ \mathsf{Simla} = 72\mathsf{cm} = 72 \times 10^{-2} \times 13600 \times 9.8 \\ \mathsf{T} \ \mathsf{at} \ \mathsf{Kalka} = 35^{\circ}\mathsf{C} = 35 + 273 = 308\mathsf{K} \\ \mathsf{P} \ \mathsf{at} \ \mathsf{Kalka} = 76\mathsf{cm} = 76 \times 10^{-2} \times 13600 \times 9.8 \\ \mathsf{PV} = \mathsf{nRT} \\ \Rightarrow \mathsf{PV} = \frac{\mathsf{m}}{\mathsf{M}} \mathsf{RT} \Rightarrow \mathsf{PM} = \frac{\mathsf{m}}{\mathsf{V}} \mathsf{RT} \\ \Rightarrow \mathsf{f} = \frac{\mathsf{PM}}{\mathsf{RT}} \frac{\mathsf{fSimla}}{\mathsf{fKalka}} = \frac{\mathsf{P}_{Simla} \times \mathsf{M}}{\mathsf{RT}_{Simla}} \times \frac{\mathsf{RT}_{kalka}}{\mathsf{P}_{Kalka} \times \mathsf{M}} \\ = \frac{72 \times 10^{-2} \times 13600 \times 9.8 \times 308}{288 \times 76 \times 10^{-2} \times 13600 \times 9.8} = \frac{72 \times 308}{76 \times 288} = 1.013 \end{array}$$

$$\frac{\text{fKalka}}{\text{fSimla}} = \frac{1}{1.013} = 0.987$$

97. A balloon partially filled with Helium has a volume of 30m³, at the earth's surface, where pressure is 76cm of Hg and temperature is 27°C. What will be the increase in volume of gas if balloon rises to a height, where pressure is 7.6cm of Hg and temperature is -54°C?

$$\begin{aligned} &\text{Ans.:} \ \frac{P_1V_1}{T_1} = \frac{PV}{T} \\ &V_1 = \frac{PVT_1}{TP_1} \\ &= \frac{76 \times 30 \times (273 - 54)}{(273 + 27) \times 7.6} \\ &= 219m^3 \end{aligned}$$

Hence increase in volume

$$= V_1 - V$$
 $= 219 - 30$
 $= 189 \text{m}^3$

98. An air bubble of radius 2.0mm is formed at the bottom of a 3.3m deep river. Calculate the radius of the bubble as it comes to the surface. Atmospheric pressure = 1.0×10^5 Pa and density of water = 1000kg/m^{-3} .

$$\begin{split} &\text{Ans.:} \ P_1 = 10^5 + fgh = 10^5 + 1000 \times 10 \times 3.3 = 1.33 \times 10^5 pa \\ &P_2 = 10^5, \ T_1 = T_2 = T, \ V_1 = \frac{4}{3}\pi (2 \times 10^{-3})^3 \\ &V_2 = \frac{4}{3}\pi r^3, r = ? \\ &\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \\ &\Rightarrow \frac{1.33 \times 10^5 \times \frac{4}{3} \times \pi \times (2 \times 10^{-3})^3}{T_1} = \frac{10^5 \times \frac{4}{3} \times \pi r^2}{T_2} \\ &\Rightarrow 1.33 \times 8 \times 10^5 \times 10^{-9} = 10^5 \times r^3 \\ &\Rightarrow r = 3\sqrt{10.64 \times 10^{-3}} = 2.19 \times 10^{-3} \approx 2.2 mm \end{split}$$

- 99. State the law of equipartition of energy of a dynamic system and use it to find the values of internal energy and the ratio of the specific heats of (a) monoatomic, (b) diatomic, (c) triatomic gas molecules.
 - **Ans.:** Law of equipartition of energy: For any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all degrees of freedom and the energy associated with each molecule per degree of freedom is $\frac{1}{2}K_BT$, where K_B is Boltzmann's constant and T is the temperature of the system.
 - a. For monoatomic gas there are only three degrees of freedom. For a gas in thermal equilibrium at temperature T, the average value of translation energy of molecule is

$$(E_1) = \left(rac{1}{2}mv_x^2
ight) + \left(rac{1}{2}mv_y^2
ight) + \left(rac{1}{2}mv_z^2
ight)$$

Therefore, energy associated with monoatomic molecule is $\frac{3}{2}K_{B}T.$

Ratio of specific heat
$$\gamma=rac{C_P}{C_V}=rac{5}{3}=5:3$$

b. In case of diatomic gases, each molecule has two rotational degrees of freedom in addition to three translation degrees of freedom. Therefore, total energy of a diatomic gas molecule is sum of translation energy E_t and rotational energy E_r ,

i.e.,
$$E_t+E_r=\left(rac{1}{2}mv_x^2+rac{1}{2}mv_y^2+rac{1}{2}mv_z^2
ight) \ +\left(rac{1}{2}I_1\omega_1^2+rac{1}{2}I_2\omega_2^2
ight)$$

 ω_1,ω_2 and I_1,I_2 are angular speed about the axes and corresponding moments of inertia.

Ratio of specific heat $\gamma = \frac{C_P}{C_V} = 7:5$

c. **Triatomic gas:** Tri - atomic gas molecule has seven degrees of freedom.

Atoms oscillates along the interatomic axis contributing a vibrational energy, term E_{ν} to the total energy,

where
$$E_v = \frac{1}{2} m \Big(\frac{\mathrm{d}y}{\mathrm{d}t}\Big)^2 + \frac{1}{2} k y^2$$

The total energy of the gas molecule

$$E = E_t + E_r + E_v \label{eq:energy}$$

each vibrational mode contributes two squared terms, one of K.E. and the other for P.E. of the molecule.

Accordingly, each vibrational mode contribute $2 imes rac{1}{2}$

$$K_BT=K_BT$$
 to the total energy.

Specific heat ratio
$$\gamma = \frac{C_P}{C_V} = 9:7.$$

100. An ideal gas is kept in a long cylindrical vessel fitted with a frictionless piston of cross-sectional area 10cm^2 and weight 1kg. The length of the gas column in the vessel is 20cm. The atmospheric pressure is 100kPa. The vessel is now taken into a spaceship revolving round the earth as a satellite. The air pressure in the spaceship is maintained at 100kPa. Find the length of the gas column in the cylinder.

$$\begin{split} &\text{Ans.:} \ P_1 V_1 = P_2 V_2 \\ &\Rightarrow \left(\frac{mg}{A} + P_0\right) A \ell \ \ P_0 A \ell \\ &\Rightarrow \left(\frac{1 \times 9.8}{10 \times 10^{-4}} + 10^5\right) 0.2 = 10^5 \ell' \\ &\Rightarrow \left(9.8 \times 10^3 + 10^5\right) \times 0.2 = 10^5 \ell' \\ &\Rightarrow 109.8 \times 10^3 \times 0.2 = 10^5 \ell' \\ &\Rightarrow \ell' = \frac{109.8 \times 0.2}{10^2} = 0.2196 \approx 0.22 m \approx 22 cm \end{split}$$

101. Two molecules of a gas have speeds of 9 x 10^{16} ms⁻¹ and 1 x 10^{6} ms⁻¹ respectively. What is the root mean square speed of these molecules?

Ans.: rms speed for w-molecules is defined as:

$$v_{rms} = \sqrt{rac{v_1^2 + v_2^2 + v_3^2 + + v_n^2}{n}} \; [v_{rms} = {
m root \, mean \, square \, velocity}]$$

Where v_1 , v_2 , v_1 ,vn are individual velocities of n-molecules of the gas.

For two molecules,

According to the problem, $v_1=9\times 10^6 \text{m/s}$ and $v_2=1\times 10^6 \text{m/s}$

$$egin{align*} \therefore ext{v}_{
m rms} &= \sqrt{rac{(9 imes 10^6)^2 + (1 imes 10^6)}{2}} \ &= \sqrt{rac{81 imes 10^{12} + 1 imes 10^{12}}{2}} \ &= 10^6 \sqrt{rac{81 + 1}{2}} = \sqrt{41} imes 10^6 ext{ms}^{-1} \end{aligned}$$

102. A gaseous mixture contains 16g of helium and 16g of oxygen, then calculate the ratio of $\frac{C_p}{C_V}$ of the mixture.

Ans. : Moles of helium
$$(\mu_{\mathrm{He}}) = \frac{16}{4} = 4$$

Moles of oxygen
$$(\mu_{\mathrm{O}_2})=rac{16}{32}=rac{1}{2}$$

As helium is monoatomic, so degrees of freedom of helium, f = 3, so $C_{V_{\rm He}}=\frac{f}{2}R=\frac{3}{2}R$. As oxygen is diatomic, so degrees of freedom of oxygen f = 5, so

$$egin{aligned} & C_{V_{O_2}} = rac{f}{2}R = rac{5}{2}R \ & \therefore C_{V\, ext{mixture}} = rac{\mu_{ ext{He}}C_{V_{ ext{He}}} + \mu_{O_2}C_{V_{O_2}}}{\mu_{ ext{He}} + \mu_{O_2}} \ & = rac{4 imes rac{3}{2}R + rac{1}{2} imes rac{5}{2}R}{4 + rac{1}{2}} = rac{29}{18}R \ & \gamma = rac{C_P}{C_V} ext{ [of mixture]} \ & \gamma = rac{1}{2} + rac{R}{2} \ & \gamma = rac{1}{2} + rac{1}{2} + rac{R}{2} \ & \gamma = rac{1}{2} + rac{1}{2} + rac{R}{2} \ & \gamma = rac{1}{2} + rac{1}$$

$$egin{aligned} \gamma_{\mathrm{mixture}} &= 1 + rac{\mathrm{R}}{\mathrm{C}_{\mathrm{V}_{\mathrm{mixture}}}} \ &= 1 + rac{\mathrm{R}}{rac{29}{18}\mathrm{R}} = 1.62 \, ext{[as C}_{ extsf{P}} ext{-} \, extsf{C}_{ extsf{V}} = extsf{R} extsf{]} \end{aligned}$$

103. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T. Neglecting all vibrational modes, calculate the total internal energy of the system. (Oxygen has two rotational modes.)

Ans.: To find total energy of a given molecule of a gas we must find its degree of freedom. In molecule of oxygen it has 2 atom.

So it has degree of freedom 3T + 2R = 5, so total internal energy $=\frac{5}{2}RT$ per mole as gas O_2 is 2 mole

So total internal energy of 2 mole oxygen $=rac{2 imes 5}{2}RT=5RT$

Neon gas is mono atomic so its degree of freedom is only 3 hence total internal energy $=\frac{3}{2}RT$ per mole.

So, total internal energy of 4 mole Ne $=rac{4 imes3}{2} ext{RT}=6 ext{RT}$

Total internal energy of 2 mole oxygen and 4 mole Ne = 5RT + 6RT = 11RT

104. Two molecules of a gas have speeds of 9 x 10^{16} ms⁻¹ and 1 x 10^{6} ms⁻¹ respectively. What is the root mean square speed of these molecules?

Ans.: rms speed for w-molecules is defined as:

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots \ldots + v_n^2}{n}} \; [v_{\text{rms}} = \text{root mean square velocity}]$$

Where $v_1,\,v_2,\,v_1,\,.....v_n$ are individual velocities of n-molecules of the gas.

For two molecules,

According to the problem, $v_1 = 9 \times 10^6 \text{m/s}$ and $v_2 = 1 \times 10^6 \text{m/s}$

$$\begin{split} & \therefore v_{\rm rms} = \sqrt{\frac{(9 \times 10^6)^2 + (1 \times 10^6)}{2}} \\ & = \sqrt{\frac{81 \times 10^{12} + 1 \times 10^{12}}{2}} \\ & = 10^6 \sqrt{\frac{81 + 1}{2}} = \sqrt{41} \times 10^6 {\rm ms}^{-1} \end{split}$$

- 105. A ballon has 5.0g mole of helium at 7°C. Calculate.
 - a. The number of atoms of helium in the balloon,
 - b. The total internal energy of the system.

Ans.: For gas helium n = 5 mole

$$T = 7 + 273 = 280k$$

- a. Number of atoms of he is 5 mole = $5 \times 6.023 \times 10^{23}$ atoms
 - $= 30.115 \times 10^{23}$ atoms
 - $= 30.115 \times 10^{24}$ atoms.
- b. He atoms is mono atomic so degree of freedom is 3 So average kinetic energy

$$=\frac{3}{2}K_{B}T$$
 per molecule

$$=rac{3}{2}\mathrm{K_{B}T} imes$$
 Number of He Atom

$$=rac{3}{2} imes1.38 imes10^{-23} imes280 imes3.0115 imes10^{24}$$

Total E of 15 mole of He = 1.74×10^4 J

106. Explain why. There is no atmosphere on moon.

Ans. : As acceleration due to gravity on moon is 1/6th of g on earth. So the escape velocity on moon $V_{es}=\sqrt{2gR}=2.38km/s$

M = Mass of hydrogen, As H_2 is lightest gas m = $1.67 \times 10^{-24} kg$

$$m{v_{rms}} = \sqrt{rac{3K_BT}{m}} = \sqrt{rac{3 imes 1.38 imes 10^{-23} imes 300}{1.67 imes 10^{-24}}}$$

= 2.72 km/s

Due to small gravitational force and v_{rms} is greater than escape velocity so molecule of air can escape out.

As the distance of moon from sun is approximately equal to that of earth so the intensity of energy of sun reaches to moon is larger due to lower density of atmosphere, distance become smaller than earth when moon is towards sun during its rotation around earth.

Due to this (sun light), rms speed of molecule increase and some of them can speed up more than escape velocity and so probability of escaping out increased.

Hence over a long time moon has lost most of its atmosphere.

107. Consider a rectangular block of wood moving with a velocity v_0 in a gas at temperature T and mass density ρ . Assume the velocity is along x-axis and the area of cross-section of the block perpendicular to v_0 is A. Show that the drag force on

the block is $4 \rho \ A v_0 \sqrt{rac{\mathrm{KT}}{\mathrm{m}}}$ where m is the mass of the gas molecule.

Ans.: Let $ho_{
m m}$ is the number of molecule per unit volume i.e. $ho_{
m m}$ is molecular density per unit volume.

 $v = v_{rms}$ is velocity of gas molecules

When box moves in gas the molecules of gas strike to front face in opposite direction and on back face in same direction as $v >> v_0$ (box) so relative velocity on back face = $(v - v_0)$

Change in momentum by a molecule on front face = $2m(v + v_0)$

Change in momentum by a molecule on back side = $2m(v - v_0)$

Number of molecule striking on front face in Δt $time = \frac{1}{2}$ volume x molecular density/ vol.

To front face

$$=rac{1}{2}[\mathrm{A.}\,(\mathrm{v}+\mathrm{v}_0)\Delta\mathrm{t}]
ho_\mathrm{m}$$

Number of molecules striking to front face,

$$N_F = \frac{1}{2}(v + v_0)A
ho_m \Delta t$$

Similarly as the speed of molecule and block are same so number of molecule striking on backend face,

$$m N_B = rac{1}{2}(v-v_0)A~
ho_m \Delta t$$

Total change in momentum due to striking the molecule on front face,

$$\mathrm{P_F} = 2\mathrm{m}(\mathrm{v} + \mathrm{v_0})\mathrm{N_F} = 2\mathrm{m}(\mathrm{v} + \mathrm{v_0}) imes rac{1}{2}(\mathrm{v} + \mathrm{v_0})\mathrm{A} \;
ho_\mathrm{m} \Delta \mathrm{t}$$

$$P_{\mathrm{F}} = -m(v+v_0)^2 \; A
ho_{\mathrm{m}} \Delta t$$
 (Backward direction)

So rate of change of momentum on front face is equal to the force,

$$F_{\mathrm{F}} = -m(v+v_0)^2 \; A
ho_{\mathrm{m}}$$
 in Backward direction.

Similarly force on back end ${
m F_B}=+{
m m}({
m v}-{
m v_0})^2~{
m A}
ho_{
m m}$

Net dragging force $=-\mathrm{m}(\mathrm{v}+\mathrm{v}_0)^2\;\mathrm{A}
ho_\mathrm{m}+\mathrm{m}(\mathrm{v}-\mathrm{v}_0)^2\;\mathrm{A}
ho_\mathrm{m}$

$$=-{
m mA}
ho_{
m m}[({
m v}+{
m v}_0)^2-({
m v}-{
m v}_0)^2]$$

$$=-\mathrm{mA}
ho_{\mathrm{m}}[\mathrm{v}^{2}+\mathrm{v}_{0}^{2}+2\mathrm{v}\mathrm{v}_{0}-(\mathrm{v}^{2}+\mathrm{v}_{0}^{2}-2\mathrm{v}.\,\mathrm{v}_{0}]$$

$$=-\mathrm{mA}
ho_\mathrm{m}\,4\mathrm{v.v_0}$$

So magnitude of dragging force due to gas molecule $=4 m v v_0 A P_m$

KE of gas molecule.

$$= \tfrac{1}{2} m v^2 = \tfrac{3}{2} K_B T$$

$$\therefore$$
 $m v = \sqrt{rac{K_B T}{m}}$ [using equ. (A) of Q.13.30]

... Dragging force becomes,

$$= 4m\; A P_m v_0 \sqrt{\frac{K_B T}{m}}$$

108. We have 0.5g of hydrogen gas in a cubic chamber of size 3cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100atm. Is one justified in assuming the ideal gas law, in the final state? (Hydrogen molecules can be consider as spheres of radius 1 A).

Ans.: Volume of 1 molecule

$$=\frac{4}{3}\pi r^3=\frac{4}{3}\times 3.14\times (10^{-10})^3$$

$$m r=1A=10^{-10} m$$
 (Given)

... Volume of 1 molecule = $4 \times 1.05 \times 10^{-30} \text{m}^3 = 4.20 \times 10^{-30} \text{m}^3$

Number of mole in $0.5 \mathrm{g~H_2gas} = \frac{0.5}{2} = 0.25$ mole [.: H $_2$ has 2 mole]

... Volume of H₂ molecules in .25 mole

$$= 0.25 \times 6.023 \times 10^{23} \times 4.2 \times 10^{-30} \text{m}^3$$

$$= 1.05 \times 6.023 \times 10^{+23-30}$$

$$= 6.324 \times 10^{+23-30}$$

Volume of H_2 molecules = $6.3 \times 10^{-7} \text{m}^3$

Now for ideal gas at constant temperature

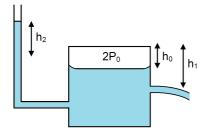
$$P_tV_t = P_fV_f$$

$$V_f=rac{P_tV_t}{P_f}=rac{1}{100} imes(3 imes10^{-2})^3$$
 [:: vol. of cube Vt = (side)3 and Pt = 1atm at NTP]

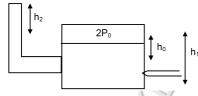
$$V_{\rm f} = rac{27 imes 10^{-5}}{100} = 2.7 imes 10^{-5-2} = 2.7 imes 10^{-7} m^3$$

Hence on compression the volume of the gas of the order of nuclear force of interaction will play the role, as in kinetic theory of gas molecules do not interact each other so gas will not obey the ideal gas behavior.

- 109. Figure. shows a large closed cylindrical tank containing water. Initially the air trapped above the water surface has a height h_0 and pressure $2p_0$ where p_0 is the atmospheric pressure. There is a hole in the wall of the tank at a depth h_1 below the top from which water comes out. A long vertical tube is connected as shown.
 - a. Find the height h₂ of the water in the long tube above the top initially.
 - b. Find the speed with which water comes out of the hole.
 - c. Find the height of the water in the long tube above the top when the water stops coming out of the hole.



Ans.:



a. $2P_0x=(h_2+h_0)fg$ [. Since liquid at the same level have same pressure] $\Rightarrow 2P_0=h_2fg+h_0fg$

$$\Rightarrow \mathrm{h_2fg} = \mathrm{2P_0} - \mathrm{h_0fg}$$

$$ho_2 = rac{2P_0}{fg} - rac{h_0 fg}{fg} = rac{2P_0}{fg} - h_0$$

b. K.E. of the water = Pressure energy of the water at that layer

$$\Rightarrow \tfrac{1}{2} m V^2 = m \times \tfrac{P}{f}$$

$$\begin{split} \Rightarrow V^2 &= \tfrac{2P}{f} = \left[\tfrac{2}{f(P_0 + fg)(h_1 - h_0)} \right] \\ \Rightarrow V &= \left[\tfrac{2}{f(P_0 + fg)(h_1 - h_0)} \right]^{\tfrac{1}{2}} \\ \mathsf{c.} \quad & (\mathbf{x} + P_0)f\mathbf{h} = 2P_0 \\ & \therefore 2P_0 + fg\left(\mathbf{h} - \mathbf{h}_0\right) = P_0 + fg\mathbf{x} \\ & \therefore \mathbf{X} = \tfrac{P_0}{fg + h_1 - h_0} = \mathbf{h}_2 + \mathbf{h}_1 \end{split}$$

 \therefore i.e. x is h_1 meter below the top

- \Rightarrow x is -h₁ above the top
- 110. A bucket full of water is placed in a room at 15°C with initial relative humidity 40%. The volume of the room is 50m³.
 - a. How much water will evaporate?
 - b. If the room temperature is increased by 5°C, how much more water will evaporate? The saturation vapour pressure of water at 15°C and 20°C are 1.6kPa and 2.4kPa respectively.

Ans.:

a. Rel. humidity
$$=rac{
m VP}{
m SVP~at~15^{\circ}C} \Rightarrow 0.4 = rac{
m VP}{1.6 imes10^{3}}$$
 $\Rightarrow {
m VP} = 0.4 imes1.6 imes10^{3}$

The evaporation occurs as along as the atmosphere does not become saturated. Net pressure change = 1.6×10^3 - $0.4 \times 1.6 \times 10^3$ = $(1.6 - 0.4 \times 1.6)10^3$ = 0.96×10^3

Net mass of water evaporated $=m\Rightarrow0.96\times10^3\times50=\frac{m}{18}\times8.3\times288$ $\Rightarrow m=\frac{0.96\times50\times18\times10^3}{8.3\times288}=361.45\approx361g$

b. At 20°C SVP = 2.4KPa, At 15°C SVP = 1.6KPa
Net pressure charge =
$$(2.4 - 1.6) \times 10^3$$
Pa = 0.8×10^3 Pa

Mass of water evaporated = $m'=0.8 \times 10^3~50=rac{m'}{18} \times 8.3 \times 293$

$$\Rightarrow$$
 m' = $\frac{0.8 \times 50 \times 18 \times 10^3}{8.3 \times 293}$ = 296.06 $pprox$ 296grams

111. Figure. shows a cylindrical tube of length 30cm which is partitioned by a tight-fitting separator. The separator is very weakly conducting and can freely slide along the tube. Ideal gases are filled in the two parts of the vessel. In the beginning, the temperatures in the parts A and B are 400K and 100K respectively. The separator slides to a momentary equilibrium position shown in the figure. Find the final equilibrium position

400 K A 100 K B

of the separator, reached after a long time.

Ans.: The middle wall is weakly conducting. Thus after a long time the temperature of both the parts will equalise.



← 3	× ←	30 – x →
Т	Ρ'	T P'

The final position of the separating wall be at distance x from the left end. So it is at a distance 30 - x from the right end Putting combined gas equation of one side of the separating wall,

$$\begin{split} &\frac{P_1 \times V_1}{T_1} = \frac{P_2 \times V_2}{T_2} \\ &\Rightarrow \frac{P \times 20 A}{400} = \frac{P' \times A}{T} \dots (1) \\ &\Rightarrow \frac{P \times 10 A}{100} = \frac{-P'(30-x)}{T} \dots (2) \\ &\text{Equating (1) and (2)} \\ &\Rightarrow \frac{1}{2} = \frac{x}{30-x} \end{split}$$

 $\Rightarrow 30 - x = 2x \Rightarrow 3x = 30 \Rightarrow x = 10cm$

The separator will be at a distance 10cm from left end.

112. Air is pumped into the tubes of a cycle rickshaw at a pressure of 2 atm. The volume of each tube at this pressure is 0.002m^3 . One of the tubes gets punctured and the volume of the tube reduces to 0.0005m^3 . How many moles of air have leaked out? Assume that the temperature remains constant at 300K and that the air behaves as an ideal gas.

Ans.:
$$P_1 = 2atm = 2 \times 10^5 pa$$
 $V_1 = 0.002m^3, T_1 = 300K$ $P_1V_1 = n_1RT_1$ $\Rightarrow n = \frac{P_1V_1}{RT_1} = \frac{2 \times 10^5 \times 0.002}{8.3 \times 300} = \frac{4}{8.3 \times 3} = 0.1606$ $P_2 = 1atm = 10^5 pa$ $V_2 = 0.0005m^3, T_2 = 300K$ $P_2V_2 = n_2RT_2$ $\Rightarrow n_2 = \frac{P_2V_2}{RT_2} = \frac{10^5 \times 0.0005}{8.3 \times 300} = \frac{5}{3 \times 8.3} \times \frac{1}{10} = 0.02$ Δn moles leaked out = 0.16 - 0.02 = 0.14.

113. The temperature and pressure at Simla are 15.0°C and 72.0cm of mercury and at Kalka these are 35.0°C and 76.0cm of mercury. Find the ratio of air density at Kalka to the air density at Simla.

Ans.: T at Simla =
$$15^{\circ}$$
C = $15 + 273 = 288$ K P at Simla = 72 cm = $72 \times 10^{-2} \times 13600 \times 9.8$ T at Kalka = 35° C = $35 + 273 = 308$ K P at Kalka = 76 cm = $76 \times 10^{-2} \times 13600 \times 9.8$ PV = nRT
$$\Rightarrow PV = \frac{m}{M}RT \Rightarrow PM = \frac{m}{V}RT$$

$$\Rightarrow f = \frac{PM}{RT} \frac{f \text{Simla}}{f \text{Kalka}} = \frac{P_{\text{Simla}} \times M}{RT_{\text{Simla}}} \times \frac{RT_{\text{kalka}}}{P_{\text{Kalka}} \times M}$$

$$= \frac{72 \times 10^{-2} \times 13600 \times 9.8 \times 308}{288 \times 76 \times 10^{-2} \times 13600 \times 9.8} = \frac{72 \times 308}{76 \times 288} = 1.013$$

$$\frac{f \text{Kalka}}{f \text{Simla}} = \frac{1}{1.013} = 0.987$$

114. The temperature and humidity of air are 27°C and 50% on a particular day. Calculate the amount of vapour that should be added to 1 cubic metre of air to saturate it. The

saturation vapour pressure at $27^{\circ}C = 3600Pa$.

Ans.:
$$\mathrm{RH} = \frac{\mathrm{VP}}{\mathrm{SVP}}$$

Given,
$$0.50=rac{
m VP}{3600}$$

$$\Rightarrow \text{VP} = 3600 \times 0.5$$

Let the Extra pressure needed be P

So,
$$P=rac{m}{M} imesrac{RT}{V}=rac{m}{18} imesrac{8.3 imes300}{1}$$

So, $P=\frac{m}{M} imes\frac{RT}{V}=\frac{m}{18} imes\frac{8.3 imes300}{1}$ Now, $\frac{m}{18} imes8.3 imes300+3600 imes0.50=3600$ [air is saturated i.e. RH = 100% = 1 or VP =

$$ightarrow \mathrm{m} = \left(rac{36-18}{8.3}
ight) imes 6 = 13\mathrm{g}$$

115. Air is pumped into an automobile tyre's tube up to a pressure of 200kPa in the morning when the air temperature is 20°C. During the day the temperature rises to 40°C and the tube expands by 2%. Calculate the pressure of the air in the tube at this temperature.

Ans.:
$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$P_1 \to 200 \text{KPa} = 2 \times 10^5 \text{pa}, P^2 = ?$$

$$T_1 = 20^{\circ}C = 293K, T_2 = 40^{\circ}C = 313K$$

$$V_2 = V_1 + 2\%V_1 = rac{102 imes V_1}{100}$$

$$\Rightarrow \frac{2 \times 10^5 \times V_1}{293} = \frac{P_2 \times 102 \times V_1}{100 \times 313}$$

$$\Rightarrow$$
 P₂ = $\frac{2 \times 10^7 \times 313}{102 \times 293}$ = 209462Pa = 209.462KPa

Case study based questions

- [12]
- 116. On a winter day, the outside temperature is 0°C and relative humidity 40%. The air from outside comes into a room and is heated to 20°C. What is the relative humidity in the room? The saturation vapour pressure at 0°C is 4.6mm of mercury and at 20°C it is 18mm of mercury.

Ans.: Relative humidity = 40%

$$SVP = 4.6$$
mm of Hg

$$0.4 = \frac{\text{VP}}{4.6} \Rightarrow \text{VP} = 0.4 \times 4.6 = 1.84$$

$$\frac{P_1V}{T_1} = \frac{P_2V}{T_2} \Rightarrow \frac{1.84}{273} = \frac{P_2}{293}$$

$$\Rightarrow$$
 P₂ = $\frac{1.84}{273}$ \times 293

Relative humidity at 20°C

$$=\frac{\text{VP}}{\text{SVP}}=\frac{1.84\times293}{273\times10}=0.109=10.9\%$$

117. The human body has an average temperature of 98°F. Assume that the vapour pressure of the blood in the veins behaves like that of pure water. Find the minimum atmospheric pressure which is necessary to prevent the blood from boiling. Use

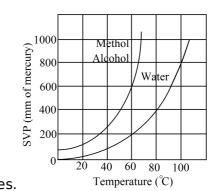


figure. of the text for the vapour pressures.

Ans.: The temp. of body is $98^{\circ}F = 37^{\circ}C$

At 37°C from the graph SVP = Just less than 50mm

B.P. is the temp. when atmospheric pressure equals the atmospheric pressure.

Thus min. pressure to prevent boiling is 50mm of Hg.

118. On a winter day, the outside temperature is 0°C and relative humidity 40%. The air from outside comes into a room and is heated to 20°C. What is the relative humidity in the room? The saturation vapour pressure at 0°C is 4.6mm of mercury and at 20°C it is 18mm of mercury.

Ans.: Relative humidity = 40%

SVP = 4.6mm of Hg

$$0.4 = \frac{\text{VP}}{4.6} \Rightarrow \text{VP} = 0.4 \times 4.6 = 1.84$$

$$\frac{P_1 V}{T_1} = \frac{P_2 V}{T_2} \Rightarrow \frac{1.84}{273} = \frac{P_2}{293}$$

$$\Rightarrow \mathrm{P}_2 = rac{1.84}{273} imes 293$$

Relative humidity at 20°C

$$=rac{ ext{VP}}{ ext{SVP}}=rac{1.84 imes293}{273 imes10}=0.109=10.9\%$$

----- "Itni shiddat se maine tumhe (success) paane ki koshish ki hai,ki har zarre ne mujhe tumse milane ki saazish ki hai -----