

**KD EDUCATION ACADEMY [9582701166] STREET NO.
21 A-1 BLOCK BENGALI COLONY SANT NAGAR
BURARI DELHI -110084**

Time : 3 Hour

STD 11 Science class 11 physics
kd 90+ questions ch-1 units and measurement

Total Marks : 120

*** Choose The Right Answer From The Given Options.[1 Marks Each]**

[45]

1. Find the value of $12.9\text{g} - 7.05\text{g}$.

(A) 5.84g

(B) 5.8g

(C) 5.86g

(D) 5.9g

Ans. :

b. 5.8g

2. Which of the following are not a unit of time?

(A) Second.

(B) Parsec.

(C) Year.

(D) Light year.

Ans. :

b. Parsec.

d. Light year.

Explanation:

Parsec and light year are those practical units which are used to measure large distances.

For example:

The distance between sun and earth or other celestial bodies. So they are the units of length not time. Here, second and year represent time.

1 light year (distance that light travels in 1 year with speed = $3 \times 10^8\text{m/s}$) = $9.46 \times 10^{11}\text{m}$ And 1 par see = $3.08 \times 10^{16}\text{m}$

3. A device which is used for measurement of length to an accuracy of about 10^{-4}m , is:

(A) Screw gauge.

(B) Spherometer.

(C) Vernier callipers.

(D) Either (a) or (b).

Ans. :

d. Either (a) or (b).

4. Pascal is the unit of:

(A) Force.

(B) Stress.

(C) Work.

(D) Energy.

Ans. :

b. Stress.

5. Which of the following is not a physical quantity?

(A) Time.

(B) Impulse.

(C) Mass.

(D) Kilogram.

Ans. :

d. Kilogram.

6. If R and L represent resistance and self-inductance respectively, which of the following combinations has the dimensions of frequency?

(A) $\frac{R}{L}$

(B) $\frac{L}{R}$

(C) $\sqrt{\frac{R}{L}}$

(D) $\sqrt{\frac{L}{R}}$

Ans. :

a. $\frac{R}{L}$

7. The dimensional formula for latent heat is:

(A) $M^0L^2T^{-1}$

(B) ML^2T^{-1}

(C) MLT^{-2}

(D) ML^2T^{-2}

Ans. :

a. $M^0L^2T^{-1}$

8. If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

(A) $\frac{(P-Q)}{R}$

(B) $\frac{PQ}{R}$

(C) $\frac{(PR-Q^2)}{R}$

(D) $\frac{(R+Q)}{P}$

Ans. :

a. $\frac{(P-Q)}{R}$

e. $\frac{(R+Q)}{P}$

Explanation:

In option (a) and (e) there is term (P - Q) and (R + Q) as different physical quantities can never be added or subtracted so option (a) and (e) can never be meaningful.

In option (b), the dimension of PQ may be equal to dimension of R so option (b) can be possible. Similarly dimensions of PR and Q^2 may be equal and gives the possibility of option (d).

In option (c), there is no addition subtraction gives the possibilities of option (c).

Hence, verifies the right option (a) and (e).

9. When 1m, 1kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system?

(A) 10^5 dyne

(B) 10^3 dyne

(C) 10^8 dyne

(D) 10^4 dyne

Ans. :

b. 10^3 dyne

Explanation:

As, dimensional formula of force = $[MLT^{-2}]$

$n_1 = 36, M_1 = 1\text{kg}, L_1 = 1\text{m}, T_1 = 1\text{min} = 60\text{s}$

$n_2 = ?, M_2 = 1\text{g}, L_2 = 1\text{cm}, T_2 = 1\text{s}$

So, conversion of 36 units into CGS system

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$n_2 = n_1 \left[\frac{1\text{Kg}}{1\text{g}} \right]^1 \left[\frac{1\text{m}}{1\text{cm}} \right]^1 \left[\frac{1\text{min}}{1\text{s}} \right]^c$$

$$= 36 \left[\frac{1000\text{g}}{1\text{g}} \right] \left[\frac{100\text{cm}}{1\text{cm}} \right]^1 \left[\frac{60\text{s}}{1\text{s}} \right]^{-2} = 10^3 \text{ dyne}$$

10. Which of the following time measuring devices is most precise?

(A) A wall clock.

(B) A stop watch.

(C) A digital watch.

(D) An atomic clock. Give reason for your answer.

Ans. :

d. An atomic clock.

Explanation:

The least count of a wall clock, stop watch, digital watch and atomic clock are 1 sec,

$\frac{1}{10}$ sec, $\frac{1}{100}$ sec and $\frac{1}{10^{13}}$ sec respectively. So atomic clock is most precise.

11. Which of the following is a dimensional constant?

(A) Refractive index.

(B) Dielectric constant

(C) Relative density.

(D) Gravitational constant.

Ans. :

d. Gravitational constant.

12. The pair(s) of physical quantities that have the same dimensions is (are):

(A) Volumetric strain and coefficient of friction.

(B) Disintegration constant of a radioactive substance and frequency of light wave.

(C) Heat capacity and gravitational potential.

(D) Planck's constant and torque.

Ans. :

a. Volumetric strain and coefficient of friction.

b. Disintegration constant of a radioactive substance and frequency of light wave.

c. Heat capacity and gravitational potential.

Explanation:

a. Volumetric strain $\frac{\Delta V}{V} = \frac{L^3}{L^3} = 1$

Coefficient of friction $\mu = \frac{F}{R} = \frac{MLT^{-2}}{MLT^{-2}} = 1$

$\lambda = \frac{0.693}{T} = T^{-1}$

b. $v = \frac{1}{T} = T^{-1}$

Both have same dimensions.

c. Heat capacity is measured in cal/kg and gravitational potential is measured in joule/ kg. Both have the same dimensions $[L^2T^{-2}]$.

13. Percentage errors in the measurement of mass and speed are 2% and 3%, respectively. The error in the estimation of kinetic energy obtained by measuring mass and speed will be:

(A) 8%

(B) 2%

(C) 12%

(D) 10%

Ans. : Kinetic energy, $K = \frac{1}{2}mv^2$

$$\therefore \frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{2\Delta v}{v} \times 100$$

$$= 2\% + 2 \times 3\% = 8\%$$

14. The length and breadth of a metal sheet are 3.124m and 3.002m respectively. The area of this sheet up to four correct significant figures is:

(A) 9.37m²

(B) 9.378m²

(C) 9.3782m²

(D) 9.378248m²

Ans. :

b. 9.378m²

Explanation:

As area = length × breadth, therefore, as per rules numerical value of area has four significant digits.

15. The dimensions of entropy are:

(A) $[M^0L^{-1}T^0K]$

(B) $[M^0L^{-2}T^0k^2]$

(C) $[MLT^{-2}K]$

(D) $[ML^2T^{-2}k^{-1}]$

Ans. :

d. $[ML^2T^{-2}k^{-1}]$

16. Which of the following has neither units nor dimensions?

(A) Angle.

(B) Energy.

(C) Relative density.

(D) Relative velocity.

Ans. :

c. Relative density.

17. The quantity having the same unit in all system of unit is:

(A) Mass.

(B) Time.

(C) Length.

(D) Temperature.

Ans. :

b. Time.

Explanation:

Time is the quantity which has the same unit in all systems of unit, i.e. second. Other three quantities, i.e. mass, length and temperature have different units in different systems.

18. Obtain the dimensional equation for universal gas constant.

(A) $[ML^2T^{-2}] \text{ mol}^{-1}K^{-1}$

(B) $[M^2LT^{-1} \text{ mol}^{-2}K^{-2}]$

(C) $[ML^2LT^{-1} \text{ mol}^{-1}K^{-1}]$

(D) $[ML^3LT^{-1} \text{ mol}^{-1}K^{-2}]$

Ans. :

a. $[ML^2T^{-2}] \text{ mol}^{-1}K^{-1}$

Explanation:

According to ideal gas equation for universal gas constant.

i.e., $pV = nRT$, where n is the number of moles of gases.

$$R = \frac{(p)(V)}{(n)(T)} = \frac{[ML^{-1}T^2][L^3]}{[mol][K]} \\ = [ML^2T^{-2} \text{ mol}^{-1}K^{-1}]$$

19. In the formula $x = 3yz^2$, x and z have dimensions of capacitance and magnetic induction, respectively. The dimensions of y in MKS system are:

(A) $[M^{-2}L^{-2}T^4A^4]$

(B) $[M^{-3}L^{-3}T^4A^5]$

(C) $[M^{-3}L^{-2}T^8A^4]$

(D) $[M^{-1}L^{-4}T^2A^4]$

Ans. :

c. $[M^{-3}L^{-2}T^8A^4]$

Explanation:

Given, $[x] = \text{capacitance} = [M^{-1}L^{-2}T^4A^2]$

$[Z] = \text{magnetic induction} [MA^{-1}T^{-2}]$

$$\text{So, } [y] = \frac{[M^{-1}L^{-2}T^4A^2]}{[MA^{-1}T^{-2}]^2} = [M^{-3}L^{-2}T^8A^4]$$

20. If the length of a rectangle $l = 10.5\text{cm}$, breadth $b = 2.1\text{cm}$ and minimum possible measurement by scale $= 0.1\text{cm}$, then the area is:

(A) 22.0cm^2 (B) 21.1cm^2
(C) 22.05cm^2 (D) 22cm^2

Ans. : Area of rectangle, $A = \text{Length} \times \text{Breadth}$

$$A = lb = 10.5 \times 2.1 = 22.05\text{cm}^2$$

Minimum possible measurement of scale $= 0.1\text{cm}$. So, area measured by scale $= 22.0\text{cm}^2$.

21. The number of particles crossing per unit area perpendicular to X-axis in unit time is: $N = -D \frac{n_2 - n_1}{x_2 - x_1}$ Where n_1 and n_2 are number of particles per unit volume for the value of x_1 and x_2 respectively. The dimensions of diffusion constant D are:

(A) M^0LT^2 (B) $M^0L^2T^{-4}$
(C) M^0LT^{-3} (D) $M^0L^2T^{-1}$

Ans. :

d. $M^0L^2T^{-1}$

Explanation:

Since N is no. of particles per unit area per unit time, thus its dimensions are $M^0L^2T^{-1}$.

n_1 and n_2 are no. of particles per unit volume. Thus they have the dimensions M^0L^{-3} .

x has dimension of length, i.e L .

Thus, D has dimensions of $\frac{Nx}{n} = \frac{M^0L^2T^{-1}L}{M^0L^{-3}} = L^2T^{-1}$

22. Which of the following statement is incorrect regarding mass?

(A) It is a basic property of matter.
(B) The SI unit of mass is candela.
(C) The mass of an atom is expressed in u.
(D) None of the above.

Ans. :

b. The SI unit of mass is candela.

23. The mass and volume of a body are 4.237g and 2.5cm^3 , respectively. The density of the material of the body in correct significant figures is:

(A) 1.6048gcm^{-3} .
(B) 1.69gcm^{-3} .

- (C) 1.7gcm^{-3} .
(D) 1.695gcm^{-3} .

Ans. :

c. 1.7gcm^{-3} .

Explanation:

The significant figures in given numbers 4.237g and 2.5cm^3 are four and two respectively so result must have only two significant figures.

Density = $\frac{\text{mass}}{\text{volume}} = \frac{4.237}{2.5}$, Density = $1.6948 = 1.7\text{gcm}^{-3}$ rounding off upto 2 significant figures.

24. The number of significant figures in 3400 is:

- (A) 3 (B) 4
(C) 2 (D) 1

Ans. :

c. 2

Explanation:

In $x = 3400$, zero are not significant. Therefore, number of significant figure = 2.

25. If the value of force is 100N and value of acceleration is 0.001ms^{-2} , what is the value of mass in this system of units?

- (A) 10^3kg (B) 10^4kg
(C) 10^5kg (D) 10^6kg

Ans. :

c. 10^5kg .

26. Which of the following has same dimension as that of Planck constant?

- (A) Work.
(B) Linear momentum.
(C) Angular momentum.
(D) Impulse.

Ans. :

c. Angular momentum.

Explanation:

$$\text{As, } E = hv \text{ or } h = \frac{E}{\nu} = \left[\frac{\text{ML}^2\text{T}^{-2}}{\text{T}^{-1}} \right] = [\text{ML}^2\text{T}^{-1}]$$

$$\text{Angular momentum} = mvr = [\text{M}][\text{LT}^{-1}][\text{L}] = [\text{ML}^2\text{T}^{-1}]$$

27. In 4700m , significant digits are:

- (A) 2 (B) 3
(C) 4 (D) 5

Ans. :

a. 2

28. If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula:

- (A) $(P^1 A^{-1} T^1)$. (B) $(P^2 A^1 T^1)$.
 (C) $(P^1 A^{\frac{-1}{2}} T^1)$. (D) $(P^1 A^{\frac{1}{2}} T^{-1})$.

Ans. :

d. $(P^1 A^{\frac{1}{2}} T^{-1})$.

Explanation:

According to the problem, fundamental quantities are momentum (p), area (A) and time (T) and we have to express energy in these fundamental quantities.

Let energy E,

$$E \propto p^a A^b T^c \Rightarrow E = k p^a A^b T^c$$

where, k is dimensionless constant of proportionality.

Dimensional formula of energy, $[E] = [ML^2 T^{-2}]$ and $[p] = [MLT^{-1}]$

$$[A] = [L^2], [T] = [T] \text{ and } [E] = [K][p]^a [A]^b [T]^c$$

Putting all the dimensions, we get

$$ML^2 T^{-2} = [MLT^{-1}]^a [L^2]^b [T]^c$$

$$M^a L^{a+2b} T^{-a+c}$$

According to the principle of homogeneity of dimensions, we get,

$$a = 1 \quad \dots (i)$$

$$a + 2b = 2 \quad \dots (ii)$$

$$-a + c = -2 \quad \dots (iii)$$

By solving these equations (i), (ii) and (iii), we get

$$a = 1, b = \frac{1}{2}, c = -1$$

Dimensional formula for E is $[p^1 A^{\frac{1}{2}} t^{-1}]$.

29. In the standard equation $S_{nth} = u + \frac{a}{2}[2n - 1]$, what dimensions do you view for S_{nth} ?

- (A) $M^0 L^1 T^0$ (B) $M^0 L^{-1} T^1$
 (C) $M^0 L^1 T^{-1}$ (D) $M^0 L^0 T^1$

Ans. :

c. $M^0 L^1 T^{-1}$

30. In the gas equation $\left(p + \frac{a}{\sqrt{v}}\right)(V - b) = RT$ the dimensions of a are:

- (A) $[ML^3 T^{-2}]$
 (B) $[M^{-1} L^3 T^{-1}]$
 (C) $[ML^5 T^{-2}]$
 (D) $[M^{-1} L^{-5} T^2]$

Ans. :

c. $[ML^5 T^{-2}]$

31. Give $\text{force} = \frac{\alpha}{\text{Density} + \beta}$ What are the dimensions of α, β

- (A) $[\text{ML}^2\text{T}^{-2}][\text{ML}^{-\frac{1}{3}}]$
- (B) $[\text{M}^2\text{L}^4\text{T}^{-2}], [\text{M}^{\frac{1}{3}}\text{L}^{-1}]$
- (C) $[\text{M}^2\text{L}^{-2}\text{T}^{-2}][\text{M}^{\frac{1}{3}}\text{L}^{-1}]$
- (D) $[\text{M}^2\text{L}^{-2}\text{T}^{-2}][\text{ML}^{-2}]$

Ans. :

c. $[\text{M}^2\text{L}^{-2}\text{T}^{-2}][\text{M}^{\frac{1}{3}}\text{L}^{-1}]$

Explanation:

Dimensions of $\beta^3 = \text{Dimensions of density} = [\text{ML}^{-3}]$

$$\beta = [\text{M}^{\frac{1}{3}}\text{L}^{-1}]$$

$$\begin{aligned}\text{Also, } \alpha &= \text{Force} \times \text{Density} = [\text{MLT}^{-2}][\text{ML}^{-3}] \\ &= [\text{M}^2\text{L}^{-2}\text{T}^{-2}]\end{aligned}$$

32. A dimensionless quantity:

- (A) May have a unit.
- (B) Never has a unit.
- (C) Always has a unit.
- (D) Doesn't exist.

Ans. :

- a. May have a unit.

Explanation:

A dimensionless quantity may have a unit. For example, angle has a unit but is dimensionless.

33. Which physical quantities have same dimension?

- (A) Force and power.
- (B) Torque and energy.
- (C) Torque and power.
- (D) Force and torque.

Ans. :

- b. Torque and energy.

34. Number of degrees present in one radian is:

- (A) 58°
- (B) 57.3°
- (C) 56.3°
- (D) 56°

Ans. :

- b. 57.3°

Explanation:

We know that,

$$\pi \text{ radian} = 180^\circ$$

$$1 \text{ radian} = \frac{180}{\pi} = \frac{180}{22} \times 7 = 57.3^\circ$$

35. 'Parsec' is the unit of:

- (A) Time.
- (B) Distance.
- (C) Frequency.
- (D) Angular acceleration.

Ans. :

- b. Distance.

36. The surface area of a solid cylinder of radius 2.0cm and height A cm is equal to $1.5 \times 10^4 (\text{mm})^2$. Here, A refers to:

- (A) 0.9cm
- (B) 10cm
- (C) 30cm
- (D) 15cm

Ans. :

- b. 10cm.

37. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give:

- (A) 2.75 and 2.74.
- (B) 2.74 and 2.73.
- (C) 2.75 and 2.73.
- (D) 2.74 and 2.74.

Ans. :

- d. 2.74 and 2.74.

Explanation:

Key concept: While rounding off measurements, we use the following rules by convention:

- i. If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
- ii. If the digit to be dropped is more than 5, then the preceding digit is raised by one.
- iii. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.
- iv. If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.
- v. If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

Units and Measurements,

Let us round off 2.745 to 3 significant figures.

Here the digit to be dropped is 5, then preceding digit is left unchanged, if it is even.

Hence on rounding off 2.745, it would be 2.74.

Now consider 2.737, here also the digit to be dropped is 5, then the preceding digit is raised by one, if it is odd. Hence on rounding off 2.735 to 3 significant figures, it

would be 2.74.

38. SI unit of capacitance is:

- (A) ohm-second.
- (B) Wb.
- (C) coulomb (volt)⁻¹
- (D) A-m²

Ans. :

- c. coulomb (volt)⁻¹

Explanation:

SI unit of capacitance is coulomb (volt)⁻¹. However, ohm-second is the unit of inductance, Wb is the unit of magnetic flux and A-m² is the unit of magnetic moment.

39. Dimensions of gravitational constant are:

- (A) M⁻¹L³T⁻²
- (B) M⁻²L³T⁻¹
- (C) M³L⁻¹T⁻²
- (D) M⁻¹L²T⁻³

Ans. :

- a. M⁻¹L³T⁻²

Explanation:

$$\text{From } F = \frac{Gm_1m_2}{r^2}$$

$$G = \frac{Fr^2}{m_1m_2} = \frac{(MLT^{-2})L^2}{M^2} = [M^{-1}L^3T^{-2}]$$

40. The number of significant figures in 0.06900 is:

- (A) 5.
- (B) 4.
- (C) 2
- (D) 3.

Ans. :

- b. 4.

Explanation:

In the number 0.06900, two zeroes before six are not significant figure and two zero on right side of 9 are significant figures. Significant figures are underlined, so verifies option (b).

41. On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct:

- a. $y = \frac{a \sin 2\pi t}{T}$.
- b. $y = a \sin vt$.
- c. $y = \frac{a}{T} \sin \left(\frac{t}{a} \right)$.
- d. $y = a\sqrt{2} \left(\sin \frac{2\pi t}{T} - \cos \frac{2\pi t}{T} \right)$.

Ans. :

- b. $y = a \sin vt$.
- c. $y = \frac{a}{T} \sin \left(\frac{t}{a} \right)$.

Explanation:

The argument of trigonometric functions (sin, cos etc.) should be dimensionless. y is displacement and according to the principle of homogeneity of dimensions LHS and RHS.

$$[Y] = [L], [a] = [L]$$

$$\left[\frac{2\pi t}{T}\right] = \frac{[T]}{[T]} = [T^0]$$

$$[vt] = [v][t] = [LT^{-1}][T] = [L]$$

$$\left[\frac{a}{T}\right] = \frac{[a]}{[T]} = \frac{[L]}{[T]} = [LT^{-1}]$$

$$\left[\frac{t}{a}\right] = [L^{-1}T]$$

$$[LHS] \neq [RHS]$$

Hence, (c) is not the correct option.

$$\Rightarrow LHS \neq RHS.$$

So, option (b) is also not correct.

42. The number of significant figures in 0.06900 is:

- a. 5.
- b. 4.
- c. 2
- d. 3.

Ans. :

- b. 4.

Explanation:

In the number 0.06900, two zeroes before six are not significant figure and two zero on right side of 9 are significant figures. Significant figures are underlined, so verifies option (b).

43. A unitless quantity:

- a. Never has a non-zero dimension.
- b. Always has a non-zero dimension.
- c. May have a non-zero dimension.
- d. Does not exist.

Ans. :

- a. Never has a non-zero dimension.

Explanation:

A unitless quantity never has a non-zero dimension.

44. The dimensions $ML^{-1}T^{-2}$ may correspond to:

- a. Work done by a force.
- b. Linear momentum.
- c. Pressure.
- d. Energy per unit volume.

Ans. :

- c. Pressure.

- d. Energy per unit volume.

Explanation:

$$[\text{Work done}] = [\text{ML}^2 \text{T}^{-2}]$$

$$[\text{Linear momentum}] = [\text{MLT}^{-1}]$$

$$[\text{Pressure}] = [\text{ML}^{-1} \text{T}^{-2}]$$

$$[\text{Energy per unit volume}] = [\text{ML}^{-1} \text{T}^{-2}]$$

From the above, we can see that pressure and energy per unit volume have the same dimension, i.e., $\text{ML}^{-1} \text{T}^{-2}$.

45. The radius of a circle is stated as 2.12cm. Its area should be written as:

- a. 14cm^2 .
- b. 14.1cm^2 .
- c. 14.11cm^2 .
- d. 14.1124cm^2 .

Ans. :

- b. 14.1cm^2 .

Explanation:

Area of a circle, $A = \pi r^2$

On putting the values, we get:

$$A = \frac{22}{7} \times 2.12 \times 2.12$$

$$\Rightarrow A = 14.1\text{cm}^2$$

The rules to determine the number of significant digits says that in the multiplication of two or more numbers, the number of significant digits in the answer should be equal to that of the number with the minimum number of significant digits. Here, 2.12cm has a minimum of three significant digits. So, the answer must be written in three significant digits.

*** Answer The Following Questions In One Sentence.[1 Marks Each] [2]**

46. The normal duration of I.Sc. Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury = $10^{-6} \times 100$ years. How many microcenturies did you sleep yesterday?

Ans. : 1 micro century = $10^4 \times 100$ years = $10^{-4} \times 365 \times 24 \times 60$ min

$$\text{So, } 100 \text{ min} = \frac{10^5}{52560} = 1.9 \text{ microcentury}$$

47. Express the power of a 100 watt bulb in CGS unit.

Ans. : In S.I. unit 100 watt = 100 Joule/sec

In C.G.S. Unit = 10^9 erg/sec

*** Given Section consists of questions of 2 marks each. [6]**

48. Name the device used for measuring the mass of atoms and molecules.

Ans. : Deflection of a charge particle or ionized atom or molecule depends on the magnitude of either magnetic or electric field. Mass and Charge of a particle by using this principle can be measured by using spectrograph or spectrometer which measures the mass of atoms and molecules.

49. The distance of a galaxy is of the order of 10^{25}m . Calculate the order of magnitude of time taken by light to reach us from the galaxy.

Ans. : According to the problem, distance of the galaxy = 10^{25}m .

Speed of light = $3 \times 10^8\text{m/s}$

Hence, time taken by light to reach us from galaxy is,

$$t = \frac{\text{Distance}}{\text{Speed}} = \frac{10^{25}\text{m}}{3 \times 10^8\text{m/s}} = \frac{1}{3} \times 10^{17}$$

$$= \frac{10}{3} \times 10^{16} = 3.33 \times 10^{16}\text{s}$$

50. Find the dimensions of Planck's constant h from the equation $E = hv$ where E is the energy and v is the frequency.

Ans. : $E = hv$, where

E = energy and v = frequency.

$$h = \frac{E}{v} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}^{-1}]} [\text{ML}^2\text{T}^{-1}]$$

* Given Section consists of questions of 3 marks each.

[42]

51. A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c . (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c . He writes: $m = \frac{m_0}{(1-v^2)^{1/2}}$. Guess where to put the missing c .

Ans. : Given the relation,

$$m = \frac{m_0}{(1-v^2)^{1/2}}.$$

Dimension of $m = \text{M}^1 \text{L}^0 \text{T}^0$

Dimension of $m_0 = \text{M}^1 \text{L}^0 \text{T}^0$

Dimension of $v = \text{M}^0 \text{L}^1 \text{T}^{-1}$

Dimension of $v^2 = \text{M}^0 \text{L}^2 \text{T}^{-2}$

Dimension of $c = \text{M}^0 \text{L}^1 \text{T}^{-1}$

The given formula will be dimensionally correct only when the dimension of L.H.S is the same as that of R.H.S. This is only possible when the factor, $(1-v^2)^{1/2}$ is dimensionless i.e., $(1 - v^2)$ is dimensionless.

This is only possible if v^2 is divided by c^2 . Hence, the correct relation is $m = m_0 \left(\frac{1-v^2}{c^2} \right)^{\frac{1}{2}}$

52. The frequency 'f' of vibration of a stretched string depends upon:

- Its length
- The mass per unit length 'm'

- iii. The Tension 'T' in the string.
- iv. Obtain dimensionally an expression for frequency 'f'.

Ans. : Let us suppose that $f = K l^a T^b m^c$ where K is a dimensionless constant and a, b, c are unknown powers to be determined. Writing the dimension of all the quantities involved, We get:

Comparing powers L, M and T on both sides, we have

$$a + b - c = 0, b + c = 0 \text{ and } -2b = -1$$

On simplifying,

We get:

$$b = \frac{1}{2}, c = -\frac{1}{2}, a = -1$$

$$\therefore f = K l^{-1} T^{\frac{1}{2}} m^{-\frac{1}{2}}$$

$$\Rightarrow f = \frac{K}{L} \sqrt{\frac{T}{m}}$$

53. If $x = at^2 + bt + c$, where x is displacement as a function of time. Write the dimensions of a b and c.

Ans. : All the terms should have the same dimension:

$$\therefore [a] = \left[\frac{x}{t^2} \right] s = [LT^{-2}]$$

$$[b] = \left[\frac{x}{t} \right] s = [LT^{-1}]$$

$$[c] = [x] = [L]$$

54. Write the dimensional formula for the following:

- i. Wein's constant.
- ii. Planck's constant.
- iii. Specific heat.
- iv. Latent heat.
- v. Rydberg's constant.

Ans. :

i. $[M^0 L T^0 K]$

ii. $[ML^2 T^{-1}]$

iii. $[M^0 L^2 T^{-2} K^{-1}]$

iv. $[M^0 L^2 T^{-2}]$

v. $[M^0 L^{-1} T^0]$.

55. Compute the following with regards to significant figures.

i. 4.6×0.128

ii. $\frac{0.9995 \times 1.53}{1.592}$

iii. $876 + 0.4382$

Ans. :

i. $4.6 \times 0.128 = 0.5888 = 0.59$

The result has been rounded off to have two significant digits (as in 4.6)

- ii. $\frac{0.9995 \times 1.53}{1.592} = 0.96057 = 0.961$
 iii. $876 + 0.4382 = 876.4382 = 876.$

As, there is no decimal point in 876, therefore, result of addition has been rounded off to no decimal point.

56. The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as, $v = \frac{\pi}{8} \frac{Pr^4}{\eta l}$ where P is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $ML^{-1}T^{-1}$. Check whether the equation is dimensionally correct.

Ans. : If dimensions of LHS of an equation is equal to dimensions of RHS, then equation is said to be dimensionally correct.

According to the problem, the volume of a liquid flowing out per second of a pipe is given

$$\text{by } V = \frac{\pi}{8} \frac{Pr^2}{\eta l}$$

(where, V = rate of volume of liquid per unit time)

Dimension of given physical quantities,

$$[V] = \frac{\text{Dimension of volume}}{\text{Dimension of time}} = \frac{[L^3]}{[T]} = [L^3T^{-1}], [p] = [ML^{-1}T^{-2}],$$

$$[\eta] = [ML^{-1}T^{-1}], [l] = [L], [r] = [L]$$

$$\text{LHS} = [V] = \frac{[L^3]}{[T]} = [L^3T^{-1}]$$

$$\text{RHS} = \frac{[ML^{-1}T^{-2}] \times [L^4]}{[ML^{-1}T^{-1}] \times [L]} = [L^3T^{-1}]$$

Dimensionally, L.H.S. = R.H.S.

Therefore, equation is correct dimensionally.

57. A planet moves around the sun in a circular orbit. The time period of revolution T of the planet depends on
- Radius of the orbit (R).
 - Mass of the sun M .
 - Gravitational constant G .

Show dimensionally that $T^2 \propto R^3$

Ans. : $T \propto R^a M^b G^c$

Or $T = KR^a M^b G^c$, Where K is constant

substituting the dimension on both sides, we have

$$[T] = [L]^a [M]^b [M^{-1}L^3T^{-2}]^c$$

$$[M^0L^0T] = [M^{b-c}L^{a+3c}T^{-2c}]$$

Comparing the power of M , L and T ,

We get:

$$b - c = 0$$

$$a + 3c = 0$$

$$-2c = 1$$

On solving,

$$\text{We get: } a = \frac{3}{2}, b = \frac{-1}{2}, c = \frac{-1}{2}$$

$$T = KR^{\frac{3}{2}} M^{\frac{1}{2}} G^{-\frac{1}{2}}$$

$$T \propto R^{\frac{3}{2}}$$

$$T^2 \propto R^3$$

58. Check by the method of dimensional analysis whether the following relations are correct. $v = \sqrt{\frac{P}{D}}$ where v = velocity of sound and P = pressure, D = density of medium $n = \frac{1}{2l} \sqrt{\frac{F}{m}}$, where n = frequency of vibration l = length of the string F = Stretching force m = mass per unit length of the string.

Ans. :

$$i. [R.H.S] = \sqrt{\frac{[P]}{[D]}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [LT^{-1}]$$

$$[L.H.S] = [v] = [LT^{-1}]$$

$$[R.H.S.] = [L.H.S.]$$

Hence, the relation is correct.

$$ii. [R.H.S.] = \frac{1}{[l]} \sqrt{\frac{[F]}{[m]}}$$

$$= \frac{1}{L} \sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \frac{1}{L} [LT^{-1}] = [T^{-1}]$$

$$[L.H.S.] = \left[\frac{1}{\text{Time}} \right] = \frac{1}{T} = [T^{-1}]$$

Hence, the relation is correct.

59. By using the method of dimension, check the accuracy of the following formula : $T = \frac{r\rho g}{2 \cos \theta}$ where T is the surface tension, h is the height of the liquid, ρ is the density of the liquid, g acceleration due to gravity θ angle of contact, and r is the radius of the tube.

Ans. : In order to find out the accuracy of the given equation we shall compare the dimensions of T and $\frac{r\rho g}{2 \cos \theta}$

$$T = \frac{\text{force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]}$$

$$\text{Dimension of } \frac{r\rho g}{2 \cos \theta} = [L][L][ML^{-3}][LT^{-2}]$$

($2 \cos \theta$ has no dimension)

The dimensions of both the sides are the same and hence the equation is correct.

60. E , m , l and G denote energy, mass, angular momentum and gravitational constant respectively. Determine the dimensions of $\frac{E^2 l}{m^2 G}$

Ans. : Dimensions of $E = [ML^2T^{-2}]$

Dimensions of $l = [ML^2T^{-2}]$

Dimensions of $m = [M]$

Dimensions of $G = [M^{-1}L^3T^{-2}]$

$$\therefore \text{Dimensions of } \frac{El^2}{m^5 G^2}$$

$$= \frac{[ML^2T^{-2}][ML^2T^{-2}]^2}{[M]^5[M^{-1}L^3T^2]^{-2}} = 1$$

Thus $\frac{El^2}{m^5 G^2}$ is dimensionless.

61. A physical quantity Q is given by $Q = \frac{A^2 B^{\frac{3}{2}}}{C^{+4} D^{\frac{1}{2}}}$. The percentage error in A, B, C, D are 1%, 2%, 4%, 2% respectively. Find the percentage error in Q.

Ans. : % error in $Q = 2\left(\frac{dA}{A} \times 100\right) + \frac{3}{2}\left(\frac{dB}{B} \times 100\right)$
 $+ 4\left(\frac{dC}{C} \times 100\right) + \frac{1}{2}\left(\frac{dD}{D} \times 100\right)$
 $= 2 \times 1 + \frac{3}{2} \times 2 + 4 \times 4 + \frac{1}{2} \times 2$
 $= 2 + 3 + 16 + 1 = 22\%$

62. If the unit of force is 100N, unit of length is 10m and unit of time is 100s, what is the unit of mass in this system of units?

Ans. : Dimension of force = $[M^1 L^1 T^2] = 100N \dots (i)$

Dimension of length = $[L^1] = 10m \dots (ii)$

Dimension of time = $[T^1] = 100s \dots (iii)$

Substituting (ii), (iii) in (i)

$$M \times (10) \times (100)^{-2} = 100$$

$$\frac{10M}{100 \times 100} = 100$$

$$M = 10^5 \text{Kg} \quad L = 10^1 \text{m}$$

$$F = 10^2 \text{N} \quad T = 10^2 \text{sec}$$

63. What are the dimensions of:

- Volume of a cube of edge a.
- Volume of a sphere of radius a.
- The ratio of the volume of a cube of edge a to the volume of a sphere of radius a?

Ans. :

a. L^3

b. L^3

c. $M^0 L^0 T^0$.

64. Let x and a stand for distance. Is $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a} \sin^{-1} \frac{a}{x}$ dimensionally correct?

Ans. : Dimension of the left side = $\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{L}{\sqrt{(L^2 - L^2)}} = [L^0]$

Dimension of the right side = $\frac{1}{a} \sin^{-1} \left(\frac{a}{x} \right) = [L^{-1}]$

So, the dimension of $\int \frac{dx}{\sqrt{a^2 - x^2}} \neq \frac{1}{a} \sin^{-1} \left(\frac{a}{x} \right)$

So, the equation is dimensionally incorrect.

* Given Section consists of questions of 5 marks each.

[25]

65. Name the physical quantity of the dimension given below:

- i. ML^0T^{-3}
- ii. $ML^{-1}T^{-1}$
- iii. $M^{-1}L^3T^{-2}$
- iv. ML^2T^{-3}
- v. ML^0T^{-2}
- vi. T^{-1}

Ans. :

- i. Energy intensity.
- ii. Coeff. of viscosity.
- iii. Gravitational Constant.
- iv. Power.
- v. Surface tension or force constant or spring factor.
- vi. Frequency.

66. An artificial satellite is revolving around a planet of mass M and radius R , in a circular orbit of radius r . From Kepler's Third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r . Show using dimensional analysis, that $T = \frac{k}{R} \sqrt{\frac{r^3}{g}}$, where k is a dimensionless constant and g is acceleration due to gravity.

Ans. : According to Kepler's third law, $T^2 \propto a^3$ i.e., square of time period (T^2) of a satellite revolving around a planet, is proportional to the cube of the radius of the orbit (a^3).

We have to apply Kepler's third law,

$$T^2 \propto r^3 \Rightarrow T \propto r^{\frac{3}{2}}$$

Also, T depends on R and g .

$$\text{Let } T \propto r^{\frac{3}{2}} g^a R^b$$

$$\Rightarrow T = k r^{\frac{3}{2}} R^a g^b \quad \dots (i)$$

Where, k is a dimensionless constant of proportionality.

Writing the dimensions of various quantities on both the sides, we get

$$\begin{aligned} [M^0 L^0 T] &= [L]^{\frac{3}{2}} [L T^{-2}]^a [L]^b \\ &= [M^0 L^{a+b+\frac{3}{2}} T^{-2a}] \end{aligned}$$

On comparing the dimensions of both sides, we get

$$a + b + \frac{3}{2} = 0 \quad \dots (ii)$$

$$-2a = 1 \Rightarrow a = -\frac{1}{2} \quad \dots (iii)$$

From Eq. (ii), we get

$$b - \frac{1}{2} + \frac{3}{2} = 0 \Rightarrow b = -1$$

Substituting the values of a and b in Eq. (i), we get

$$T = kr^{\frac{3}{2}} R^{-1} g^{\frac{-1}{2}} \Rightarrow T = \frac{k}{R} \sqrt{\frac{r^3}{g}}$$

67. A physical quantity X is related to four measurable quantities a, b, c and d as follows: $X = a^2 b^3 c^{\frac{5}{2}} d^{-2}$. The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity X? If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result.

$$\text{Ans.} \therefore \frac{\Delta X}{X} \times 100 = \pm \left[2 \frac{\Delta a}{a} + 3 \frac{\Delta b}{b} + \frac{5}{2} \frac{\Delta c}{c} + 2 \frac{\Delta d}{d} \right] \times 100$$

$$\frac{\Delta X}{X} \times 100 = \pm \left[\frac{2 \times 1}{100} + \frac{3 \times 2}{100} + \frac{5}{2} \times \frac{3}{100} + \frac{2 \times 4}{100} \right] \times 100$$

$$= \pm \frac{100}{100} \left[2 + 6 + \frac{15}{2} + 8 \right]$$

$$\frac{\Delta x}{x} \times 100 = \pm \left[16 + \frac{15}{2} \right] = \pm \left[\frac{32+15}{2} \right] = \pm \frac{47}{2} = \pm 23.5\%$$

$$\text{Mean absolute error} = \pm \frac{23.5}{100} = \pm 0.235$$

$$= 0.24 \text{ (rounding off in significant figure)}$$

$$\text{Again rounding off } X = 2.763 \text{ in two significant figure} = 2.8.$$

68. Test if the following equations are dimensionally correct:

a. $h = \frac{2S \cos \theta}{\rho g}$

b. $u = \sqrt{\frac{P}{\rho}}$

c. $V = \frac{\pi P r^4 t}{8 \eta l}$

d. $v = \frac{1}{2\pi} \sqrt{\frac{mgl}{I}}$

where h = height, S = surface tension, ρ = density, P = pressure, V = volume, η = coefficient of viscosity, v = frequency and I = moment of inertia.

Ans. :

a. $h = \frac{2S \cos \theta}{\rho g}$

$$\text{LHS} = [L]$$

$$\text{Surface tension} = S = \frac{F}{l} = \frac{MLT^{-2}}{L} = [MT^{-2}]$$

$$\text{Density} = \rho = \frac{M}{V} = [ML^{-3}T^0]$$

$$\text{Radius} = r = [L], g = [LT^{-2}]$$

$$\text{RHS} = \frac{2S \cos \theta}{\rho g} = \frac{[MT^{-2}]}{[ML^{-3}T^0][L][LT^{-2}]} = [M^0L^1T^0] = [L]$$

$$\text{LHS} = \text{RHS}$$

So, the relation is correct

b. $v = \sqrt{\frac{P}{\rho}}$ where v = velocity

$$\text{LHS} = \text{Dimension of } v = [LT^{-1}]$$

$$\text{Dimension of } p = \frac{F}{A} = [ML^{-1}T^{-2}]$$

$$\text{Dimension of } \rho = \frac{m}{v} = [ML^{-2}]$$

$$\text{RHS} = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{[\text{ML}^{-1}\text{T}^{-2}]}{[\text{ML}^{-3}]}} = [\text{L}^2\text{T}^{-2}]^{\frac{1}{2}} = [\text{LT}^{-1}]$$

So, the relation is correct.

$$\text{c. } V = \frac{(\pi p r^4 t)}{(8 \eta l)}$$

$$\text{LHS} = \text{Dimension of } V = [\text{L}^3]$$

$$\text{Dimension of } p = [\text{ML}^{-1}\text{T}^{-2}], r^4 = [\text{L}^4], t = [\text{T}]$$

$$\text{Coefficient of viscosity} = [\text{ML}^{-1}\text{T}^{-1}]$$

$$\text{RHS} = \frac{\pi p r^4 t}{8 \eta l} = \frac{[\text{ML}^{-1}\text{T}^{-2}][\text{L}^4][\text{T}]}{[\text{ML}^{-1}\text{T}^{-1}][\text{L}]}$$

So, the relation is correct.

$$\text{d. } v = \frac{1}{2\pi} \sqrt{(mgI/I)}$$

$$\text{LHS} = \text{dimension of } v = [\text{T}^{-1}]$$

$$\text{RHS} = \sqrt{(mgI/I)} = \sqrt{\frac{[\text{M}][\text{LT}^{-2}][\text{L}]}{[\text{ML}^2]}} = [\text{T}^{-1}]$$

$$\text{LHS} = \text{RHS}$$

So, the relation is correct.

69. The frequency of vibration of a string depends on the length L between the nodes, the tension F in the string and its mass per unit length m . Guess the expression for its frequency from dimensional analysis.

Ans. : Frequency $f = KL^a F^b M^c$ M = Mass/unit length, L = length, F = tension (force)

$$\text{Dimension of } f = [\text{T}^{-1}]$$

Dimension of right side,

$$L^a = [\text{L}^a], F^b = [\text{MLT}^{-2}]^b, M^c = [\text{ML}^{-1}]^c$$

$$\therefore [\text{T}^{-1}] = K[\text{L}]^a [\text{MLT}^{-2}]^b, [\text{ML}^{-1}]^c$$

$$M^0 L^0 T^{-1} = K L^{b+c} L^{a+b-c} T^{-2b}$$

Equating the dimensions of both sides,

$$\therefore b + c = 0 \dots (1)$$

$$-c + a + b = 0 \dots (2)$$

$$-2b = -1 \dots (3)$$

Solving the equations we get

$$a = -1, b = \frac{1}{2} \text{ and } c = \frac{-1}{2}$$

$$\therefore \text{So frequency } f = KL^{-1} F^{\frac{1}{2}} M^{-\frac{1}{2}} = \frac{K}{L} F^{\frac{1}{2}} M^{-\frac{1}{2}} = \frac{K}{L} = \sqrt{\frac{F}{M}}$$

----- मंज़िल उन्हीं को मिलती है, जिनके सपनों में जान होती है!! पंख से कुछ नहीं होता, हौसलों से उड़ान होती है!
