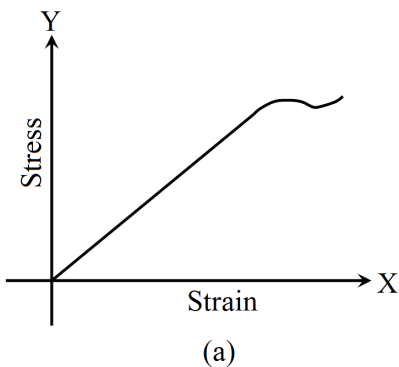


\* Choose The Right Answer From The Given Options.[1 Marks Each]

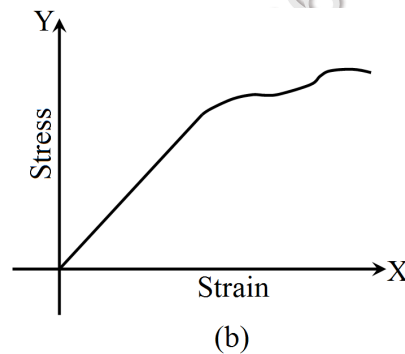
[46]

1. Following are the graphs of elastic materials. Which one corresponds to that of brittle material?

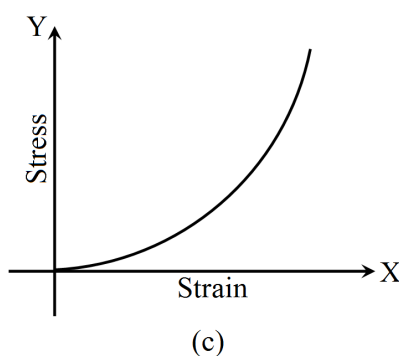
(A)



(B)

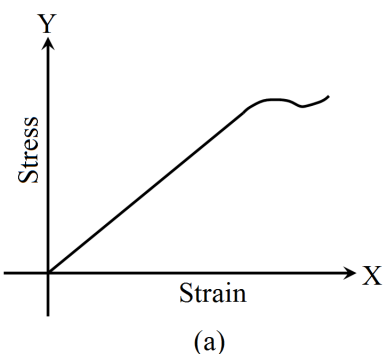


(C)



Ans. :

a.



2. A copper and a steel wire of the same diameter are connected end to end. A deforming force  $F$  is applied to this composite wire which causes a total elongation of 1cm. The two wires will have

(A) The same stress.  
(C) The same strain.

(B) Different stress.  
(D) Different strain.

**Ans. :**

- a. The same stress.
- d. Different strain.

**Explanation:**

$$\therefore \text{stress} = \frac{F}{A}$$

$\therefore$  area of cross section for both wire same and stretched by same force. So their stress are equal verifies option (a).

$$\text{strain} = \frac{\text{stress}}{Y}$$

As stress for both wires are same, so

$$\text{strain}_{\text{steel}} \propto \frac{1}{Y_{\text{steel}}} \text{ and } (\text{strain})_{\text{Al}} \propto \frac{1}{Y_{\text{Al}}}$$

$$\frac{\text{strain}_{\text{steel}}}{(\text{strain})_{\text{Al}}} = \frac{Y_{\text{Al}}}{Y_{\text{steel}}}$$

$$Y_{\text{Al}} < Y_{\text{s}} \text{ So } \frac{\text{Al}}{\text{steel}}$$

$$\text{or } (\text{strain})_{\text{steel}} < (\text{strain})_{\text{Al}}$$

Verifies option (d).

3. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is:

- (A) 0.01                      (B) 0.06                      (C) 0.02                      (D) 0.03

**Ans. :**

- d. 0.03

4. A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.

- (A) Tensile stress at any cross section A of the wire is  $F/A$ .
- (B) Tensile stress at any cross section is zero.
- (C) Tensile stress at any cross section A of the wire is  $2F/A$ .
- (D) Tension at any cross section A of the wire is F.

**Ans. :**

- a. Tensile stress at any cross section A of the wire is  $F/A$ .
- d. Tension at any cross section A of the wire is F.

**Explanation:**

$$\text{Stress} = \frac{F}{A} \text{ verifies option (a).}$$

Here, Tension is balanced by force F. Hence,  $T = F$  verifies option (d).

5. A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of  $2 \times 10^{-3}$ , then the percentage change in volume is:

- (A) 0.6                      (B) 0.4                      (C) 0.2                      (D) Zero.

**Ans. :**

- d. Zero.

**Explanation:**

As, the Poisson's ratio of material is 0.5, so there is no change in volume.

6. On applying a stress of  $20 \times 10^8 \text{Nm}^{-2}$ , the length of a perfectly elastic wire is doubled. Its Young's modulus will be:
- (A)  $40 \times 10^8 \text{Nm}^{-2}$  (B)  $20 \times 10^8 \text{Nm}^{-2}$   
 (C)  $10 \times 10^8 \text{Nm}^{-2}$  (D)  $5 \times 10^8 \text{Nm}^{-2}$

**Ans. :**

b.  $20 \times 10^8 \text{Nm}^{-2}$

7. Young's modulus of a material has the same unit as:
- (A) Stress. (B) Energy.  
 (C) Compressibility. (D) Pressure.

**Ans. :**

a. Stress.  
 d. Pressure.

8. The property of a body by virtue of which it tends to regain its original size and shape of a body when applied force is removed, is known as:
- (A) Fluidity. (B) Elasticity. (C) Plasticity. (D) Rigidity.

**Ans. :**

b. Elasticity.

**Explanation:**

The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity and the deformation caused is known as elastic deformation.

9. On suspending a weight  $Mg$ , the length  $l$  of elastic wire having area of cross-section  $A$ , becomes double the initial length. The instantaneous stress action on the wire is:
- (A)  $\frac{Mg}{A}$  (B)  $\frac{Mg}{2A}$  (C)  $\frac{2Mg}{A}$  (D)  $\frac{4Mg}{A}$

**Ans. :**

c.  $\frac{2Mg}{A}$

10. A steel ring of radius  $r$  and cross-section area  $A$  is shifted on to a wooden disc of radius  $R (R > r)$ . If Young's modulus be  $E$ , then the force with which the steel ring is expanded is:
- (A)  $\frac{AER}{T}$  (B)  $\frac{AE(R-r)}{r}$  (C)  $\frac{E(R-r)}{Ar}$  (D)  $\frac{Er}{AR}$

**Ans. :**

b.  $\frac{AE(R-r)}{r}$

11. The length of a wire increases by 1% by a load of 2kg-wt. The linear strain produced in the wire will be:
- (A) 0.02 (B) 0.001 (C) 0.01 (D) 0.002

**Ans. :**

c. 0.01

12. A copper and a steel wire of the same diameter are connected end to end. A deforming force  $F$  is applied to this composite wire which causes a total elongation of 1cm. The two wires will have

(A) The same stress and strain.

(B) The same stress but different strain.

(C) The same strain but different stress.

(D) Different strains and stress.

**Ans. :**

b. The same stress but different strain.

13. The upper end of a wire of radius 4mm and length 100cm is clamped and its other end is twisted through an angle of  $30^\circ$ . Then, angle of shear is:

(A)  $12^\circ$

(B)  $0.12^\circ$

(C)  $1.2^\circ$

(D)  $0.012^\circ$

**Ans. :**

b.  $0.12^\circ$

14. A long spring is stretched by 2cm and its potential energy is V. If the spring is stretched by 10cm, its potential energy will be:

(A)  $\frac{V}{5}$

(B)  $\frac{V}{25}$

(C) 5V

(D) 25V

**Ans. :**

d. 25V

**Explanation:**

P.E. of a stretched spring,  $V = \frac{1}{2}kx^2$ , where k is the spring constant,

$$\therefore V = \frac{1}{2}k \times 2^2 \text{ or } k = \frac{V}{2}$$

And Now, P.E.,  $V' = \frac{1}{2}k \times 10^2$

$$= \frac{1}{2} \left( \frac{V}{2} \right) \times 100 = 25V$$

15. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will

(A) Be double.

(B) Be half.

(C) Be four times.

(D) Remain same.

**Ans. :**

d. Remain same.

**Explanation:**

$$\text{Breaking stress} = \frac{\text{Breaking force}}{\text{Area of cross-section}}$$

Since breaking force doesn't depend on length, hence changing the cross section has no effect.

So the breaking force remain same.

16. A rod elongates by l when a body of mass M is suspended from it. The work done is:

(A) Mgl

(B)  $\frac{1}{2}mgl$

(C) 2Mgl

(D) Zero.

**Ans. :**

b.  $\frac{1}{2}mgl$

**Explanation:**

$$\text{Work done} = \frac{1}{2}F \times \Delta l = \frac{1}{2}Mgl.$$

17. A and B are two wires. The radius of A is twice that of B. They are stretched by the same load. Then, the stress on B is:

(A) Equal to that on A.

(B) Four times that on A.

(C) Two times that on A.

(D) Half that on A.

**Ans. :**

b. Four times that on A.

18. A wire of length  $L$  and radius  $r$  is rigidly fixed at one end. On stretching the other end of the wire with a force  $F$ , the increase in its length is  $l$ . If another wire of same material but of length  $2L$  and radius  $2r$  is stretched with a force of  $2F$ , the increase in its length will be:

(A)  $l$

(B)  $2l$

(C)  $\frac{1}{2}l$

(D)  $\frac{1}{4}l$

**Ans. :**

a.  $l$

19. A wire suspended vertically from one end, is stretched by attaching a weight  $200\text{N}$  to the lower end. The weight stretches the wire by  $1\text{mm}$ . The energy gained by the wire is:

(A)  $0.1\text{J}$

(B)  $0.2\text{J}$

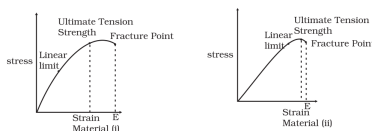
(C)  $0.4\text{J}$

(D)  $10\text{J}$

**Ans. :**

a.  $0.1\text{J}$

20. The stress - strain graphs for two materials are shown in (assume same scale).



- (A) Material (ii) is more elastic than material (i) and hence material (ii) is more brittle.  
(B) Material (i) and (ii) have the same elasticity and the same brittleness.  
(C) Material (ii) is elastic over a larger region of strain as compared to (i).  
(D) Material (ii) is more brittle than material (i).

**Ans. :**

- c. Material (ii) is elastic over a larger region of strain as compared to (i).  
d. Material (ii) is more brittle than material (i).

**Explanation:**

On comparing ultimate tensile strength of the materials, (ii) is greater than (i). Hence, material (ii) is elastic over larger region as compare to (i) so the material (ii) is elastic over a larger region of strain as compared to (i) (verifies option c).

As the fracture point of material (ii) is nearer than (i), hence the material (ii) is more brittle than material (i).

21. Two wires of the same material and length but diameter in the ratio  $1 : 2$  are stretched by the same load. The ratio of elastic potential energy per unit volume for the two wires is:

(A)  $1 : 1$

(B)  $2 : 1$

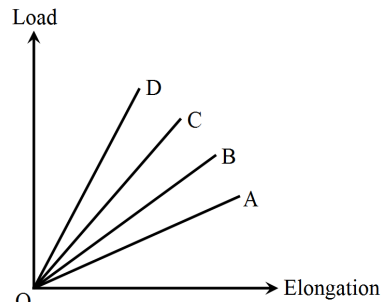
(C)  $4 : 1$

(D)  $16 : 1$

**Ans. :**

d.  $16 : 1$

22. The load versus elongation graph for four wires of the same material is shown in the fig.



The thinnest wire is represented by the line. O

- (A) OC (B) OD (C) OA (D) OB

**Ans. :**

- c. OA

**Explanation:**

For a thinnest wire, the elongation in the wire will be maximum for a given load, which is so corresponding to line OA.

23. Modulus of rigidity of ideal liquids is

- (A) Infinity. (B) Zero.  
(C) Unity. (D) Some finite small non - zero constant value.

**Ans. :**

- b. Zero.

**Explanation:**

As the liquid is ideal, hence it does not have frictional force among its layers, thus the tangential forces are zero as there is no stress developed. This verifies.

24. Two wires A and B of the same material have radii in the ratio 2 : 1 and lengths in the ratio 4 : 1. The ratio of the normal forces required to produce the same change in the lengths of these two wires is:

- (A) 1 : 1 (B) 2 : 1 (C) 1 : 2 (D) 1 : 4

**Ans. :**

- a. 1 : 1

25. A steel rod of length 1m and radius 10mm is stretched by a force 100kN along its length. The stress produced in the rod is  $Y_{\text{steel}} = 2 \times 10^{11} \text{Nm}^{-2}$ .

- (A)  $3.18 \times 10^6 \text{Nm}^{-2}$  (B)  $3.18 \times 10^7 \text{Nm}^{-2}$   
(C)  $3.18 \times 10^8 \text{Nm}^{-2}$  (D)  $3.18 \times 10^9 \text{Nm}^{-2}$

**Ans. :**

- c.  $3.18 \times 10^8 \text{Nm}^{-2}$

26. The nature of molecular forces resembles with the nature of the:

- (A) Gravitational force. (B) Nuclear force.  
(C) Electromagnetic force. (D) Weak force.

**Ans. :**

- c. Electromagnetic force.

27. When a pressure of 100 atmosphere is applied on a spherical ball of rubber, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber in dyne  $\text{cm}^{-2}$  is:

(A)  $10 \times 10^{12}$  (B)  $100 \times 10^{12}$  (C)  $1 \times 10^{12}$  (D)  $20 \times 10^{12}$

Ans. :

c.  $1 \times 10^{12}$

**Explanation:**

$$1\text{atm} = 10^5 \text{Nm}^{-2}$$

$$\therefore 100\text{atm} = 10^7 \text{Nm}^{-2} \text{ and } \Delta V = 0.01\%V$$

$$\therefore \frac{\Delta V}{V} = 0.0001$$

$$B = \frac{P}{\frac{\Delta V}{V}} = 1 \times 10^{11} \text{Nm}^{-2}$$

$$= 1 \times 10^{12} \frac{\text{dyne}}{\text{cm}^2}$$

28. The upper end of a wire of radius 4mm and length 100cm is clamped and its other end is twisted through an angle of  $30^\circ$ . The angle of shear is:

(A)  $12^\circ$  (B)  $1.2^\circ$  (C)  $0.12^\circ$  (D)  $0.012^\circ$

Ans. :

c.  $0.12^\circ$

**Explanation:**

Angle of twist at free end,

$$= 30^\circ = \frac{30}{180} \times \pi \text{ rad} = \frac{\pi}{6} \text{ rad}$$

Displacement of the free surface,

$$\Delta L = \frac{2\pi r}{2\pi} \times \frac{\pi}{6} = \frac{\pi r}{6} = \frac{\pi \times 0.4}{6} \text{ cm}$$

$$\text{Angle of shear or shearing strain} = \frac{\Delta L}{L}$$

$$= \frac{\pi \times \frac{0.4}{6}}{100} \text{ rad}$$

$$= \frac{\pi \times 0.4}{6 \times 100} \times \frac{180}{\pi} \text{ degree} = 0.12^\circ$$

29. When an elastic material with Young's modulus Y is subjected to stretching stress S, the elastic energy stored per unit volume of the material is:

(A)  $\frac{YS}{2}$  (B)  $\frac{YS^2}{2}$  (C)  $\frac{S^2}{2Y}$  (D)  $\frac{2}{2Y}$

Ans. :

c.  $\frac{S^2}{2Y}$

**Explanation:**

Elastic energy per unit volume,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{\text{Young modulus}}$$

$$= \frac{1}{2} S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

30. The temperature of a wire is doubled. The Young's modulus of elasticity  
 (A) Will also double. (B) Will become four times.  
 (C) Will remain same. (D) Will decrease.

Ans. :

d. Will decrease.

**Explanation:**

**Key concept:** Young's modulus ( $Y$ ).

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

The fractional change in length of any material is defined as

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

where  $\Delta T$  is change in the temperature,  $L_0$  is original length,  $\alpha$  is the coefficient of linear expansion of the given material and  $L_0$  is the original length of material.

So, simply change in length is due to change in temperature.

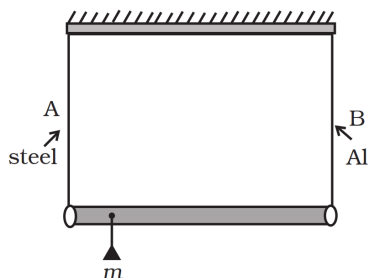
$$\Delta L = L_0 \alpha \Delta T$$

And Young's modulus

$$(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{FL_0}{A \times \Delta L} = \frac{FL_0}{AL_0 \alpha \Delta T} \propto \frac{1}{\Delta T}$$

$$\text{As } Y \propto \frac{1}{\Delta T}$$

31. A rod of length  $l$  and negligible mass is suspended at its two ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths. The cross-sectional areas of wires A and B are  $1.0\text{mm}^2$  and  $2.0\text{mm}^2$ , respectively. ( $Y_{\text{Al}} = 70 \times 10^9\text{Nm}^{-2}$  and  $Y_{\text{steel}} = 200 \times$



$10^9\text{Nm}^{-2}$ )

- (A) Mass  $m$  should be suspended close to wire A to have equal stresses in both the wires.  
 (B) Mass  $m$  should be suspended close to B to have equal stresses in both the wires.  
 (C) Mass  $m$  should be suspended at the middle of the wires to have equal stresses in both the wires.  
 (D) Mass  $m$  should be suspended close to wire A to have equal strain in both wires.

Ans. :

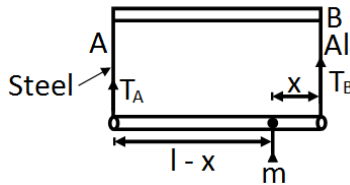
- b. Mass  $m$  should be suspended close to wire A to have equal stresses in both the wires.  
 d. Mass  $m$  should be suspended close to wire A to have equal strain in both wires.

**Explanation:**

According to the diagram a massless rod is suspended at its two ends by two wires of steel (wire A) and aluminum (wire B) of equal lengths.



Let the mass is suspended at  $x$  from the end B, which develop equal stress in wires. Let  $T_A$  and  $T_B$  be the tensions in wire A and wire B respectively.



$$\text{stress in steel wire A, } S_A = \frac{T_A}{A_A} = \frac{T_A}{10^{-6}}$$

$$\text{stress in Al wire } S_B = \frac{T_B}{A_B} = \frac{T_B}{2 \times 10^{-6}}$$

where  $A_A$  and  $A_B$  are cross-sectional areas of wire A and B respectively. Also, from rotational equilibrium, net torque is zero, i.e.  $T_B x - T_A (l - x) = 0$

$$\Rightarrow \frac{T_B}{T_A} = \frac{l-x}{x} \dots (i)$$

For equal stress,  $S_A = S_B$

$$\Rightarrow S_A = S_B \Rightarrow \frac{T_A}{10^{-6}} = \frac{T_B}{2 \times 10^{-6}}$$

$$\Rightarrow \frac{l-x}{x} = 2 \Rightarrow \frac{l}{x} - 1 = 2$$

$$\Rightarrow x = \frac{l}{3} \Rightarrow l - x = l - \frac{l}{3} = \frac{2l}{3}$$

Hence, mass  $m$  should be suspended close to wire B (Al wire).

We know,  $\text{Strain} = \frac{\text{Stress}}{Y}$

So, for equal strain in the wires,

$$\Rightarrow \frac{S_A}{Y_{\text{Steel}}} = \frac{S_B}{Y_{\text{Al}}}$$

$$\Rightarrow \frac{Y_{\text{Steel}}}{T_A/a_A} = \frac{Y_{\text{Al}}}{T_B/a_B}$$

$$\Rightarrow \frac{Y_{\text{Steel}}}{Y_{\text{Al}}} = \frac{T_A}{T_B} \times \frac{a_B}{a_A} = \left( \frac{x}{l-x} \right) \left( \frac{2a_A}{a_A} \right)$$

$$\Rightarrow \frac{200 \times 10^9}{70 \times 10^9} = \frac{2x}{l-x} \Rightarrow \frac{20}{7} = \frac{2x}{l-x}$$

$$\Rightarrow 17x = 10l \Rightarrow x = \frac{10l}{17}$$

$$\Rightarrow l - x = l - \frac{10l}{17} = \frac{7l}{17}$$

Hence, mass  $m$  should be suspended close to wire A (steel wire).

32. A rigid bar of mass  $M$  is supported symmetrically by three wires each of length  $l$ . Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to

(A)  $Y_{\text{copper}}/Y_{\text{iron}}$ .

(B)  $\sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$ .

(C)  $\frac{Y_{\text{iron}}^2}{Y_{\text{copper}}^2}$ .

(D)  $\frac{Y_{\text{iron}}}{Y_{\text{copper}}}$ .

Ans. :

b.  $\sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$ .

**Explanation:**

As the bar is supported symmetrically by the three wires, therefore extension in each wire is same. Let  $T$  be the tension in each wire and diameter of the wire is  $D$ , then Young's modulus is

$$(Y) = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L}$$

$$= \frac{F}{\pi(D/2)^2} \times \frac{L}{\Delta L} = \frac{4FL}{\pi D^2 \Delta L}$$

$$\Rightarrow D^2 = \frac{4FL}{\pi \Delta L Y} \Rightarrow D = \sqrt{\frac{4FL}{\pi \Delta L Y}}$$

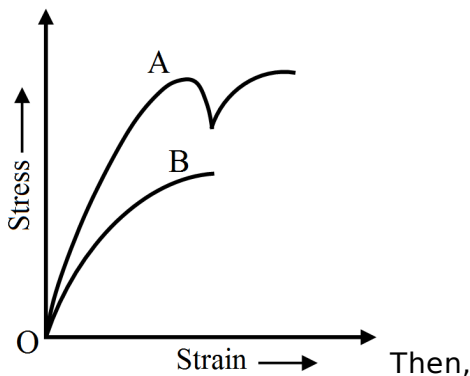
As  $F$  and  $\frac{L}{\Delta L}$  are constants.

Hence,  $D \propto \sqrt{\frac{1}{Y}}$

or  $D = \frac{K}{\sqrt{Y}}$  ( $K$  is the proportionality constant)

Now we can find ratio as  $\frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$

33. Stress-strain curves for the material A and B are shown below:



(A) A is brittle material.

(B) B is ductile material.

(C) B is brittle material.

(D) Both (a) and (b).

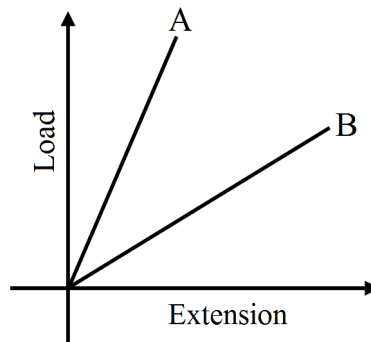
Ans. :

c. B is brittle material.

**Explanation:**

B is brittle as there is no plastic region. However, A is ductile as it has large plastic range of extension.

34. In the given figure, if the dimension of the wire are the same and materials are



different, Young's modulus is more for:

(A) A

(B) B

(C) Both.

(D) None of these.

Ans. :

a. A

35. Young's modulus of a wire depends on:

(A) Its material.

(B) Its length.

(C) Its area of cross-section.

(D) Both (b) and (c).

**Ans. :**

- a. Its material.

36. Two rods of different materials having coefficient of thermal expansion  $\alpha_1, \alpha_2$  and Young's modulus  $Y_1, Y_2$ , respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If  $\alpha_1 : \alpha_2 = 2 : 3$ , the thermal stresses developed in the two rods are equal provided  $Y_1 : Y_2$  is equal to:

(A) 2 : 3 (B) 1 : 1 (C) 3 : 2 (D) 4 : 9

**Ans. :**

- c. 3 : 2

**Explanation:**

Expansion in rod due to rise in temperature = compression in rod,

$$\therefore \frac{\alpha_1}{\alpha_2} = \frac{Y_1}{Y_2} \text{ or } \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

37. Dimensional formula of stress is same as that of:

(A) Impulse. (B) Strain. (C) Force. (D) Pressure.

**Ans. :**

- d. Pressure.

38. A wire of diameter 1mm breaks under a tension of 1000N. Another wire of same material as that of the first one, but of diameter 2mm breaks under a tension of:

(A) 500N (B) 1000N (C) 10000N (D) 4000N

**Ans. :**

- d. 4000N

39. Wire A and B are made from the same material A has twice the diameter and three times the length of B. If the elastic limits are not reached, when each is stretched by the same tension, the ratio of energy stored in A to that in B is:

(A) 2 : 3 (B) 12 : 1 (C) 3 : 2 (D) 6 : 1

**Ans. :**

- b. 12 : 1

**Explanation:**

Given  $D_A = 2D$ ;  $l_A = 3l$ ,  $D_B = D$ ,  $l_B = l$

$F_A = F = F_B$ ,  $Y_A = Y_B = Y$

Energy stored (E) =  $\frac{1}{2} \times \frac{(\text{Stress})^2}{Y} \times \text{Volume}$

$$\therefore E_A = \frac{\frac{F}{\pi} (2D)^2}{Y} \times \frac{\pi (2D)^2}{4} \times 3l$$

$$E_B = \frac{1}{2} \times \frac{\frac{F}{\pi} D^2}{Y} \times \frac{\pi D^2}{4} \times l$$

$$\frac{E_A}{E_B} = \frac{12}{1}$$

40. A solid sphere falls with a terminal velocity of 20m/s in air. If it is allowed to fall in vacuum,

- a. Terminal velocity will be 20m/s  
b. Terminal velocity will be less than 20m/s

- c. Terminal velocity will be more than 20m/s
- d. There will be no terminal velocity.

**Ans. :**

- d. There will be no terminal velocity.

**Explanation:**

In vacuum, no viscous force exists. The sphere therefore, will have constant acceleration because of gravity. An accelerated motion implies that it won't have uniform velocity throughout its motion. In other words, there will be no terminal velocity.

41. The force of viscosity is:
- a. Electromagnetic.
  - b. Gravitational.
  - c. Nuclear.
  - d. Weak.

**Ans. :**

- a. Electromagnetic.

**Explanation:**

The force of viscosity arises from molecular interaction between different layers of fluids that are in motion. Molecular forces are electromagnetic in nature. Therefore, viscosity must also be electromagnetic.

42. A wire can sustain the weight of 20kg before breaking. If the wire is cut into two equal parts, each part can sustain a weight of.
- a. 10kg
  - b. 20kg
  - c. 40kg
  - d. 80kg.

**Ans. :**

- b. 20kg

**Explanation:**

As the wire is cut into two equal parts, both have equal cross-sectional areas. Therefore, a weight of 20kg exerts a force of 20g on both the pieces. Breaking stress depends upon the material of the wire.

Since 20g of force is exerted on wires with equal cross-sectional areas, both the wires can sustain a weight of 20kg.

43. A liquid is contained in a vertical tube of semicircular cross-section. The contact angle is zero. The forces of surface tension on the curved part and on the flat part are in ratio:
- a. 1 : 1
  - b. 1 : 2
  - c.  $\pi$  : 2
  - d. 2 :  $\pi$

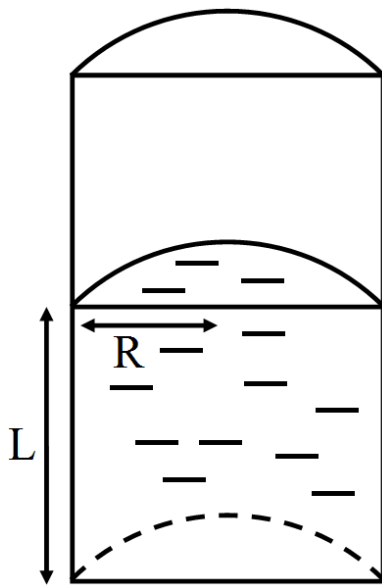


Ans. :

c.  $x : 2$

**Explanation:**

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Let the height of the liquid filled column be  $L$

Let the radius be denoted by  $R$

Total perimeter of the curved part = semi - circumference of upper area =  $\pi R$

Total surface tension force =  $\pi R S$

Total perimeter of the flat part =  $2R$

Total surface tension force =  $2RS$

Ratio of curved surface force to flat surface force =  $\frac{\pi R S}{2RS} = \frac{\pi}{2}$

44. A solid sphere moves at a terminal velocity of  $20\text{m/s}$  in air at a place where  $g = 9.8\text{m/s}^2$ . The sphere is taken in a gravity free hall having air at the same pressure and pushed down at a speed of  $20\text{m/s}$ .
- Its initial acceleration will be  $9.8\text{m/s}^2$  downward.
  - Its initial acceleration will be  $9.8\text{m/s}^2$  upward.
  - The magnitude of acceleration will decrease as the time passes.
  - It will eventually stop.

**Ans. :**

- Its initial acceleration will be  $9.8\text{m/s}^2$  upward.
- The magnitude of acceleration will decrease as the time passes.
- It will eventually stop.

**Explanation:**

There is no gravitational force acting downwards. However, when the starting velocity is  $20\text{m/s}$ , the viscous force, which is directly proportional to velocity, becomes maximum and tends to accelerate the ball upwards.

When the ball falls under gravity,

neglecting the density of air:

Mass of the sphere =  $m$

Radius =  $r$

Viscous drag coefficient =  $\eta$

Terminal velocity is given by:

$$mg = 6\pi\eta r v_T$$

$$\Rightarrow \frac{6\pi\eta r v T}{m} = g \dots (1)$$

Now, at terminal velocity, the acceleration of the ball due to the viscous force is given by:

$$a = \frac{6\pi\eta r v T}{m}$$

Comparing equations (1) and (2), we find that:

$$a = g$$

- b. Thus, we see that the initial acceleration of the ball will be  $9.8\text{m/s}^{-2}$ .
- c. The velocity of the ball will decrease with time because of the upward viscous drag. As the force of viscosity is directly proportional to the velocity of the ball, the acceleration due to the viscous force will also decrease.
- d. When all the kinetic energy of the ball is radiated as heat due to the viscous force, the ball comes to rest.

45. A rope 1cm in diameter breaks if the tension in it exceeds 500N. The maximum tension that may be given to a similar rope of diameter 2cm is:

- a. 500N
- b. 250N
- c. 1000N
- d. 2000N.

**Ans. :**

- d. 2000N

**Explanation:**

$$F_1 = 500\text{N}$$

Let the required breaking force on the 2cm wire be F.

$$\text{Breaking stress in 1cm wire} = \frac{F_1}{A_1} = \frac{500}{\pi \left(\frac{0.01}{2}\right)^2}$$

$$\text{Breaking stress in 2cm wire} = \frac{F_2}{A_2} = \frac{F_2}{\pi \left(\frac{0.02}{2}\right)^2}$$

The breaking stress is the same for a material.

$$\Rightarrow \frac{500}{\pi \left(\frac{0.01}{2}\right)^2} = \frac{F_2}{\pi \left(\frac{0.02}{2}\right)^2}$$

$$\Rightarrow F_2 = 2000\text{N}$$

46. A wire elongates by 1.0mm when a load W is hung from it. If this wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be:

- a. 0.5mm
- b. 1.0mm
- c. 2.0mm
- d. 4.0mm.

**Ans. :**

- b. 1.0mm

**Explanation:**

Let the Young's modulus of the material of the wire be  $Y$ .

Force = Weight =  $W$  (given)

Let C.S.A. =  $A$

$x = 1\text{mm}$  = Elongation in the first case

Length =  $L$

$$Y = \frac{\frac{W}{A}}{\frac{x}{L}} = \frac{WL}{Ax}$$

Let  $y$  be the elongation on one side of the wire when put in a pulley. When put in a pulley, the length of the wire on each side =  $\frac{L}{2}$

$$\frac{\frac{W}{A}}{\frac{y}{\frac{L}{2}}} = Y$$

$$\Rightarrow \frac{\frac{W}{A}}{y \frac{L}{2}} = \frac{WL}{Ax}$$

$$\Rightarrow Y = \frac{x}{2}$$

Total elongation in the wire =  $2y = 2\left(\frac{x}{2}\right) = x = 1\text{mm}$

**\* Answer The Following Questions In One Sentence.[1 Marks Each]**

**[9]**

47. A wire increases by  $10^{-3}$  of its length when a stress of  $10^8 \text{Nm}^{-2}$  is applied to it. What is the Young's modulus of the material of the wire?

**Ans. :** Given,  $\Delta L = 10^{-3}L$ , with  $L$  as the original length,

$$\text{Strain} = \frac{\Delta L}{L} = 10^{-3}$$

$$\text{Stress} = \frac{F}{A} = 10^8 \text{Nm}^2$$

$$\therefore Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$Y = \frac{1 \times 10^8}{10^{-3}} = 10^{11} \text{Nm}^2$$

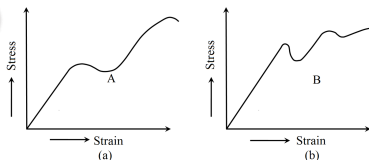
48. What is the value of bulk modulus for an incompressible liquid?

**Ans. :** Infinite.

49. What does Hooke's law essentially define?

**Ans. :** Elastic limit.

50. The stress versus strain graphs for two materials A and B are shown below: (The graphs



are to the same scale).

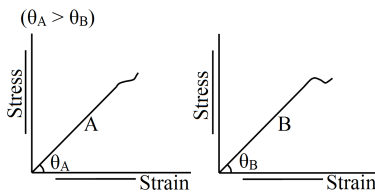
- Which material has greater Young's modulus?
- Which material is more ductile?
- Which is more brittle?

**Ans. :**



- i. A, because for producing the same strain, more stress is required in case of the material A.
- ii. A, because it has a greater plastic range.
- iii. B, because it has a lesser plastic range.

51. The stress-strain graph for material A and B are shown in the figure (drawn on same scale). Which of the two is stronger material? Justify your answer.



**Ans. :** Material A is stronger because for producing the same strain, more stress is required in case of material A.

52. Bridges are declared unsafe after long use. Why?

**Ans. :** A bridge undergoes alternating stress and strain for a large number of times during its use. When bridge is used for long time, it loses its elastic strength. Therefore, the amount of strain in the bridge for a given stress will become large and ultimately, the bridge will collapse. So, they are declared unsafe after long use.

53. Two persons pull a rope towards themselves. Each person exerts a force of 100N on the rope. Find the Young's modulus of the material of the rope if it extends in length by 1cm. Original length of the rope = 2m and the area of cross-section =  $2\text{cm}^2$ .

**Ans. :** Area of cross-section  $A = 2\text{cm}^2 = \frac{2}{1000}\text{m}^2$

Force  $F = 100\text{N}$  Stress

$$\begin{aligned}\sigma &= \frac{F}{A} \\ &= \frac{100}{\left(\frac{2}{1000}\right)} \\ &= \frac{1000000}{2} \\ &= 500000\end{aligned}$$

$$= 5 \times 10^5 \text{N/m}^2$$

$$l = 1\text{cm} = \frac{1}{100}\text{m} = 0.01\text{m}$$

$$\text{Strain } \epsilon = \frac{l}{L} = \frac{0.01}{2} = 0.005$$

$$\text{Young's modulus of the material } Y = \frac{\sigma}{\epsilon}$$

$$= 5 \times \frac{10^5}{0.005} \text{N/m}^2$$

$$= 1 \times 10^8 \text{N/m}^2$$

54. The elastic limit of steel is  $8 \times 10^8 \text{N/m}^2$  and its Young's modulus  $2 \times 10^{11} \text{N/m}^2$ . Find the maximum elongation of a half meter steel wire that can be given without exceeding the elastic limit.

**Ans. :**

It means the maximum stress in the wire  $\sigma = 8 \times 10^8 \text{N/m}^2$

$$Y = 2.0 \times 10^{11} \text{N/m}^2$$

Length of the wire  $L = 0.50\text{m}$

Let corresponding maximum elongation  $= l\text{m}$

$$\text{Strain } \epsilon = \frac{l}{L} = \frac{1}{0.50} = 2l$$

we have  $\frac{\sigma}{\epsilon} = Y$

$$\Rightarrow \epsilon = \frac{\sigma}{Y}$$

$$\Rightarrow 2l = 8 \times \frac{10^8}{2.0} \times 10^{11}$$

$$\Rightarrow l = \frac{2}{100}\text{m} = 2\text{mm}$$

55. A 5.0cm long straight piece of thread is kept on the surface of water. Find the force with which the surface on one side of the thread pulls it. Surface tension of water = 0.076N/m.

**Ans. :** Given:

Length of thread  $l = 5\text{cm} = 5 \times 10^{-2}\text{m}$

Surface tension of water  $T = 0.76\text{N/m}$

We know that:

$$F = T \times l = 0.76 \times 5 \times 10^{-2}$$

$$= 3.8 \times 10^{-3}\text{N}$$

Therefore, the water surface on one side of the thread pulls it with a force of  $3.8 \times 10^{-3}\text{N}$ .

**\* Given Section consists of questions of 2 marks each.**

**[10]**

56. A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of.
- The stresses developed in the two wires.
  - The strains developed.  $Y$  of steel  $= 2 \times 10^{11}\text{N/m}^2$ .  $Y$  of copper  $= 1.3 \times 10^{11}\text{N/m}^2$ .

**Ans. :** Given:

Young's modulus of steel  $= 2 \times 10^{11}\text{ N m}^{-2}$

Young's modulus of copper  $= 1.3 \times 10^{11}\text{ N m}^{-2}$

Both wires are of equal length and equal cross-sectional area. Also, equal tension is applied on them.

As per the question:

$$L_{\text{steel}} = L_{\text{Cu}}$$

$$A_{\text{steel}} = A_{\text{Cu}}$$

$$F_{\text{Cu}} = F_{\text{steel}}$$

Here:  $L_{\text{steel}}$  and  $L_{\text{Cu}}$  denote the lengths of steel and copper wires, respectively.  $A_{\text{steel}}$  and  $A_{\text{Cu}}$  denote the cross-sectional areas of steel and copper wires, respectively.

$F_{\text{steel}}$  and  $F_{\text{Cu}}$  denote the tension of steel and copper wires, respectively.

$$\text{a. } \frac{\text{Stress of cu}}{\text{Stress of steel}} = \frac{F_{\text{Cu}}}{A_{\text{Cu}}} \frac{A_{\text{steel}}}{F_{\text{steel}}} = 1$$

$$b. \quad \frac{\text{Strain of cu}}{\text{Strain of steel}} = \frac{\frac{\Delta L_{\text{steel}}}{L_{\text{steel}}}}{\frac{\Delta L_{\text{Cu}}}{L_{\text{Cu}}}} = \frac{F_{\text{steel}} L_{\text{steel}} A_{\text{Cu}} Y_{\text{Cu}}}{A_{\text{steel}} Y_{\text{steel}} F_{\text{Cu}} L_{\text{Cu}}}$$

$$\left( \text{Using } \frac{\Delta L}{L} = \frac{F}{AY} \right)$$

$$\Rightarrow \frac{\text{Strain of cu}}{\text{Strain of steel}} = \frac{Y_{\text{Cu}}}{Y_{\text{steel}}} = \frac{1.3 \times 10^{11}}{2 \times 10^{11}}$$

$$\Rightarrow \frac{\text{Strain of cu}}{\text{Strain of steel}} = \frac{13}{20}$$

$$\Rightarrow \frac{\text{Strain of steel}}{\text{Strain of Cu}} = \frac{20}{13}$$

Hence, the required ratio is 20 : 13.

57. A load of 10kg is suspended by a metal wire 3m long and having a cross-sectional area 4mm<sup>2</sup>. Find.

- The stress.
- The strain and.
- The elongation. Young's modulus of the metal is  $2.0 \times 10^{11} \text{ N/m}^2$ .

**Ans. :** Length of wire  $L = 3\text{m}$ ,

Load  $F = 10 \times 10\text{N} = 100\text{N}$ , {Taking  $g = 10\text{m/s}^2$ }

Area of cross-section  $A = 4\text{mm}^2$

$$\Rightarrow A = 4 \times 10^{-6} \text{m}^2$$

- The stress = Load (force) on unit area of cross-section =  $\frac{F}{A}$

$$= \frac{100}{4} \times 10^{-6} \text{N/m}^2$$

$$= 25 \times 10^6 \text{N/m}^2$$

$$= 2.5 \times 10^7 \text{N/m}^2$$

- Let the elongation of the wire under this stress be  $l$ , The strain =  $\frac{l}{L}$ , Young's modulus of the metal  $Y = 2.0 \times 10^{11} \text{N/m}^2$  We

have  $\frac{\text{Stress}}{\text{Strain}} = Y$  (constant)

$$\Rightarrow \frac{\left(\frac{F}{A}\right)}{\text{Strain}} = Y$$

$$\Rightarrow \text{Strain} = \frac{\left(\frac{F}{A}\right)}{Y} = 2.5 \times \frac{10^7}{2.0} \times 10^{11}$$

$$\Rightarrow \text{strain} = 1.25 \times 10^{-4}$$

- strain =  $\frac{l}{L} = 1.25 \times 10^{-4}$

$$\Rightarrow l = 3.0 \times 1.25 \times 10^{-4} \text{m}$$

$$= 3.75 \times 10^{-4} \text{m}$$

58. A wire forming a loop is dipped into soap solution and taken out so that a film of soap solution is formed. A loop of 6.28cm long thread is gently put on the film and the film is pricked with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution = 0.030N/m.

**Ans. :** Given:

Surface tension of soap solution  $T = 0.030 \text{N/m}^{-1}$

Let the radius of the thread loop be  $r$ .

$$\Rightarrow 2\pi r = 6.28\text{cm}$$

$$\Rightarrow r = \frac{6.28}{2 \times 3.14} = 1\text{cm}$$

The excess pressure inside the loop is expressed as follows:

$$\Delta P = \frac{4T}{r}$$

Tension in the thread:

$$T' = \Delta P \times (\text{area of loop})$$

$$\Rightarrow T = \frac{4T}{r} \times \pi r^2$$

$$\Rightarrow T' = 4\pi r$$

$$= 4 \times 0.030 \times 3.14 \times 10^{-2}\text{N}$$

$$= 3.8 \times 10^{-3}\text{N}$$

59. A copper wire of cross-sectional area  $0.01\text{cm}^2$  is under a tension of  $20\text{N}$ . Find the decrease in the cross-sectional area. Young's modulus of copper  $= 1.1 \times 10^{11}\text{N/m}^2$  and Poisson's ratio  $= 0.32$ . [Hint:  $\frac{\Delta A}{A} = 2\frac{\Delta r}{r}$ ]

**Ans. :**

Given:

$$\text{Cross-sectional area of copper wire } A = 0.01\text{cm}^2 = 10^{-6}\text{m}^2$$

$$\text{Applied tension } T = 20\text{N}$$

$$\text{Young modulus of copper } Y = 1.1 \times 10^{11}\text{N/m}^2$$

$$\text{Poisson ratio } \sigma = 0.32$$

We know that:

$$Y = \frac{FL}{A\Delta L}$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{F}{AY}$$

$$= \frac{20}{10^{-6} \times 1.1 \times 10^{11}} = 18.18 \times 10^{-5}$$

$$\text{Poisson's ratio, } \sigma = \frac{\frac{\Delta d}{d}}{\frac{\Delta L}{L}} = 0.32$$

Where  $d$  is the transverse length

$$\text{So, } \frac{\Delta d}{d} = (0.32) \times \frac{\Delta L}{L}$$
$$= 0.32 \times (18.18) \times 10^{-5} = 5.81 \times 10^{-5}$$

$$\text{Again, } \frac{\Delta A}{A} = \frac{2\Delta r}{r} = \frac{2\Delta d}{d}$$

$$\Rightarrow \Delta A = \frac{2\Delta d}{d} A$$

$$\Rightarrow \Delta A = 2 \times (5.8 \times 10^{-5}) \times (0.01)$$

$$= 1.165 \times 10^{-6}\text{cm}^2$$

Hence, the required decrease in the cross-sectional area is  $1.164 \times 10^{-6}\text{cm}^2$ .

60. The contact angle between pure water and pure silver is  $90^\circ$ . If a capillary tube made of silver is dipped at one end in pure water, will the water rise in the capillary?

**Ans. :** No, the water will neither rise nor fall in the silver capillary.

According to Jurin's law, the level of water inside a capillary tube is given by

$$h = \frac{2T \cos \theta}{r \rho g}$$

Here,  $\theta = 90^\circ$

$$\Rightarrow h = \frac{2T \cos 90^\circ}{r \rho g}$$

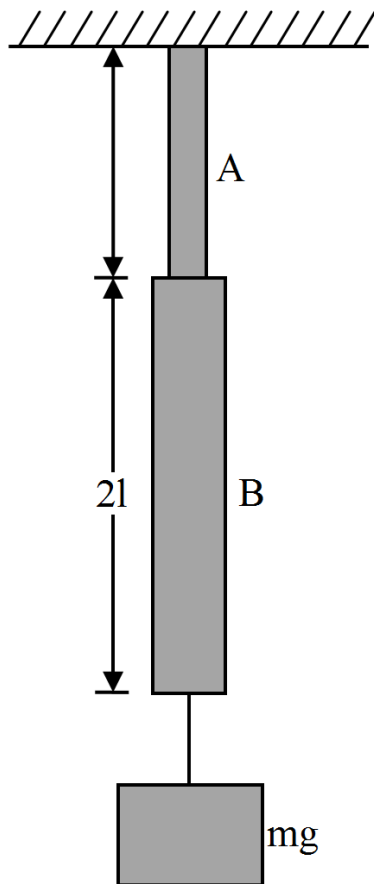
$$\Rightarrow h = 0$$

Thus, the water level neither rises nor falls.

\* **Given Section consists of questions of 3 marks each.**

[21]

61. Two wires A and B of length  $l$ , radius  $r$  and length  $2l$ , radius  $2r$  having same Young's modulus  $Y$  are hung with a weight  $mg$ , see fig. What is the net elongation in the two



wires?

**Ans. :** Here, the pulling force  $F (= mg)$  is same on both the wires. Let  $\Delta l_1, \Delta l_2$  be the elongations in the two wires.

$$\text{As, } Y = \frac{Fl}{\pi r^2 \Delta l} \text{ or } \Delta l = \frac{Fl}{Y \pi r^2}$$

$$\text{For wire 'A' } \Delta l_1 = \frac{mgl}{Y \pi r^2}$$

$$\text{For wire 'B' } \Delta l_2 = \frac{mg(2l)}{Y \pi (2r)^2} = \frac{mgl}{2Y \pi r^2}$$

$$\text{Total elongation } \Delta l_1 + \Delta l_2$$

$$= \frac{mgl}{Y \pi r^2} + \frac{1}{2} \frac{mgl}{Y \pi r^2} = \frac{3}{2} \frac{mgl}{Y \pi r^2}.$$

62. Determine the volume contraction of a solid copper cube, 10cm on an edge, when subjected to a hydraulic pressure of  $7.0 \times 10^6$  Pa.

**Ans. :** Given:  $L = 10\text{cm} = 0.1\text{m}$

$K = \text{bulk modulus of Cu}$

$$= 140 \times 10^9 \text{ Pa}$$

$$P = 7 \times 10^6 \text{ Pa}$$

$\Delta V = \text{Volume contraction of solid copper cube} = ?$

$$\therefore V = L^3 = (0.1)^3 = 0.001\text{m}^3.$$

Using formula,  $K = \frac{P}{\left(\frac{\Delta V}{V}\right)}$

$$\text{We get } \Delta V = -\frac{PV}{K} = \frac{7 \times 10^6 \times 0.001}{140 \times 10^9} \text{m}^3$$

$$= -\frac{1}{20} \times 10^{-6} \text{m}^3$$

$$= -0.05 \times 10^{-6} \text{m}^3$$

$$= -5 \times 10^{-2} \text{cm}^3$$

Here negative sign shows volume contraction.

63. A steel wire of length 4m is stretched through 2mm. The cross-section area of the wire is  $2.0\text{mm}^2$ . If Young's modulus of steel is  $2.0 \times 10^{11} \text{N/m}^2$ , find:

- The energy density of the wire,
- The elastic potential energy stored in the wire.

**Ans. :**

- i. **Energy density:**

$$= \frac{1}{2} \left( \frac{Yl}{L} \right) \cdot \frac{1}{L}$$

$$= \frac{1}{2} \left[ \frac{2 \times 10^{11} \times 2 \times 10^{-3}}{4} \right] \times \left[ \frac{2 \times 10^{-3}}{4} \right]$$

$$= \frac{1}{2} \times 10^8 \times \frac{1}{2} \times 10^{-3}$$

$$= 0.25 \times 10^5 = 2.5 \times 10^4 \text{J/m}^3$$

- ii. **Potential energy stored in the wire:**

$$u = \frac{1}{2} \left( \frac{YAl}{L} \right) \cdot l$$

$$= \frac{1}{2} \left[ \frac{2 \times 10^{11} \times 2 \times 10^{-6} \times 2 \times 10^{-3}}{4} \right] \times 2 \times 10^{-3}$$

$$= 10^2 \times 2 \times 10^{-3} = 0.2\text{J}$$

64. Explain why steel is more elastic than rubber.

**Ans. :** Consider two pieces of wires, one of steel and the other of rubber. Suppose both are of equal length ( $L$ ) and of equal area of cross-section ( $a$ ).

Let each be stretched by equal forces, each being equal to  $F$ . We find that the change in length of the rubber wire ( $l_r$ ) is more than that of the steel ( $l_s$ ) i.e.  $l_r > l_s$ .

If  $Y_s$  and  $Y_r$  the Young's moduli of steel and rubber respectively, then from the definition of Young's modulus,

$$Y_s = \frac{F.L}{a.l_s} \text{ and } Y_r = \frac{F.L}{a.l_r}$$

$$\therefore \frac{Y_s}{Y_r} = \frac{l_r}{l_s}.$$

As  $l_r > l_s \therefore \frac{Y_s}{Y_r} > 1$  or  $Y_r$

i.e., the Young's modulus of steel is more than that of rubber. Hence steel is more elastic than rubber.

**OR**

Any material which offers more opposition to the deforming force to change its configuration is more elastic.

65. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The bulk modulus of rubber is  $9.8 \times 10^8 \text{ N m}^{-2}$ , and the density of sea water is  $10^3 \text{ kg m}^{-3}$ .)

**Ans. :** According to the problem, Bulk modulus of rubber ( $B$ ) =  $9.8 \times 10^8 \text{ N m}^{-2}$  Density of sea water ( $\rho$ ) =  $10^3 \text{ kg m}^{-3}$  Percentage decrease in volume

$$\frac{\Delta V}{V} = 0.1\% = \frac{0.1}{100} = 10^{-3}$$

$$\rho = 10^3 \text{ kg m}^{-3}, h = ?$$

Let the rubber ball be taken up to depth  $h$ .

$\therefore$  Change in pressure ( $P$ ) =  $h\rho g$

$$\text{we know, } B = \frac{\Delta P}{(\Delta V/V)} \Rightarrow \Delta P = B \times \frac{\Delta V}{V}$$

$$\Rightarrow \Delta P = 9.8 \times 10^8 \times 10^{-3} = 9.8 \times 10^5 \text{ Nm}^{-2}$$

Also,  $\Delta P = \rho gh$

$$h = \frac{\Delta P}{\rho g} = \frac{9.8 \times 10^5}{10^3 \times 9.8} \Rightarrow h = 10^2 \text{ m} = 100 \text{ m}$$

66. A sphere of mass 20kg is suspended by a metal wire of unstretched length 4m and diameter 1mm. When in equilibrium, there is a clear gap of 2mm between the sphere and the floor. The sphere is gently pushed aside so that the wire makes an angle  $\theta$  with the vertical and is released. Find the maximum value of  $\theta$  so that the sphere does not rub the floor. Young's modulus of the metal of the wire is  $2.0 \times 10^{11} \text{ N/m}^2$ . Make appropriate approximations.

**Ans. :**  $m = 20 \text{ Kg}$

$$L = 4 \text{ m}$$

$$2r = 1 \text{ mm}, r = 5 \times 10^{-4} \text{ m}$$

At equilibrium  $T = mg$

When it moves at an angle  $\theta$  and released the tension  $T$  at lowest point is

$$T = mg + \frac{mv^2}{r}$$

The change in tension is due to centrifugal force

$$\Delta T = \frac{mv^2}{r} \dots (1)$$

Again by work energy principle

$$\frac{1}{2}mv^2 - 0 = mgr(1 - \cos \theta)$$

$$v^2 = 2gr(1 - \cos \theta) \dots (2)$$

$$\text{So, } \Delta T = \frac{m[2gr(1 - \cos \theta)]}{r} \\ = 2mg(1 - \cos \theta)$$

$$\Rightarrow F = \Delta T$$

$$\Rightarrow F = \frac{Y A \Delta L}{L}$$

$$= 2mg \cos \theta = 2mg - \frac{Y A \Delta L}{L}$$

$$\Rightarrow \cot \theta = 1 - \frac{Y A \Delta L}{L(2mg)}$$

$$\Rightarrow \cos \theta = 1 - \left[ \frac{2 \times 10^{11} \times 4 \times 3.14 \times (5)^2 \times 10^{-8} \times 2 \times 10^{-3}}{4 \times 2 \times 20 \times 10} \right]$$

$$\Rightarrow \cos \theta = 0.80$$

$$\theta = 36.4^\circ$$

67. Water near the bed of a deep river is quiet while that near the surface flows. Give reasons.

**Ans. :** The motion of any liquid is dependent upon the amount of stress acting on it. The motion of one layer of liquid is resisted by the other due to the property of viscosity. A river bed remains in a static state. Therefore, any immediate layer of liquid in contact with the river bed will also remain static due to the frictional force. However, the next layer of liquid above this static layer will have a greater velocity due to lesser resistance offered by the static layer. Moving upwards, subsequent layers provide lesser and lesser resistance to the movement of the layers above it. Finally, the topmost layer acquires the maximum velocity. Therefore, for a river, the surface waters flow the fastest.

\* **Given Section consists of questions of 5 marks each.**

**[120]**

68. A rigid bar of mass 15kg is supported symmetrically by three wires each 2.0m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

**Ans. :** Tension force acting on each wire is the same. Hence, the extension is the same for each wire.

Since, the wires are of the same length, the strain will also be same.

Young's modulus is given by,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{F}{\pi d^2 L}} = \frac{4FL}{\pi d^2} \dots (1)$$

where,

F = Tension force

A = Area of cross-section

d = Diameter of the wire

From equation (1), we have

$$Y \propto \left( \frac{1}{d^2} \right)$$

Young's modulus for iron,  $Y_1 = 190 \times 10^9 \text{ Pa}$

Diameter of the iron wire =  $d_1$

Young's modulus for copper,  $Y_2 = 120 \times 10^9 \text{ Pa}$

Diameter of the copper wire =  $d_2$

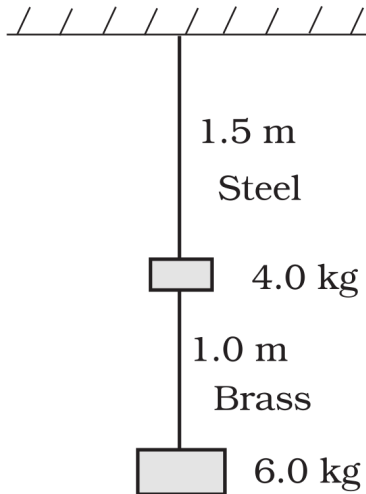
Therefore,

Ratio of their diameters is given by,



$$\frac{d_1}{d_2} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{120 \times 10^9}} = \sqrt{\frac{19}{12}}$$

69. Two wires of diameter 0.25cm, one made of steel and the other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5m and that of brass wire is 1.0m. Compute the elongations of the steel and the brass wires.



**Ans. :** Elongation of the steel wire =  $1.49 \times 10^{-4} \text{m}$

Elongation of the brass wire =  $1.3 \times 10^{-4} \text{m}$

Diameter of the wires,  $d = 0.25 \text{m}$  Hence, the radius of the wires,  $r = d/2 = 0.125 \text{cm}$

Length of the steel wire,  $L_1 = 1.5 \text{m}$  Length of the brass wire,  $L_2 = 1.0 \text{m}$

Total force exerted on the steel wire:

$$F_1 = (4 + 6)g = 10 \times 9.8 = 98 \text{N}$$

Young's modulus for steel:

$$Y_1 = \frac{\left(\frac{F_1}{A_1}\right)}{\left(\frac{\Delta L_1}{L_1}\right)}$$

Where,

$\Delta L_1$  = Change in the length of the steel wire

$A_1$  = Area of cross-section of the steel wire =  $\pi r_1^2$

Young's modulus of steel,  $Y_1 = 2.0 \times 10^{11} \text{ Pa}$

$$\begin{aligned} \therefore \Delta L_1 &= \frac{F_1 \times L_1}{A_1 \times Y_1} = \frac{F_1 \times L_1}{\pi r_1^2 \times Y_1} \\ &= \frac{98 \times 1.5}{\pi (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4} \text{m} \end{aligned}$$

Total force on the brass wire:

$$F_2 = 6 \times 9.8 = 58.8 \text{N}$$

Young's modulus for brass:

$$Y_2 = \frac{\left(\frac{F_2}{A_2}\right)}{\left(\frac{\Delta L_2}{L_2}\right)}$$

Where,

$\Delta L_2$  = Change in length

$A_2$  = Area of cross-section of the wire

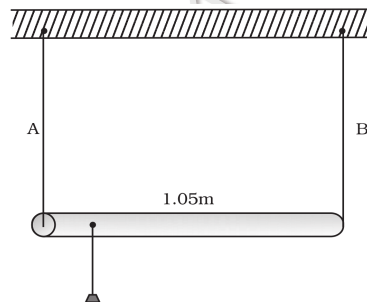
$$\therefore \Delta L_2 = \frac{F_2 \times L_2}{A_2 \times Y_2} = \frac{F_2 \times L_2}{\pi r_2^2 \times Y_2}$$

$$= \frac{58.8 \times 1.0}{\pi \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})} = 1.3 \times 10^{-4} \text{m}$$

Elongation of the steel wire =  $1.49 \times 10^{-4} \text{m}$

Elongation of the brass wire =  $1.3 \times 10^{-4} \text{m}$

70. A rod of length 1.05m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. The cross-sectional areas of wires A and B are  $1.0 \text{mm}^2$  and  $2.0 \text{mm}^2$ , respectively. At what point along the rod should a mass  $m$  be suspended in order to produce (a) equal stresses and



(b) equal strains in both steel and aluminium wires.

**Ans. :**

a. 0.7m from the steel-wire end

Let a small mass  $m$  be suspended to the rod at a distance  $y$  from the end where wire A is attached.

$$\text{Stress in wire} = \frac{\text{Force}}{\text{Area}} = \frac{F}{a}$$

If the two wires have equal stresses, then:

$$\frac{F_1}{a_1} = \frac{F_2}{a_2}$$

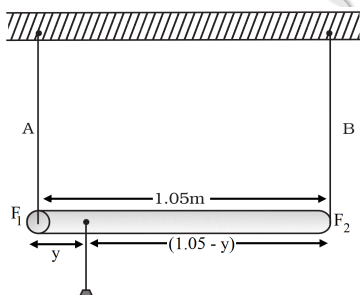
Where

$F_1$  = Force exerted on the steel wire

$F_2$  = Force exerted on the aluminum wire

$$\frac{F_1}{F_2} = \frac{a_1}{a_2} = \frac{1}{2} \dots (1)$$

The situation is shown in the following figure.



Taking torque about the point of suspension, we have:

$$F_1 y = F_2 (1.05 - y)$$

$$\frac{F_1}{F_2} = \frac{1.05 - y}{y} \dots (2)$$

Using equations (1) and (2), we can write:

$$\frac{1.05-y}{y} = \frac{1}{5}$$

$$2(1.05 - y) = y$$

$$2.1 - 2y = y$$

$$3y = 2.1$$

$$\therefore y = 0.7\text{m}$$

In order to produce an equal stress in the two wires, the mass should be suspended at a distance of 0.7m from the end where wire A is attached.

b. 0.432m from the steel-wire end

Cross-sectional area of wire A,  $a_1 = 1.0\text{mm}^2 = 1.0 \times 10^{-6}\text{m}^2$

Cross-sectional area of wire B,  $a_2 = 2.0\text{mm}^2 = 2.0 \times 10^{-6}\text{m}^2$

Young's modulus for steel,  $Y_1 = 2 \times 10^{11}\text{Nm}^{-2}$

Young's modulus for aluminium,  $Y_2 = 7.0 \times 10^{10}\text{Nm}^{-2}$

Young's modulus =  $\frac{\text{Stress}}{\text{Strain}}$

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{a}{Y}$$

$$\frac{\frac{F_1}{a_1}}{Y_1} = \frac{\frac{F_2}{a_2}}{Y_2}$$

$$\frac{F_1}{F_2} = \frac{a_1}{a_2} \frac{Y_1}{Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7} \dots\dots (3)$$

Taking torque about the point where mass  $m$ , is suspended at a distance  $y_1$  from the side where wire A attached, we get:

$$F_1 y_1 = F_2 (1.05 - y_1)$$

$$\frac{F_1}{F_2} = \frac{1.05 - y_1}{y_1} \dots\dots (4)$$

Using equations (3) and (4), we get:

$$\frac{(1.05 - y_1)}{y_1} = \frac{10}{7}$$

$$7(1.05 - y_1) = 10y_1$$

$$17y_1 = 7.35$$

$$\therefore y_1 = 0.432$$

In order to produce an equal strain in the two wires, the mass should be suspended at a distance of 0.432m from the end where wire A is attached.

71. The edge of an aluminium cube is 10cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25G Pa. What is the vertical deflection of this face?

**Ans. :** Edge of the aluminium cube,  $L = 10\text{cm} = 0.1\text{m}$

The mass attached to the cube,  $m = 100\text{kg}$

Shear modulus ( $\eta$ ) of aluminium = 25G Pa =  $25 \times 10^9$  Pa

Shear modulus,  $\eta = \text{Shear stress} / \text{Shear strain} = (F/A) / (L/\Delta L)$

Where,

$F = \text{Applied force} = mg = 100 \times 9.8 = 980\text{N}$

$A = \text{Area of one of the faces of the cube} = 0.1 \times 0.1 = 0.01\text{m}^2$

$\Delta L = \text{Vertical deflection of the cube}$

$$\begin{aligned}\therefore \Delta L &= \frac{FL}{A\eta} \\ &= 980 \times 0.1 / [10^{-2} \times (25 \times 10^9)] \\ &= 3.92 \times 10^{-7} \text{m}\end{aligned}$$

The vertical deflection of this face of the cube is  $3.92 \times 10^{-7} \text{m}$ .

72. A 14.5kg mass, fastened to the end of a steel wire of unstretched length 1.0m, is whirled in a vertical circle with an angular velocity of 2rev/s at the bottom of the circle. The cross-sectional area of the wire is  $0.065 \text{cm}^2$ . Calculate the elongation of the wire when the mass is at the lowest point of its path.

**Ans. :** Mass,  $m = 14.5 \text{kg}$

Length of the steel wire,  $l = 1.0 \text{m}$

Angular velocity,  $\omega = 2 \text{ rev/s} = 2 \times 2\pi \text{ rad/s}$   
 $= 12.56 \text{rad/s}$

Cross-sectional area of the wire,  $a = 0.065 \text{cm}^2 = 0.065 \times 10^{-4} \text{m}^2$

Let  $\Delta l$  be the elongation of the wire when the mass is at the lowest point of its path.

When the mass is placed at the position of the vertical circle, the total force on the mass is:

$$\begin{aligned}F &= mg + m\omega^2 \\ &= 14.5 \times 9.8 + 14.5 \times 1 \times (12.56)^2 \\ &= 2429.53 \text{N}\end{aligned}$$

Young's modulus =  $\frac{\text{Stress}}{\text{Strain}}$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F}{A} \frac{l}{\Delta l}$$

$$\therefore \Delta l = \frac{Fl}{AY}$$

Young's modulus for steel =  $2 \times 10^{11} \text{ Pa}$

$$\begin{aligned}\Delta l &= \frac{2429.53 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}} \\ \Rightarrow \Delta l &= 1.87 \times 10^{-3} \text{m}\end{aligned}$$

Hence, the elongation of the wire is  $1.87 \times 10^{-3} \text{m}$ .

73. The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about  $1.1 \times 10^8 \text{ Pa}$ . A steel ball of initial volume  $0.32 \text{m}^3$  is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

**Ans. :** Water pressure at the bottom,  $p = 1.1 \times 10^8 \text{ Pa}$

Initial volume of the steel ball,  $V = 0.32 \text{m}^3$

Bulk modulus of steel,  $B = 1.6 \times 10^{11} \text{Nm}^{-2}$

The ball falls at the bottom of the Pacific Ocean, which is 11km beneath the surface.

Let the change in the volume of the ball on reaching the bottom of the trench be  $\Delta V$ .

$$\text{Bulk modulus, } B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

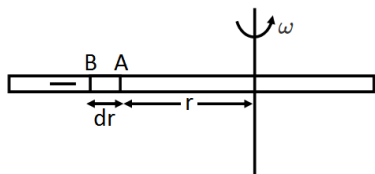
$$\Delta V = \frac{B}{pV}$$

$$= 1.1 \times 10^8 \times 0.32 / (1.6 \times 10^{11}) = 2.2 \times 10^{-4} \text{m}^3$$

Therefore, the change in volume of the ball on reaching the bottom of the trench is  $2.2 \times 10^{-4} \text{m}^3$ .

74. A steel rod of length  $2l$ , cross sectional area  $A$  and mass  $M$  is set rotating in a horizontal plane about an axis passing through the centre. If  $Y$  is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform.)

**Ans. :** Consider in given figure an element ( $dr$ ) of rod at a distance  $r$  from the centre.



Let  $T(r)$  and  $T(r + dr)$  are the tensions external force to rod extend at A and B ends of element (small) respectively. Centrifugal force on element  $dr$  due to tension difference  $= T(r + dr) - T(r)$

Centrifugal force  $= -dT$  (outward)

Centripetal Force due to rotation on element  $dr = dm\omega^2 r$

$\therefore -dT = dm \omega^2 r$  (Let  $\mu$  = mass per unit length)

then  $-dT \omega^2 r (dr \cdot \mu)$

$$-dT = \mu \omega^2 r \cdot dr$$

Integrating both sides

$$-\int_0^T dT = \mu \omega^2 \int_r^l r dr$$

Tension in rod at distance  $r$  from centre so limits will varies from  $r$  to  $l$

$$\therefore -T(r) = \mu \omega^2 \left[ \frac{r^2}{2} \right]_r^l = \frac{\mu \omega^2}{2} (l^2 - r^2) \quad (I)$$

Let the increase in length of  $dr$  element at distance  $r$  from centre is  $\delta r$  then

$$Y = \frac{\text{stress}}{\text{Strain}} = \frac{T(r)/A}{\delta r/dr} = \frac{T(r)}{A} \cdot \frac{dr}{\delta r}$$

$$\frac{\delta r}{dr} = \frac{T(r)}{AY} = \frac{-\mu \omega^2}{2AY} (l^2 - r^2)$$

$\therefore$  Negative sign shows only that the direction of extension is opposite to restoring force

$$\therefore \delta r = \frac{\mu \omega^2}{2AY} (l^2 - r^2) dr$$

$$\int_0^\delta \delta r = \int_0^l \frac{\mu \omega^2}{2AY} (l^2 - r^2) dr \quad (\text{for rod one from centre})$$

$$\delta = \frac{\mu \omega^2}{2AY} \left( l^3 - \frac{l^3}{3} \right) = \frac{\mu \omega^2}{2AY} \frac{2}{3} l^3$$

$$\delta = \frac{\mu \omega^2}{3AY} l^3$$

$$\therefore \text{Total extension in rod both sides} = 2\delta = \frac{2\mu \omega^2 l^3}{3AY}$$

75.

- a. A steel wire of mass  $\mu$  per unit length with a circular cross section has a radius of 0.1cm. The wire is of length 10m when measured lying horizontal, and hangs from a hook on the wall. A mass of 25kg is hung from the free end of the wire. Assuming the wire to be uniform and lateral strains  $\ll$  longitudinal strains, find the extension in the length of the wire. The density of steel is  $7860 \text{ kg m}^{-3}$  (Young's modulus  $Y = 2 \times 10^{11} \text{ Nm}^{-2}$ ).
- b. If the yield strength of steel is  $2.5 \times 10^8 \text{ Nm}^{-2}$ , what is the maximum weight that can be hung at the lower end of the wire?

**Ans. :**

- a. Consider an element  $dx$  at a distance  $x$  from the load ( $x = 0$ ).

If  $T(x)$  and  $T(x + dx)$  are tensions on the two cross sections a distance  $dx$  apart, then

$$T(x + dx) - T(x) = \mu g dx \text{ (where } \mu \text{ is the mass/ length)}$$

$$\left( \frac{dT}{dx} \right) dx = \mu g dx$$

$$\Rightarrow T(x) = \mu g x + C$$

$$\text{At } x=0, T(0)=Mg \Rightarrow C \Rightarrow Mg$$

$$T(x) = \mu g x + Mg$$

Let the length  $dx$  at  $x$  increase by  $dr$ , then

$$\frac{T(x)/A}{dr/dx} = Y$$

$$\text{or, } \frac{dr}{dx} = \frac{1}{YA} T(x)$$

$$\Rightarrow r = \frac{1}{YA} \int_0^L (\mu g x + mg) dx$$

$$= \frac{1}{YA} \left[ \frac{\mu g x^2}{2} + mgx \right]_0^L$$

$$= \frac{1}{YA} \left[ \frac{mgl}{2} + mgL \right]$$

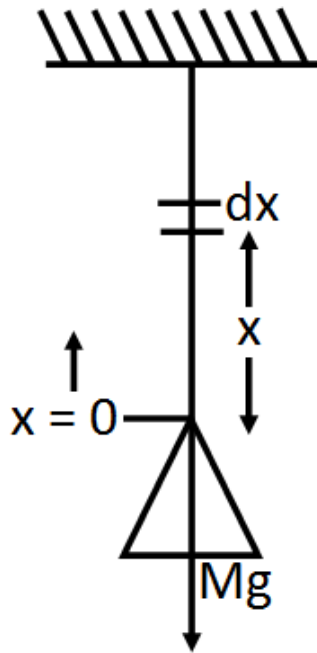
( $m$  is the mass of the wire)

$$A = \pi \times (10^{-3})^2 \text{ m}^2, Y = 200 \times 10^9 \text{ Nm}^{-2}$$

$$m = \pi \times (10^{-3})^2 \times 10 \times 7860 \text{ kg}$$

$$\therefore r = \frac{1}{2 \times 10^{11} \times \pi \times 10^{-6}} \left[ \frac{\pi \times 786 \times 10^{-7} \times 10 \times 10}{2} + 25 \times 10 \times 10 \right]$$

$$= [196.5 \times 10^{-6} + 3.98 \times 10^{-3}] \sim 4 \times 10^{-3} \text{ m}$$



- b. The maximum tension would be at  $x = L$ .

$$T = \mu g L + Mg = (m + M)g$$

The Yield force

$$= 250 \times 10^6 \times \pi \times (10^{-3})^2 = 250 \times \pi \text{ N} =$$

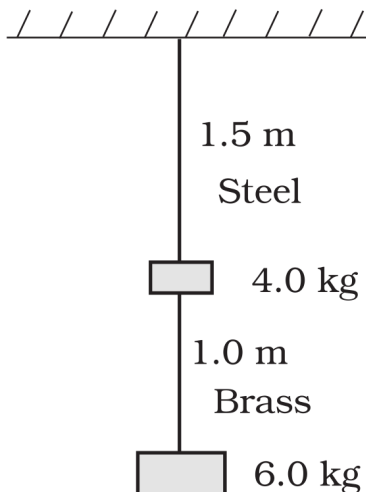
At Yield

$$(m + M)g = 250 \times \pi$$

$$m = \pi \times (10^{-3})^2 \times 10 \times 7860 \ll M \therefore Mg \sim 250 \times \pi$$

$$\text{Hence, } M = \frac{250 \times \pi}{10} = 25 \times \pi \sim 75 \text{ kg.}$$

76. Two wires of diameter 0.25cm, one made of steel and the other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5m and that of brass wire is 1.0m. Compute the elongations of the steel and the brass wires.



**Ans. :** Elongation of the steel wire  $= 1.49 \times 10^{-4} \text{ m}$

Elongation of the brass wire  $= 1.3 \times 10^{-4} \text{ m}$

Diameter of the wires,  $d = 0.25 \text{ m}$  Hence, the radius of the wires,  $r = d/2 = 0.125 \text{ cm}$

Length of the steel wire,  $L_1 = 1.5 \text{ m}$  Length of the brass wire,  $L_2 = 1.0 \text{ m}$

Total force exerted on the steel wire:

$$F_1 = (4 + 6)g = 10 \times 9.8 = 98\text{N}$$

Young's modulus for steel:

$$Y_1 = \frac{\left(\frac{F_1}{A_1}\right)}{\left(\frac{\Delta L_1}{L_1}\right)}$$

Where,

$\Delta L_1$  = Change in the length of the steel wire

$A_1$  = Area of cross-section of the steel wire =  $\pi r_1^2$

Young's modulus of steel,  $Y_1 = 2.0 \times 10^{11}$  Pa

$$\begin{aligned}\therefore \Delta L_1 &= \frac{F_1 \times L_1}{A_1 \times Y_1} = \frac{F_1 \times L_1}{\pi r_1^2 \times Y_1} \\ &= \frac{98 \times 1.5}{\pi (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4} \text{m}\end{aligned}$$

Total force on the brass wire:

$$F_2 = 6 \times 9.8 = 58.8\text{N}$$

Young's modulus for brass:

$$Y_2 = \frac{\left(\frac{F_2}{A_2}\right)}{\left(\frac{\Delta L_2}{L_2}\right)}$$

Where,

$\Delta L_2$  = Change in length

$A_2$  = Area of cross-section of the wire

$$\begin{aligned}\therefore \Delta L_2 &= \frac{F_2 \times L_2}{A_2 \times Y_2} = \frac{F_2 \times L_2}{\pi r_2^2 \times Y_2} \\ &= \frac{58.8 \times 1.0}{\pi \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})} = 1.3 \times 10^{-4} \text{m}\end{aligned}$$

Elongation of the steel wire =  $1.49 \times 10^{-4} \text{m}$

Elongation of the brass wire =  $1.3 \times 10^{-4} \text{m}$

77. What is meant by elastic potential energy? Derive an expression for the elastic potential energy of a stretched wire. Prove that its elastic energy density is equal to  $\frac{1}{2} \text{stress} \times \text{strain}$ .

**Ans. :** When a wire is stretched, some work is done against the internal restoring forces acting between particles of the wire. This work done appears as elastic potential energy in the wire.

Consider a wire of length  $l$  and area of cross-section  $a$ . Let  $F$  be the stretching force applied on the wire and  $\Delta l$  be the increase in length of the wire.

Initially, the internal restoring force was zero but when the length is increased by  $\Delta l$ , the internal force increases from 0 to  $F$  (applied force). Thus, average initial force on an increase in length ( $\Delta l$ ,) of the wire,

$$= \frac{0+F}{2} = \frac{F}{2}.$$

Hence, work done on the wire,  $W = \text{Average force} \times \text{Increase in length}$ ,



$$= \frac{F}{2} \times \Delta l$$

This is stored as elastic potential energy  $U$  in the wire,

$$\therefore U = \frac{1}{2} F \times \Delta l = \frac{1}{2} \frac{F}{a} \times \frac{\Delta l}{l} \times al$$

$$= \frac{1}{2} (\text{Stress}) \times (\text{Strain}) \times \text{Volume of the wire}$$

$\therefore$  Elastic potential energy per unit volume of the wire is given by,

$$u = \frac{U}{al} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

78. A wire loaded by a weight of density  $7.6 \text{ g/cm}^3$  is found to measure  $90 \text{ cm}$ . On immersing the weight in water, the length decreased by  $0.18 \text{ cm}$ . Find the original length of wire.

**Ans. :** Let  $L$  be the original length of the wire,  $A$  be its area of cross-section and  $W$  be the load attached to the wire. Then, Young's modulus of the wire is given by,

$$Y = \frac{W \times L}{A \times \Delta L} (\because F = W)$$

$$\text{Since } \Delta L = 90 - L$$

$$\therefore Y = \frac{W \times L}{A(90 - L)} \dots (1)$$

Volume of weight attached,

$$= \frac{W}{\text{Density of weight}} = \frac{W}{7.6} \text{ cm}^3$$

Weight of water displaced,

$$= \frac{W}{7.6} \times \text{density of water} = \frac{W}{7.6} \times 1 = \frac{W}{7.6}$$

$\therefore$  Net weight after immersing in water is,

$$W' = W - \frac{W}{7.6} = \frac{6.6W}{7.6}$$

Length of wire after immersing in water,

$$= (90 - 0.18) = 89.82 \text{ cm}$$

$\therefore$  Change in length on immersing in water,

$$\Delta L' = (89.82 - L) \text{ cm}$$

$$\therefore Y = \frac{W'L}{A\Delta L'} = \frac{6.6W \times L}{7.6 \times A \times (89.82 - L)}$$

Comparing eqns. (1) and (2), we get,

$$\frac{W \times L}{A(90 - L)} = \frac{6.6W \times L}{7.6 \times A \times (89.82 - L)}$$

$$682.632 - 7.6L = 594 - 6.6L$$

$$L = 88.632 \text{ cm}$$

79. A steel wire of cross-sectional area  $0.5 \text{ mm}^2$  is held between two fixed supports. If the tension in the wire is negligible and it is just taut at a temperature of  $20^\circ \text{C}$ , determine the tension when the temperature falls to  $0^\circ \text{C}$ . Young's modulus of steel is  $21 \times 10^{11} \text{ dyne cm}^{-2}$  and the coefficient of linear expansion of steel is  $12 \times 10^{-6} \text{ per } ^\circ \text{C}$ . Assume that the distance between the supports remains unchanged.

**Ans. : Numerical:** Let  $l$  be the length of the wire at  $20^\circ \text{C}$  and  $l_0$ , the length at  $0^\circ \text{C}$ . Then,

$$l - l_0 = \alpha l_0 \Delta T = 20\alpha \cdot l_0$$

$$\text{Compressive strain} = \frac{l - l_0}{l_0} = 20\alpha = 20 \times 12 \times 10^{-6} = 2.4 \times 10^{-4}$$

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = Y \times \text{strain} = \frac{F}{A}$$

$$\text{Hence, tension } T = YA \times \text{Strain} = 21 \times 10^{11} \times 0.5 \times 10^{-2} \times 2.4 \times 10^{-4} \\ = 2.52 \times 10^6 \text{ dyne} = 25.2 \text{ N}$$

This is the tension in the wire when the temperature falls to  $0^\circ\text{C}$ .

80. Two wires, one of steel and the other of aluminium, each 2m long and of diameter 2.0mm, are joined end to end to form a composite wire of length 4.0m. What tension in the wire will produce a total extension of 0.90mm?  $Y$  for steel =  $2 \times 10^{11} \text{ Nm}^{-2}$ ;  $Y$  for aluminium =  $7 \times 10^{11} \text{ Nm}^{-2}$

**Ans. :**  $Y$  for steel,  $Y_1 = 2 \times 10^{11} \text{ Nm}^{-2}$

$Y$  for aluminium,  $Y_2 = 7 \times 10^{11} \text{ Nm}^{-2}$

Length of each wire,  $L = 2 \text{ m}$ ;

$d = 2.0 \text{ mm}$

$\therefore$  Radius,  $r = 1 \text{ mm} = 10^{-3} \text{ m}$

Area of cross-section,

$$A = \pi r^2 = \pi (10^{-3})^2 = \pi \times 10^{-6} \text{ m}^2$$

Total extension  $(\Delta l_1 + \Delta l_2) = 9 \times 10^{-4} \text{ m}$

We know,  $Y = \frac{FL}{A\Delta l}$

$$\therefore (\Delta l_1 + \Delta l_2) = \frac{FL}{A} \left( \frac{1}{Y_1} + \frac{1}{Y_2} \right)$$

$$\text{or } 9 \times 10^{-4} = \frac{F \times 2}{\pi \times 10^{-6}} \left( \frac{1}{2 \times 10^{11}} + \frac{1}{7 \times 10^{11}} \right)$$

$$\text{or } F = \frac{9 \times 10^{-4} \times 3.14 \times 10^{-6} \times 14 \times 10^{11}}{2 \times 9} \\ = 219.8 \text{ N}$$

81. A load of 31.4kg is suspended from a wire of radius  $10^{-3} \text{ m}$  and density  $9 \times 10^3 \text{ kg/m}^3$ . Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat capacity of the material of the wire are  $9.8 \times 10^{10} \text{ N/m}^2$  and  $490 \text{ J/kg}^\circ\text{C}$  respectively.

**Ans. :** Volume of wire  $V = \pi r^2 l$

$$\therefore \text{Density} = \frac{\text{Mass}}{V}$$

$$9 \times 10^3 = \frac{31.4}{\pi r^2 \times L}$$

$$\therefore L = \frac{31.4}{\pi^2 \times 9 \times 10^3}$$

$$L = \frac{31.4}{3.14 \times 10^{-6} \times 9 \times 10^3}$$

$$\therefore L = \frac{10}{9} \times 10^3 \text{ m} = \frac{10^4}{9} \text{ m}$$

Now,  $Y = \frac{mg \cdot L}{\pi r^2 l}$

$$\therefore l = \frac{mg \cdot L}{\pi r^2 \cdot Y} = \frac{31.4 \times 9.8 \times 10^4}{3.14 \times 10^{-6} \times 9 \times 9.8 \times 10^{10}} = \frac{10}{9} \text{ m}$$

Now the work done,  $= \frac{1}{2} F \cdot l = \frac{1}{2} \times 3.14 \times 9.8 \times \frac{10}{9}$

75% of the work done is converted into heat energy.

$$\therefore \text{Heat energy} = \frac{1}{2} \times 31.4 \times 9.8 \times \frac{10}{9} \times \frac{75}{100}$$

But heat energy = mass  $\times$  S.P heat  $\times$  temp difference,

$$= 31.4 \times 490 \times t$$

$$\therefore 31.4 \times 490 \times t$$

$$= \frac{1}{2} \times 31.4 \times 9.8 \times \frac{10}{9} \times \frac{75}{100}$$

$$\therefore t = \frac{\frac{1}{2} \times 31.4 \times 9.8 \times \frac{10}{9} \times \frac{75}{100}}{31.4 \times 490}$$

$$t = \frac{1}{120} \text{K or } 0.0083^\circ \text{C.}$$

82. An elastic spring of force constant  $K$  is compressed by an amount  $x$ . Show that its potential energy is  $\frac{1}{2} Kx^2$ .

**Ans. :** From Hook's law, when spring is compressed or elongated, it tends to recover its original length.

Restoring force  $\propto$  Stretch or compression,

$$-F \propto x$$

$$\text{or } F = -kx$$

$K$  = constant of the spring,

Let the body be displaced further through an infinitesimally small distance  $dx$ , against the restoring force.

$\therefore$  Small amount of work done in increasing the length of the spring by  $dx$  is  $dW = -F, dx = Kx, dx \dots(i)$

Total work done in giving displacement  $x$  to the body can be obtained by (i) from  $x = 0$  to  $x = x$ ,

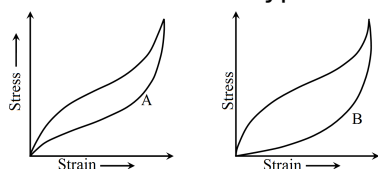
$$\text{i.e., } W = \int_{x=0}^{x=x} Kx \cdot dx$$

$$= K \left[ \frac{x^2}{2} \right]_{x=0}^{x=x}$$

$$\Rightarrow W = K \left[ \frac{x^2}{2} - 0 \right]$$

$$\Rightarrow W = \frac{1}{2} Kx^2$$

83. For two different type of rubber stress - strain curves are shown:



- To make a shock absorber which rubber will you prefer and why?
- To make car tyre which of the two rubber would you prefer and why?

**Ans. :** The area of hysteresis loop is proportional to the energy dissipated by the material as heat when material undergoes loading and unloading.

- B material having larger loop area would absorb more energy when subjected to vibration/ shock.
- A material is preferred for car type because, the energy dissipation must be minimised to avoid excessing heating of the car type.

84. A wire of length  $L$  and radius  $r$  is clamped rigidly at one end. When the other end of the wire is pulled by a force  $f$ , its length increases by  $l$ . Another wire of the same material of length  $2L$  and radius  $2r$ , is pulled by a force  $2f$ . Find the increase in length of this wire.

**Ans. :** We have to apply Hooke's law to compare the extension in each wire. According to the diagram which shows the situation.

Now, Young's modulus

$$(Y) = \frac{f}{A} \times \frac{L}{l}$$

First case, length of wire =  $L$ , radius of wire =  $r$

Force applied =  $f$ , increase in length =  $l$

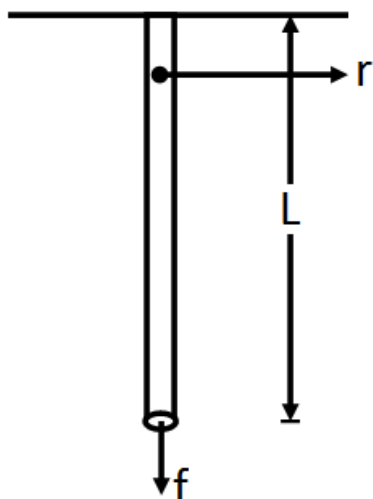
$$Y_1 = \frac{\frac{f}{\pi r^2}}{\frac{l}{L}} = \frac{fL}{\pi r^2 l} \dots (i)$$

In second case,

length of wire =  $2L$ , radius of wire =  $2r$ ,

force applied =  $2f$ , increase in length =  $x$  (say)

$$Y_2 = \frac{\frac{2f}{\pi (2r)^2}}{\frac{x}{2L}} = \frac{fL}{\pi r^2 x} \dots (ii)$$



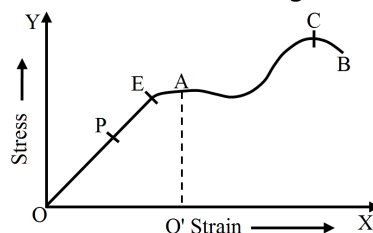
Both the wires are of same material, so Young's modulus will be same.

From Eqs. (i) and (ii),

$$\frac{f}{\pi r^2} \times \frac{L}{l} = \frac{f}{\pi r^2} \times \frac{L}{x}$$

Hence,  $x = l$ .

85. The Stress-Strain graph for a metal wire is shown in the figure upto the point E. The wire returns to its original state O along the curve EPO when it is gradually unloaded. Point B



corresponds to the fracture of the wire:

- Upto what point of the curve is Hooke's law obeyed?
- Which point on the curve corresponds to the elastic limit or yield point of the wire?

- iii. Indicate the elastic and plastic regions of the Stress-Strain graph.
- iv. Describe what happens when the wire is loaded upto a stress corresponding to the point A on the graph, and then unloaded gradually. In particular, explain, the dotted curve.
- v. What is peculiar about the portion of the Stress-Strain graph from C to B? Upto what stress can the wire be subjected without causing fracture?

**Ans. :**

- i. Upto the point P. It has to be slightly below E.
- ii. Point E.
- iii. Elastic region: O to E.  
Plastic region: E to B.
- iv. Strain increases in proportion to the load upto P. But beyond P, it increases by an increasingly greater amount for a given increase in the load. Beyond the elastic limit E, it does not retrace the curve backward. The wire is unloaded but returns to 'O' along the dotted line 'AO'. Point 'O' corresponding to zero load which implies a permanent strain in wire.
- v. From C to B, strain increases even if the wire is being unloaded and at B it fractures. Stress upto that corresponding to C can be applied without causing fracture.

86. Why a hollow shaft is stronger than a solid shaft made from the same and equal amounts of material?

**Ans. :** The torque required to produce a unit twist in a solid shaft of radius  $r$  is given by,

$$\tau = \frac{\pi \eta r^4}{2l}, \dots (1)$$

where  $\eta$  is the modulus of rigidity of the material and  $l$  is the length of the shaft.

The torque required to produce a unit twist in a hollow shaft of inner and outer radii  $r_i$  and  $r_0$  is given by,

$$\tau = \frac{\pi \eta (r_0^4 - r_i^4)}{2l} = \frac{\pi \eta (r_0^2 - r_i^2)(r_0^2 + r_i^2)}{2l}$$

Dividing (2) by (1) we get,

$$\frac{\tau'}{\tau} = \frac{(r_0^2 - r_i^2)(r_0^2 + r_i^2)}{r^4}$$

Since the two shafts are made of the same material and the amounts of material are equal,

$$\therefore \pi r^2 l = \pi (r_0^2 - r_i^2) l \text{ or } r^2 = r_0^2 - r_i^2$$

$$\text{From (3), } \frac{\tau'}{\tau} = \frac{r_0^2 + r_i^2}{r^2} \text{ or } \frac{\tau'}{\tau} > 1 \text{ or } \tau' > \tau$$

The torque required to twist a hollow shaft is clearly more than the torque required to twist a solid shaft. Thus, hollow shaft is stronger than a solid shaft.

87. When a load on a wire is increased from 3kg wt to 5kg wt., the elongation increases from 0.61mm to 1.02mm. How much work is done during the extension of the wire?

$$\text{Ans. : } W_1 = \frac{1}{2} \cdot F \times l$$

$$= \frac{1}{2} \times 3 \times 9.8 \times 0.61 \times 10^{-3} \text{ J}$$

$$W_2 = \frac{1}{2} \cdot F \times l$$

$$= \frac{1}{2} \times 5 \times 9.8 \times 1.02 \times 10^{-3} \text{ J}$$

∴ Net work done during the extensions,

$$W = W_2 - W_1$$

$$= \left( \frac{1}{2} \times 5 \times 9.8 \times 1.02 \times 10^{-3} \right)$$

$$- \left( \frac{1}{2} \times 3 \times 9.8 \times 0.61 \times 10^{-3} \right)$$

$$= \frac{1}{2} \times 9.8 \times 10^{-3} [5 \times 1.02 - 3 \times 0.61]$$

$$= \frac{1}{2} \times 9.8 \times 10^{-3} [5.10 - 1.83]$$

$$= \frac{1}{2} \times 9.8 \times 10^{-3} \times 3.27 = 16.023 \times 10^{-3} \text{ J}$$

88. A 14.5kg mass, fastened to the end of a steel wire of unstretched length 1.0m, is whirled in a vertical circle with an angular velocity of 2rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065cm<sup>2</sup>. Calculate the elongation of the wire when the mass is at the lowest point of its path.

**Ans. :** Mass,  $m = 14.5 \text{ kg}$

Length of the steel wire,  $l = 1.0 \text{ m}$

Angular velocity,  $\omega = 2 \text{ rev/s} = 2 \times 2\pi \text{ rad/s}$   
 $= 12.56 \text{ rad/s}$

Cross-sectional area of the wire,  $a = 0.065 \text{ cm}^2 = 0.065 \times 10^{-4} \text{ m}^2$

Let  $\Delta l$  be the elongation of the wire when the mass is at the lowest point of its path.

When the mass is placed at the position of the vertical circle, the total force on the mass is:

$$F = mg + ml\omega^2$$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times (12.56)^2$$

$$= 2429.53 \text{ N}$$

Young's modulus =  $\frac{\text{Stress}}{\text{Strain}}$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F}{A} \frac{l}{\Delta l}$$

$$\therefore \Delta l = \frac{Fl}{AY}$$

Young's modulus for steel =  $2 \times 10^{11} \text{ Pa}$

$$\Delta l = \frac{2429.53 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}}$$

$$\Rightarrow \Delta l = 1.87 \times 10^{-3} \text{ m}$$

Hence, the elongation of the wire is  $1.87 \times 10^{-3} \text{ m}$ .

89. Two cylinders A and B of radii  $r$  and  $2r$  are soldered co-axially. The free end of A is clamped and the free end of B is twisted by an angle  $\phi$ . Find twist at the junction taking the material of two cylinders to be same and of equal length.

**Ans. :** Let  $\tau$  be the torque applied at the free end and  $\phi'$  be the angle of twist at the junction.

$$\text{Then, } \tau = \frac{\pi \eta r^4 \phi'}{2l} = \frac{\pi \eta (2r)^4 (\phi - \phi')}{2l}$$

or

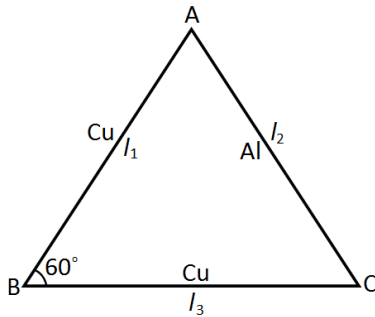
$$\phi' = \frac{16\phi}{17}$$

90. An equilateral triangle ABC is formed by two Cu rods AB and BC and one Al rod. It is heated in such a way that temperature of each rod increases by  $\Delta T$ . Find change in the angle ABC. [Coeff. of linear expansion for 1 Cu is  $\alpha_1$  Coeff. of linear expansion for 2Al is  $\alpha_2$ ]

**Ans. :** By trigonometry

$$\cos \theta = \frac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}$$

$$2l_1 l_3 \cos \theta = l_1^2 + l_3^2 - l_2^2$$



Differentiating both sides

$$2[l_1 l_3 \cdot \cos \theta + l_1 l_3 d(\cos \theta)] = 2l_1 dl_1 + 2l_3 dl_3 - 2l_2 dl_2$$

$$2[(l_1 dl_3 + l_3 dl_1) \cos \theta - l_1 l_3 \sin \theta d\theta] = 2(l_1 dl_1 + l_3 dl_3 - l_2 dl_2)$$

$$(l_1 dl_3 + l_3 dl_1) \cos \theta - l_1 l_3 \sin \theta d\theta = l_1 dl_1 + l_3 dl_3 - l_2 dl_2 \quad (i)$$

$$L_t = L_0(1 + \alpha \Delta t)$$

$$L_t - L_0 = L_0 \alpha \Delta t$$

$$\Delta L = L \alpha \cdot \Delta t$$

$$dl_1 = l_1 \alpha_1 \Delta t \quad dl_3 = l_3 \alpha_1 \Delta t$$

$$\text{and } dl_2 = l_2 \alpha_2 \Delta t$$

$$l_1 = l_2 = l_3 = l$$

$$\therefore dl_1 = l \alpha_1 \Delta t \quad dl_3 = l \alpha_1 \Delta t$$

$$\text{and } dl_2 = l \alpha_2 \Delta t$$

Substitute their value in (i)

$$\cos \theta (l^2 \alpha_1 \Delta t + l^2 \alpha_1 \Delta t) - l^2 \sin \theta d\theta = l^2 \alpha_1 \Delta t + l^2 \alpha_1 \Delta t - l^2 \alpha_2 \Delta t$$

$$2l^2 \alpha_1 \Delta t \cos \theta - l^2 [\sin \theta \cdot d\theta] = l^2 [\alpha_1 + \alpha_1 - \alpha_2] \Delta t$$

$$l^2 [2\alpha_1 \Delta t \cos 60^\circ - \sin 60^\circ d\theta] = l^2 [2\alpha_1 - \alpha_2] \Delta t$$

$$2\alpha_1 \Delta t \times \frac{1}{2} - 2\alpha_1 \Delta t + \alpha_2 \Delta t = \frac{\sqrt{3}}{2} d\theta$$

$$\frac{\sqrt{3}}{2} d\theta = [\alpha_1 - 2\alpha_1 + \alpha_2] \Delta t$$

$$d\theta = \frac{2(\alpha_2 - \alpha_1) \Delta t}{\sqrt{3}}$$

$$[\therefore \Delta t = \Delta T \text{ (given)}]$$

$$d\theta = \frac{2(\alpha_2 - \alpha_1) \Delta T}{\sqrt{3}}$$

91. A stone of mass  $m$  is tied to an elastic string of negligible mass and spring constant  $k$ . The unstretched length of the string is  $L$  and has negligible mass. The other end of the

string is fixed to a nail at a point P. Initially the stone is at the same level as the point P. The stone is dropped vertically from point P.

- Find the distance  $y$  from the top when the mass comes to rest for an instant, for the first time.
- What is the maximum velocity attained by the stone in this drop?
- What shall be the nature of the motion after the stone has reached its lowest point?

**Ans. :** A stone is tied at P with string of length  $L$ . String is fixed with nail at 'O'. Stone is lifted upto height  $L$ , so that string is stretched as shown in given fig. When stone fall under gravity. It tries to follow path PP' but due to elastic string it will go a part of circular path P to Q. This is like a centrifugal force that stretches the string outward and increases its length ( $\Delta L$ ). So the change in Potential energy of stone at Q' and p converts into mechanical energy in string of spring constant  $K$ . So P.E of stone = mechanical Energy of string.

$$mgy = \frac{1}{2}K(y - L)^2$$

$$mgy = \frac{1}{2}K(y^2 + L^2 - 2yL)$$

$$2mgy = K[y^2 + L^2 - 2yL]$$

$$2mgy = Ky^2 - 2KyL + KL^2$$

$$\text{or } Ky^2 - 2KyL - 2mgy + KL^2 = 0$$

$$Ky^2 - 2(KL + mg)y + KL^2 = 0$$

- Solving this equation by quadratic formula we get,

$$D = b^2 - 4ac \quad (a = K \quad b = -2(KL + mg) \quad c = KL^2)$$

$$D = [-2(KL + mg)]^2 - 4(K)(KL^2)$$

$$D = +4[(KL)^2 + (mg)^2 + 2(KL)(mg)] - 4K^2L^2$$

$$D = 4[K^2L^2 + m^2g^2 + 2KLmg] - 4K^2L^2$$

$$= 4K^2L^2 + 4m^2g^2 + 8KLmg - 4K^2L^2$$

$$\sqrt{D} = \sqrt{4mg[mg + 2KL]} = 2\sqrt{mg(mg + 2KL)}$$

$$\therefore y = \frac{-b \pm \sqrt{D}}{2a} = \frac{+2(KL + mg) \pm 2\sqrt{mg(2KL + mg)}}{2K}$$

$$y = \frac{2[(KL + mg)] \pm \sqrt{mg(2KL + mg)}}{2K}$$

$$y = \frac{(KL + mg) \pm \sqrt{mg(2KL + mg)}}{K}$$

- At maximum velocity as its lowest point acceleration is zero.

$$\therefore F = 0$$

So, the spring or string force  $Kx$  is balanced by gravitational force  $mg$ . so, these two forces will be equal and opposite.

$$\therefore mg = kx \dots (i) \text{ where } x \text{ is extension in string}$$

Let  $v$  be the maximum velocity of stone at bottom of journey.

By law of conservation of energy,

KE of stone + PE gain by string = P.E. lost stone from p to Q'

$$\frac{1}{2}mv^2 + \frac{1}{2}Kx^2 = mg(L + x)$$



$$mv^2 + Kx^2 = 2mg(L + x)$$

$$mv^2 = 2mgL + 2mgx - Kx^2$$

$$mg = Kx \text{ (from i)}$$

$$x = \frac{mg}{K}$$

$$\therefore mv^2 = 2mgL + 2mg \cdot \frac{mg}{K} - K \frac{m^2 g^2}{K^2}$$

$$= 2mgL + \frac{2m^2 g^2}{K} - \frac{m^2 g^2}{K}$$

$$mv^2 = m \left[ 2gL + \frac{mg^2}{K} \right]$$

$$\therefore v = \left[ 2gL + \frac{mg^2}{K} \right]^{\frac{1}{2}}$$

c. At lowest point from figure in part (a)

$$F = mg \downarrow -K(y - L) \uparrow \text{ (by string)}$$

$$\therefore m \frac{d^2 z}{dt^2} = mg - K(y - L)$$

$$\frac{d^2 z}{dt^2} - g + \frac{K}{m}(y - L) = 0$$

$$\frac{d^2 y}{dt^2} + \frac{K}{m} \left[ (y - L) - \frac{mg}{K} \right]$$

make a transformation of variables:

$$z = \left[ (y - L) - \frac{mg}{K} \right]$$

$$\text{then } \frac{d^2 z}{dt^2} + \frac{K}{m} z = 0$$

it is differential equation of second order which represents S.H.M.

$$\therefore \frac{d^2 z}{dt^2} + \omega^2 z = 0$$

Where  $\omega$  is angular frequency so  $\omega = \sqrt{\frac{K}{m}}$

Solution of above differential equation is of type

$$z = A \cos(\omega t + \theta)$$

Where  $\omega = \sqrt{\frac{K}{m}}$  and  $\theta$  is phase difference.

$$z = \left( L + \frac{m}{K} g \right) + A' \cos(\omega t + \theta)$$

So the stone performs SHM with angular frequency  $\omega$  about the point at  $y = 0$

$$|z_0| = \left| - \left( L + \frac{mg}{K} \right) \right| \text{ [from (i)]}$$

$$\therefore z_0 = \left( L + \frac{mg}{K} \right)$$

### \* Case study based questions

[24]

92. Read the passage given below and answer the following questions from 1 to 5. When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as stress. If  $F$  is the force applied normal to the cross-section and  $A$  is the area of cross section of the body. Magnitude of the stress

$= \frac{F}{A}$  The SI unit of stress is  $\text{N-m}^{-2}$  or Pascal (Pa) and its dimensional formula is  $[\text{ML}^{-1} \text{T}^{-2}]$ . The restoring force per unit area in this case is called tensile stress. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as compressive stress. Tensile or compressive stress can also be termed as longitudinal stress. In both the cases, there is a change in the length of the cylinder. The change in the length  $\Delta L$  to the original length  $L$  of the body is known as longitudinal strain. The restoring force per unit area developed due to the applied tangential force is known as tangential or shearing stress.

- i. Restoring force per unit area is called as:
  - a. Stress
  - b. Strain
  - c. Modulus of elasticity
  - d. None of these
- ii. Ratio of change in dimension to original dimension is called:
  - a. Stress
  - b. Strain
  - c. Modulus of elasticity
  - d. None of these
- iii. Define shear stress.
- iv. Define stress. Give its SI unit and dimension.
- v. Define strain. Give its SI unit and dimension

**Ans. :**

- i. (a) Stress
- ii. (b) Strain
- iii. The tangential restoring force per unit area developed known as tangential or shearing stress.
- iv. When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as stress.

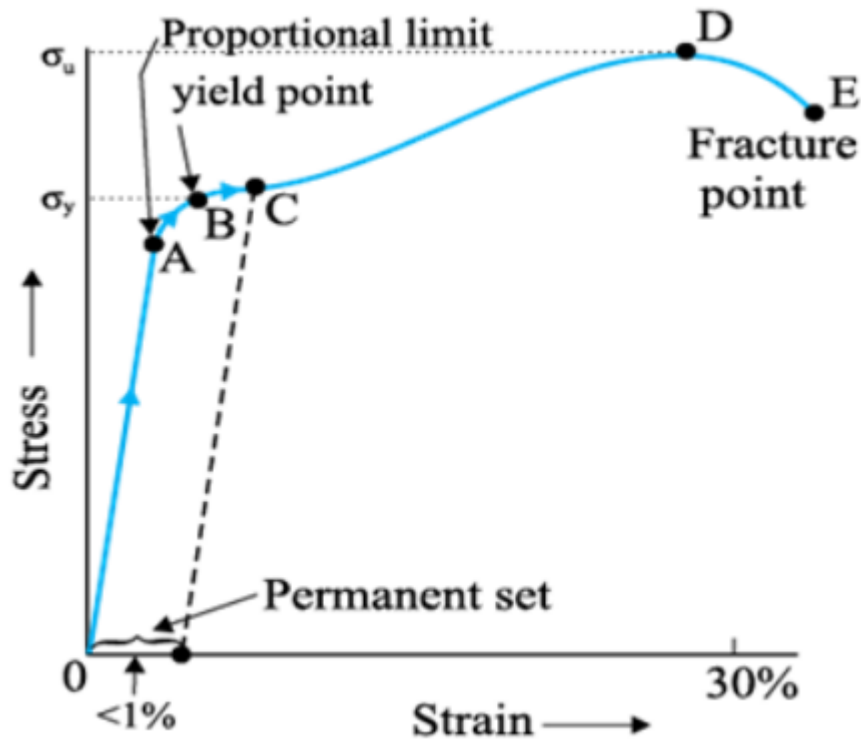
If  $F$  is the force applied normal to the cross-section and  $A$  is the area of cross section of the body.

Magnitude of the stress  $= \frac{F}{A}$

The SI unit of stress is  $\text{N-m}^{-2}$  or Pascal (Pa) and its dimensional formula is  $[\text{ML}^{-1} \text{T}^{-2}]$ .

- v. Ratio of change in dimension to original dimension is called strain. As it is ratio of similar quantities so it carries no unit and hence no dimensions.

93. Read the passage given below and answer the following questions from 1 to 5. For small deformations within elastic limit the stress and strain are proportional to each other. This is known as Hooke's law. Thus, stress  $\propto$  strain Stress =  $k \times$  strain Where  $k$  is the proportionality constant and is known as modulus of elasticity. Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibit this linear relationship.



In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as yield point (also known as elastic limit) and the corresponding stress is known as yield strength ( $\sigma_y$ ) of the material. If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The material is said to have a permanent set. The deformation is said to be plastic deformation. The point D on the graph is the ultimate tensile strength ( $\sigma_u$ ) of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E. If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.

- i. Stress is directly proportional to strain this is valid:
  - a. Above elastic limit
  - b. Within elastic limit
  - c. Above plastic limit
  - d. None of these
- ii. SI unit of modulus of elasticity is:
  - a.  $\text{N/m}^2$
  - b. N
  - c. No unit
  - d. None of these
- iii. Define modulus of elasticity.
- iv. State hooks law.
- v. Write note on stress strain curve for ductile material.

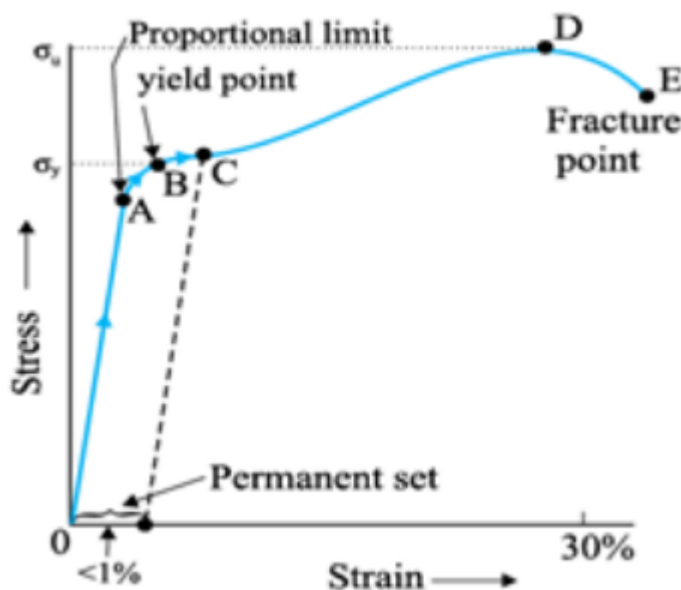
**Ans. :**

- i. (b) Within elastic limit
- ii. (a)  $\text{N/m}^2$
- iii. Modulus of elasticity is defined as ratio of stress to strain.
- iv. For small deformations within elastic limit the stress and strain are proportional to each other. This is known as

Hooke's law. Thus, stress  $\propto$  strain

$$\text{Stress} = k \times \text{strain}$$

Where  $k$  is the proportionality constant and is known as modulus of elasticity. Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibit this linear relationship.



v.

In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as yield point (also known as elastic limit) and the corresponding stress is known as yield strength ( $\sigma_y$ ) of the material.

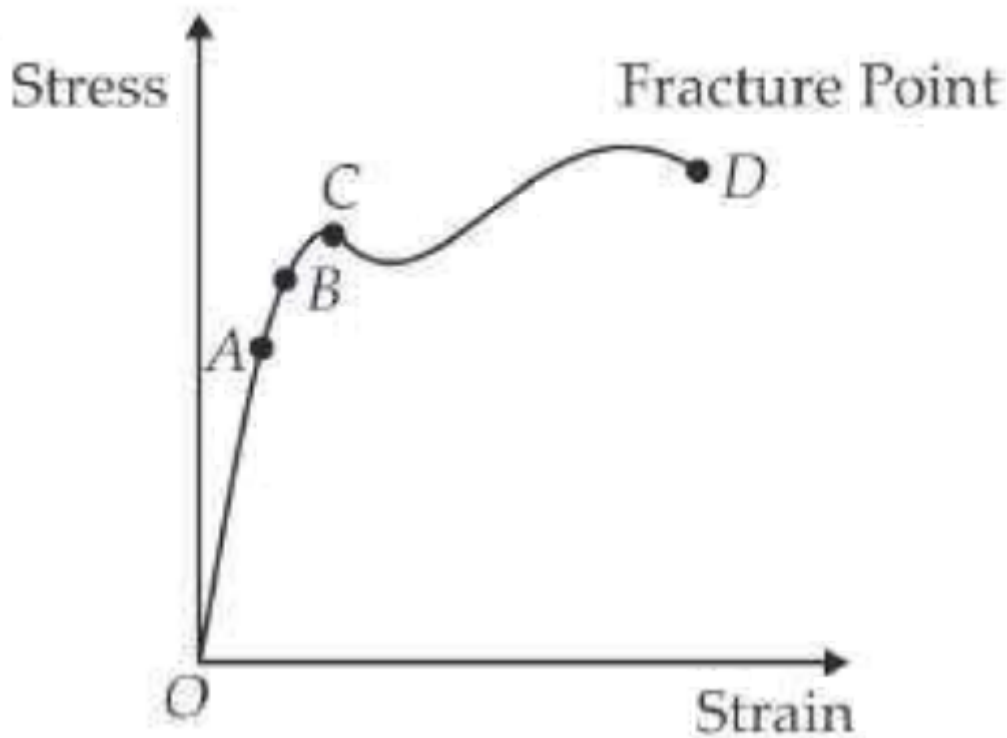
If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and

D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The material is said to have a permanent set. The deformation is said to be plastic deformation. The point D on the graph is the ultimate tensile strength ( $\sigma_u$ ) of the material.

Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E. If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be ductile

94. Read the passage given below and answer the following questions from 1 to 5. Stress-Strain Curve The graph shown below shows qualitatively the relation between the stress and the strain as the deformation gradually increases. Within Hooke's limit for a certain region stress and strain relation is linear. Beyond that up to a certain value of strain the

body is still elastic and if deforming forces are removed the body recovers its original



shape.

- i. If deforming forces are removed up to which point the curve will be retraced?
  - a. Upto OA only
  - b. Upto OB
  - c. Upto C
  - d. Never retraced its path
- ii. In the above question, during loading and unloading the force exerted by the material are conservative up to:
  - a. OA only
  - b. OB only
  - c. OC only
  - d. OD only
- iii. During unloading beyond B, say C, the length at zero stress is now equal to:
  - a. Less than original length
  - b. Greater than original length
  - c. Original length
  - d. Can't be predicted
- iv. The breaking stress for a wire of unit cross - section is called:
  - a. Yield point
  - b. Elastic fatigue
  - c. Tensile strength
  - d. Young's modulus
- v. Substances which can be stretched to cause large strains are called:
  - a. Isomers
  - b. Plastomers
  - c. Elastomers
  - d. Polymers

**Ans. :**

- i. (b) Upto OB

- ii. (b) OB only

**Explanation:**

Point B is the elastic limit.

- iii. (b) Greater than original length

**Explanation:**

Beyond B even if deforming forces are removed still some deformation is left.

- iv. (c) Tensile strength

**Explanation:**

The breaking stress for a wire of unit cross-section is called tensile strength.

- v. (c) Elastomers

**Explanation:**

Substances which can be stretched to cause large Strains are called elastomers.

95. A steel blade placed gently on the surface of water floats on it. If the same blade is kept well inside the water, it sinks. Explain.

**Ans. :** It floats because of the surface tension of water. The surface of water behaves like a stretched membrane. When a blade is placed on the water surface, it's unable to pierce the stretched membrane of water due to its low weight and remains floating.

However, if the blade is placed below the surface of water, it no longer experiences the surface tension and sinks to the bottom as the density of the blade is greater than that of water.

96. When some wax is rubbed on a cloth, it becomes waterproof. Explain.

**Ans. :** A liquid wets a surface when the angle of contact of the liquid with the surface is small or zero. Due to its fibrous nature, cloth produces capillary action when in contact with water. This makes clothes have very small contact angles with water. When wax is rubbed over cloth, the water does not wet the cloth because wax has a high contact angle with water.

97. If a mosquito is dipped into water and released, it is not able to fly till it is dry again. Explain.

**Ans. :** A mosquito thrown into water has its wings wetted. Now, wet wing surfaces tend to stick together because of the surface tension of water. This does not let the mosquito fly.

----- if talent doesn't work hard then hardwork beat the talent -----