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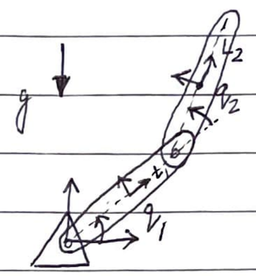
HW 12 - Trajectory Optimization



$m = l = 1$

$I_l = \frac{1}{12}$

$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$

1.1) The optimization problem can be defined as follows:

$$\min \sum_{k=0}^{N-1} \frac{1}{2}(\tau(t_k) + \tau(t_{k+1}))(t_{k+1} - t_k)$$

wrt $q(t_k)$, $\dot{q}(t_k)$, $\tau(t_k)$ $\Big\}$ $\boxed{\text{6 Decision variables } (q_1, q_2, \dot{q}_1, \dot{q}_2, \tau_1, \tau_2)}$

s.t.

$$\left.\begin{aligned}
\dot{q}(t_{k+1}) &= \dot{q}(t_k) + \left(\frac{\ddot{q}(t_k) + \ddot{q}(t_{k+1})}{2}\right)(t_{k+1} - t_k) \\
q(t_{k+1}) &= q(t_k) + \left(\frac{\dot{q}(t_k) + \dot{q}(t_{k+1})}{2}\right)(t_{k+1} - t_k) \\
q(t_0) &= \begin{bmatrix} -\pi/2 \\ 0 \end{bmatrix} \\
\dot{q}(t_0) &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
q(t_f) &= \begin{bmatrix} \pi/2 \\ 0 \end{bmatrix} \\
\dot{q}(t_f) &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}\right\} \boxed{\text{6 Constraint Eq}^n}$$

Here, trapezoidal collocation (linear interpolation) is used for $\ddot{q}$ (dynamics) and τ (torque)

$t_{k+1} - t_k = 20\text{ ms}$ (Given)

$t_0 = 0$ $t_f = 1.5\text{s}$

$$\ddot{q}(t) = M^{-1}(\tau - C\dot{q} - N)$$ (We plug ~~these~~ this ~~values~~ below)

$$\approx \ddot{q}(t_k) + \frac{\left(\ddot{q}(t_{k+1}) - \ddot{q}(t_k)\right)}{(t_{k+1} - t_k)}(t - t_k)$$

$$\tau(t) \approx \tau(t_k) + \frac{\left(\tau(t_{k+1}) - \tau(t_k)\right)}{(t_{k+1} - t_k)}(t - t_k)$$

M, C, N, τ are calculated below:

$$M = J_{sl_1}^{b\,T} M_{l_1} J_{sl_1}^{b} + J_{sl_2}^{b\,T} M_{l_2} J_{sl_2}^{b}$$

$$J_{sl_1}^{b} = \begin{bmatrix} -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}-l/2\\0\\0\end{bmatrix} & \begin{matrix}0\\0\\0\end{matrix} \\ \begin{matrix}0\\0\\1\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ l/2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_{sl_2}^{b} = \begin{bmatrix} -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}-\frac{l}{2} - l\cos\theta_2\\ l\sin\theta_2\\0\end{bmatrix} & -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}-l/2\\0\\0\end{bmatrix} \\ \begin{matrix}0\\0\\1\end{matrix} & \begin{matrix}0\\0\\1\end{matrix} \end{bmatrix} = \begin{bmatrix} l\sin\theta_2 & 0 \\ l\cos\theta_2 + \frac{l}{2} & l/2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$M_{l_1} = M_{l_2} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix}$$

$$J_{sl_1}^{b\,T} M_{l_1} J_{sl_1}^{b} = \begin{bmatrix} \frac{ml^2}{4} + I_z & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{se_2}^{b\,T} M_{e_2} J_{se_2}^{b} = \begin{bmatrix} ml^2\sin^2\theta_2 + m\left(\frac{l}{2} + l\cos\theta_2\right)^2 + I_z & \frac{ml}{2}\left(l\cos\theta_2 + \frac{l}{2}\right) + I_z \\ \frac{ml}{2}\left(l\cos\theta_2 + \frac{l}{2}\right) + I_z & \frac{ml^2}{4} + I_z \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta_2 + \left(\frac{1}{2} + \cos\theta_2\right)^2 + \frac{1}{12} & \frac{1}{2}\left(\cos\theta_2 + \frac{1}{2}\right) + \frac{1}{12} \\ \frac{1}{2}\left(\cos\theta_2 + \frac{1}{2}\right) + \frac{1}{12} & \frac{1}{3} \end{bmatrix}$$

$$M = J_{se_1}^{b\,T} M_{e_1} J_{se_1}^{b} + J_{se_2}^{b\,T} M_{e_2} J_{se_2}^{b}$$

$$M = \begin{bmatrix} \sin^2\theta_2 + \left(\frac{1}{2} + \cos\theta_2\right)^2 + \frac{5}{12} & \frac{1}{2}\left(\cos\theta_2 + \frac{1}{2}\right) + \frac{1}{12} \\ \frac{1}{2}\left(\cos\theta_2 + \frac{1}{2}\right) + \frac{1}{12} & \frac{1}{3} \end{bmatrix}$$

C would be a 2x2 matrix s.t.

$$C_{ij} = \frac{1}{2} \sum_{k=1}^{2} \left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{kj}}{\partial q_i} \right) \dot{q}_k$$

Solving using MATLAB,

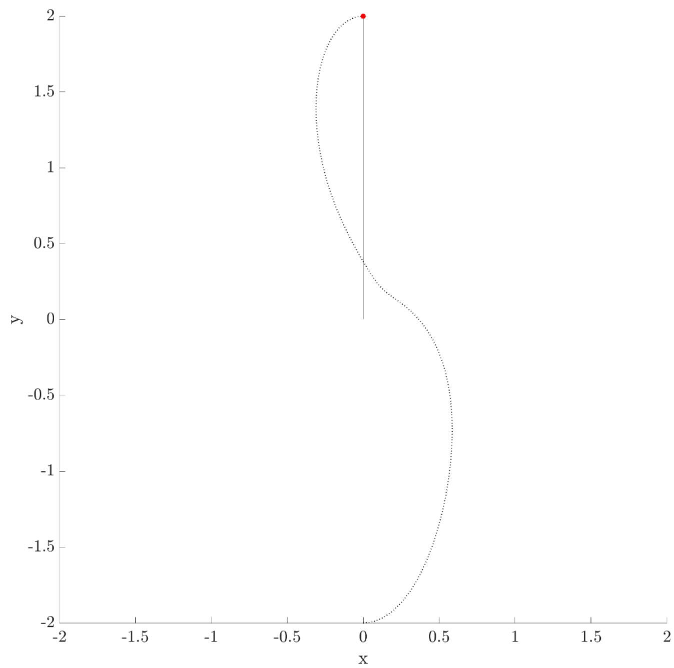
$$C = \begin{bmatrix} -\frac{1}{2}\left(\dot{q}_2 \sin(q_2)\right) & -\frac{1}{2}(\dot{q}_1 + \dot{q}_2)\sin(q_2) \\ \frac{1}{2}\dot{q}_1 \sin(q_2) & 0 \end{bmatrix}$$

$$V = mg\frac{l}{2}\sin q_1 + mg\left(l\sin q_1 + \frac{l}{2}\sin(q_1+q_2)\right)$$

$$N = \frac{\partial V}{\partial q} = \begin{bmatrix} \frac{3mgl}{2}\cos q_1 + mg\frac{l}{2}\cos(q_1+q_2) \\ mg\frac{l}{2}\cos(q_1+q_2) \end{bmatrix} = \begin{bmatrix} \frac{g}{2}\left(3\cos q_1 + \cos(q_1+q_2)\right) \\ \frac{g}{2}\cos(q_1+q_2) \end{bmatrix}$$

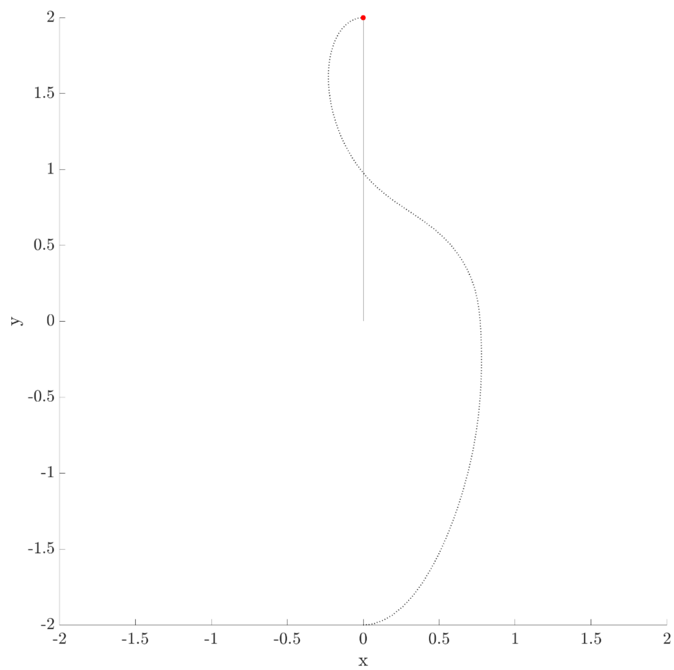
$$\Upsilon = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

1.3) (a) When there are no bounds on q:



Value of the cost function: 79.0438

(b) When q1 and q2 are restricted to lie in the interval -3*pi/4 to 3*pi/4



Value of the cost function: 119.7581

The optimal trajectory changes when lower and upper bounds are imposed on q. The value of the cost function increases for the 2nd case in which q is bounded.