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Exercise 1:

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix} \qquad s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} \qquad s_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

Let $w_1 = y,\ w_2 = \dot{y},\ w_3 = \psi,\ w_4 = \dot{\psi}$

$$\dot{w}_1 = w_2$$

$$\dot{w}_2 = -w_4\dot{x} + \frac{2C_\alpha}{m}\left(\cos u_1\left(u_1 - \frac{(w_2 + l_f w_4)}{\dot{x}}\right) - \left(\frac{w_2 - l_r w_4}{\dot{x}}\right)\right)$$

$$\dot{w}_3 = w_4$$

$$\dot{w}_4 = \frac{2l_f C_\alpha}{I_z}\left(u_1 - \left(\frac{w_2 + l_f w_4}{\dot{x}}\right)\right) - \frac{2l_r C_\alpha}{I_z}\left(-\left(\frac{w_2 - l_r w_4}{\dot{x}}\right)\right)$$

$$\dot{w} = f(w, u)$$

At equilibrium, $\dot{w} = 0$

$$\dot{w}_1 = 0 \Rightarrow \overline{w}_2 = 0$$

$$\dot{w}_3 = 0 \Rightarrow \overline{w}_4 = 0$$

$$\dot{w}_2 = 0 \Rightarrow \frac{2C_\alpha}{m}\left(\cos u_1 (u_1)\right) = 0 \Rightarrow u_1\cos(u_1) = 0 \Rightarrow \bar{u}_1 = 0 \text{ or } \frac{(2k+1)\pi}{2} \quad k \in \mathbb{Z}$$

$$\dot{w}_4 = 0 \Rightarrow \frac{2l_f C_\alpha}{I_z} u_1 = 0 \Rightarrow \bar{u}_1 = 0$$

$$A_1 = \left[\frac{\partial f}{\partial w}\right] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_\alpha}{m}\frac{(\cos(u_1)+1)}{\dot{x}} & 0 & -\dot{x} - \frac{2C_\alpha}{m\dot{x}}\left[\cos(u_1)l_f - l_r\right] \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2l_f C_\alpha}{I_z\dot{x}} + \frac{2l_r C_\alpha}{I_z\dot{x}} & 0 & -\frac{2l_f C_\alpha}{I_z}\frac{l_f}{\dot{x}} - \frac{2l_r C_\alpha l_r}{I_z\dot{x}} \end{bmatrix}_{(\overline{w}, \bar{u})}$$

$$B_1 = \left[\frac{\partial f}{\partial u}\right] = \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m}\left(-u_1\sin u_1 + \cos u_1 + \frac{(w_2 + l_f w_4)}{\dot{x}}\sin(u_1)\right) & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix}_{(\overline{w}, \bar{u})}$$

Thus, the linearized system is :

$$\dot{S}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \cancel{\frac{2C_\alpha}{m\dot{x}}}\; \frac{-4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_\alpha(l_r - l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha(l_r - l_f)}{I_z\dot{x}} & 0 & -\frac{2C_\alpha(l_r^2 + l_f^2)}{I_z\dot{x}} \end{bmatrix} S + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix} u$$

Using $C_\alpha = 20{,}000$ $\quad m = 1888.6$ $\quad l_r = 1.39$ $\quad l_f = 1.55$ $\quad I_z = 25854$

$$\dot{S}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{42.36}{\dot{x}} & 0 & -\dot{x} - \frac{3.39}{\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{0.247}{\dot{x}} & 0 & -\frac{6.706}{\dot{x}} \end{bmatrix} S + \begin{bmatrix} 0 & 0 \\ 21.18 & 0 \\ 0 & 0 \\ 2.398 & 0 \end{bmatrix} u$$

Let $t_1 = x, \quad t_2 = \dot{x}$

$$\dot{t}_1 = t_2$$

$$\dot{t}_2 = \dot{\psi}\dot{y} + \frac{1}{m}(u_2 - fmg)$$

$$\dot{t} = f(t, u)$$

$$\delta\dot{t} = \left.\frac{\partial f}{\partial t}\right|_{(\bar{t},\bar{u})} \delta t + \left.\frac{\partial f}{\partial u}\right|_{(\bar{t},\bar{u})} \delta u$$

The equilibrium points are :

$$\dot{t}_1 = 0 \Rightarrow \bar{t}_2 = 0$$

$$\dot{t}_2 = 0 \Rightarrow \dot{\psi}\dot{y} + \frac{1}{m}(\bar{u}_2 - fmg) = 0$$

$$\Rightarrow \bar{u}_2 = \dot{\psi}\dot{y}m + fmg$$

$$A_2 = \frac{\partial f}{\partial t} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

Substituting $m = 1888.6$

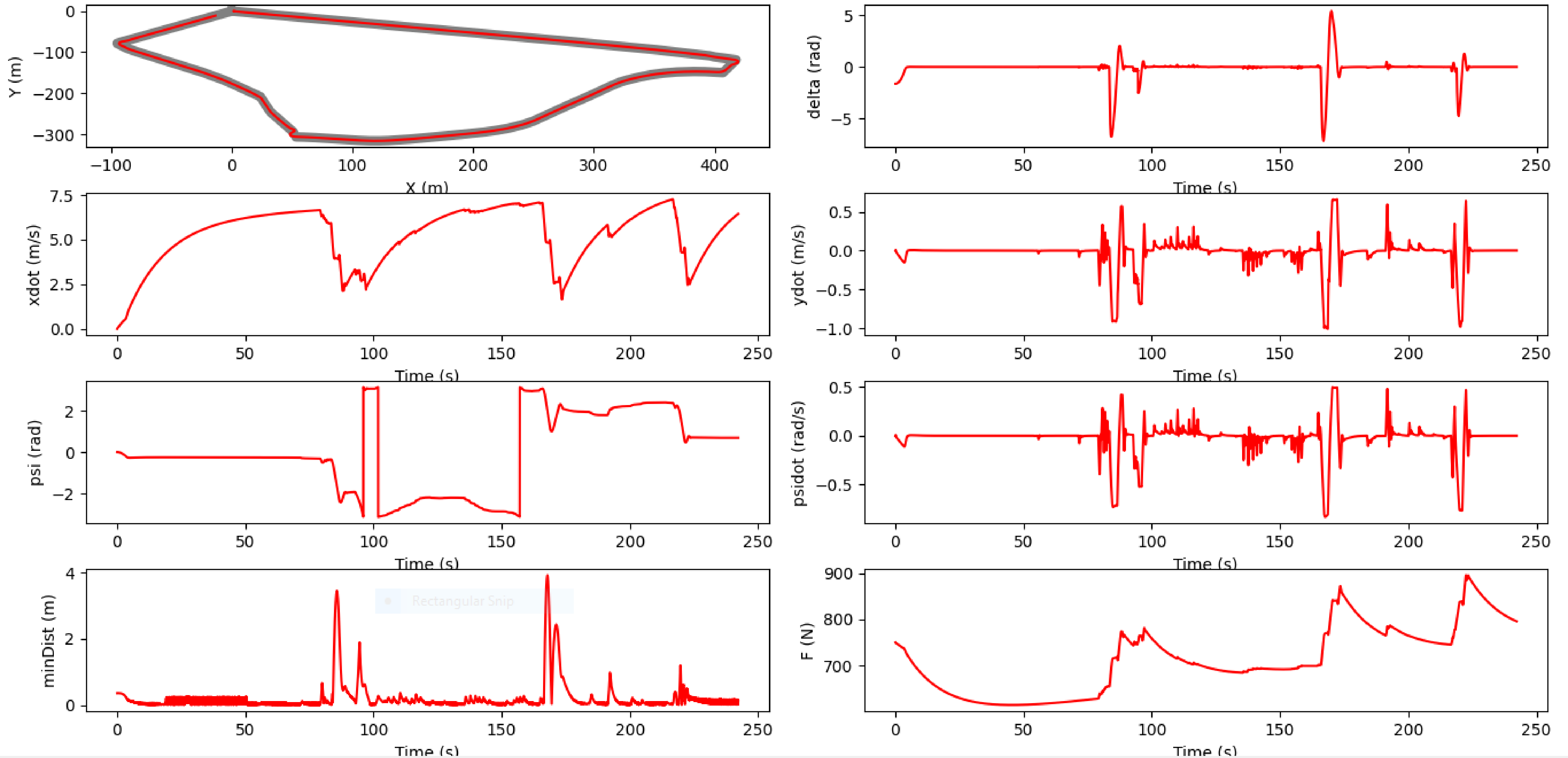
$$B = \begin{bmatrix} 0 & 0 \\ 0 & 5.29 \times 10^{-4} \end{bmatrix}$$

$$\therefore \dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} S_2 + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} u$$

OR

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} S_2 + \begin{bmatrix} 0 & 0 \\ 0 & 5.29\times10^{-4} \end{bmatrix} u$$

Figure 1



x=129.9 y=−0.88