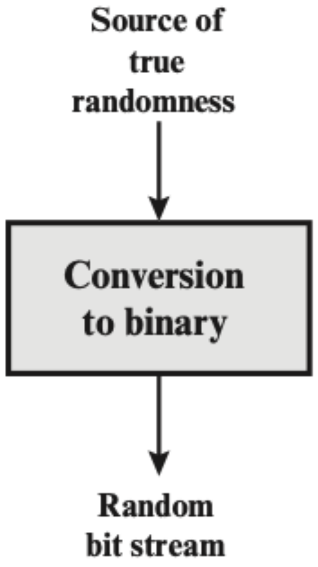
Lecture 9

True random numbers generators

- Several sources of randomness – natural sources of randomness
  - decay times of radioactive materials
  - electrical noise from a resistor or semiconductor
  - radio channel or audible noise
  - keyboard timings
  - disk electrical activity
  - mouse movements
  - Physical unclonable function (PUF)
- Some are better than others



(a) TRNG

Combining sources of randomness

- Suppose r1, r2, …, rk are random numbers from different sources. E.g.,

  r1 = electrical noise from a resistor or semiconductor

  r2 = sample of hip-hop music on radio

  r3 = clock on computer

  b = r1$\oplus$r2$\oplus$…$\oplus$rk

  If any one of r1, r2, …, rk is truly random, then so is b

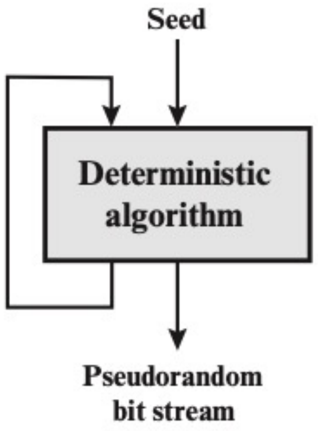
  Many poor sources + 1 good source = good entropy

Pseudorandom Number Generators (PRNGs)

- True randomness is expensive
- **Pseudorandom number generator** (**PRNGs**): An algorithm that uses a little bit of true randomness to generate a lot of random-looking output
    - Also called **deterministic random bit generators** (**DRBGs**)
- PRNGs are deterministic: Output is generated according to a set algorithm
    - However, for an attacker who can't see the internal state, the output is *computationally indistinguishable* from true randomness

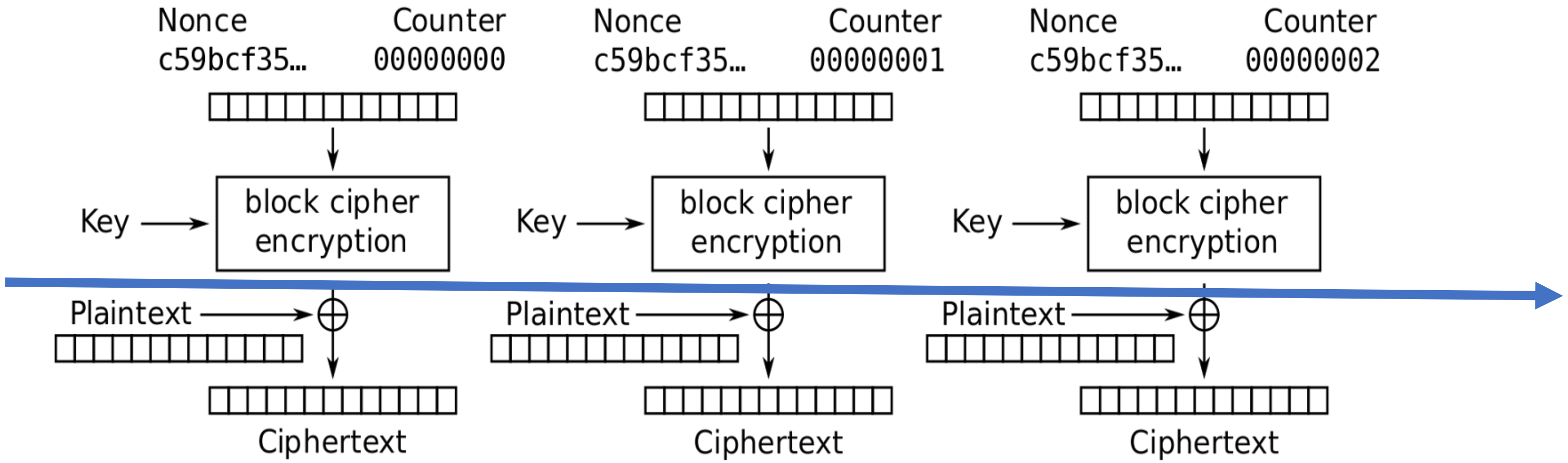
PRNG: Definition

- A PRNG has two functions:
  - PRNG.Seed(randomness): Initializes the internal state using the entropy
    - Input: Some truly random bits
  - PRNG.Generate(*n*): Generate *n* pseudorandom bits
    - Input: A number *n*
    - Output: *n* pseudorandom bits
    - Updates the internal state as needed
- Properties
  - **Correctness**: Deterministic
  - **Efficiency**: Efficient to generate pseudorandom bits
  - **Security**: Indistinguishability from random
  - **Rollback resistance**: cannot deduce anything about any previously-generated bit



Example construction of PRNG

- Using block cipher in Counter (CTR) mode:
- If you want m random bits, and a block cipher with E_k has n bits, apply the block cipher m/n times and concatenate the result:
- PRNG.Seed(K | IV) = E_k(IV, 1) | E_k(IV, 2) | E_k(IV, 3) … E_k(IV, ceil(m/n)),
  - | is concatenation
  - Initialization vector (IV) / Nonce – typically is random or pseudorandom



Randomness, PRNG output

Counter (CTR) mode encryption

PRNG: Security

- Can we design a PRNG that is truly random?
- A PRNG cannot be truly random
  - The output is deterministic given the initial seed
- A secure PRNG is computationally indistinguishable from random to an attacker
  - Game: Present an attacker with a truly random sequence and a sequence outputted from a secure PRNG
  - An attacker should be able to determine which is which with probability ≈ 0
- Equivalence: An attacker cannot predict future output of the PRNG

Create pseudorandom numbers

- Truly random numbers are impossible with any program!
- However, we can generate seemingly random numbers, called pseudorandom numbers
- The function rand() returns a non-negative number between 0 and RAND_MAX
- For C, it is defined in stdlib.h

PRNGs: Summary

- True randomness requires sampling a physical process
- PRNG: An algorithm that uses a little bit of true randomness to generate a lot of random-looking output
    - Seed(entropy): Initialize internal state
    - Generate(n): Generate n bits of pseudorandom output
- Security: computationally indistinguishable from truly random bits