DAA- HANDS ON3

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```
function x = f(n)
   x = 1;
   for i = 1:n
        for j = 1:n
             x = x + 1;
```

1. Find the runtime of the algorithm mathematically (I should see summations).

DAA

Hands-On3

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Function 1:

```
function x = f(n)
     x = 1;
   for i = 1:n
      for j = 1:n
          x = x+1;
```

i) Find the runtime of the algorithm mathematically

→ The above algorithm has 2 nested loops;

The outer loop runs from $i=1$ to $i=n$,

The inner loop runs from $j=1$ to $j=n$,

So in each inner loop iteration, the variable 'x' is incremented by 1

→ Now, total no. of iterations is;

$$T(n) = 1 + \sum_{i=1}^{n} \sum_{j=1}^{n} 1$$

The inner summation $\sum_{j=1}^{n} 1$ says the no. of inner loop iterations i.e., 'n' since the inner loop runs n times for each outer loop iteration;

Substituting in outer summation,

$$T(n) = 1 + \sum_{i=1}^{n} n = 1 + n \cdot \sum_{i=1}^{n} 1$$

$1 + \sum_{i=1}^{n} 1 = n$, as there are 'n' terms in the sum

$$T(n) = 1 + (n \cdot n) = 1 + n^2$$

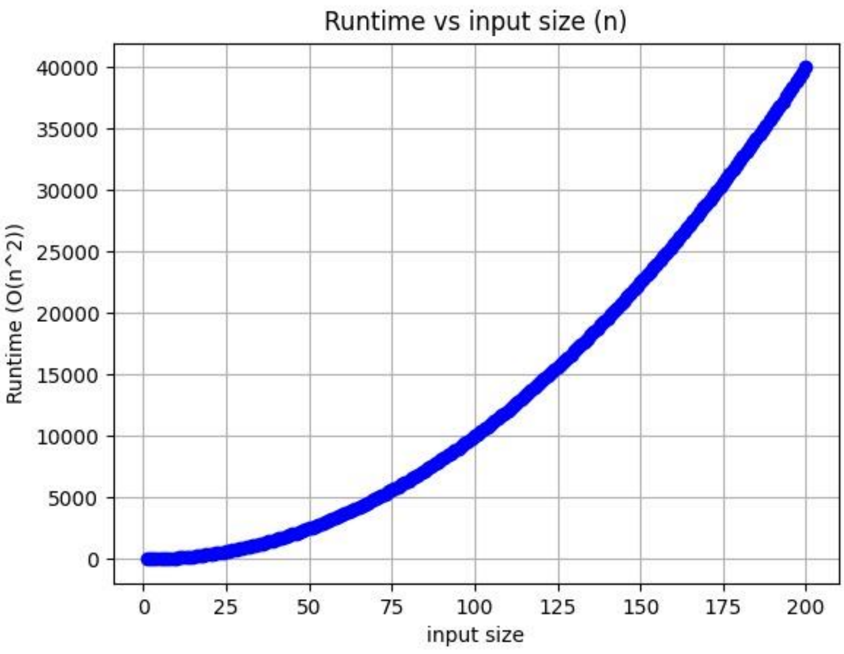
∴ Runtime for the algorithm is $\underline{O(n^2)}$

2. Time this function for various n e.g. n = 1,2,3.... You should have small values of n all the way up to large values. Plot "time" vs "n" (time on y-axis and n on x-axis). Also, fit a curve to your data, hint it's a polynomial.

Code:

```
import matplotlib.pyplot as plt

def runtime(n):

    return n**2

values = list(range(1, 201))

runtime = [runtime(n) for n in values]

# Plotting

plt.plot(values, runtime, marker='o', linestyle='-', color='b')

plt.xlabel('input size')

plt.ylabel('Runtime (O(n^2))')

plt.title('Runtime vs input size (n)')

plt.grid(True)

plt.show()
```

Plotting Graph:



3. Find polynomials that are upper and lower bounds on your curve from #2. From this specify a big-O, a big-Omega, and what big-theta is.

3) Given function, $T(n) = n^2+1$

Considering the upper bound $U(n) = 2n^2$

Lower bound $L(n) = 0.8n^2$

1) Big O (O) :

$T(n) = n^2+1$ is $O(n^2)$

because there exists the constants c & n_0

i.e, $T(n) \le c \cdot n^2 \; \forall \; n \ge n_0$

2) Big-Omega (Ω) :

$T(n) = n^2+1$ is $\Omega(n^2)$

because there exists the constants c & n_0

i.e $T(n) \ge c \cdot n^2 \; \forall \; n \ge n_0$

3) Big-Theta (θ) :

$T(n) = n^2+1$ is $\Theta(n^2)$

because there exists the constants c_1, c_2, n_2, n_1

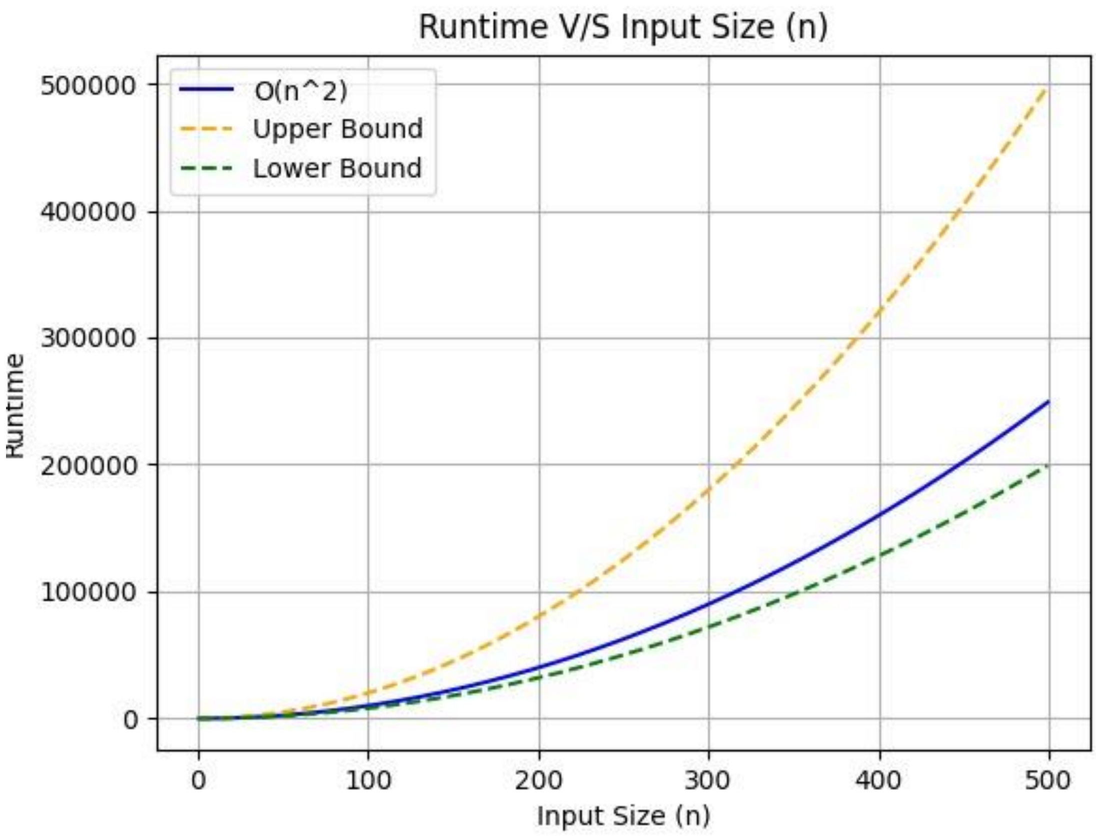
i.e, $c_1 \cdot n^2 \le T(n) \le c_2 \cdot n^2 \; \forall \; n \ge \max(n_1, n_2)$

Code:

```
import matplotlib.pyplot as plt

def quadratic_function(n):

    return n**2

input_values = list(range(1, 500))
```

```
runtime = [quadratic_function(n) for n in input_values]

plt.plot(input_values, runtime, label='O(n^2)', color='blue')

upper_bound = [2 * n ** 2 for n in input_values]

lower_bound = [0.8 * n ** 2 for n in input_values]

plt.plot(input_values, upper_bound, label='Upper Bound', linestyle='--', color='orange')

plt.plot(input_values, lower_bound, label='Lower Bound', linestyle='--', color='green')

plt.xlabel('Input Size (n)')

plt.ylabel('Runtime')

plt.title('Runtime V/S Input Size (n)')

plt.legend()

plt.grid(True)

plt.show()
```

Graph:

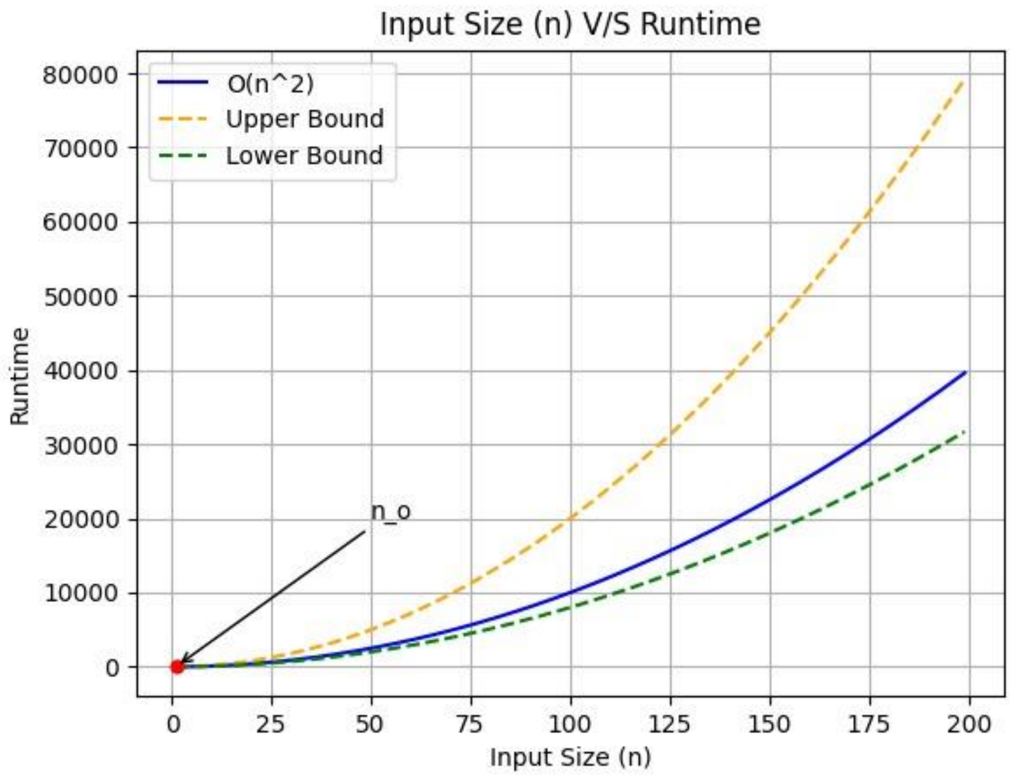


4. Find the approximate (eye ball it) location of "n_0" . Do this by zooming in on your plot and indicating on the plot where n_0 is and why you picked this value. Hint: I should see data that does not follow the trend of the polynomial you determined in #2.

Code:

```
import matplotlib.pyplot as plt

def quadratic_function(n):

    return n**2

input_values = list(range(1, 200))

runtime = [quadratic_function(n) for n in input_values]

plt.plot(input_values, runtime, label='O(n^2)', color='blue')

upper_bound = [2 * n ** 2 for n in input_values]

lower_bound = [0.8 * n ** 2 for n in input_values]

plt.plot(input_values, upper_bound, label='Upper Bound', linestyle='--', color='orange')

plt.plot(input_values, lower_bound, label='Lower Bound', linestyle='--', color='green')

plt.xlabel('Input Size (n)')

plt.ylabel('Runtime')

plt.title('Input Size (n) V/S Runtime')

plt.legend()

plt.plot(1, 1, marker='o', markersize=5, color='red')

plt.annotate('n_o', xy=(1, 1), xytext=(50, 20000),arrowprops=dict(facecolor='black', arrowstyle='->'))

plt.grid(True)

plt.show()
```

Graph:



Reason to pick n_0 = 1:

I have selected n_0=1 because at this particular input size, a significant change in the runtime behaviour is apparent. Beyond this threshold, the deviation from the expected polynomial trend becomes more pronounced, indicating that the quadratic term starts to exert a more noticeable deviation on the runtime.

If I modified the function to be:

```
x = f(n)
   x = 1;
   y = 1;
   for i = 1:n
        for j = 1:n
             x = x + 1;
             y = i + j;
```

4. Will this increate how long it takes the algorithm to run (e.x. you are timing the function like in #2)?

Function 2 :

The modified function to be:

```
x = f(n)
x = 1;
y = 1;
for i = 1:n
     for j = 1:n
          x = x + 1;
          y = i + j;
```

4) Ans: In this function 2, there are 2 nested loops ranges from 1 to 'n'. However, there are additional constant time operations inside the inner loop i.e, (y = i+j). The dominant factor is still the nested loops, so, the time complexity remains $O(n^2)$.

And yes, the addition of an extra operation within the nested loop results in increasing the overall runtime.

5. Will it effect your results from #1?

5) No, the runtime for the function #2 is still $\underline{O(n^2)}$

$$= c + \sum_{i=1}^{n} \sum_{j=1}^{n} 1 \qquad \text{w.k.t} \sum_{i=1}^{n} 1 = n$$

$$= c + n(n)$$

$$= c + n^2$$

$$= \underline{\underline{O(n^2)}}$$

6. Implement merge sort, upload your code to github and show/test it on the array [5,2,4,7,1,3,2,6].