Line Assignment

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CONTENTS

I Problem 1

II Solution 1

I. PROBLEM

In figure below, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ). [Hint : Join AC.]

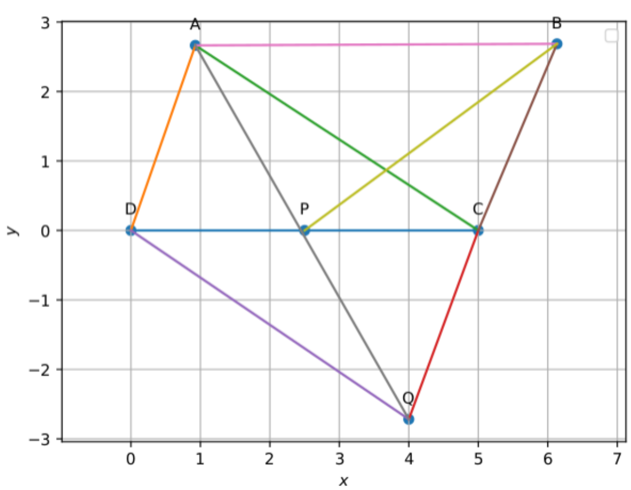

Figure of Construction

II. SOLUTION

Construction: Join AC
Theoretical Proof:
To Prove: ar(Δ BPC) = ar(Δ DPQ)
Since ABCD is a paralellogram, $AD \,||\, BC$
and BC is extended to point Q

$\therefore$ $AD \,||\, CQ$
and AD=CQ (given)

A quadrilateral is said to be paralellogram if a pair of its opposite sides are parallel and equal

$\therefore$ ACQD is a paralellogram

By the theorem,
Theorem: Two triangles on the same base and between same parallels are equal in area

ar(Δ BPC) = ar(Δ APC)......(1)

In Δ APC and Δ DPQ,

$\angle$ PAC = $\angle$ PQD [Alternate interior angles are equal]
AC=QD [Opposite sides of a paralellogram are equal]
$\angle$ ACP = $\angle$ PDQ [Alternate interior angles are equal]

By ASA rule,Δ APC and Δ DPQ are congruent

$\therefore$ ar(Δ APC)=ar(Δ DPQ)......(2) [Since congruent triangles have equal area]

From 1 and 2 ar(Δ BPC)=ar(Δ DPQ)
hence proved

To Prove: ar(Δ BPC) = ar(Δ DPQ)

Given ABCD is a paralellogram, and BC is produced to point Q such that AD=CQ. AQ intersect DC at P.
We need to prove that ar(BPC)=ar(DPQ)
Let,

$$\boldsymbol{A\text{-}B{=}p1}$$
$$\boldsymbol{D\text{-}C{=}s1}$$
$$\boldsymbol{A\text{-}D{=}p2}$$
$$\boldsymbol{B\text{-}C{=}s2}$$

By paralellogram property

$$\|p1\| = \|s1\| \;\&\&\; \|p2\| = \|s2\| \qquad [1]$$

Since P lies on DC,

DP=PC=k x DC
DP=PC=$\boldsymbol{k(s1)}$

Area of triangle BPC

=1/2 x [PC x BC]
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(s1)}$ x $\boldsymbol{s2}]$
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(p1)}$ x $\boldsymbol{p2}]$ (from [1])

Area of triangle BPC
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(p1)}$ x $(\boldsymbol{p2})]$ [2]
Given that BC is extended to Q so,

$AD \,||\, CQ$
and AD=CQ(given)
So, ACQD is a parallelogram

AC=DQ. [Since opposite sides of a paralellogram are equal] [3]
In triangle ABC,

AC=AB+BC
AC=(A-B)+(B-C)
AC=$\boldsymbol{p1}$ + $\boldsymbol{s2}$
AC=$\boldsymbol{p1}$ + $\boldsymbol{p2}$ (from [1])
From [3]
AC=DQ=$\boldsymbol{p1}$ + $\boldsymbol{p2}$ [4]

Area of a triangle DPQ

=1/2 x [DP x DQ]
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(s1)}$ x $(\boldsymbol{p1}$ + $\boldsymbol{p2})]$ (from [4])
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(p1)}$ x $(\boldsymbol{p1}$ + $\boldsymbol{p2})]$
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(p1)}$ x $(\boldsymbol{p1})$ + $\boldsymbol{k(p1)}$ x $(\boldsymbol{p2})]$
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(p1)}$ x $\boldsymbol{p2}]$

Area of triangle DPQ
=$\boldsymbol{1/2}$ x $[\boldsymbol{k(p1)}$ x $\boldsymbol{p2}]$ [5]
from [2] and [5]
ar(BPC)=ar(DPQ)
hence proved

Input parameters for this construction

Symbol	Value	Description
r0	5	DC
r1,r2	3,2.8	DA
r3,r4	4,173	CQ
r5,r6	10,3.4	CB
θ_1	$2\pi/5$	$\angle$ADC
θ_2	$\pi/200$	$\angle$QDC
θ_3	$2\pi/3.45$	$\angle$BDC