Optimization Assignment

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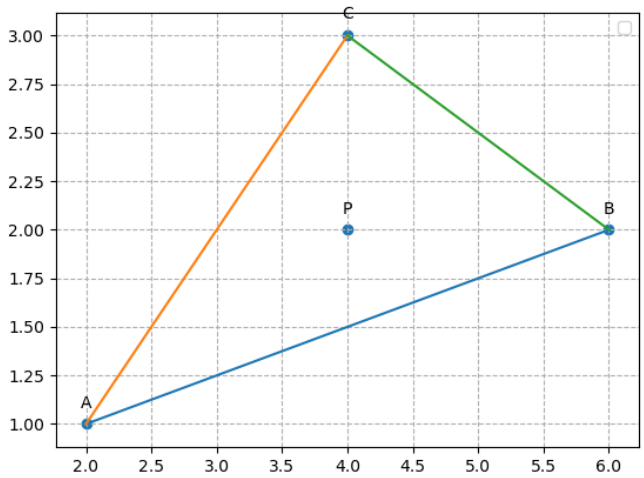
I. Problem

Show that if ABC be a triangle, and P any point $PA^2 + PB^2 + PC^2$ will be a minimum when P is at the centroid.



Graph

II. Solution

Let the centroid be P,

$$P = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1}$$

And the function be,

$$f(x, y) = 3x^2 + 3y^2 - 24x - 12y + 70 \tag{2}$$

Using Gradient descent method,

$$x_n = x_{n-1} - \mu \frac{\partial f}{\partial x} \tag{3}$$

$$\frac{\partial f}{\partial x} = 6x - 24 \tag{4}$$

$$y_n = y_{n-1} - \mu \frac{\partial f}{\partial y} \tag{5}$$

$$\frac{\partial f}{\partial y} = 6y - 12 \tag{6}$$

Substituiting (4) in (3),

$$x_n = x_{n-1} - \mu(6x_{n-1} - 24) \tag{7}$$

Substituiting (6) in (5),

$$y_n = y_{n-1} - \mu(6y_{n-1} - 1)2 \tag{8}$$

Obtained values are,

$$\boxed{\text{Minima Point} = 3.999, 1.999 \approx 4, 2}$$