Algorithms Worksheet 2

For each part of a question write the answer and include workings. Questions are worth two marks each, there are also two marks for attendance.

1. Solve
$$T(n) = -T(n-1) + 4 \tag{1}$$
with $T(0) = 1$.

   Solution:

   Substitute in $T(n) = A(-1)^n + B$ to get
$$A(-1)^n + B = -A(-1)^{n-1} - B + 4 \tag{2}$$
   so $2B = 4$ or $B = 2$; now $T(n) = A(-1)^n + 2$ so $1 = T(0) = A + 2$ and hence $A = -1$.

2. Solve for $T(n)$ using the ansatz $T(n) = r^n$ for the following two step recursion relations. Solving for r will give two values r_1 and r_2, this means that the general solution will be $T(n) = Ar_1^n + Br_2^n$. Use the two base values to find A and B.

   a) $T(n) = 2T(n-1) + 3T(n-2)$ with $T(0) = 0$ and $T(1) = 4$.

   b) $T(n) = T(n-2)$ with $T(0) = 0$ and $T(1) = 2$.

   Solution:

   For (a) we have
$$r^2 = 2r + 3 \tag{3}$$
   so $r^2 - 2r - 3 = 0$ or $(r-3)(r+1) = 0$ so
$$T(n) = 3^n A + (-1)^n B \tag{4}$$
   and the initial conditions give $A + B = 0$ and $3A - B = 4$ so
$$T(n) = 3^n - (-1)^n \tag{5}$$
   For (b) we get $r^2 = 1$ so
$$T(n) = A + (-1)^n B \tag{6}$$
   and the initial conditions give $A + B = 0$ and $A - B = 2$ so
$$T(n) = 1 - (-1)^n \tag{7}$$

3. This question is about the master theorem. Use it to calculate big-Theta for $T(n)$ in each case.

   a) $T(n) = 25T(n/5) + 4n^2$

   b) $T(n) = 20T(n/5) + 4n$

   c) $T(n) = 16T(n/2) + 2n^4$

   Solution: for the first one $\log_5 25 = 2$ and $c = 2$ so this is the middle case and $T(n) \in \Theta(n^2 \log n)$, for the second $\log_5 20 > 1$ so it is the first case and $T(n) \in \Theta(n^{\log_5 20})$; the last one is in the middle case as well since $\log_2 16 = 4$ and $T(n) \in \Theta(n^4 \log n)$.

4. Bubble sort $(3, 5, 2, 8, 4)$ showing each step.

   Solution:

3	5	2	8	4
3	2	5	8	4
3	2	5	4	8
2	3	5	4	8
2	3	4	5	8

Extra question to do at home if you want: write a recursive version of quicksort to show that in the worst case

$$T(n) = T(n-1) + cn \tag{8}$$

Solution:

```
1  void swap(int a[],int i, int j)
2  {
3     int temp=a[i];
4     a[i]=a[j];
5     a[j]=temp;
6  }
7
8
9  void bubble(int a[], int n)
10 {
11
12    if n=0 return;
13
14    for(i=0;i<n-1;i++){
15       if(a[i]>a[i+1]){
16          swap(a,i,i+1);
17       }
18    }
19    bubble(int a[], int n-1)
20 }
```