5 Recursion - another example

Here is another tricky example, this appeared in a worksheet in this course in the past but it was left out this year; I include it for interest only.

Consider $T(n) = T(n-1) + 3n$ with $T(1) = 1$. Now telescoping gives bits that look like $3n + 3(n-1) + 3(n-2) + \ldots$, in other words, you seem to get an n in every iteration of the telescope. Since there are n iterations of the telescope you might guess

$$T(n) = An^2 + Bn + C \tag{1}$$

so substituting that in gives

$$An^2 + Bn + C = A(n-1)^2 + B(n-1) + C + 3n = An^2 - 2An + A + Bn - B + C + 3n \tag{2}$$

or, after cancelling

$$-2An + A - B + 3n = 0 \tag{3}$$

so $A = 3/2$ and $B = A$. Thus

$$T(n) = \frac{3}{2}n^2 - \frac{3}{2}n + C \tag{4}$$

and the initial condition means $C = 1$.