

TACHEOMETRIC SURVEYING

Tacheometry or tachemetry or telemetry is a branch of angular surveying in which the horizontal and Vertical distances of points are obtained by optical means as opposed to the ordinary slower process of measurements by tape or chain.

- The method is very rapid and convenient.
- It is best adapted in obstacles such as steep and broken ground, deep revines, stretches of water or swamp and so on, which make chaining difficult or impossible,
- The primary object of tacheometry is the preparation of contoured maps or plans requiring both the horizontal as well as Vertical control. Also, on surveys of higher accuracy, it provides a check on distances measured with the tape.

Tacheometry (from Greek, quick measure), is a system of rapid surveying, by which the positions, both horizontal and vertical, of points on the earth surface relatively to one another are determined without using a chain or tape or a separate leveling instrument.

Uses of Tacheometry

The tacheometric methods of surveying are used with advantage over the direct methods of measurement of horizontal distances and differences in elevations. Some of the uses are:

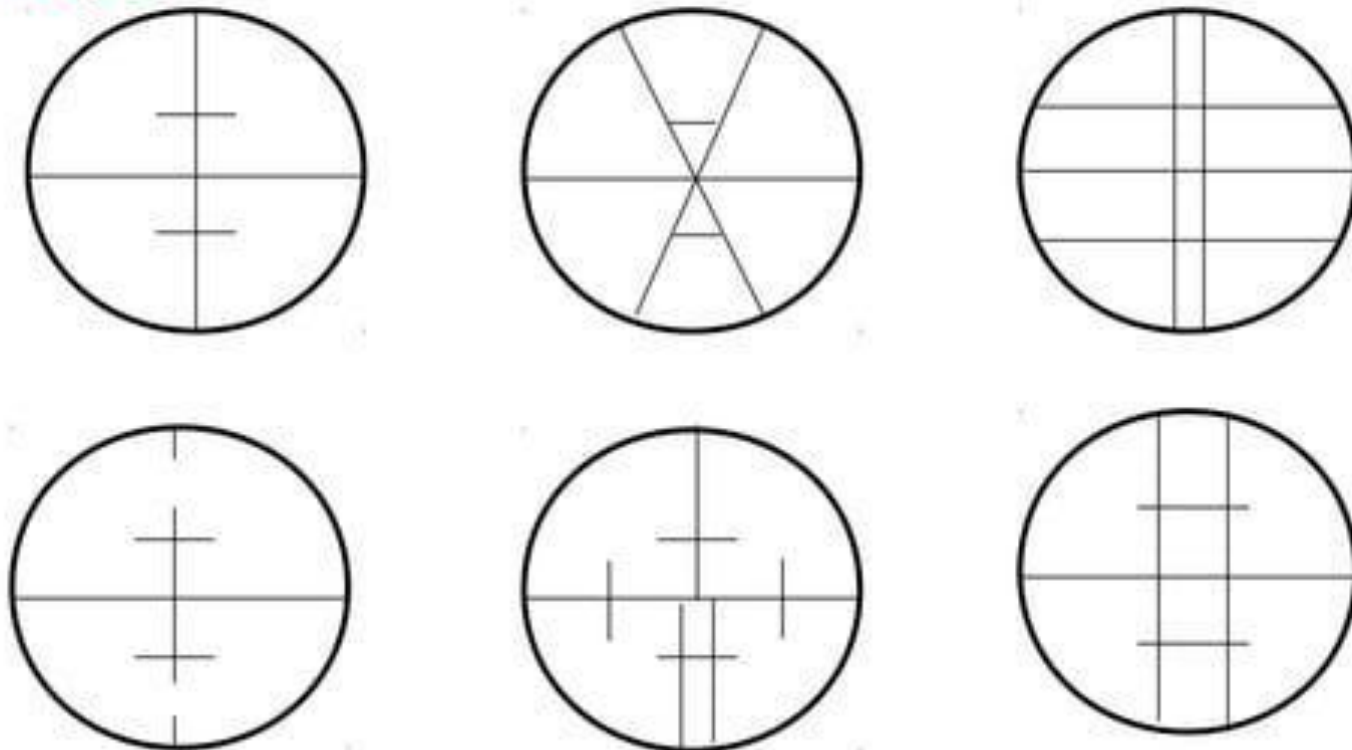
- ✓ Preparation of topographic maps which require both elevations and horizontal distances.
- ✓ Survey work in difficult terrain where direct methods are inconvenient
- ✓ Detail filling
- ✓ Reconnaissance surveys for highways, railways, etc.
- ✓ Checking of already measured distances
- ✓ Hydrographic surveys and
- ✓ Establishing secondary control.

INSTRUMENTS

-An ordinary transit theodolite fitted with a stadia diaphragm is generally used for tacheometric survey.

- The stadia diaphragm essentially consists of one stadia hair above and the other an equal distance below the horizontal cross-hair, the stadia hairs being mounted in the ring and on the same vertical plane as the horizontal and vertical cross-hairs.

Stadia is a tacheometric form of distance measurement that relies on fixed angle intercept.



Different forms of stadia diaphragm commonly used

The telescope used in stadia surveying are of three kinds:

- (1) The simple external-focusing telescope**
- (2) the external-focusing anallactic telescope (Possor's telescope)**
- (3) the internal-focusing telescope.**

A tacheometer must essentially incorporate the following features:

- (i) The multiplying constant should have a nominal value of 100 and the error contained in this value should not exceed 1 in 1000.**
 - (ii) The axial horizontal line should be exactly midway between the other two lines.**
 - (iii) The telescope should be truly anallactic.**
 - (iv) The telescope should be powerful having a magnification of 20 to 30 diameters.**
- The aperture of the objective should be 35 to 45 mm in diameter to have a sufficiently bright image.**
 - For small distances (say upto 100 meters), ordinary levelling staff may be used. For greater distances a stadia rod may be used.**
 - A stadia rod is usually of one piece, having 3 – 5 meters length.**
 - A stadia rod graduated in 5 mm (i.e. 0.005 m) for smaller distances and while for longer distances, the rod may be graduated in 1 cm (i.e. 0.01 m).**

10' 7"

6.9"

6.5"

6.1"



FOOT

GRADUATION READ
TO 0.01 FT

A



METRIC

GRADUATION READ
TO 1 CM

B

Different systems of Tacheometric Measurement:

The various systems of tacheometric survey may be classified as follows:

- **The stadia System**
 - (a) **Fixed Hair method of Stadia method**
 - (b) **Movable hair method, or Subtense method**
- **The tangential system**
- **Measurements by means of special instruments**

The principle common to all the systems is to calculate the horizontal distance between two points A and B and their distances in elevation, by observing

- (i) **The angle at the instrument at A subtended by a known short distance along a staff kept at B, and**
- (ii) **the vertical angle to B from A.**

(a) Fixed hair method

- In this method, *the angle at the instrument at A subtended by a known short distance along a staff kept at B* is made with the help of a stadia diaphragm having stadia wires at fixed or constant distance apart.
- The readings are on the staff corresponding to all the three wires taken.
- The staff intercept, i.e., the difference of the readings corresponding to top and bottom stadia wires will therefore depend on the distance of the staff from the instrument.
- When the staff intercept is more than the length of the staff, only half intercept is read.
- For inclined sight, readings may be taken by keeping the staff either vertical or normal to the line of sight.
- This is the most common method is tacheometry and the same 'stadia method' generally bears reference to this method.

Subtense Method

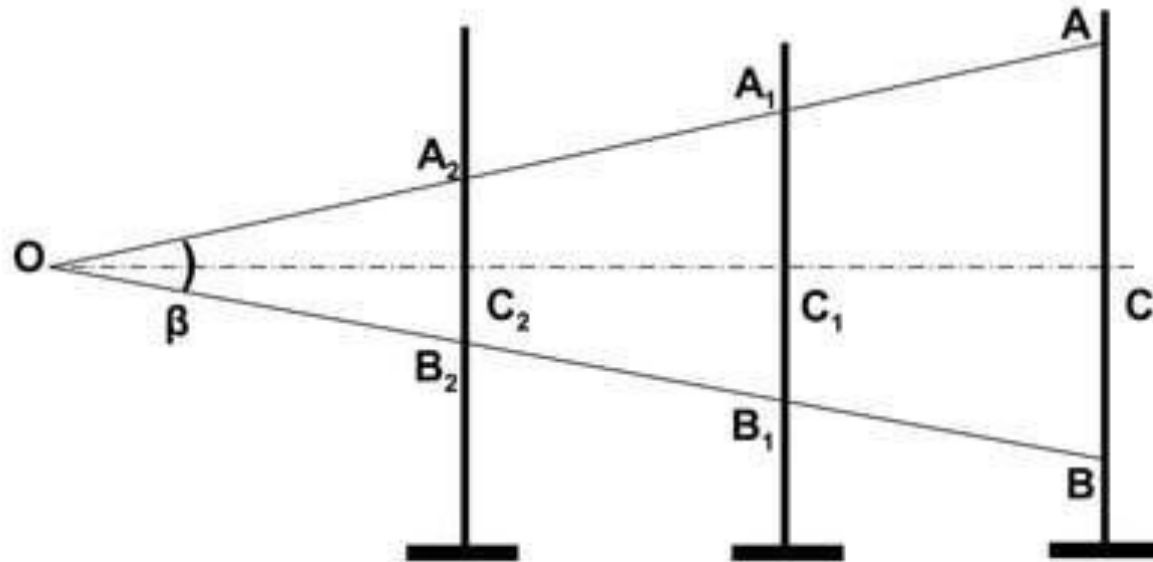
- ❖ This method is similar to the fixed hair method except that the stadia interval is variable.
- ❖ Suitable arrangement is made to vary the distance between the stadia hair as to set them against the two targets on the staff kept at the point under observation.
- ❖ Thus, in this case, the staff intercept, i.e., the distance between the two targets is kept fixed while the stadia interval, i.e., the distance between the stadia hair is variable.
- ❖ As in the case of fixed hair method, inclined sights may also be taken.

Tangential Method

- In this method, the stadia hairs are not used, the readings being taken against the horizontal cross-hair.
- To measure the staff intercept, two pointings of the instruments are, therefore, necessary.
- This necessitates measurement of vertical angles twice for one single observation.

PRINCIPLE OF STADIA METHOD

The stadia method is based on the principle that the ratio of the perpendicular to the base is constant in similar isosceles triangles.



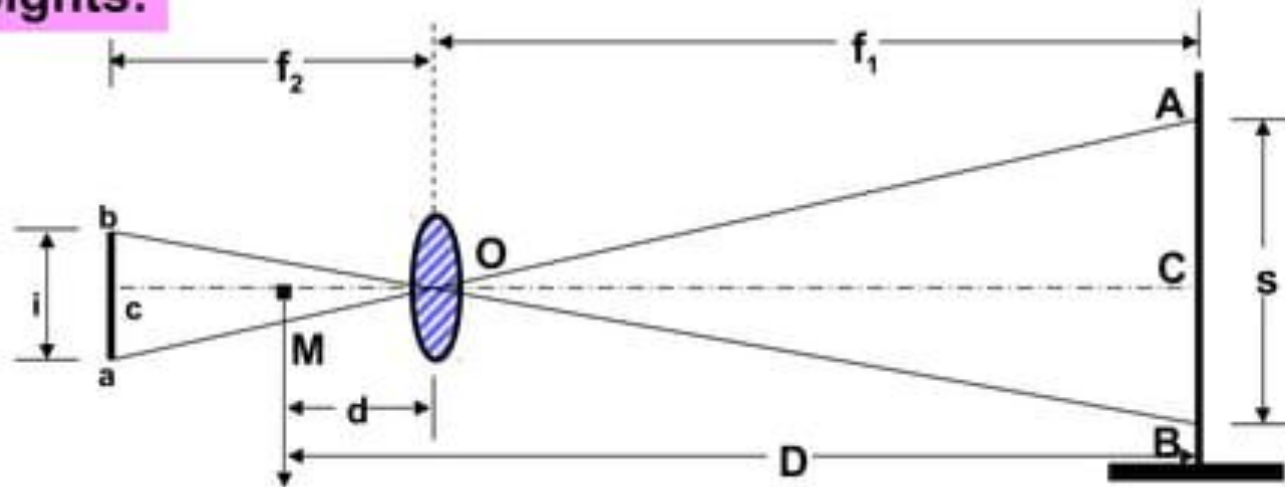
In figure, let two rays OA and OB be equally inclined to central ray OC. Let A_2B_2 , A_1B_1 and AB be the staff intercepts. Evidently,

$$\frac{OC_2}{A_2B_2} = \frac{OC_1}{A_1B_1} = \frac{OC}{AB}$$
$$= \text{constant } k = \frac{1}{2} \cot \frac{\beta}{2}$$

This constant k entirely depends upon the magnitude of the angle β .

- ✓ In actual practice, observations may be made with either horizontal line of sight or with inclined line of sight.
- ✓ In the later case the staff may be kept either **vertically** or **normal to the line of sight**.
- ✓ First the *distance-elevation* formulae for the horizontal sights should be derived.

Horizontal Sights:



Consider the figure, in which O is the optical centre of the objective of an *external focusing telescope*.

Let A , C , and B = the points cut by the three lines of sight corresponding to three wires.

b , c , and a = top, axial and bottom hairs of the diaphragm.

$ab = i$ = interval b/w the stadia hairs (stadia interval)

$AB = s$ = staff intercept;

f = focal length of the objective

f_1 = horizontal distance of the staff from the optical centre of the objective

f_2 = horizontal distance of the cross-wires from O.

d = distance of the vertical axis of the instrument from O.

D = horizontal distance of the staff from the vertical axis of the instruments.

M = centre of the instrument, corresponding to the vertical axis.

Since the rays BOb and AOa pass through the optical centre, they are straight so that AOB and aOb are similar. Hence,

$$\frac{f_1}{f_2} = \frac{s}{i}$$

Again, since f_1 and f_2 are conjugate focal distances, we have from lens formula,

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1}$$

Multiplying throughout by ff_1 , we get $f_1 = \frac{f_1}{f} f + f$

Substituting the values of $\frac{f_1}{f_2} = \frac{s}{i}$ in the above, we get

$$f_1 = \frac{s}{i} f + f$$

Horizontal distance between the axis and the staff is $D = f_1 + d$

$$D = \frac{f}{i} s + (f + d) = k \cdot s + C$$

Above equation is known as the **distance equation**. In order to get the horizontal distance, therefore, the staff intercept s is to be found by subtracting the staff readings corresponding to the top and bottom stadia hairs.

The constant $k = f/i$ is known as the **multiplying constant** or *stadia interval factor* and the constant $(f + d) = C$ is known as the **additive constant** of the instrument.

Determination of constant k and C

The values of the multiplying constant k and the additive constant C can be computed by the following methods:

1st method:

In this method, the additive constant $C = (f + d)$ is measured from the instrument while the multiplying constant k is computed from field observations:

1. Focus the instrument to a distant object and measure along the telescope the distance between the objective and cross-hairs,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

2. The distance d between the instrument axis and the objective is variable in the case of external focusing telescope, being greater for short sights and smaller for long sights. It should, therefore be measured for average sight. Thus, the additive constant $(f + d)$ is known.

3. To calculate the multiplying constant k , measure a known distance D_1 and take the intercept s_1 on the staff kept at that point, the line of sight being horizontal. Using the equation,

$$D_1 = ks_1 + C \quad \text{or} \quad k = \frac{D_1 - C}{s_1}$$

For average value, staff intercepts, s_2, s_3 etc., can be measured corresponding to distance D_2, D_3 etc., and mean value can be calculated.

Note: *In case of some external focusing instruments, the eye-piece-diaphragm unit moves during focusing. For such instruments d is constant and does not vary while focusing.*

2nd method:

In this method, both the constants are determined by field observations as under:

1. Measure a line, about 200m long, on fairly level ground and drive pegs at some interval, say 50 meters.
2. Keep the staff on the pegs and observe the corresponding staff intercepts with horizontal sight.
3. Knowing the values of D and s for different points, a number of simultaneous equations can be formed by substituting the values of D and s in equation $D = k.s + C$. The simultaneous solution of successive pairs will give the values of k and C , and the average of these can be found.

For example, if s_1 is the staff intercept corresponding to distance D_1 and s_2 corresponding to D_2 we have,

$$D_1 = k.s_1 + C \dots\dots (i) \quad \text{and} \quad D_2 = k.s_2 + C \dots\dots (ii)$$

Subtracting (i) from (ii), we get

$$k = \frac{D_2 - D_1}{s_2 - s_1} \dots\dots\dots (1)$$

Substituting the values of k in (i), we get

$$\begin{aligned} C &= D_1 - \frac{D_2 - D_1}{s_2 - s_1} s_1 \\ &= \frac{D_1 s_2 - D_2 s_1}{s_2 - s_1} \dots\dots\dots (2) \end{aligned}$$

Thus equation (1) and (2) give the values of k and C .

Distance and Elevation formulae for Staff Vertical : Inclined Sight

Let P = Instrument station;

Q = Staff station

M = position of instruments axis;

O = Optical centre of the objective

A, C, B = Points corresponding to the readings of the three hairs

$s = AB$ = Staff intercept;

i = Stadia interval

θ = Inclination of the line of sight from the horizontal

L = Length MC measured along the line of sight

$D = MQ'$ = Horizontal distance between the instrument and the staff

V = Vertical intercept at Q , between the line of sight and the horizontal line

h = height of the instrument;

r = central hair reading

β = angle between the two extreme rays corresponding to stadia hairs.

- [illegible]

Since $\beta/2$ is very small (its value being equal to $17' 11''$ for $k = 100$), angle $AA'C$ and angle $BB'C$ may be approximately taken equal to 90° .

$$\angle AA'C = \angle BB'C = 90^\circ$$

From $\triangle ACA'$, $A'C = AC \cos \theta$ or $A'B' = AB \cos \theta = s \cos \theta$ (a)

Since the line $A'B'$ is perpendicular to the line of sight OC , equation $D = k s + C$ is directly applicable. Hence, we have

$$MC = L = k \cdot A'B' + C = k s \cos \theta + C \quad \dots \dots \dots (b)$$

The horizontal distance

$$D = L \cos \theta = (k s \cos \theta + C) \cos \theta$$

$$D = k s \cos^2 \theta + C \cos \theta \quad \dots \dots \dots (1)$$

Similarly, $V = L \sin \theta = (k s \cos \theta + C) \sin \theta = k s \cos \theta \cdot \sin \theta + C \sin \theta$

$$\dots \dots \dots (2)$$

$$V = k s \frac{\sin 2\theta}{2} + C \sin \theta$$

Thus equations (1) and (2) are the distance and elevation formulae for inclined line of sight.

(a) **Elevation of the staff station for angle of elevation**

If the line of sight has an angle of elevation θ , as shown in the figure, we have

$$\text{Elevation of staff station} = \text{Elevation of instrument station} + h + V - r.$$

(b) **Elevation of the staff station for the angle of depression:**

$$\text{Elevation of Q} = \text{Elevation of P} + h - V - r$$

Distance and Elevation formulae for Staff Normal : Inclined Sight

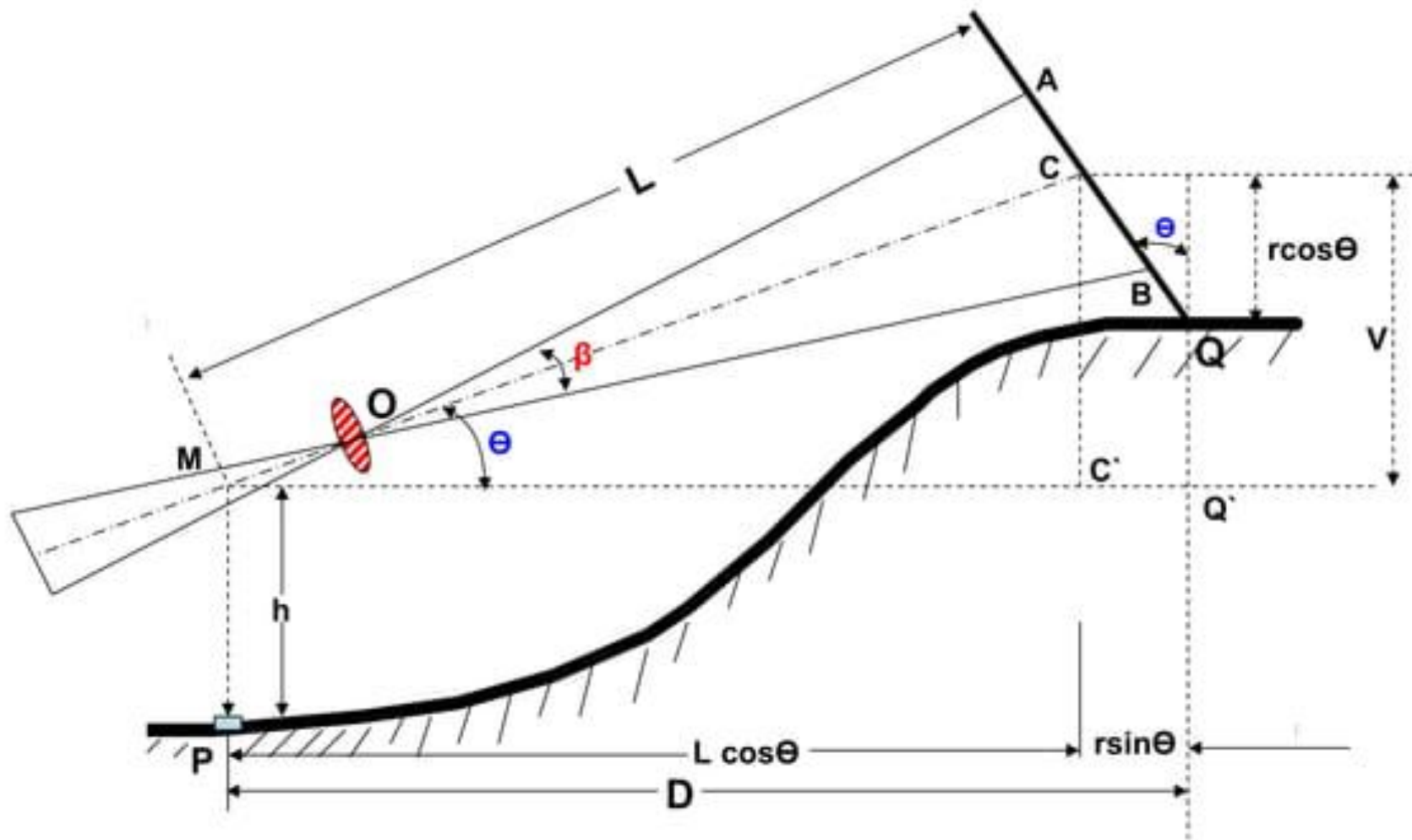


Figure shows the case when the staff is held normal to the line of sight.

Case (a): Line of Sight at an angle of elevation θ

Let $AB = s =$ staff intercept;

$CQ = r =$ axial hair reading

With the same notations as in the last case, we have

$$MC = L = K s + C$$

The horizontal distance between P and Q is given by

$$\begin{aligned} D &= MC' + C'Q' = L \cos\theta + r \sin\theta \\ &= (k s + C) \cos\theta + r \sin\theta \end{aligned} \quad \dots\dots (3)$$

Similarly, $V = L \sin\theta = (k s + C) \sin\theta \quad \dots\dots (4)$

Case (a): Line of Sight at an angle of depression θ

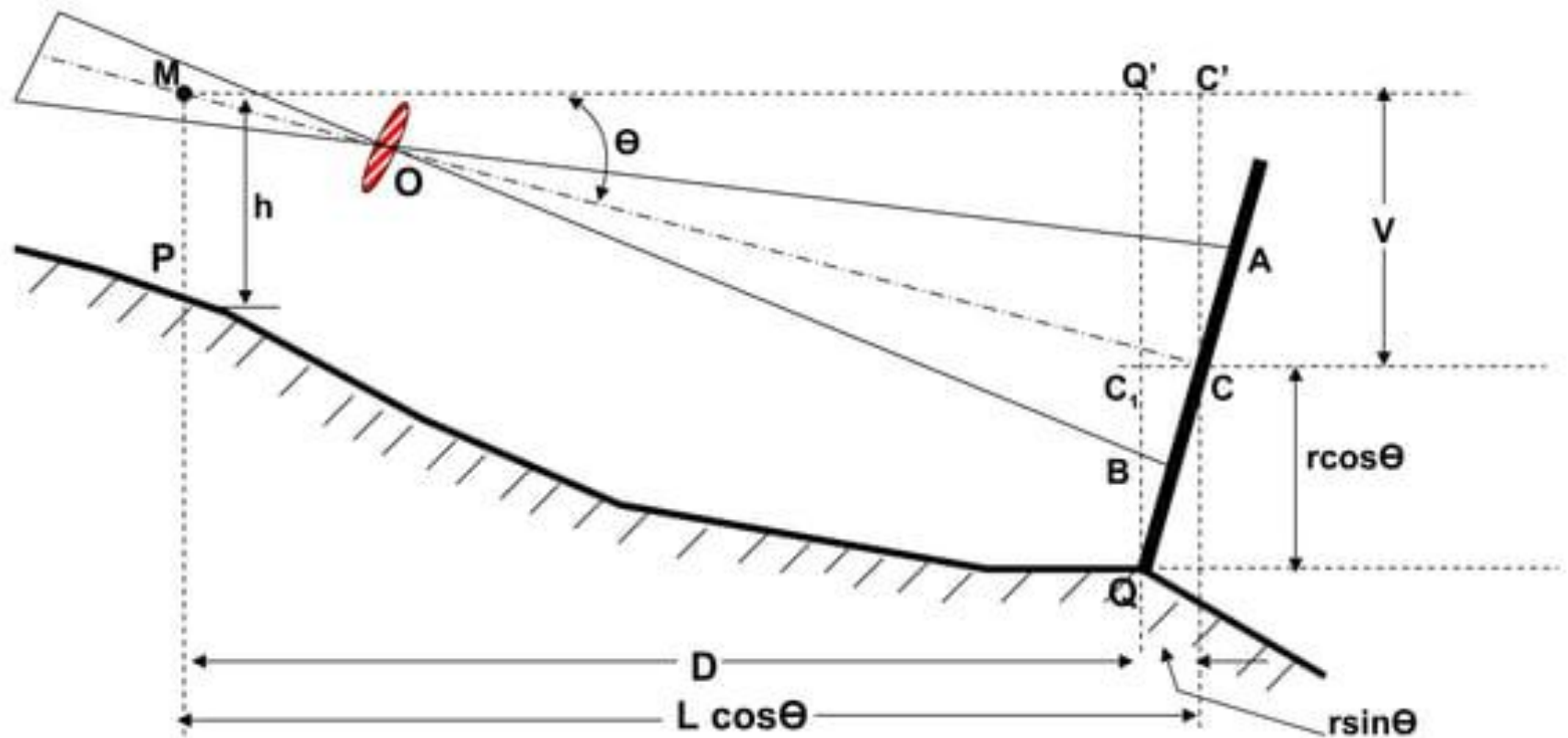


Figure shows the line of sight depressed downwards,

$$MC = L = k s + C$$

$$D = MQ' = MC' - Q'C'$$

$$= L \cos \theta - r \sin \theta$$

$$D = (k s + C) \cos \theta - r \sin \theta \quad \dots\dots (5)$$

$$V = L \sin \theta = (k s + C) \sin \theta \quad \dots\dots (6)$$

$$\text{Elevation of } Q = \text{Elevation of } P + h - V - r \cos \theta$$