UNIT-1

SIMPLE STRESS AND STRAIN

Elasticity:- when a forces acts on a body it under goes some deformation and the molecules often some resistances to the deformation. when the external forces is removed the forces of resistances also vanish and body sprish back to its original position. but it is only position of the deformatio caused by the external forces of with in a external such limit is called elastic limit. (the property of certain materials of return bags together original position after removing the external forces is known as elasticity)

A body is such to be perfectly elastic if it is returns back completely to its original space and size after that removed of external forces.

stress:- when extoral system of forces acts on a body which in elastic nature under goes some deformation as the body. under goes deformation it's molecules setup some resistancer to deformation. the resistances per unit area to deformation is known as stress. Matimatically stress may be defind as the forces per unit area.

$$\sigma = \frac{P}{A}$$

σ = stress

P = forces

A = Area

In S.I system the unit of stress is pascal (pa) which is equal to Pa = $1 N/m^2$

(mega pascal) Mpa $= 10^6\ N/m^2$

$$= 10^6 \frac{N}{(10^3 mm)^2}$$

$$Mpa = N/mm^2$$

strain :- when a forces acts on a body it under goes some deformation this deformation for unit length is known as strain mathmatically strain defind as the deformation for unit length

$$\varepsilon = \frac{\delta l}{l}$$

ε is strain

δl is chaning length

l is original length

Types of stress :-

The following are that two types of stress based on the axial forces

1) Tensile stress $P \leftarrow \boxed{- - - -} \rightarrow P$

2) compressive stress $P \rightarrow \boxed{- - - - -} \leftarrow P$

1) **Tensile stress** :- when a body is subjected to two equal and opposite forces (pull) and the body tends to increases its length, the stress, induced is called Tensile stress

2) **compressive stress** :- when a body is subjected to two equal and opposite forces (push) and the body tends to decreases its length, the stress induced is called compressive stress.

Hooks law :- It's states the when a material is loaded with in It's elastic limit the stress is proportional to the strain mathmatically

$$\frac{stress}{strain} = constant$$

$$\frac{\sigma}{\varepsilon} = E$$

Young's Modulus :- when a material is loaded with in elastic limit the stress is proportional to the strain. ratio of stress to the strain is called young's modu (or) Modulus of elasticity.

$$\frac{\sigma}{\varepsilon} = E = \left(\frac{P/A}{\delta l / l}\right) = \frac{P \cdot l}{A \cdot \delta l}$$

$$E = \frac{P \cdot l}{A \cdot \delta l}$$

The values of young's modulus :-

S.NO	Material	young's Modulus
1.	steel	200 - 220 GPa
2.	worugh iron	190 - 200 Gpa
3.	cast iron	100 - 160 Gpa
4.	copper	90 - 110 Gpa
5.	Brass	80 - 90 GPa
6.	Alluminium	60 - 80 GPa
7.	Timber	10 GPa

① A steel rod 1m long and 20mm x 20mm cross section is subjected to a tensile forces to 40 determine the elongation of the rod it body los of the elasticity for the rod material 200 Gpa.

sol: Given data :-

$L = 1m = 1000mm$

$b = 20mm$

$d = 20mm$

$A = 20 \times 20 =$

$= 400 mm^2$

$P = 40\ kN$

$E = 200\ Gpa = 200 \times 10^3\ N/mm^2$

$$\delta l = \frac{P.L}{A.E}$$

$$\delta l = \frac{40 \times 10^3 \times 1000}{400 \times 200 \times 10^3}$$

$$\delta l = 0.5mm$$

(2) A hollow cylinder 2m long as an outside diameter 50 mm and inside diameter 30 mm. if the cylinder carrying a load of 25 kN. find stress in the cylinder. Also find the deformation of the cylinder if the value of modulus of velocity for the cylinder the material is 100 GPa.

Sol Given data :-

length $= 2m = 2000mm$

width (b) $= 50mm$

depth (d) $= 30mm$

Area of cylinder (A) $= \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(50^2 - 30^2)$

$= 1256.62\ mm^2$

force (P) $= 25\ kN = 25 \times 10^3 N$

Young's modulus (E) $= 100\ Gpa = 100 \times 10^3\ N/mm^2$

stress $\sigma = \frac{P}{A}$

$= \frac{25 \times 10^3}{1256.62}$

$\sigma = 1989\ N/mm^2$

Deformation $\delta l = \frac{PL}{AE}$

$$= \frac{25\times10^3 \times (2000)}{1256.62 \times (100 \times 10^3)}$$

$$= 0.397\ mm$$

③ A hollow steel tube 3.5m long as an external diameter of 120mm in order to determine the internal diameter the tube was subjected to tensile load of 400kN and extanes was measured to be 2mm if the modulus of elasicity for the tube material is 200 Gpa. determine the internal diameter of the tube.

Sol Given data :-

length $= 3.5m = 3500\ mm$

~~depth (D)~~ external diameter (D) = 120mm

forces $(P) = 400kN = 400 \times 10^3 N$

$\delta l = 2mm$

young's modulus (E) = 200 GPa $= 200 \times 10^3 N/mm^2$

$$\text{Area } (A) = \frac{PL}{E\delta l}$$

$$A = \frac{400 \times 10^3 (3500)}{200 \times 10^3 \times 2}$$

$$A = 3500 mm^2$$

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$3500 = \frac{\pi}{4}(120^2 - d^2)$$

$$d^2 = \frac{\pi}{4}(120^2)$$

$$d = 99.71\ mm$$

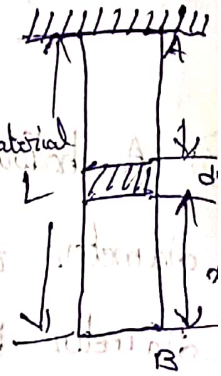
Deformation of a body due to self weight :- Consider a

AB hanging freely under it's own weight. let L = le

of the bar, A = cross sectional area of the bar

E = young's Modulus of the bar Material

γ = specific weight of the bar Material



now, consider a small section dx on the bar at a distances 'x' from 'B'. the weight of the Bar for a length 'x'. $P = \gamma A x$

alongastion of the small section of bar, give to the weigh the bar for a small section of length 'dx':

soln w

$$E = \frac{PL}{AE}$$

$$\delta l = \frac{P.L}{A.E} = \frac{(\gamma A x)\,dx}{AE}$$

$$\therefore \text{alongation} = \frac{\gamma x}{E}.dx$$

Total alongation of the bar may be find out by integrati the above equation b/w 0 and L

$$\therefore \text{total alongation } \delta l = \int_0^L \frac{\gamma x}{E}\,dx$$

$$= \frac{\gamma}{E}\int_0^L x.dx$$

$$= \frac{\gamma}{E}\left[\frac{x^2}{2}\right]_0^L$$

Specific weight $\boxed{\delta l = \frac{\gamma L^2}{2E}}$

$$\text{Density}(\gamma) = \frac{\text{weight}}{\text{volume}}$$

$$\gamma = \frac{W}{AL}$$

$$W = \gamma AL$$

now, subsuite the equation

$$\delta L = \frac{\gamma L^2}{2E}$$

$$= \left(\frac{W}{A\cancel{L}}\right)\left(\frac{L^{\cancel{2}}}{2E}\right)$$

Total weight. $\boxed{\delta L = \frac{WL}{2AE}}$

Q) A copper alloy wire of 1.5 mm diameter and 30 meters long is hanging 30 from a tower what will be alongation due to self weight. take specific weight of the copper and it's modulues of elasicity 89.2 kN/m³ and 90 GPa respecitiely

Sol: Given data :-

Diameter (d) = 1.5 mm

length (L) = 30m = 30×10^3 mm

specific weight (γ) = 89.2 kN/m³ = 89.2×10^{-6} N/mm³

yoding's modulus (E) = 90 GPa = 90×10^3 N/mm²

specific weight $(\delta L) = \frac{\gamma L^2}{2E}$

$$= \frac{89.2 \times 10^{-6} \times (30 \times 10^3)}{2 \times 90 \times 10^3}$$

$$\delta l = 0.446 \text{ mm}$$

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In alloy wire of $2mm^2$ ~~and~~ cross sectional area and 12N weight free under its own weight find the maxi-mum length of the wire if its extention is not to exceeds 0.6mm take 'E' for the wire modulus is 150 GPa

Sol Given data :-

$$\text{Area } (A) = 2mm^2$$

$$\text{weight } (W) = 12\,N$$

$$\text{young's modulus } (E) = 150\,GPa = 150 \times 10^3\, N/mm^2$$

$$\delta l = 0.6\,mm$$

Total weight. $\delta l = \frac{WL}{2AE}$

$$0.6 = \frac{12 \times L}{2 \times 2 \times 150 \times 10^3}$$

$$L = 30 \times 10^3\, mm$$

$$= 30m$$

principal of super position :-

A body is subjected to a no. of forces acting on it's outer edges as well as some other sections along the length of the body. In such a case the forces are split out and their effects are considered on individual sections. the resulting deformation of body is equal to algebric some of deformation of individual section such a principle of finding of resultant deformat.

The relation resulting deformation may be defind as

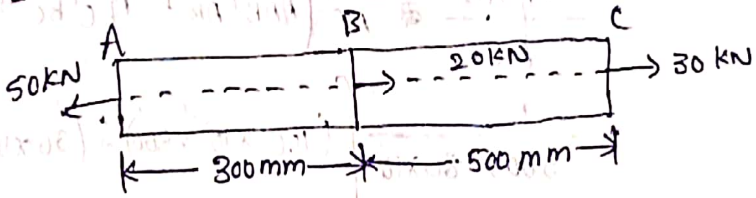
$$\delta l = \frac{PL}{AE}$$

$$= \frac{L}{AE}(P_1 + P_2 + P_3 + \ldots)$$

(or)

$$= \frac{1}{AE}(P_1L_1 + P_2L_2 + P_3L_3 + \ldots)$$

1) A steel bar of cross sectional area 200mm² is loaded as shown in figure. find the changing length of the bar take E as 200 GPa.



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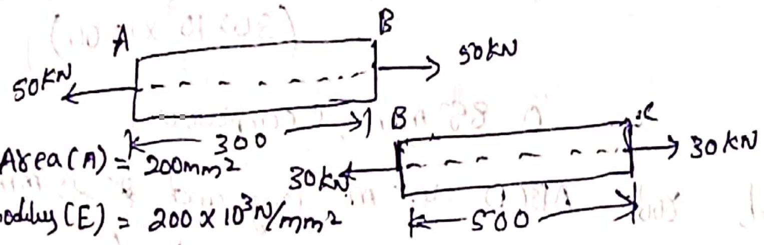


Area (A) = 200mm²

young's modulus (E) = $200 \times 10^3 N/mm^2$

$$\delta l = \frac{PL}{AE}$$

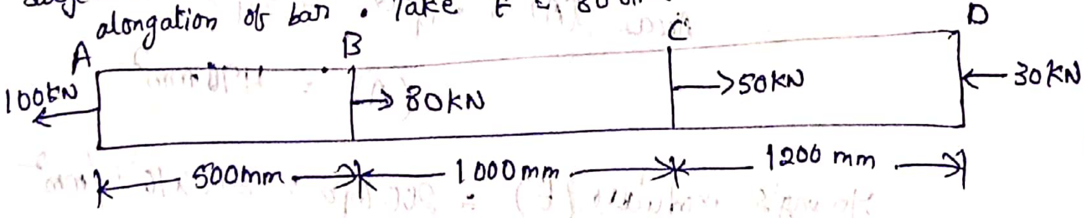
$$= \frac{P_{AB} L_{AB}}{AE} + \frac{P_{BC} L_{BC}}{AE}$$

$$= \frac{50 \times 10^3 \times 300}{200 \times (200 \times 10^3)} + \frac{30 \times 10^3 \times 500}{200 \times (200 \times 10^3)}$$

$$= \frac{50 \times 10^3 \times 300 + 30 \times 10^3 \times 500}{200 \times (200 \times 10^3)}$$

$= 0.75mm$ ✱.

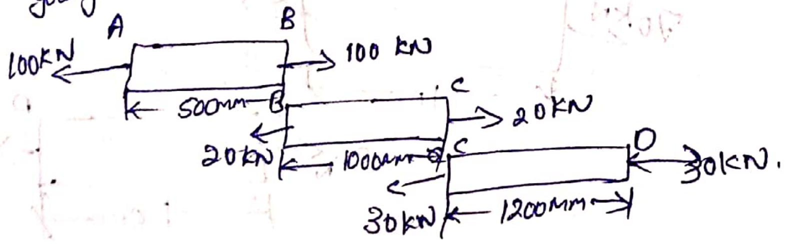
Brass bar having cross sectional area of 500mm² is subjected to axial forces as shown in figure. Find the total elongation of bar. Take E = 80 GPa.



Sr Given data:

Area (A) = 500 mm²

young's modulus (E) = 80 GPa = $80 \times 10^3 N/mm^2$



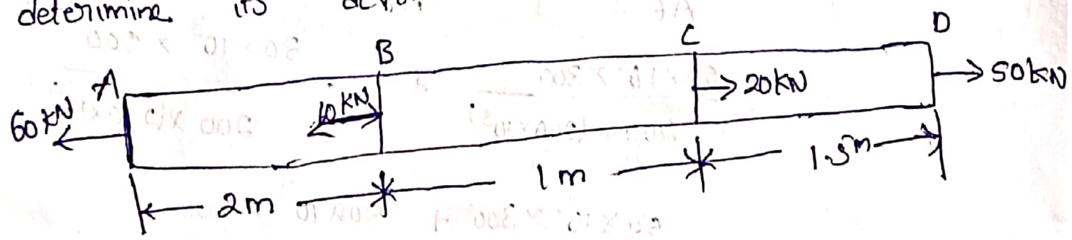
$$\delta l = \frac{PL}{AE}$$

$$= \cancel{\frac{P_{AB} L_{AB}}{AE}} = \frac{1}{AE}\left(P_{AB}L_{AB} + P_{BC}L_{BC} + P_{CD}L_{CD}\right)$$

$$= \frac{1}{500 \times 80 \times 10^3}\left[(100 \times 10^3 \times 500) + (20 \times 10^3 \times 1000) + (30 \times 10^3 \times 1200)\right]$$

$$= 0.85 \text{ mm} \text{ (compression)}$$

(Q) A steel rod ABCD 4.5 m long and ~~25~~ 25 mm diameter is subjected to the forces as shown in figure. if the value of young's modulus for the steel is 200 GPa determine it's deformation.



Sol: Given data :-

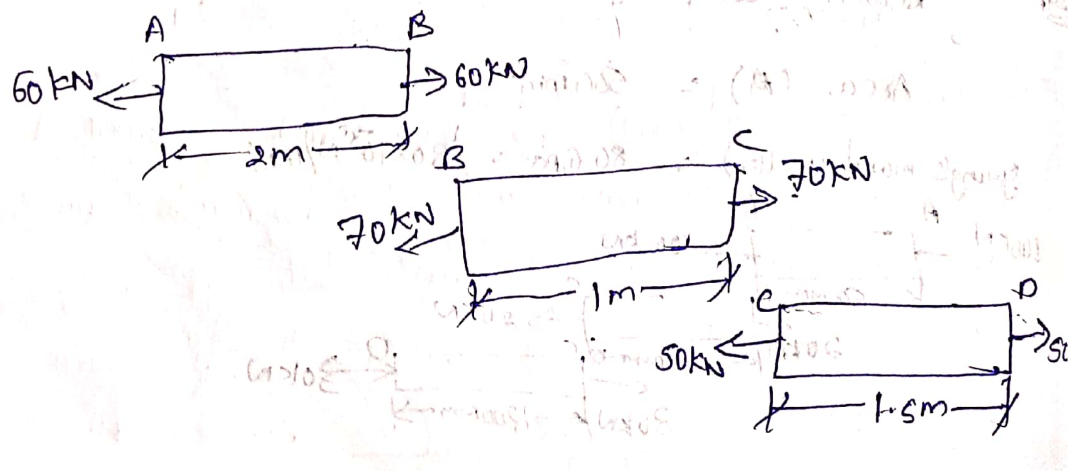
~~Area~~ length (l) = 4.5 m = 4500 mm

diameter (d) = 25 mm

Area (A) = $\frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2$

(A) = 490 mm^2

Young's modulus (E) = 200 Gpa = 200×10^3 N/mm^2



$$\delta l = \frac{PL}{AE}$$

$$= \frac{1}{AE}\left[P_{AB}L_{AB} + P_{BC}L_{BC} + P_{CD}L_{CD}\right]$$

$$= \frac{1}{490 \times 200 \times 10^3}\left[(60 \times 10^3 \times 2000) + (70 \times 10^3 \times 1000) + (50 \times 10^3 \times 1\right.$$

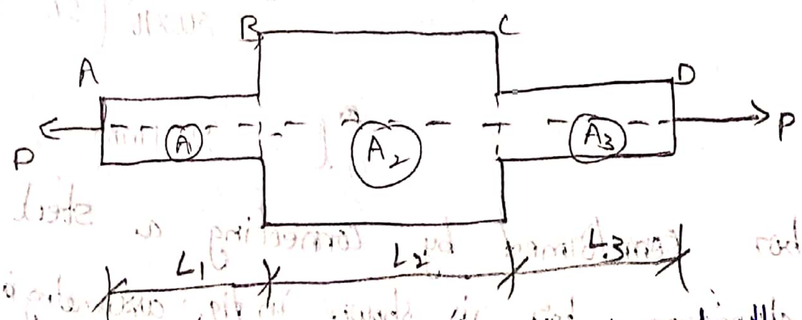
Types of bars of varying sections:-

There are many types of bars of varying section. the following are that important bars of varying sections.

1) Bars of different sections
2) Bars of uniformly tapering section
 a) circular sections
 b) Rectangular sections
3) Bars of opposite sections

stress in the Bars of different sections:-

A Bar is made up of different lens haveing different cross sectional area. as shown in fig.



In such cases the total changing ~~length is equal to~~ the sum of changes of length of indu sections

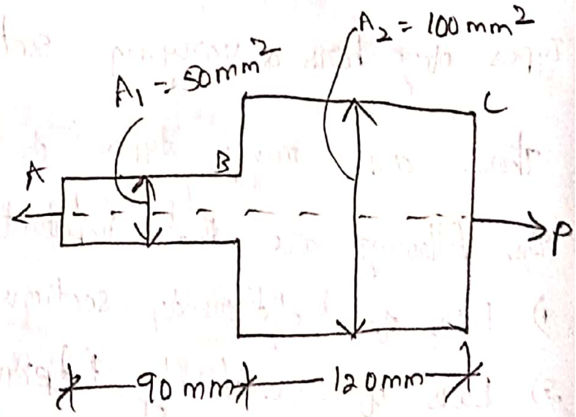
$$\delta l_1 = \frac{PL_1}{A_1 E}, \quad \delta l_2 = \frac{PL_2}{A_2 E}, \quad \delta l_3 = \frac{PL_3}{A_3 E}$$

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

If different materials of cross section $= P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right.$

Q) A Auto mobile compent as shown in fig is subject to a tensile load of 160 kN. determine the total elongation of the component. If modulus of elasicity is 200 Gpa.



Sol Given data :-

$L_1 = 90mm$

$L_2 = 120mm$

$A_1 = 50mm^2$

$A_2 = 100\,mm^2$

$P = 160\,kN = 160 \times 10^3\,N$

$E = 200\,GPa = 200 \times 10^3\,N/mm^2$

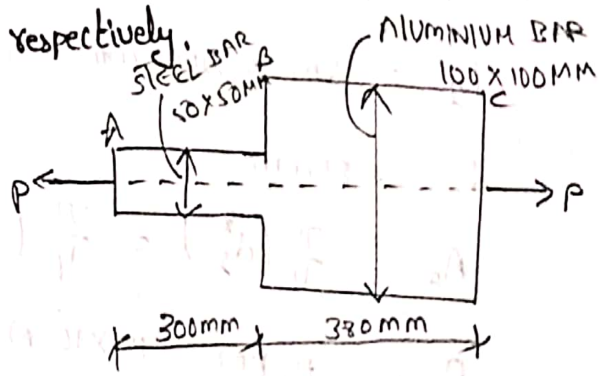
The total elongation of Bar $(\delta l) = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right)$

$$= \frac{160 \times 10^3}{200 \times 10^3}\left(\frac{90}{50} + \frac{120}{100}\right)$$

$$\delta l = 2.4mm$$

Q) A member formed by connecting a steel bar & an aluminum bar is shown in fig. assuming that the Bars are prevented from Buckling sidewise. calculate the magnitude of forces P. that will cause that total length of member to decrease by 0.25mm. the value

Gpa and 70 Gpa respectively.



Sol: Given data:-

$L_1 = 300\,mm$

$L_2 = 380\,mm$

$A_1 = 50 \times 50 = 2500\,mm^2$

$A_2 = 100 \times 100 = 10000\,mm^2$

$E_1 = 210\,Gpa = 210 \times 10^3\,N$

$E_2 = 70\,Gpa = 70 \times 10^3\,N$

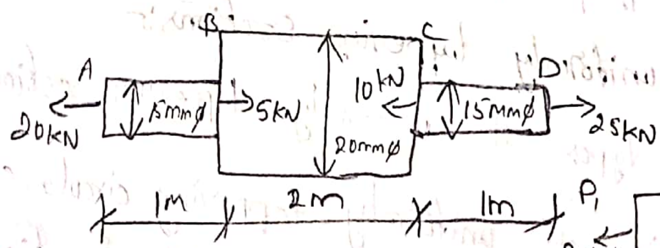
Total elongation, $\delta l = 0.25\,mm$

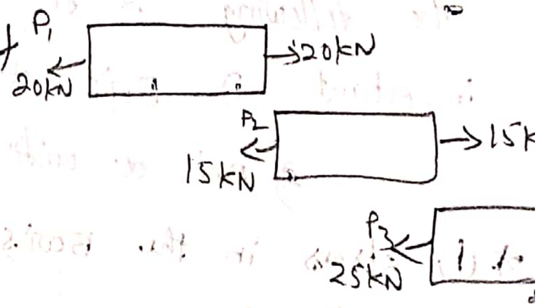
$$\delta l = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right]$$

$$0.25 = P\left(\frac{300}{2500 \times 210 \times 10^3} + \frac{380}{10000 \times 70 \times 10^3}\right)$$

$$P = 224.4 \times 10^3\,N.$$

(6) A steel bar ABCD 4 m long is subjected to forces as shown in fig. find out the total elongation of the bar take young's modulus for the steel 200 Gpa.





Given data:

Section ①

$L_1 = 1\,m = 1000\,mm$

$d_1 = 15\,mm$

$A_1 = \frac{\pi}{4}(d_1^2) = \frac{\pi}{4}(15^2) = 1707\,mm^2$

$P_1 = 20\,kN = 20 \times 10^3\,N$

Section - 2

$L_2 = 2m = 2000\,mm$

$d_2 = 20mm$

$A_2 = \frac{\pi}{4}(d_2^2) = \frac{\pi}{4}(20^2) = 314\,mm^2$

$P_2 = 15\,kN = 15\times10^3\,N$

Section - 3

$L_3 = 1m = 1000\,mm$

$d_3 = 15mm$

$A_3 = \frac{\pi}{4}(d_3^2) = \frac{\pi}{4}(15^2)$

$P_3 = 25\,kN = 25\times10^3\,N$

$$\delta l = \frac{1}{E}\left[\frac{P_1L_1}{A_1} + \frac{P_2L_2}{A_2} + \frac{P_3L_3}{A_3}\right]$$

$$\delta l = \frac{1}{200\times10^3}\left(\frac{20\times10^3\times1000}{.177} + \frac{15\times10^3\times2000}{314} + \frac{25\times10^3\times}{177}\right.$$

$$\delta l = \frac{\cancel{55\times10^6}}{5.0}\left[112994.3503 + 95541.401 + 141242.\right.$$

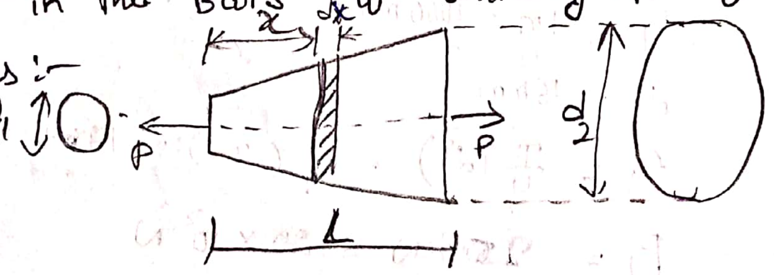
$\delta l = 1.74\,mm$

stress in bars of uniformly tappering Sections:-

The following 2 types of uniformly tappering sections important 1) Bar's of uniformly tappering circular sec

2) Bar's of uniformly tappering rectangular section

* 1, stress in the Bar's of uniformly tappering circula sections :-



consider a circular bar AB of uniformly tappering circular section as shown in fig. let 'p' is pulled on bar. 'L' = length of the bar. 'd_1' = diameter of one end. d_2 = diameter of other end.

the total alongation of bar = $\delta l = \frac{4PL}{\pi E d_1 d_2}$

① ~~A~~ Circular ~~#~~ bars ~~two~~ 2m long uniformly tappers from 13 mm diameter to 20mm diameter. calculate the alongation allowey of the bar under an axial forces of 50 kN. Take E bar of as 140 GPa.

Sol: Given data..

diameter (d_1) = 13 mm

diameter (d_2) = 20 mm

length (L) = 2m = 2000 mm

young's Modulus (G) = 140 GPa = 140×10^3 N/mm²

Total alongation of bar is = $\frac{4PL}{\pi G d_1 d_2}$

$$= \frac{4(50 \times 10^3)(2000)}{\pi (140 \times 10^3)(13)(20)}$$

= 1.52 mm

② Two circular bars A and B of the same material are subjected to the same pull 'P'. and deformed by the same amount. what is the ratio of their length If one of them a constant diameter of 60mm and the other uniformly tappers from 80mm from one end and 40 mm to other end.

Sol: Given data

Bar A: Pull (P) = P

length (L) = LA

diameter (d) = 60mm

$$\text{Area} = \frac{\pi}{4}(60)^2 = 2827.43\, mm^2$$

Young's modulus (E) = E

Bar 'B':

Pull = P

length (l) = L_B

young's modulus = E

diameter (d_1) = 80mm

diameter (d_2) = 40mm

The total elongation Bar A = $\delta l = \frac{P L_A}{A_A E}$

$$\frac{\cancel{P} L_A}{A_A \cancel{E}} = \frac{4 \cancel{P} L_B}{\pi \cancel{E} d_1 d_2}$$

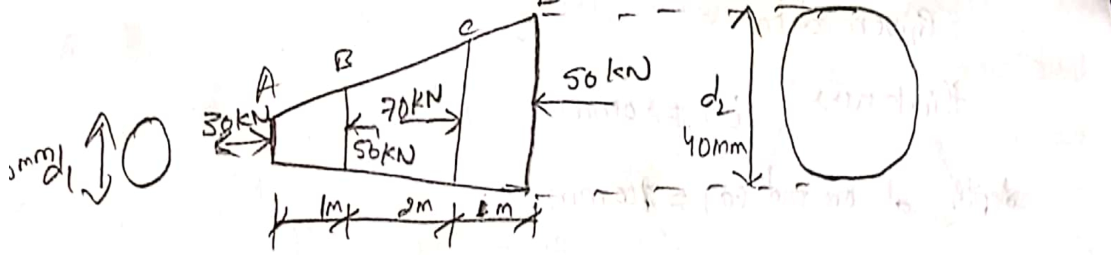
$$\frac{L_A}{A_A} = \frac{4 L_B}{\pi d_1 d_2}$$

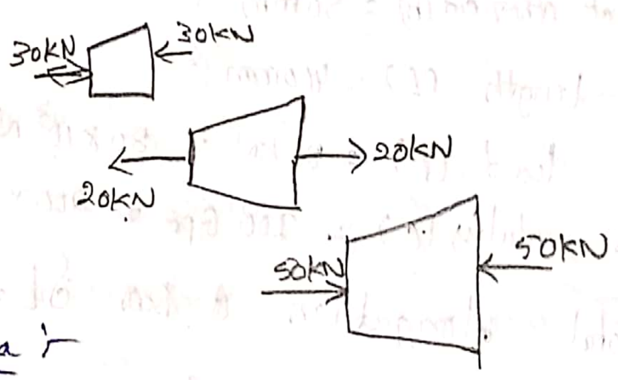
$$\frac{L_A}{L_B} = \frac{4 A_A}{\pi d_1 d_2}$$

$$\frac{L_A}{L_B} = \frac{\cancel{4}\left(\frac{\cancel{\pi}}{\cancel{4}} D^2\right)}{\cancel{\pi} d_1 d_2} = \frac{D^2}{d_1 d_2}$$

$$\frac{L_A}{L_B} = \frac{60^2}{80 \times 40} = \frac{9}{8} = 1.125$$

③ A round tappered alloy bar 4 m. Long is subjected to load as shown in fig find the change length of the bar. take E for the bar material as 120 GPa.





Given data :-

~~diameter (d_1) = 20mm~~

~~diameter (d_2) = 40mm~~

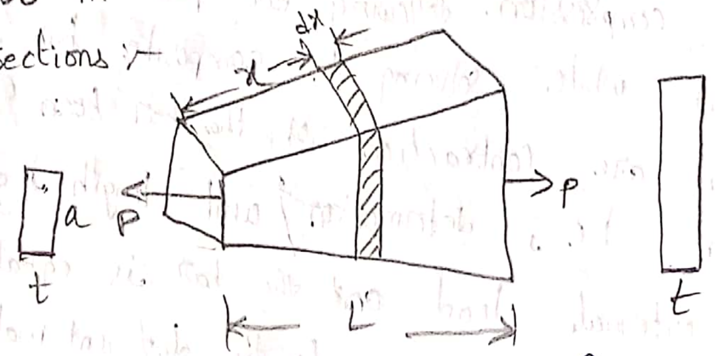
$$\delta L = -\delta l_{AB} + \delta l_{BC} - \delta l_{CD}$$

$$\delta l = \frac{1}{190\times10^3}\left\{-\frac{4\times30\times10^3\times1000}{\pi\times20\times25} + \frac{4\times20\times10^3\times2000}{\pi\times25\times35} - \frac{4\times30\times10^3\times1000}{\pi\times35\times40}\right\}$$

$$\delta L = -0.530 \text{ mm}$$

The total ~~alongation~~ contraction = 0.53 mm.

) stress in the Bar's of uniformly tappering rectangular sections :-



the total alongation of Bar, $\delta L = \frac{PL}{E\, t(a-b)} \ln\left(\frac{a}{b}\right)$

3) A steel plate of 20mm thikness tappers uniformly from 100 mm to 50mm in a length of 400mm. what is the alongation of the plate if an axial forces of 80 kN ~~acts~~ on it. take E = 200 Gpa.

Given data :-

thickness $(t) = 20mm$

depth at one end $(a) = 100 mm$

depth at other end $(b) = 50mm$

length $(L) = 400mm$

load $(P) = 80 kN = 80 \times 10^3 N$

Young's modulus $(E) = 200 Gpa = 200 \times 10^3 N/mm^2$

Total elongation of Bar $\delta L = \frac{PL}{E\, t(a-b)} \ln($

$$= \frac{80 \times 10^3 \times 400}{200 \times 10^3 \times 20 (100-50)} \ln\left(\frac{100}{50}\right)$$

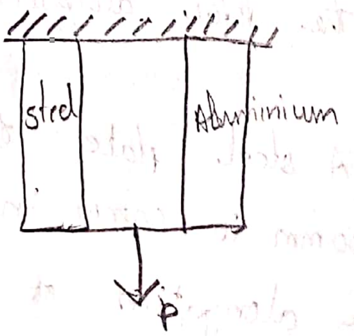
$\delta L = 0.11 mm.$

* Stress in Bars of composite sections :-

A bar made up of two or more different materia joined together is called composite bar. the bars are joined in such a manner that the system extendes are contractes as 1unit, equally when subjected t tension or compression. following two points should al kept in new while solveing the composite bars.

1) Extension are Contraction of the bar been equal the strain i.e., deformation / unit length is also c

2) the total external load and the bar is equal to the sum of the loads carrying by the different materia



$P = P_1 + P_2$

$\delta L = \delta L_1 = \delta L_2$

(or)

$\varepsilon = \varepsilon_1 = \varepsilon_2$

⑪ A renifcced concrete circular section of ~~1500m~~ 50000 mm cross sectional area carry's 6 renifoing bars whose total area is 500 mm^2. find the safe load the column can to carry if the concrete is not to be stress more than 3.5 mpa. take moduler ratio for steel and concrete as 18.

Sol

Given data: = $A_c + A_s$

Total area $(A) = 50,000 \text{ mm}^2$

Area of steel $(A_s) = 500 \text{ mm}^2$

$A_c = A - A_s = 50,000 - 500$

$A_c = 49,500 \text{ mm}^2$

stress $(\sigma_c) = \leq 3.5 \text{ mpa} = 3.5 \text{ N/mm}^2$

$1 \text{mpa} = 1 \times 10^6 \text{ N/m}^2$
$= 10^6 \times \frac{N}{(10^3 \text{mm})^2}$
$= 1 \text{ N/mm}^2$

Safe load $(P) = ?$

modulor ratio $(m) = \frac{E_s}{E_c} = 18$.

★ modulor ratio (M) = modulor ratio is defind as the ratio of young's modulus of steel and young's modulus of concrete.

$\sigma = \frac{P}{A}$

$\varepsilon = \frac{\delta L}{L}$

$E = \frac{\sigma}{\varepsilon}$

$\varepsilon = \frac{\sigma}{E}$

Total load $= P = P_c + P_s$ —— ①

$\frac{\text{load}}{\text{Area}} = \frac{P}{A} = \text{stress} =$

$\delta L = \delta l_c = \delta l_s$ —— ②

$\varepsilon = \varepsilon_c = \varepsilon_s$

$\varepsilon_c = \varepsilon_s$

$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$

$\sigma_s = \frac{\sigma_c}{E_c} \times E_s$

$= 3.5 \times 18$

$= 63 \text{ N/mm}^2$

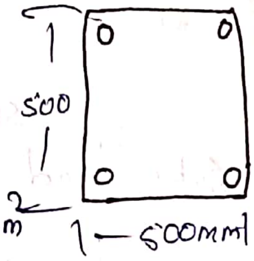
$$P = P_c + P_s$$

$$= (\sigma_c \cdot A_c) + (\sigma_s \cdot A_s)$$

$$= (3.5 \times 49.500) + (.63 \times 500)$$

$$= 204.75 \times 10^3\ N$$

(2) A reinforced concrete column 500×500 mm in section is reinforced with 4 steel bar of 25 mm di one in each corner. the column is carrying a load 1000 KN. find the stress in concrete in steel bars. Take $E = 210$ GPa and $E_c = 14$ GPa



Sol Given data :-

Total area $(A) = 500 \times 500$

$$= 250000\ mm^2 = 25 \times 10^4\ mm^2$$

Area of steel $(A_s) = 4 \times \frac{\pi}{4} (25)^2 = 1963.49\ mm^2$

Area of concrete $(A_c) = 250000 - 1963.49$

$$= 248036.51\ mm^2$$

load $(P) = 1000 \times 10^3\ N$

Young's modulus of steel $(E_s) = 210$ Gpa $= 210 \times 10^3\ N/mm^2$

Young's modulus of concrete $(E_c) = 14$ Gpa $= 14 \times 10^3\ N/mm^2$

$\sigma_c = ?$ 3.6

$\sigma_s = ?$ 54

$$P = P_c + P_s \quad \text{——— (1)}$$

$$1000 \times 10^3 = (\sigma_c \cdot A_c) + (\sigma_s \cdot A_s)$$

$$\varepsilon_c = \varepsilon_s$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

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$$\frac{\sigma_c}{14\times10^3} \times 210\times10^3 = \sigma_s$$

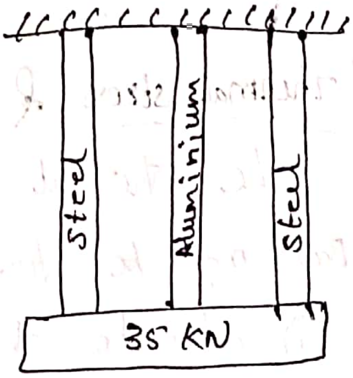
$$1000\times10^3 = (\sigma_c \cdot A_c) + (\sigma_s \cdot A_s) \quad \text{--- (1)}$$

$$1000\times10^3 = (\sigma_c \cdot 248036.51) + \left[\left(\frac{\sigma_c}{14\times10^3}\times 210\times10^3\right)\times 1963.49\right]$$

$$\therefore \sigma_c = 3.60 \ N/mm^2$$

$$\sigma_s = 54 \ N/mm^2$$

(3) A block shown in fig weing 35kN is supported by 3 wires the outer 2 wires are steel and have in area of $100mm^2$ each. where as the middle wire of aluminium and has an area of $200mm^2$. If the elastic modulus of steel and aluminium ~~are~~ is 200 GPa and 80 GPa respectvely. Calculate the aluminium and steel wires.



Sol: Given data :-

Total ~~lad~~ load $(P) = 35\times10^3 N$

Area of steel $(A_s) = 100 mm^2$

Area of aluminium $(A_A) = 200 \ mm^2$

young's modulus of steel $(E_s) = 200$ GPa $= 200\times10^3 \ N/mm^2$

young's modulus of Aluminium $(E_A) = 80$ GPa $= 80\times10^3 \ N/mm^2$

$$(\sigma_s) = ?$$

$$(\sigma_c) = ?$$

$$P = P_s + P_c + P_s \cdot \quad \text{--- (1)}$$

$$35\times10^3 = \cancel{(\sigma_s \cdot A_s)} (\sigma_c \cdot A_c) + (\sigma_s \cdot A_s)$$

$$\varepsilon_S = \varepsilon_A = \varepsilon_B$$

$$\frac{\sigma_S}{200\times10^3} = \frac{\sigma_A}{80\times10^3} \times \frac{\sigma_S}{200\times10^3}$$

$$\sigma_S = 200\times10^3 \times \frac{\sigma_A}{80\times10^3}$$

$$\boxed{\sigma_S = 2.5\,\sigma_A}$$

$$P = P_S + P_A + P_S$$

$$35\times10^3 = (\sigma_S \cdot A_S) + (\sigma_A \cdot A_A) + (\sigma_S \cdot A_S).$$

$$35\times10^3 = (2.5\sigma_A \cdot 100) + (\sigma_A \cdot 200) + (2.5\sigma_A \cdot 100)$$

$$35\times10^3 = 700\,\sigma_A$$

$$\sigma_A = 50\ N/mm^2$$

$$\sigma_S = 2.5\times\sigma_A$$

$$= 2.5\times50$$

$$\sigma_S = 125\ N/mm^2$$

Thermal stress & strains :-

The thermal stresses (or) strains may be simp[l]... in a Bar may be found out as below

1) calculate the amount of deformation due to change of temperature with the assumption that bar is free to expand or contraction.

2) calculate the load are force required to being the deformed bar's to original length.

3) calculate the stress and strain the bar caused by this load

Now consider a body subjected to an increasing temperature, let

let L = original length of the body.

Δt (or) t = increase in temperature.

α = coefficient linear expansion.

The increasing length due to the increasing temperature

$$\delta L = L\alpha t \text{ (or) } \Delta t$$

If the ends of bar are fixed to rigid support so that it's expansion is prevented.

The compressive strain induced in the bar

$$\varepsilon = \frac{\delta l}{L} = \frac{L\alpha\Delta t}{L}$$

$$\varepsilon = \alpha\Delta t$$

$$\sigma = \varepsilon \cdot E = \alpha\Delta t \cdot E$$

If support the yield by an amount $= \Delta$ then the actual expansion is the actual

$$\delta L = L\alpha\Delta t - \Delta$$

$$\varepsilon = \frac{L\alpha\Delta t - \Delta}{L}$$

$$\boxed{\varepsilon = \alpha\Delta t - \frac{\Delta}{L}}$$

$$\sigma = \alpha\Delta t \cdot E - \left(\frac{\Delta}{L}\right)E$$

The value of co-efficient of

S.NO	material	co.efficient of linear expansion.
1.	steel	$11.5\times10^{-6} - 13\times10^{-6}$
2.	Iron	$11\times10^{-6} - 12\times10^{-6}$
3.	Aluminium	$23\times10^{-6} - 24\times10^{-6}$
4.	copper, brass braze	$17\times10^{-6} - 18\times10^{-6}$

① An aluminium alloy bar fixed ~~at its end~~ both Ends he is heated to 20 k. Point the stresses developed in the bar's take modulus of elasicity and coefficient of expension of the linear expension for the bar material has E 80 Gpa an α 24 $\times 10^{-6}$/K

sol: Given data :-

$$\Delta t = 20K$$

$$E = 80\ Gpa = 80\times10^{3}\ N/mm^2$$

$$\alpha = 24\times10^{-6}/k$$

$$\sigma = \alpha \cdot \Delta t \cdot E$$

$$\sigma = 24\times10^{-6} \times 20 \times 80\times10^{3}$$

$$\sigma = 38.4\ N/mm^2$$

② A Brass rod 2m long is fixed at both ends if the thermal stress is not to exceed 76.5 mpa calculate the temperature through which the rod should be heated. take α and E has 17×10^{-6}/K and 90 Gpa respectively.

Given data :-

$$\Delta t = ?$$

$$E = 90\ Gpa$$

$$\alpha = 17\times10^{-6}/k$$

$\sigma = 76.5\ mpa = 76.5 \times N/mm^2$

$\sigma = \alpha \cdot \Delta t \cdot E$

$76.5 = 17 \times 10^{-6} \times 90 \times 10^{3} \times \Delta t$

$$\Delta t = \frac{76.5}{17 \times 10^{-6} \times 90 \times 10^{3}}$$

$\Delta t = 50K$

③ Two parellel walls 6m apart stayed together by a steel rod of 25mm diameter passing through metal plates and nut at each other. the nuts are tighting then the rod is at a temperature of $100^\circ c$. determine the stress in the rod when the temperature falls down to $60^\circ c$. ~~If first consi~~

i) the ends do not yield.

2) The ends yield by 1mm.

take $E = 200\ Gpa$ and $\alpha = 12 \times 10^{-6}/k$

Sol: changing temperature $\Delta t = 40^\circ c$

$E = 200\ Gpa = 200 \times 10^3\ N/mm^2$

$\alpha = 12 \times 10^{-6}/k$

length $(L) = 6\ m = 6000\ mm$

diameter $(d) = 25\ mm$

case - 1 : do not yield.

stress in the rod when the ends

$\sigma = \alpha \cdot \Delta t \cdot E$

$\cancel{12 \times 10^{-6}}\ \sigma = 12 \times 10^{-6} \times 40 \times 200 \times 10^{3}$

$\sigma = 96\ N/mm^2$

Case - 2 :

stress in the rod when the end yield by 1mm.

$\Delta = 1mm$

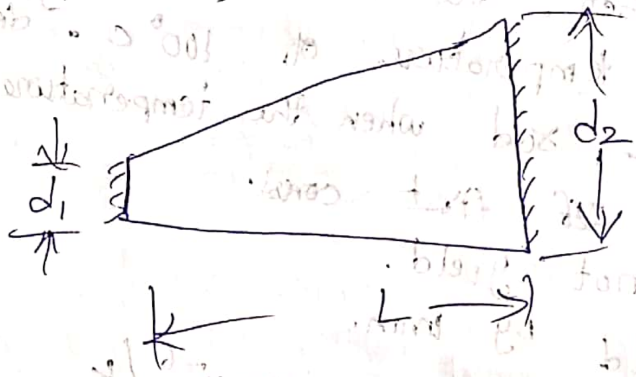
$$\sigma = \alpha t E - \left[\frac{\Delta}{L}\right] E$$

$$\sigma = 12 \times 10^{-6} \times 40 \times 200 \times 10^{3} - \left(\frac{1}{6000}\right) 200 \times 10^{3}$$

$$\sigma = 96 - \left[\frac{1}{6000}\right] 200 \times 10^{3}$$

$$\sigma = 62.66 \ N/mm^{2}$$

* Thermal stresses in bars of circular tappering section :-



$$\delta l = \frac{4PL}{\pi E d_1 d_2}$$

$$\alpha (\Delta t) \cancel{L} = \frac{4P\cancel{L}}{\pi E d_1 d_2}$$

$$P = \frac{\alpha (\Delta t) \pi E d_1 d_2}{4}$$

$$\sigma = \frac{\alpha \Delta t \cdot \cancel{\pi} E \cdot \cancel{d_1} d_2}{\cancel{4} \times \frac{\cancel{\pi}}{\cancel{4}} (d_1)^{\cancel{2}}}$$

$$\sigma = \alpha \Delta t \ E \left(\frac{d_2}{d_1}\right)$$

① A rigidly fixed circular bar 1.75m long uniformly tapper's from 125mm dia at one end ~~and 2~~ 100mm dia at other. If the maximum stress in the bar is need to not exceed 108 mpa. find the temperature to which it can be heated,

Take E and α has 100 Gpa and $18 \times 10^{-6}/°c$ respectively

Sol Given data.

$(L) = 1.75m = 1750 \, mm$

$(d_1) = 125mm$

$(d_2) = 100 \, mm$

$E = 100 \, Gpa = 100 \times 10^{3} \, N/mm^2$

$\sigma = 108 \, mpa = 108 \, N/mm^2$

$\alpha = 18 \times 10^{-6} / °C$

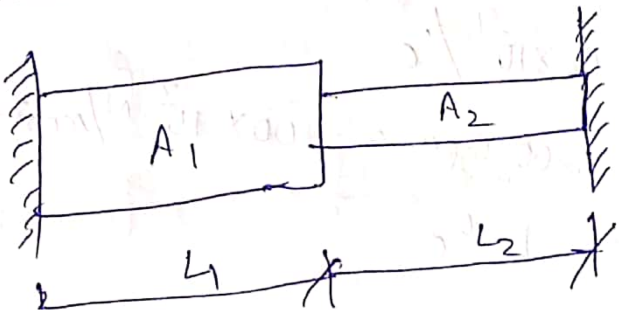
$$\sigma = \alpha \Delta t \, E \left[\frac{d_2}{d_1}\right]$$

$$108 = 18 \times 10^{-6} \cdot \Delta t \times 100 \times 10^{3} \left[\frac{125}{100}\right]$$

$$\frac{108}{18 \times 10^{-6} \times 100 \times 10^{3} \left(\frac{125}{100}\right)} = \Delta t$$

$$\Delta t = 48°C$$

② thermal stress in the bars of ~~an~~ an uniform sections :-



consider a bar ABC fixed at it's both ends 'A' & 'C' subjected to increase of temperature :-

* $\delta l_0 = \delta l_1 + \delta l_2$

* $P_0 = P_1 = P_2$

$$\delta l = \alpha_1 \Delta t L_1 + \alpha_2 \Delta t L_2$$

(or)

$$\delta l = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E_2}$$

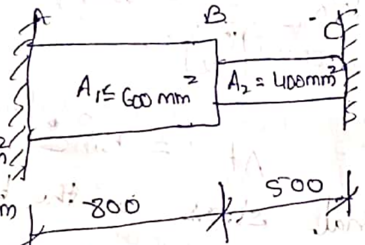
case-1

$$\boxed{\delta l = \frac{\sigma_1 L_1}{E_1} + \frac{\sigma_2 L_2}{E_2}} \quad \text{--- (1)}$$

case-2

$$\boxed{\sigma_1 \cdot A_1 = \sigma_2 \cdot A_2} \quad \text{--- (2)}$$

(1) A steel rod ABC formly held b/w two rigid supports A & C as shown in fig. find the stresses developed in the two positions in the rod. when it is heated through 15°C. take $\alpha = 12\times10^{-6}/°c$ and $E = 200$ Gpa.



Given data :- Bar-1

Length $(L_1) = 800mm$

$(A_1) = 600mm^2$

Bar-2

$(L_2) = 500mm$

$A_2 = 400mm^2$

$\alpha = 12\times10^{-6}/°c$

$E = 200$ Gpa $= 200\times10^{3}\ N/mm^2$

$t = 15°c$

Total deformation due to :-

$$\delta l = \alpha_1 \Delta t L_1 + \alpha_2 (\Delta t) L_2$$

$$\delta l = \alpha_1 \Delta t (L_1 + L_2)$$

$$\delta l = 12\times10^{-3} \times 15 \times [800 + 500]$$

$$\delta l = 0.234\ mm$$

$$\sigma_1 A_1 = \sigma_2 A_2$$

$$\Rightarrow \sigma_1 = \frac{2}{3}\sigma_2$$

$$\delta l = \frac{1}{E}[\sigma_1 l_1 + \sigma_2 l_2]$$

$$0.234 = \frac{1}{200\times10^3}\left[\left(\frac{2}{3}\sigma_2\right)800 + (\sigma_2)\,500\right]$$

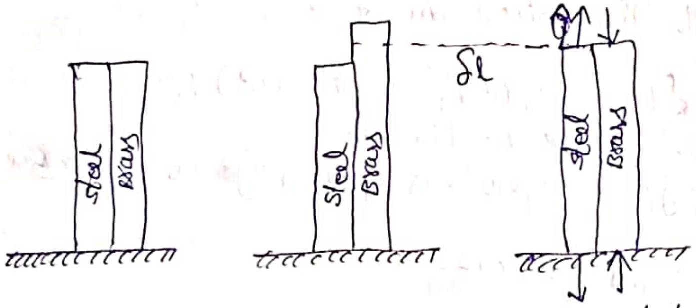
$$0.234 = 5166.66\,\sigma_2$$

$$\sigma_2 = 45.3\ N/mm^2$$

$$\sigma_1 = \frac{2}{3}(45.3)$$

$$\sigma_1 = 30.2\ N/mm^2$$

* Thermal stresses in composite Bars :- increase (or) decrease in the bar consising of two or more different materials. It temperature causes the bar to expand are contract on accou of different coefficient's of linear expansions. that to material do not expand are contract by the same amount but the expand or contract by different amount. now considering in a composite bar consisting of two members a bar of steel and another of Brass. As shown in fig.



As the compressive load on the brass is equal to the tensi-e load on the steel.

$\therefore P_1 = P_2$

$$\boxed{\sigma_1 A_1 = \sigma_2 A_2} \quad \text{——} (1)$$

however the total deformation of the bar

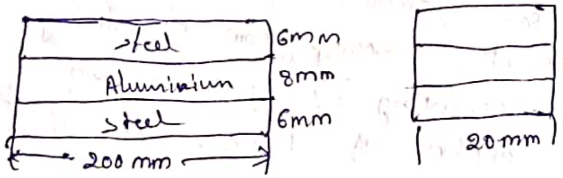
$\delta l = \delta l_1 = \delta l_2$

$$\boxed{\delta l = (\alpha_1 - \alpha_2)\Delta t\, L}$$

$$\delta l = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2}$$

$$\boxed{(\alpha_1 - \alpha_2)\Delta t = \frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2}} \quad \text{——} (2)$$

(1) A flat steel bar 200 x 20 x 8 mm is placed b/w 2 aluminium bars of 200 x 20 x 6 mm as to form a composite bar as shown in fig. all the three bars are fastened together at room temperature. find the stresses in each bar where the temperature is raised by 50°C. take 'E' for steel 200 Gpa. 'E' for alluminium 80 Gpa, α_s for steel $12 \times 10^{-6}/°C$, α for aluminium $24 \times 10^{-6}/°C$.



Given data :-

$E_s = 200 \text{ Gpa} = 200 \times 10^3 \text{ N/mm}^2$

$E_A = 80 \text{ Gpa} = 80 \times 10^3 \text{ N/mm}^2$

$\alpha_s = 12 \times 10^{-6} /°C$

$\alpha_A = 24 \times 10^{-6} /°C$

$\Delta t = 50°C$

Aluminium (A₁) $= 2 \times 20 \times 6$
$= 240 \text{ mm}^2$

steel $(A_2) = 20 \times 8$
$= 160 \text{ mm}^2$

$$\sigma_A A_A = \sigma_S A_S$$

$$\sigma_A = \frac{\sigma_S A_S}{A_A}$$

$$\sigma_A = \left(\frac{20 \times 8}{2 \times 20 \times 6}\right) \sigma_S$$

$$\boxed{\sigma_A = \left(\frac{2}{3}\right)\sigma_S}$$

$$(\alpha_A - \alpha_S)\Delta t = \frac{\sigma_A}{E_A} + \frac{\sigma_S}{E_S}$$

$$(24 \times 10^{-6} - 12 \times 10^{-6})\, 50 = \left[\frac{\left(\frac{2}{3}\right)\sigma_S}{(3 \times 80 \times 10^3)}\right] + \left[\frac{\sigma_S}{200 \times 10^3}\right]$$

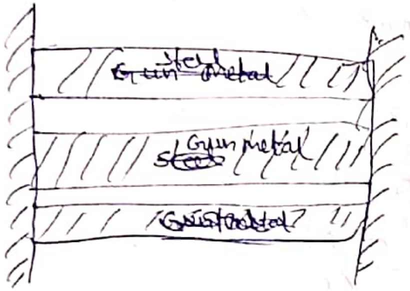
$$\sigma_S = 45 \text{ N/mm}^2$$

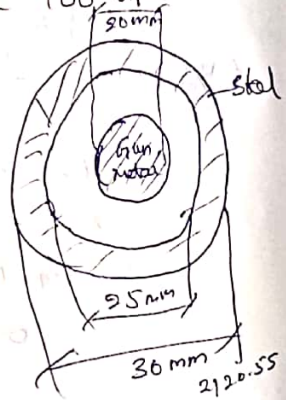
$$\sigma_A = \frac{2}{3}(45)$$

$$\sigma_A = 30 \text{ N/mm}^2$$

(2) Gun metel rod 20 m diameter screwed at the ends pass through a steel tube 25mm and 30mm internal and external respectively. the nuts on the

rod are screw tightly on the ends of the tube. find the intensity of stress in each metal when the temperature risies by 200°F. take coefficient of expansion for steel = $6\times10^{-6}/°F$. coefficient of expansion of Gun metal $10\times10^{-6}/°F$. modulus of elasticity of steel 200GPa. modulus of elasticity of Gun metal 100 GPa.





Sol Given data :-

dia meter of Gun metal (D) = 20mm

internal dia meter of steel = 25 mm

external dia meter of steel = 30 mm

$\Delta t = 200°F$

$(\alpha_s) = 6\times10^{-6}/°F$

$(\alpha_{Gm}) = 10\times10^{-6}/°F$

$(E_s) = 200\,GPa = 200\times10^3\,N/mm^2$

$(E_{Gm}) = 100\,GPa = 100\times10^3\,N/mm^2$

$\sigma_s A_s = \sigma_{Gm} A_{Gm}$

$\sigma_s = \dfrac{\sigma_{Gm} A_{Gm}}{A_s}$

$\sigma_s = \sigma_{Gm}\left(\dfrac{314.15}{706.85}\right)$

$\boxed{\sigma_s = 1.544\,\sigma_{Gm}}$

$(\alpha_{Gm} - \alpha_s)\Delta t = \dfrac{\sigma_s}{E_s} + \dfrac{\sigma_{Gm}}{E_{Gm}}$

$(10\times10^{-6} - 6\times10^{-6})200 = \dfrac{1.54\,\sigma_{Gm}}{200\times10^3} + \dfrac{\sigma_{Gm}}{100\times10^3}$

$0.08 =$

$\sigma_{Gm} = 46.4\ mpa$

$\sigma_s = 1.544(46.4)$

$\sigma_s = 71.45\ mpa$.

* elastic consistant.

1) poissions ratio :-

$= \dfrac{\text{lateral strain}}{\text{linear strain}} = \dfrac{1}{m}$ (or) μ

2) volumatric strain $= \varepsilon_v = \dfrac{\delta v}{v} = \dfrac{\text{changing volume}}{\text{original volume}}$

$\varepsilon_v = \varepsilon(1-2\mu)$

3) volumatric strain of a rectangular body or a subjucted to a axial forces

$\varepsilon_v = \dfrac{\delta v}{v} = \varepsilon(1-2\mu)$

4) volumatric strain of a rectangular body or a subjucted to a mutually perpendicular forces.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu\sigma_y - \mu\sigma_z\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu\sigma_x - \mu\sigma_z\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu\sigma_x - \mu\sigma_y\right]$$

$$\varepsilon_V = \frac{\delta_V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

5) **Bulk modulus** :-

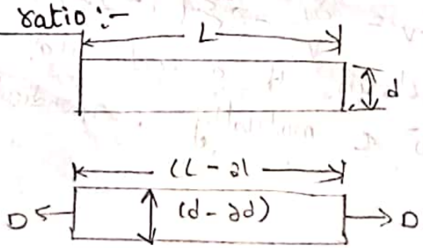
$$K = \frac{\text{direct stress}}{\text{volumetric stress}} = \frac{\sigma}{\varepsilon_V} = \frac{\sigma}{\left(\frac{\delta_V}{V}\right)}$$

~~1) ed Relation b/w Bulk modulus and young's modulus~~ :-

- **linear strain (or) primary strain** :- the deformation of the bar per unit length in the direction of the force that is $\left(\frac{\delta l}{l}\right)$ is known as linear strain (or) primary strain.

- **lateral strain (or) secondary strain** :- The direct stress is always accompanied by a strain in its own direction and an opposite kind of strain in every direction at right angles to it. such a strain is known as secondary (or) lateral strain.

- **poissons ratio** :-



If a body is stressed within ... the lateral strain bears a constant ratio to the linear strain. mathematically = $\frac{\text{lateral strain}}{\text{linear strain}}$ = constant

this constant is known as poission ratio and is denoted by $\frac{1}{m}$ (or) $\mu = \frac{\text{lateral strain}}{\text{linear strain}}$

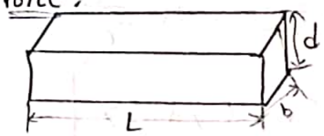
S.NO	material	poisson's ratio
1	steel	0.25 to 0.33
2	cast iron	0.23 to 0.27
3	copper	0.31 to 0.34
4	Brass	0.32 to 0.42
5	Aluminum	0.32 to 0.36
6	concrete	0.08 to 0.18
7	Rubber	0.45 to 0.50

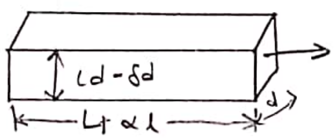
Volumetric strain :- The ratio of change in volume to the original volume is known as volumetric strain, mathamatically volumetric strain $\varepsilon_V = \frac{\delta V}{V}$ where

δV = change in volume

V = original volume

volumetric strain of a rectangular body subjected an axial force :-





original volume (V) = LBD

final volume = $(L+\partial l)(B-\delta b)(D-\delta d)$

$$\approx LBD\left(1+\frac{\partial l}{L}-\frac{\partial b}{B}-\frac{\partial d}{D}\right)$$

change in volume (δV) = final volume − original volume

$$= LBD\left(\frac{\partial L}{L}-\frac{\partial b}{B}-\frac{\partial d}{D}\right)$$

$$\mu = \frac{\left(\frac{\partial b}{B}\right)}{\left(\frac{\partial l}{L}\right)} \text{ (or) } \frac{\left(\frac{\partial d}{D}\right)}{\left(\frac{\partial L}{L}\right)}$$

$$\varepsilon_V = \frac{\partial V}{V}$$

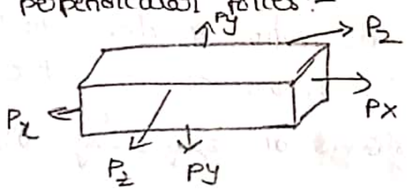
$$= \frac{\partial l}{L} - \frac{\partial b}{B} - \frac{\partial d}{D}$$

$$= \frac{\partial l}{L} - \mu\frac{\partial l}{L} - \mu\frac{\partial l}{L}$$

$$= \frac{\partial l}{L}(1-2\mu)$$

$$\varepsilon_v = \varepsilon(1-2\mu)$$

Volumetric strain of a rectangular body subjected to three mutually perpendicular forces:-



$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$\boxed{\frac{\delta}{V} = \sigma V = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

Bulk modulus:- when a body is subjected to three mutually perpendicular stress of equal intensity the ratio of direct stress to the corresponding volumetric strain is known as Bulk modulus It is denoted by 'k' mathematically Bulk modulus (k) = $\frac{\text{direct stress}}{\text{Volumetric strain}}$

$$k = \frac{\sigma}{\varepsilon_V} = \frac{\sigma}{\left(\frac{\partial V}{V}\right)}$$

* **Relation between bulk modolus and young's modolus:-**

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_y = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_z = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

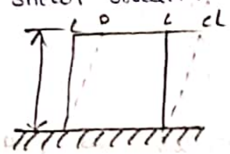
$$k = \frac{\sigma}{\varepsilon_V} = \frac{\sigma}{\varepsilon_x+\varepsilon_y+\varepsilon_z}$$

$$k = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$k = \frac{E}{3(1-2\mu)}$$

$$\boxed{E = 3k(1-2\mu)}$$

shear stress :- when a section is subjected to two equal and opposite forces acting tangentially across the resisting section. as a use of which, the body tense to shear off across the section, the stress is induced is called shear stress. the corresponding strain is called as shear strain.

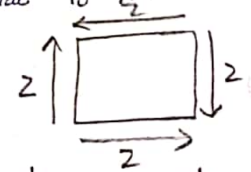


mathamatically shear strain = $\dfrac{\text{deformation}}{\text{original length}}$

$$= \frac{cc l}{l} = \phi$$

Principal of shear stress :-

It states a shear stress across a plane, is always accontineed by balance in shear stress across the plane and normal to it.



$$\boxed{Z = Z}$$

shear moddlus (or) moddulus of rigidity :- It has been experimantally found that within the elastic limit the shear stress is proportional to the shear strain, mathamatically.

$$Z \propto \phi$$
$$Z = C\phi$$

where the constant 'C' is called shear modolus (or) modulus of rigidity

Z is shear stress

ϕ is shear strain

Relation b/w modolus of elasticity and modulus of rigidity

$$\boxed{E = 2C(1+\mu)}$$

* Relation b/w elastic constant

$$E = 3K(1-2\mu)$$

$$\boxed{E = \frac{9KC}{3K+C}}$$

$$\boxed{\mu = \frac{3K-2C}{2C-6K}}$$