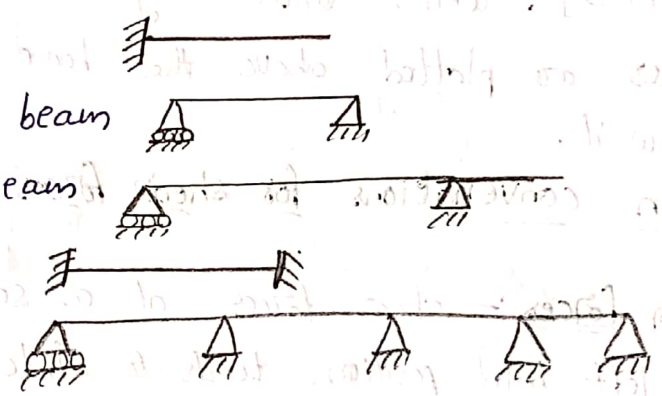
2. Shear force and Bending moment

A structural member which is acted upon a system of external loads at right angle to it axis is known as BEAM

Types of Beams :-

Types of Beams classified are as under.

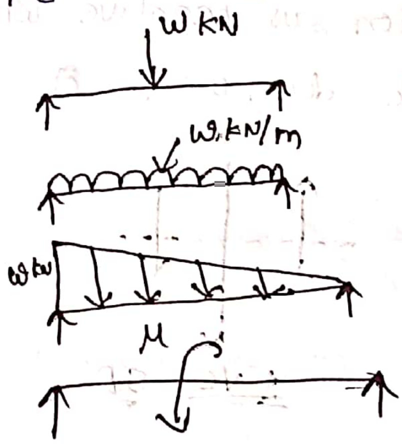
1. cantilever beam.
2. simply supported beam
3. over hanging beam
4. fixed Beam
5. continous Beam



Types of loadings :-

Beam may be subjeted to i.e or in combinatoformation of following loads.

1) concentrated (or) point load
2) uniformlly distributed load
3) uniformlly varying load
4) concentrated moment



shear forces :-

The shear force at cross section of a beam is defind as the un balanced vertical forces to the right or left of the sections. The abbrivation of shear forces is S.F.

Bending moment :-

Bending moment at the cross section of beam is defind as the algribraic sum of the moments of forces to the righ or left of the sections. the abrivation of Bending moment is B.M.

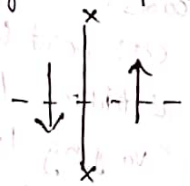
Shear forces and Bending moment diagram:

The shear forces and bending moment can be calculated numerically at any particular sections. The diagram show in the variation of shear forces (or) Bending moment along the length of the beam is called shear forces diagram [S.F.D] (or) Bending moment diagram [B.M.D]. while ~~dra~~drawing SFD and BMD all positive values are plotted above the base line and negative value below it.

Sign convenations for shear forces and Bending moment:-

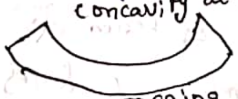
1) **shear forces:-** shear forces at a section is positive when the left hand position tends to slide upwords (or) the right hand position silds downwards. similarly the shear forces at a section is negative when the left hand position tends to slide down wards or the right hand position silds to upwards

+ve SF



-ve S.F

2) **Bending moment:-** The Bending moment at a section is positive ~~when~~ If ~~that~~ tends to Bend the beam at that point to a curvature having concavity at the top. similarly bending moment at a section negative If it's tends to Bend the beam at that point to a curvature having convexity at the top. the positive B.M is also concavity at top called as sagging. convexity at top.



sagging

+ve B.M

Hogging

- Ve B.M

and negative B.M as Hogging moment

Relation b/w ~~shear~~ load S.F and B.M:-

1) The rate of change of Bending moment is called shear force
2) The rate of change of shear forces is intensity of load.

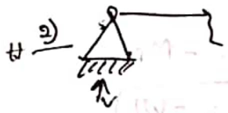
$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = \omega$$

case-1:- cantiliver beam with a point load at it's free end:-

Types of supports:-

1) roller support :- It consisting of only vertical reaction's.
2) ~~pr~~ hinged support : It consisting of vertical and horizontal reactions
3) fixed support :- It consisting of vertical / horizontal and moment reacti

1) 
2) 
3) 

Equalibrium equations:-

The following equailibrium equations can be used.

1. The sum of all horizontal forces and reactions must be zero

$$\boxed{\Sigma H = 0}$$

2. The sum of all vertical forces and reaction must be zero

$$\boxed{\Sigma V = 0}$$

3. The sum of all moments/forces and reactions must be zero.

$$\boxed{\Sigma M = 0}$$

case-1:- cantiliver beam with a point load at it's free-end:

Reactions :-

$\Sigma V = 0$

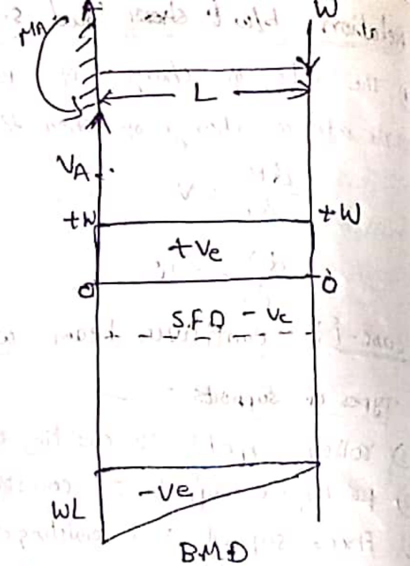
$V_A - W = 0$ —— ①

$\boxed{V_A = W}$

$\Sigma M_A = 0$

$(WL) + (V_A \times 0) - M_A = 0$

$\boxed{M_A = WL}$



Shear force :-

S.f At A :-

$LHS = V_A = 0$

$RHS = V_A = +V_A = +W$

S.f At B

$L.H.S = V_B = +W$

$R.H.S = V_B = 0$

Bending moment :-

B.M at A : $M_A = -M_A$

$\boxed{M_A = -WL}$

B.M At B :- $M_B = -M_A + V_A L = 0$

*** Reactions :-**

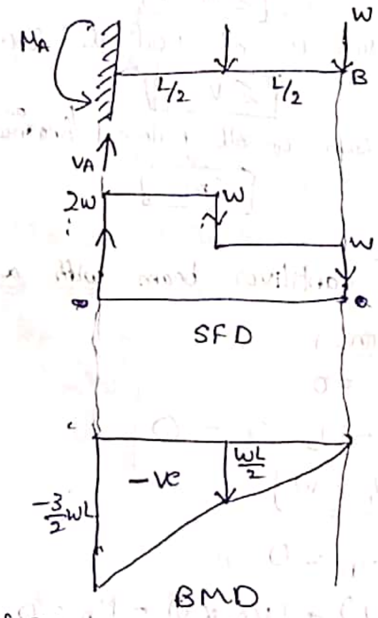
$\boxed{\Sigma V = 0}$

Σ upward = Σ downward

$V_A = W + W$

$\boxed{V_A = 2W}$

$\boxed{\Sigma M = 0}$



At point A :-

~~left hand side~~ :

$M_A = -\left(\frac{WL}{2}\right) - (WL)$

$\boxed{M_A = -\frac{3WL}{2}}$

S.F. calculations :-

At point A :- $L.H.S = 0$

$R.H.S = +2W$

At point C :-

$L.H.S = +2W$

$R.H.S = +W$

At point B :-

$L.H.S = +W$

$R.H.S = 0$

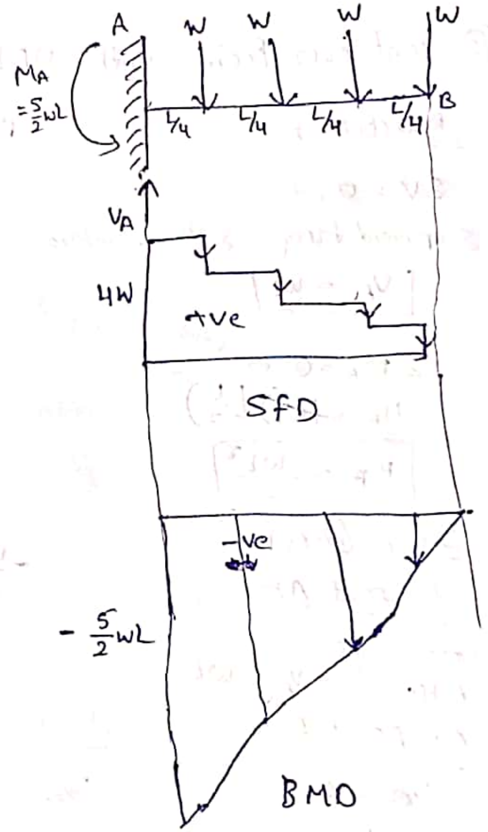
B.M :-

$M_A = -\frac{3}{2}WL$

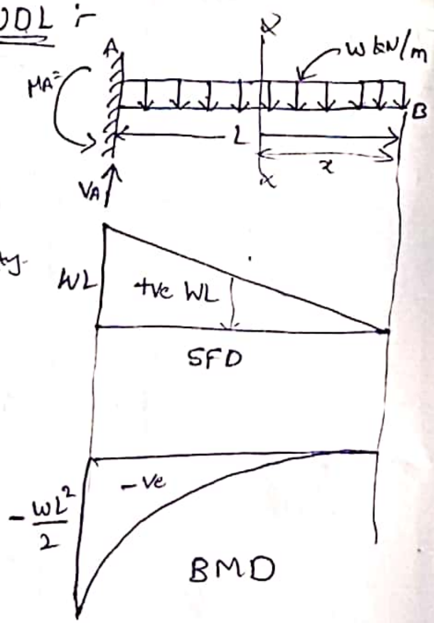
$M_C = -\frac{WL}{2}$

$M_B = 0$

*** Reactions :-**



② cantilever Beam with UDL :-



Reactions :-

$\Sigma V = 0$

Σ upward forces = Σ downward forces

$\boxed{V_A = wL}$

$\Sigma M_A = 0$ area of diagram → center of gravity

$M_A = (wL)\left(\frac{L}{2}\right)$

$\boxed{M_A = \frac{wL^2}{2}}$

Shear forces :-

At point A :-

L.H.S = 0

R.H.S = $+V_A = wL$

At point B

$V_B = 0$

at section xx

$V_{xx} = wx$

Bending Moment :

At point A :-

$M_A = -\frac{wL^2}{2}$

At point B :-

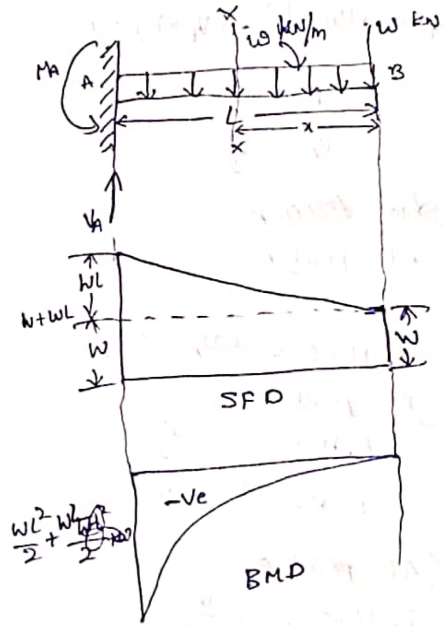
$M_B = 0$

at section xx

$M_x = -(wx)\left(\frac{x}{2}\right)$

$\boxed{M_x = -\frac{wx^2}{2}}$

③ cantilever Beam with combination of point load and UDL :-



Reactions :-

$\Sigma V = 0$

Σ upward forces = Σ downward forces

$\boxed{V_A = W + wL}$

$\Sigma M_A = 0$

$M_A = (WL) + (wL)\frac{L}{2}$

$\boxed{M_A = WL + \frac{wL^2}{2}}$

S.F :-

At point A :-

L.H.S = 0

R.H.S = $+V_A = W + wL$

At point B :-

L.H.S = +W

R.H.S = 0

At section xx :-

$V_{xx} = wx + W$

M.B :-

at point A

$M_A = -WL - \frac{wL^2}{2}$

at point B

$M_B = 0$

at section xx

$M_x = -(wx)\left(\frac{x}{2}\right) - Wx$

$\boxed{M_x = -\frac{wx^2}{2} - Wx}$

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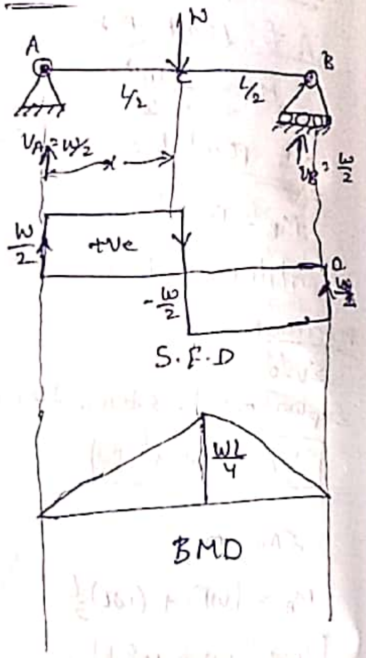
Simply supported beam with



$\boxed{\Sigma V = 0}$

Σ upward forces = Σ downward forces

$\boxed{V_A + V_B = W}$ — (1)

$\Sigma M_A = 0$

$(V_A \times 0) + (W \times \frac{L}{2}) - (V_B \times L) = 0$

$V_B = \frac{W}{2}$

$V_A = \frac{W}{2}$

Shear forces :-

At point A

L.H.S = 0

R.H.S = $+V_A = +\frac{W}{2}$

At point C :-

L.H.S = $+\frac{W}{2}$

R.H.S = $-\frac{W}{2}$

At point B :-

L.H.S = $-\frac{W}{2}$

R.H.S = 0

Bending moment :-

$M_A = M_B = 0$

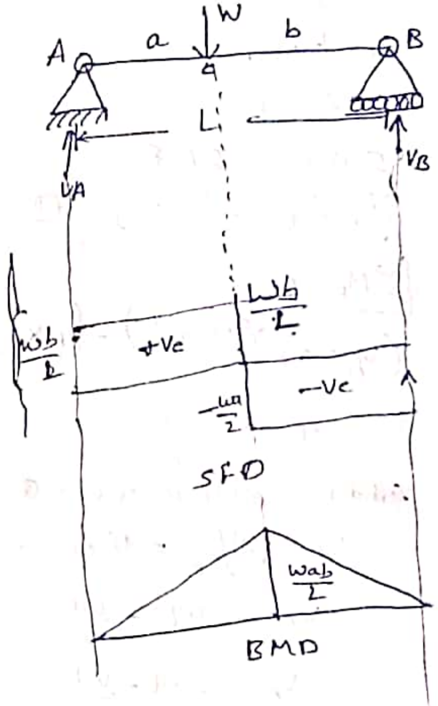
At point C :-

$M_C = (V_A)\frac{L}{2} = \frac{WL}{4}$

At section XX :- (A-C)

$M_x = V_A x = \frac{Wx}{2}$

simply supported beam carrying at point load (unsymmetric)



Reactions :-

$\Sigma V = 0$

Σ UF = Σ D.W.F

$\boxed{V_A + V_B = W}$ — (1)

$\Sigma M_A = 0$

$(V_A \times 0) + (W \times a) - (V_B \times L) = 0$

$V_B = \frac{Wa}{L}$, $V_A = \frac{Wb}{L}$

Substitute in equation (1)

$V_A + V_B = W$

$V_A + (\frac{Wa}{L}) = W$

Shear forces :-

At point A :-

L.H.S = 0

R.H.S = $+V_A = \frac{Wb}{L}$

At point C :-

L.H.S = $+\frac{Wb}{L}$

R.H.S = $-\frac{Wa}{L}$

At point B :-

L.H.S = $-\frac{Wa}{L}$

R.H.S = 0

Bending moment :-

$M_A = M_B = 0$

At point C :-

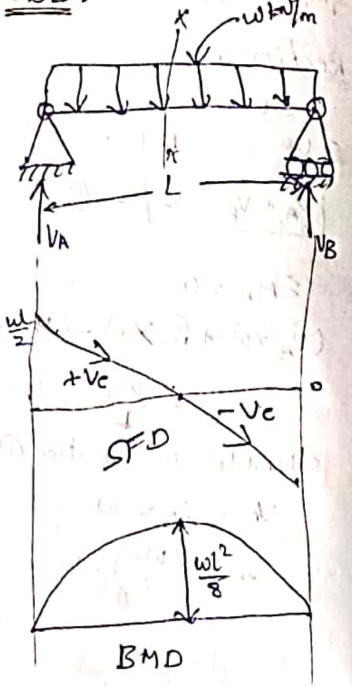
$M_C = +V_A a = \frac{Wab}{L}$

At section XX (A-C)

$M_x = V_A x = \frac{Wb}{L} x$

$M_x = (\frac{Wb}{L}) x$

Simply supported beam with UDL :-



Reactions :-

$\boxed{\Sigma V = 0}$

$\Sigma U.F = \Sigma D.F$

$\boxed{V_A + V_B = W \times L} \rightarrow ①$

$\boxed{\Sigma M_A = 0}$

$(V_A \times 0) \cdot (wL \times \frac{L}{2}) - (V_B \times L) = 0$

$\boxed{V_B = \frac{WL}{2}}$

~~Substi~~ Subsitute the equation ①

$V_A + \left(\frac{wL}{2}\right) = W \times L$

$V_A = WL - \frac{WL}{2}$

$V_A = \frac{2WL - WL}{2}$

$\boxed{V_A = \frac{WL}{2}}$

Shear forces :- At point A :-

$L.H.S = 0$

$R.H.S = +V_A = +\frac{wL}{2}$

At point B

$L.H.S = -V_B = -\frac{wL}{2}$

$R.H.S = 0$

at section xx :-

$V_x = +V_A - (wx)$

$\boxed{V_x = \frac{wL}{2} - wx}$

Bending moment :-

$M_A = M_B = 0$

At point C :-

$M_C = (V_A \frac{L}{2}) - (w\frac{L}{2}) \times \frac{L}{4}$

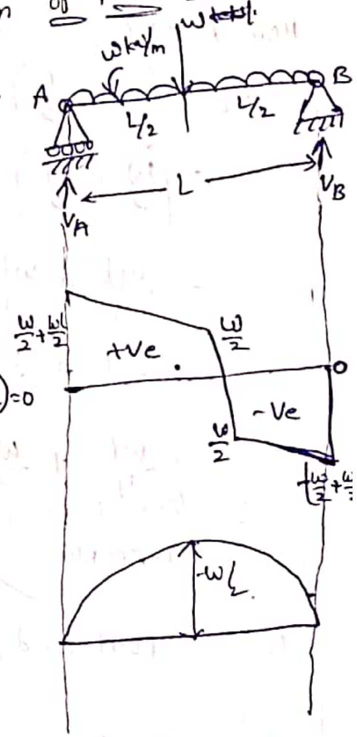
$\boxed{M_C = \frac{WL^2}{8}}$

at section xx :-

$M_x = +V_A x - (wx)\frac{x}{2}$

$M_x \boxed{\left(\frac{wL}{2}\right)x - \left(\frac{w}{2}\right)x^2}$

Simply supported with combination of point load and UDL :



Reactions :-

$\boxed{\Sigma u = 0}$

sum of upward forces = sum of down ward forces

$\boxed{V_A + V_B = W + (wL)} \rightarrow ①$

$\boxed{\Sigma M_A = 0}$

$(V_A \times 0) + (W \times \frac{L}{2}) + (wL) \times \frac{L}{2} - (V_B \times L) = 0$

$\boxed{V_B = \frac{W}{2} + \frac{wL}{2}}$

$\boxed{V_A = \frac{W}{2} + \frac{wL}{2}}$

Shear forces :-

At point A :-

$L.H.S = 0$

$R.H.S = +V_A$

At point C :-

$L.H.S = V_A - \frac{wL}{2}$

$= \frac{W}{2} + \frac{wL}{2} - \frac{wL}{2} = \frac{W}{2}$

$R.H.S = +V_A - \frac{wL}{2} - W = -\frac{W}{2}$

$= \frac{W}{2} + \frac{wL}{2} - \frac{wL}{2} - W$

$= \frac{W}{2} - W = \frac{W-2W}{2} = -\frac{W}{2}$

At point B :-

$L.H.S = +V_A - (wL) - W = -\frac{W}{2} - \frac{wL}{2}$

$R.H.S = 0$

Bending Moment :-

$M_A = M_B = 0$

Moment at C :-

$M_C = \left(V_A \times \frac{L}{2}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right)$

$= \left[\frac{W}{2} + \frac{wL}{2}\right]\frac{L}{2} - \frac{wL^2}{8}$

$= \frac{WL}{4} + \frac{wL^2}{4} - \frac{wL^2}{8}$

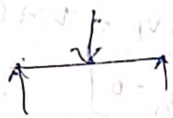
$= \frac{WL}{4} + \frac{2wL^2 - wL^2}{8}$

$= \frac{WL}{4} + \frac{wL^2}{8}$

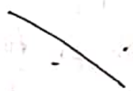
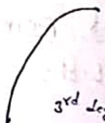
section XX :-

$M_x = (V_A x) - wx \cdot \frac{x}{2}$

$\cancel{M_x = \left[\frac{W}{2} + \frac{wL}{2}\right]}$



cantilever beam :-

S.NO	LOADING	S.F.D	B.M.D
1.	Point load		
2.	UDL		
3.	UVL	 2nd degree Parabola	 3rd degree Parabolic

S.NO	loading	S.F.D	B.M.D
1.	Point load	$V_x = V_A$	$M_x = V_A \cdot x$
2.	UDL	$V_x = wx$	$M_x = \frac{w \cdot x^2}{2}$

Simply supported beam :-

S.NO	loading	S.F.D	BMD		

Cantilever With U.V.L :-

Reactions :-

$\Sigma V = 0$

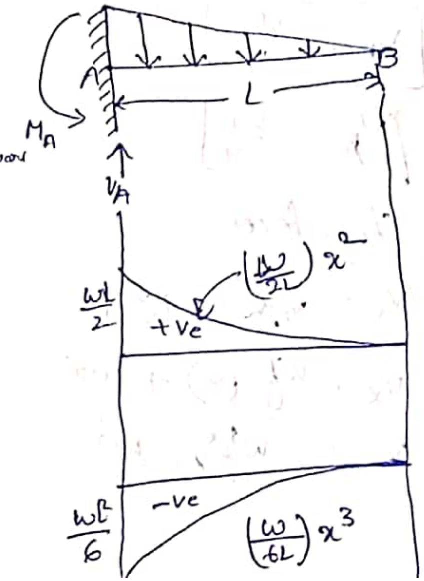
Sum of upward forces = Sum of downward forces

$\boxed{V_A = \frac{1}{2} wL}$

$\Sigma M_A = 0$

$M_A = \left(\frac{wL}{2}\right) \times \frac{L}{3}$

$\boxed{M_A = \frac{wL^2}{6}}$



Shear forces

$V_x = \frac{1}{2}\left(\frac{wx}{L}\right)x$

$V_x = \frac{wx^2}{2L}$

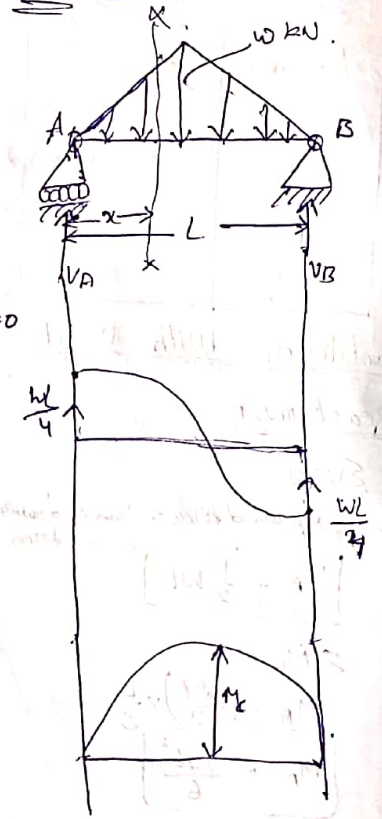
$V_x = \left(\frac{w}{2L}\right)x^2$

Bending moment

$M_x = \left(\frac{w}{2L}\right)x^2 \times \frac{x}{3}$

$$\boxed{M_x = \left(\frac{w}{6L}\right)x^3}$$

Simply supported beam with U.V.L :-



$\Sigma V = 0$

$\Sigma U.F = \Sigma D.F$

$$\boxed{V_A + V_B = \frac{1}{2}wL}$$

$M_A = 0$

$(V_A \times 0) + \left(\frac{1}{2}wL\right)\times\frac{L}{2} - (V_A \times L) = 0$

$$\boxed{V_B = \frac{wL}{4}}$$

$$\boxed{V_A = \frac{wL}{4}}$$

Shear forces :-

section x x :-

$V_x = V_A + wx\left(\frac{x}{2}\right)$

$V_x = \frac{wL}{4}(x) + \frac{wx^2}{2}$

$V_x = \frac{wL}{4}x + \frac{wx^2}{2}$

Bending moment.

Draw Shear forces and Bending moment diagrams for given cantilever beam :-

standed cases :-

cantilever beam :-

load	Max S.F	Max B.M
Point	W @ fixed end	WL @ fixed end.
UDL	WL	$\frac{WL^2}{2}$
UVL (◺)	$\frac{WL}{2}$	$\frac{WL^2}{6}$

S.S beam :-

Load	Max S.F	Max B.M
Point	$\frac{W}{2}$ @ support	$\frac{WL}{4}$ @ mid span
UDL	$\frac{WL}{2}$	$\frac{WL^2}{8}$
UVL (△)	$\frac{WL}{4}$	$\frac{WL^2}{12}$

Reactions:

$\Sigma V = 0$

Σ upward forces = Σ downward forces

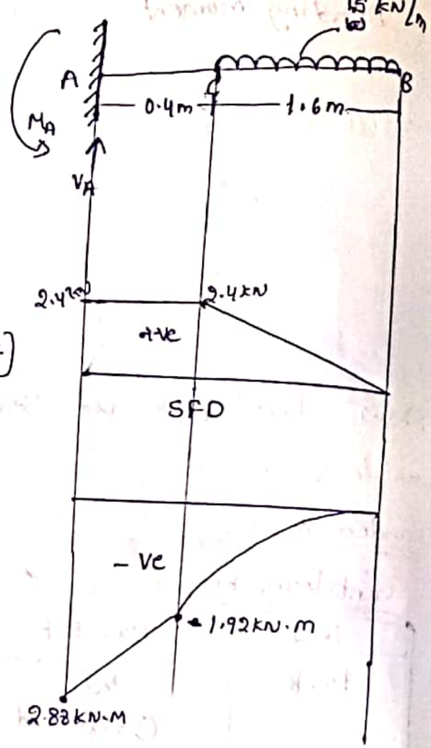
$V_A = (1.5 \times 1.6)$

$\boxed{V_A = 2.4\ kN}$

$\Sigma M_A = 0$

$M_A = (1.5 \times 1.6) \times \left[0.4 + \frac{1.6}{2}\right]$

$\boxed{M_A = 2.88\ kN.m}$



shear forces:

$V_A = +V_A = 2.4\ kN$

$V_C = +V_A = 2.4\ kN$

$V_B = 0.$

Bending moment:

$M_A = -M_A = -2.88\ kN.m$

(L.H.S) $M_C = -M_A + (V_A \times 0.4)$

$M_C = -2.88 + 2.4 \times 0.4$

$= -1.92\ kN.m$

R.H.S: $M_C = -(1.5 \times 1.6) \times \left(\frac{1.6}{2}\right)$

$= -1.92\ kN.m.$

Draw the shear forces and B.M.D for given cantilever beam:

Reactions: $\Sigma V = 0$

Σ upward forces = Σ downward forces

$V_A = (2 + (1 \times 1))$

$\boxed{V_A = 3\ kN}$

$\Sigma M_A = 0$

$M_A = (2 \times 1.5) + (1 \times 1) \times \frac{1}{2}$

$\boxed{M_A = 3.5\ kN.m}$

Shear forces:

$V_A = +V_A = 3\ kN$

$V_C = +V_A - (1 \times 1)$

$= 3 - 1$

$V_C = 2\ kN$

$V_B = +V_A - (1 \times 1)$

$= 3 - 1$

$V_B = 2\ kN$

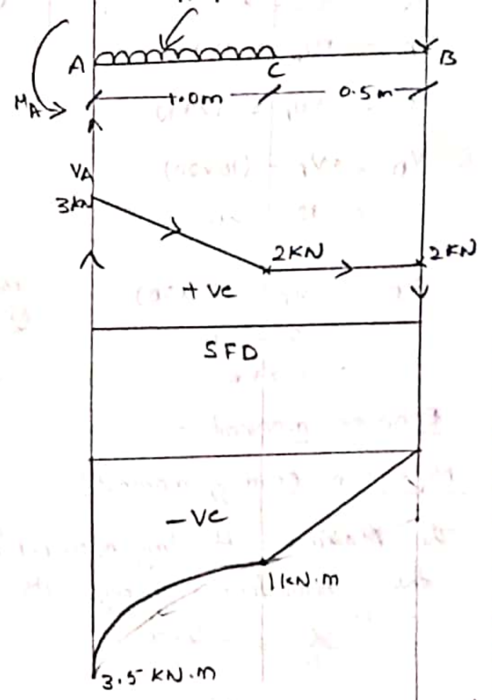
Bending moment:

$M_A = -M_A$

$= \cancel{3k} -3.5\ kN.m$

$M_C = -(2 \times 0.5) = -1\ kN.m$

$M_B = 0.$



BMD

Draw the shear forces and bending moment diagram for given ~~cantilever~~ ~~beam~~ simply supported beam.

Sol: (1) Reactions:

$\Sigma V = 0$

Σ up ward forces = Σ down ward forces

$\boxed{V_A + V_B = (10 \times 2)}$ — (1)

$\Sigma M_A = 0$

$(V_A \times 0) + (10 \times 2) \times \left(1 + \frac{2}{2}\right) - (V_B \times 5) = 0$

$(20 \times 2) - (V_B \times 5) = 0$

$40 = V_B \times 5$

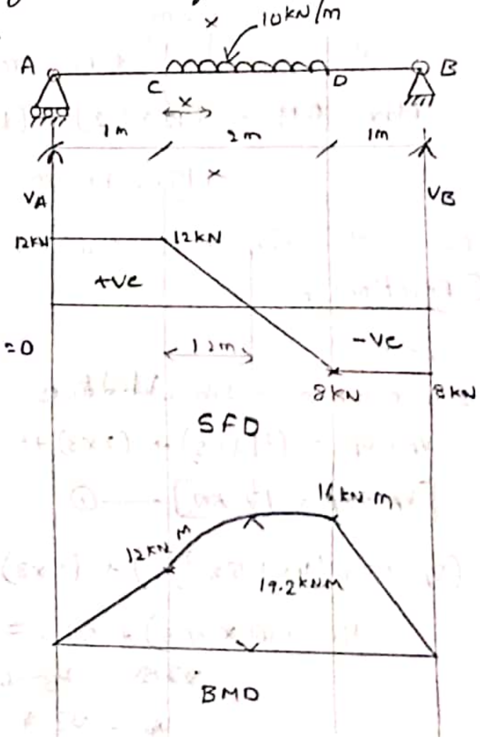
$\boxed{V_B = 8\ kN}$

V_B substitute in equation (1)

$V_A + V_B = 20$

$V_A + 8 = 20$

$\boxed{V_A = 12\ kN}$



② Shear forces :-

$V_A = +V_A = 12\ kN$

$V_C = +V_A = 12\ kN$

$V_D = +V_A - (10\times 20)$
$= 12 - 20$
$= -8\ kN$

$V_B = +V_A - (10\times 20)$
$= 12 - 20$
$= -8\ kN$

Bending moment :-

Maximum Bending moment :-

the Maximum Bending moment occurs when the shear forces change its signs

case (ii) $\frac{x}{12} = \frac{2-x}{8}$

$x = \frac{24}{20}$

$x = 1.2\ m$

∴ the maximum B.M occurred at 1.2m from 'C'.

Max B.M $= (12\times 2.2) - (10\times 1.2)\times\frac{1.2}{2}$
$= 19.2\ kN.m$

③ Bending moment

$M_A = M_B = 0$

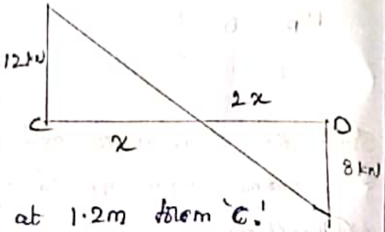
$M_C = +(V_A\times 1) = +12\ kN.m$

$M_D = +(V_B\times 2)$
$= 8\times 2$
$= 16\ kN.m$

~~Maximum~~

④ Section xx :- case (ii)

$V_x = +V_A - (10\times x) = 0$
$= 12 - 10x = 0$
$\frac{12}{10} = x$
$x = 1.2\ m$



Draw the shear forces and B.M.D for given simply supported beam.

① Reactions :-

$\Sigma V = 0$

Σ upward forces = Σ downward forces

$V_A + V_B = (4\times 1.5) + (2\times 3) + 5$

$\boxed{V_A + V_B = 17\ kN}$ — ①

$(V_A\times 0) + (4\times 1.5\times\frac{1.5}{2}) + (2\times 3)\times(3+\frac{3}{2}) + (5\times 4.5) - (V_B\times 6) = 0$

$4.5 + (6\times 4.5) + 22.5 = V_B\times 6.$

$54 = V_B\times 6$

$V_B = \frac{54}{6} = \boxed{V_B = 9\ kN}$

V_B is substitute in equation ①.

$V_A + V_B = 17$

$V_A + 9 = 17$

$V_A = 17 - 9$

$\boxed{V_A = 8\ kN}$

② Shear forces :-

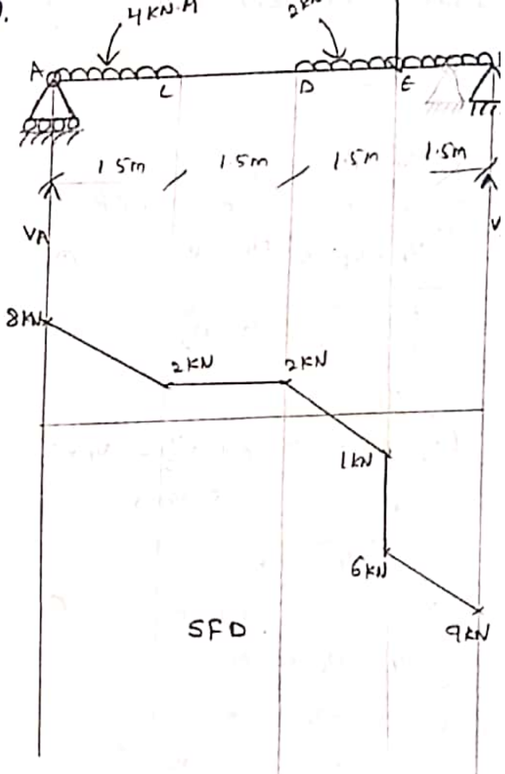
$V_A = +V_A = 8\ kN$

$V_C = +V_A - (4\times 1.5) = 2\ kN$

$V_D = +V_A - (4\times 1.5) = 2\ kN$

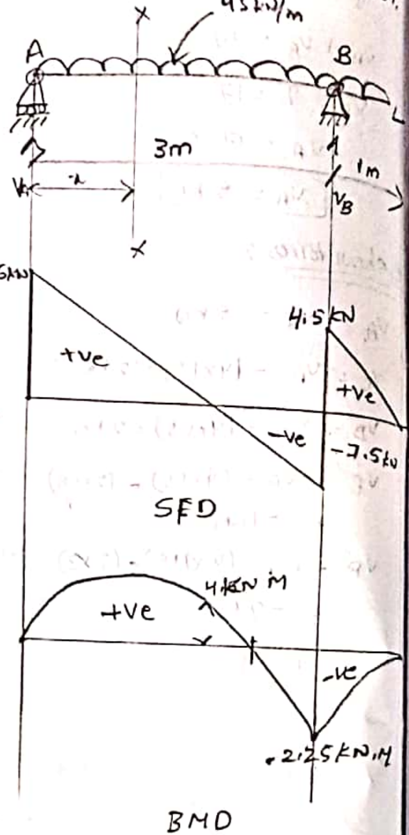
$V_E = +V_A - (4\times 1.5) - (2\times 1.5)$
$= -1\ kN$

$V_B = +V_A - (4\times 1.5) - (2\times 3) - 5$
$= -9\ kN.$



~~Draw~~ the shear forces and B.M.D for

Draw the S.F and B.M.D for given simply supported beam.



Sol ① Reactions :-

$\Sigma V = 0$

Σ upward forces = Σ down ward forces

$\boxed{V_A + V_B = (4.5 \times 4)}$ —— ①

$\boxed{V_A + V_B = 18\ kN}$

$\Sigma M_A = 0$

$(V_A \times 0) + (4.5 \times 4) \times \frac{4}{2} - (V_B \times 3) = 0$

$= V_B \times 3$

$\boxed{V_B = 12\ kN}$

V_B is substitute equation ①

$V_A + V_B = 18$

$V_A + 12 = 18$

$V_A = 18 - 12$

$\boxed{V_A = 6\ kN}$

② shear forces :-

$V_A = +V_A = 6kN$ } At point A.

$V_B = +V_A - (4.5 \times 3) = -7.5 kN$

$V_B = +V_A - (4.5 \times 3) + V_B = 4.5$ } At point B

$V_C = 0$

③ Bending moment :-

$M_A = M_C = 0$

$M_B = -(4.5 \times 1) \times \frac{1}{2}$

$M_B = -2.25\ kN.M$

④ for maximum bending moment :-

case (i) section xx :- (or)

$V_x = +V_A - (4.5 \times x) = 0$

$6 = 4.5 \times x$

$x = \frac{6}{4.5}$

$x = 1.33M$

case (ii)

$\frac{x}{6} = \frac{(3-x)}{7.5}$

$7.5x = 18 - 6x$

$x = \frac{18}{13.5}$

$x = 1.33m$

⑤ Max. B.M $= M_x = (V_A x) - (4.5 \times x \times \frac{x}{2})$

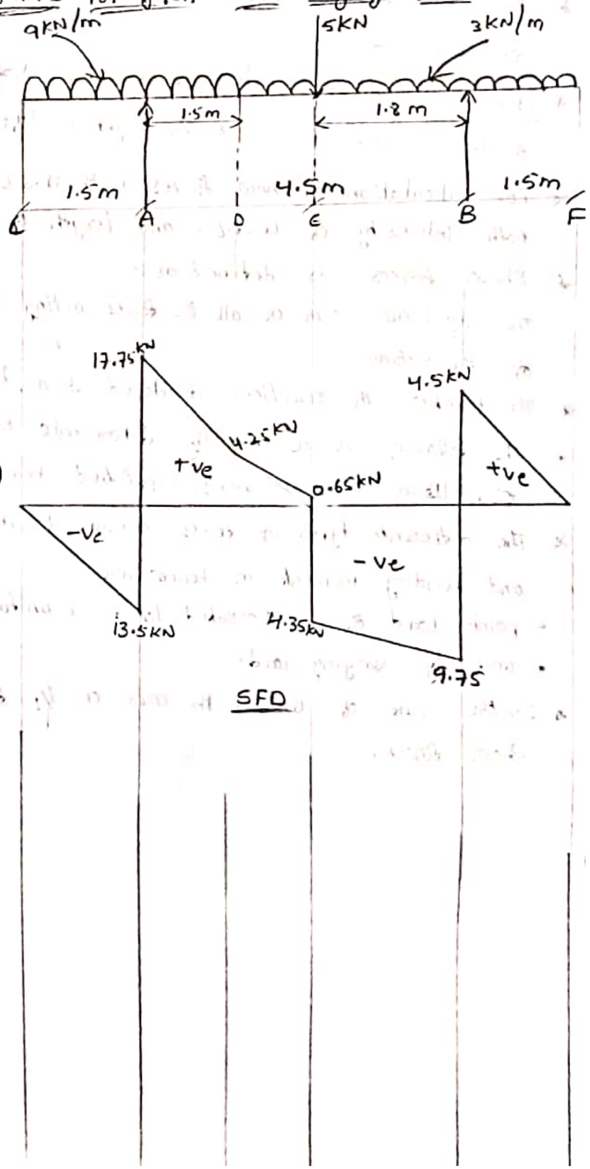
$= 6(1.33) - (4.5 \times 1.33 \times \frac{1.33}{2})$

$= 4 kN.$

Point of contraflexture (POC) :-

The Bending moment is change sign from -ve to +ve or vice versa. such a point where the bending moment changes sign is known as point of contraflexure.

Draw the S.F and B.M.D for given over hanging beam :-



① Reactions :-

$\Sigma V = 0$

$\Sigma UF = \Sigma DF$

$V_A + V_B = (9 \times 3) + (3 \times 4.5) + 5$

$\boxed{V_A + V_B = 45.5\ kN}$ —— ①

$\Sigma M_A = 0$

$= (V_A \times 0) + (9 \times 1.5 \times \frac{1.5}{2}) + (3 \times 4.5) \times [1.5 + \frac{4.5}{2}] + (5 \times 2.7) - (V_B \times 4.5) - (9 \times 1.5 \times \frac{1.5}{2})$

$= 10.125 + 13.5 \times [3.75] + 13.5 - (V_B \times 4.5) - [10.125]$

$= 64.125 = V_B \times 4.5$

$V_B = \frac{64.125}{4.5}$

$\boxed{V_B = 14.25\ kN}$

V_B is substitute in equation ①

$V_A + V_B = 45.5$

$V_A + 14.25 = 45.5$

$V_A = 45.5 - 14.25$

$\boxed{V_A = 31.25\ kN}$