4.11.22

EE25BTECH11045 - P.Navya Priya

Question:

Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are P $(4, 2, -6)$, Q $(5, -3, 1)$, R $(12, 4, 5)$ and S $(11, 9, -2)$.Use these equations to find the point of intersection of diagonals.

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.
Variables **T**aken:

P	$\begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$
Q	$\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$
R	$\begin{pmatrix} 12 \\ 4 \\ 5 \end{pmatrix}$
S	$\begin{pmatrix} 11 \\ 9 \\ -2 \end{pmatrix}$

The equation of the diagonal $\mathbf{R} - \mathbf{P}$ is

$$\mathbf{x} = \mathbf{P} + \lambda_1 (\mathbf{R} - \mathbf{P}) \tag{1}$$

The equation of the diagonal $\mathbf{S} - \mathbf{Q}$ is

$$\mathbf{x} = \mathbf{Q} + \lambda_2 (\mathbf{S} - \mathbf{Q}) \tag{2}$$

The centre of the parallelogram is the point of intersection of the diagonals.We solve the above two equations to find the centre using elemenetary transformations

From (1) & (2)

$$\mathbf{P} + \lambda_1 (\mathbf{R} - \mathbf{P}) = \mathbf{Q} + \lambda_2 (\mathbf{S} - \mathbf{Q}) \tag{3}$$

$$\lambda_1 (\mathbf{R} - \mathbf{P}) - \lambda_2 (\mathbf{S} - \mathbf{Q}) = \mathbf{Q} - \mathbf{P} \tag{4}$$

$$\begin{pmatrix} \mathbf{R} - \mathbf{P} & \mathbf{Q} - \mathbf{S} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \mathbf{Q} - \mathbf{P} \tag{5}$$

$$\begin{pmatrix} 8 & -6 \\ 2 & -12 \\ 11 & 3 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} \tag{6}$$

As there are only two variables, we consider the first two rows to compute λ_1,λ_2

$$\begin{pmatrix} 8 & -6 \\ 2 & -12 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \tag{7}$$

Forming Augmented matrix from (7),

$$\left(\begin{array}{cc|c} 8 & -6 & 1 \\ 2 & -12 & -5 \end{array}\right) \tag{8}$$

Using Gaussian Elimination,

$$\left(\begin{array}{cc|c} 8 & -6 & 1 \\ 2 & -12 & -5 \end{array}\right) \xleftrightarrow[R_2 \to \frac{8}{21} R_2]{R_2 \to R_2 - \frac{1}{4} R_1} \left(\begin{array}{cc|c} 4 & -3 & -\frac{1}{2} \\ 0 & -2 & -1 \end{array}\right) \tag{9}$$

From (9)

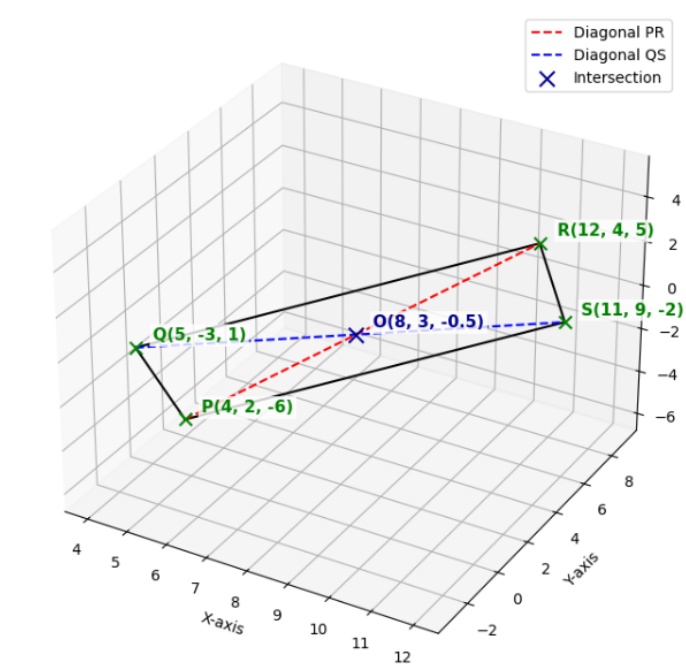
$$\lambda_2 = \frac{1}{2} \tag{10}$$

Substituting the value of λ_2 in (2)

$$\text{x} = \begin{pmatrix} 8 \\ 3 \\ -\frac{1}{2} \end{pmatrix} \tag{11}$$

$$\therefore \text{The point of intersection of the diagonals is} \begin{pmatrix} 8 \\ 3 \\ -\frac{1}{2} \end{pmatrix}$$

From the graph, theoretical solution matches with the computational solution.



Parallelogram PQRS with diagonals and intersection)