MATHEMATICS FOR ELECTRONIC ENGINEERS
Complex Analysis

Ravikumar. R

Science and Humanities

TOPICS

- Limit, Continuity, Differentiability

LIMIT OF A COMPLEX FUNCTION

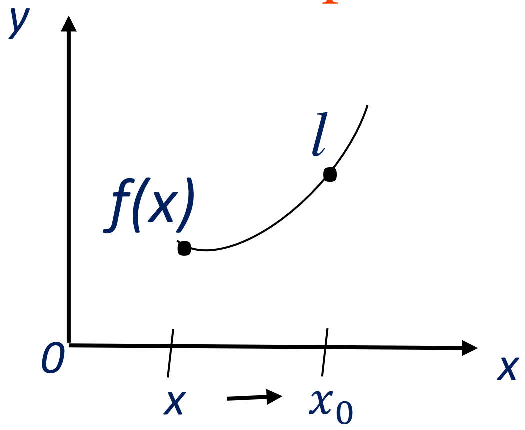
A function $w = f(z)$ is said to tend to a limit l as z approaches a point z_0 , if for every ε, we can find a +ve real number δ such that $|f(z) - l| < \varepsilon$ for $|z - z_0| < \delta$. i.e. for every $z \neq z_0$ in the δ-disc of z-plane, $f(z)$ has a value lying in the ε -disc of w-plane and is written as

$$\lim_{z \to z_0} f(z) = l$$

Note: In real calculus x approaches x_0 only along the real line, where as in complex calculus z-approaches z_0 in different paths in z-plane.

LIMITS: INTERPRETATION

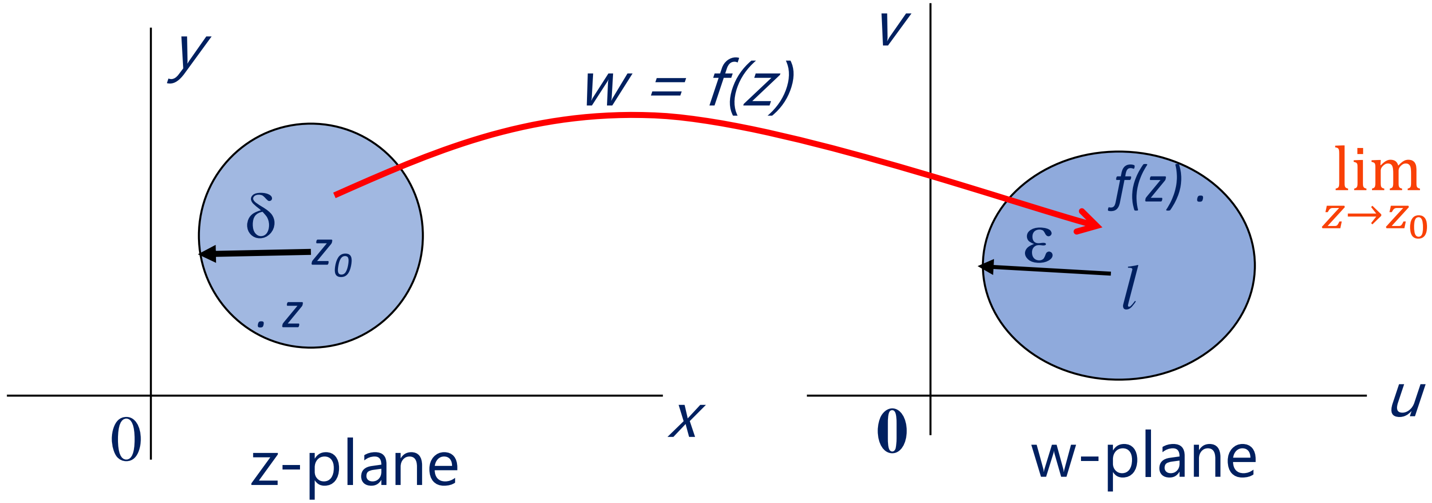
The Real plane



$$\lim_{x \to x_0} f(x) = l$$

If $|x - x_0| < \delta \ \exists \ \varepsilon$ such that $|f(x) - l| < \varepsilon$.

The complex plane



$$\lim_{z \to z_0} f(z) = l$$

CONTINUITY (Real valued function)

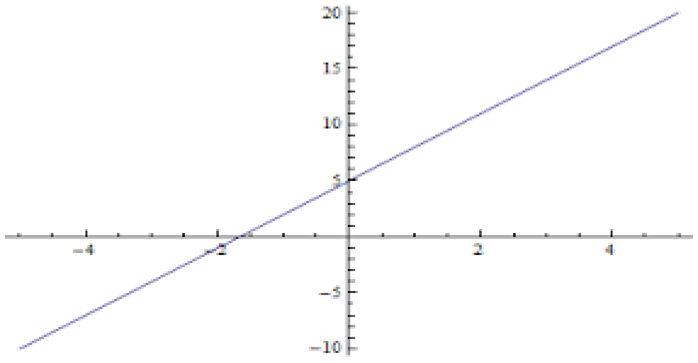
Let *f(x)* be a function such that *f: R→R*, *f(x)* is said to be continuous *at* x_0 iff:

- ✓ f is defined in a neighbourhood of x_0 i.e., $f(x_0)$ exists.
- ✓ The limit exists
- ✓ $\lim_{x \to x_0} f(x) = f(x_0)$

<u>Geometrical meaning of continuous function:</u>

If the function is continuous and if we draw the graph , there is no break in the curve.

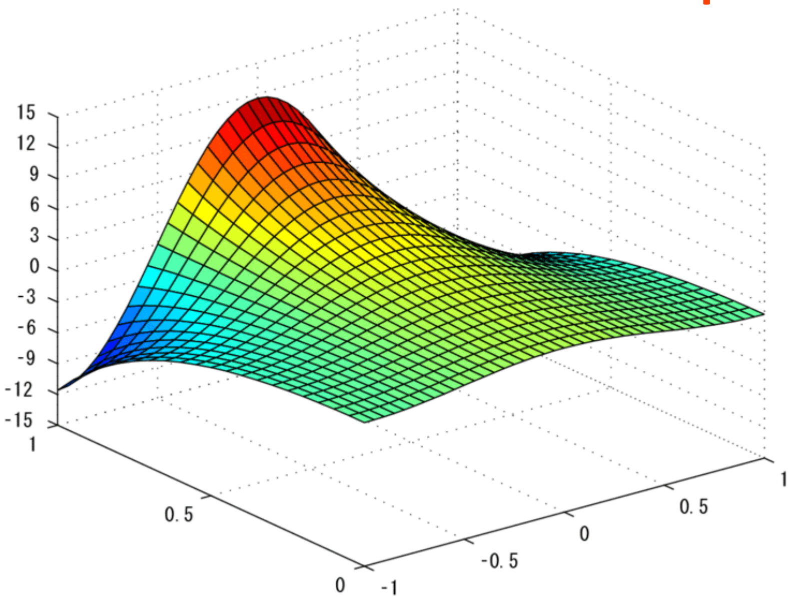
For example: $f(x)=3x$.

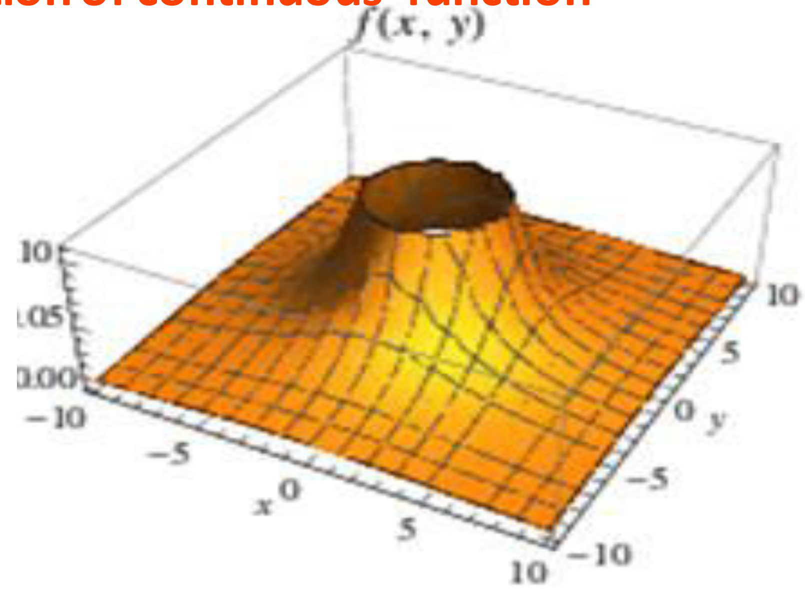


CONTINUITY (COMPLEX VALUED FUNCTION)

- Let $f(z)$ be a function such that $f: C \to C$. $f(z)$ *is said to be* continuous at z_0 iff:
  - ✓ f is defined in a neighbourhood of z_0 i.e., $f(z_0)$ exists.
  - ✓ The limit exists
  - ✓ $\lim_{z \to z_0} f(z) = f(z_0)$

Geometrical interpretation of continuous function





Continuity **Discontinuity**

DERIVATIVE OF *f(Z)*:

Let *f(x)* be a real-valued function defined in a neighbourhood of x_0. Then the derivative of *f(x)* at x_0 is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Let *f(z)* be a complex-valued function defined in a neighbourhood of z_0. Then the derivative of *f(z)* at z_0 is given by

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

OR

$$\frac{dw}{dz} = f'(Z) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \quad (\text{with } z = z_0 + \Delta z)$$

Provided this limit exists. The limit $f'(z)$ *is known as the derivative of f(z) at* z_0.

The above limit should be unique along every path from z to z_0.

Given $f(z) = \begin{cases} \frac{xy(y-ix)}{x^2+y^2} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ Discuss differentiability of $f(z)$ at $z=0$.

w.k.t $f'(z) = \lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$

$$f'(z) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$= \lim_{z \to 0} \frac{f(z)-0}{z-0}$$

$$= \lim_{z \to 0} \frac{\frac{xy(y-ix)}{x^2+y^2}}{x+iy}$$

I Path: $\lim_{x \to 0} \frac{\frac{xy(y-ix)}{x^2+y^2}}{x+iy} = 0,$

II path: $\lim_{y \to 0} \frac{\frac{xy(y-ix)}{x^2+y^2}}{x+iy} = 0,$

Continued…..

III path: $y = mx$

$$= \lim_{\substack{x\to 0\\ y\to 0}} \frac{x(mx)(mx-ix)}{(x^2+(mx)^2)(x+imx)}$$

$$= \lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3(m)(m-i)}{x^3(1+(m)^2)(1+im)}$$

$\Rightarrow f'(z)$ is not unique, as $f'(z)$ takes different values for different m.

Therefore $f(z)$ is not differentiable at origin.

THANK YOU

Ravikumar. R
Science and Humanities

Ravikumarr@pes.edu