4.11.6

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Question:

Find the equation of the plane passing through the intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

and

$$\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

and parallel to the X-axis. Hence, find the distance of the plane from the X-axis.

Solution:

Let the equations of the given planes be:

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \tag{1}$$

$$\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 4 \tag{2}$$

Any plane passing through their intersection can be written as:

$$\left(\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1\right) + \lambda \left(\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) - 4\right) = 0 \tag{3}$$

Solution:

Expanding:

$$\mathbf{r} \cdot \left((\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k}) \right) = 1 + 4\lambda \tag{4}$$

The normal vector of the plane is:

$$\mathbf{N} = (1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k} \tag{5}$$

Since the plane is parallel to the X-axis, its normal $\mathbf{N}$ must be perpendicular to the X-axis direction $\hat{i}$:

$$(\text{Coefficient of } \hat{i} \text{ in } \mathbf{N}) = 0 \quad \implies \quad 1 + 2\lambda = 0 \implies \lambda = -\frac{1}{2} \tag{6}$$

Solution:

Substitute $\lambda = -\frac{1}{2}$:

$$\mathbf{N} = 0 \cdot \hat{i} + \left(1 + 3\left(-\frac{1}{2}\right)\right)\hat{j} + \left(1 - \left(-\frac{1}{2}\right)\right)\hat{k} = -\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \tag{7}$$

Equation of the plane (using the scalar form $\mathbf{r} \cdot \mathbf{N} = D$):

$$\mathbf{r} \cdot \left(-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right) = 1 + 4\left(-\frac{1}{2}\right) = -1 \tag{8}$$

$$-\frac{1}{2}y + \frac{3}{2}z = -1 \tag{9}$$

$$-\frac{1}{2}y + \frac{3}{2}z + 1 = 0 \quad \Rightarrow \quad -y + 3z + 2 = 0 \tag{10}$$

Solution:

The X-axis is the line $y = 0, z = 0$.
Distance from the plane to the X-axis (taking point $(0, 0, 0)$) is:

$$D = \frac{|-0 + 3 \cdot 0 + 2|}{\sqrt{(-1)^2 + 3^2}} = \frac{2}{\sqrt{10}} \tag{11}$$

- Required plane: $-y + 3z + 2 = 0$
- Distance from X-axis: $\dfrac{2}{\sqrt{10}}$

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Plane: y - 3z + 6 = 0 => y = 3z - 6
# Create grid for plane
z = np.linspace(-5, 5, 20)
x = np.linspace(-5, 5, 20)
X, Z = np.meshgrid(x, z)
Y = 3*Z - 6

# Create figure
fig = plt.figure(figsize=(10,7))
ax = fig.add_subplot(111, projection='3d')

ax.plot_surface(X, Y, Z, alpha=0.5, color='cyan', rstride=1,
    cstride=1, edgecolor='k')
```

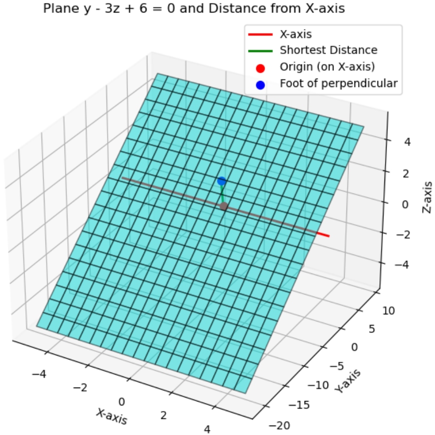
Python Code

```
# Plot X-axis (y=0, z=0)
ax.plot([-5,5],[0,0],[0,0], color='red', linewidth=2, label='X-
    axis')

# Point on X-axis (origin)
origin = np.array([0,0,0])

# Distance point on plane from origin (perpendicular foot)
# Plane: 0*x + 1*y - 3*z + 6 = 0 => normal = (0,1,-3)
normal = np.array([0,1,-3])
d = 6
# Formula for projection of origin onto plane
t = -(np.dot(normal, origin) + d) / np.dot(normal, normal)
foot = origin + t*normal

# Plot perpendicular distance line
ax.plot([origin[0], foot[0]], [origin[1], foot[1]], [origin[2],
    foot[2]],
```

Python Code

```
    color='green', linewidth=2, label='Shortest Distance')
# Plot origin and foot point
ax.scatter(*origin, color='red', s=50, label='Origin (on X-axis)'
    )
ax.scatter(*foot, color='blue', s=50, label='Foot of
    perpendicular')

# Labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Plane y - 3z + 6 = 0 and Distance from X-axis')
ax.legend()
plt.savefig ("fig8.png")
plt.show()
```

Plot-Using Python



C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Coefficients of the final plane: y - 3z + 6 = 0
    double A = 0, B = 1, C = -3, D = 6;

    double x0 = 0, y0 = 0, z0 = 0;

    double numerator = fabs(A*x0 + B*y0 + C*z0 + D);
    double denominator = sqrt(A*A + B*B + C*C);
    double distance = numerator / denominator;

    printf("Equation of the plane: y - 3z + 6 = 0\n");
    printf("Distance of plane from X-axis: %.4f\n", distance);

    return 0;
}
```

Python and C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D


lib = ctypes.CDLL("./libplane.so")

# Variables to store plane coefficients
a = ctypes.c_double()
b = ctypes.c_double()
c = ctypes.c_double()
d = ctypes.c_double()
```

Python and C Code

```
# Call the C function to get coefficients
lib.get_plane(ctypes.byref(a), ctypes.byref(b), ctypes.byref(c),
    ctypes.byref(d))

# Extract values
a, b, c, d = a.value, b.value, c.value, d.value
print(f"Plane equation: {a}x + {b}y + {c}z + {d} = 0")


x = np.linspace(-5, 5, 30) # X range
z = np.linspace(-5, 5, 30) # Z range
X, Z = np.meshgrid(x, z)

# Solve plane eqn for Y: y = (-ax - cz - d)/b
Y = (-a*X - c*Z - d) / b
```

Python and C Code

```
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection="3d")

ax.plot_surface(X, Y, Z, alpha=0.5, color="cyan", label="Plane")

ax.plot([-5, 5], [0, 0], [0, 0], color="red", linewidth=3, label=
    "X-axis")

# Take a point on X-axis (0,0,0)
P = np.array([0,0,0])

# Formula: foot of perpendicular from point to plane
num = a*P[0] + b*P[1] + c*P[2] + d
den = a*a + b*b + c*c

Px = P[0] - a*num/den
Py = P[1] - b*num/den
Pz = P[2] - c*num/den
```

Python and C Code

```
# Plot distance line
ax.plot([P[0], Px], [P[1], Py], [P[2], Pz],
        color="black", linewidth=3, label="Shortest distance")

# Mark points
ax.scatter(P[0], P[1], P[2], color="red", s=50, label="Origin (on
     X-axis)")
ax.scatter(Px, Py, Pz, color="blue", s=50, label="Foot on Plane")

ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("Plane y - 3z + 6 = 0 and Distance from X-axis")
ax.legend()

plt.show()
```