# 4.3.16

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**Question:**
Find the equation of the plane through the points

$$(2,1,0), \quad (3,-2,-2), \quad (3,1,7).$$

**Solution:** The given points are

$$P_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}.$$

*Step 1: General plane equation*

$$ax + by + cz + d = 0 \tag{1}$$

*Step 2: Substitution of points*

Substituting each point:

$$2a + b + d = 0 \tag{2}$$

$$3a + 2b - 2c + d = 0 \tag{3}$$

$$3a + b + 7c + d = 0 \tag{4}$$

*Step 3: Matrix form*

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 3 & 2 & -2 & 1 \\ 3 & 1 & 7 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

*Step 4: Solving the system*

From (2):

$$d = -2a - b \tag{6}$$

Substitute in (3):

$$3a + 2b - 2c - 2a - b = 0 \quad \Rightarrow \quad a + b - 2c = 0 \tag{7}$$

Substitute in (4):

$$3a + b + 7c - 2a - b = 0 \quad \Rightarrow \quad a + 7c = 0 \tag{8}$$

From (8):

$$a = -7c \tag{9}$$

From (7):

$$-7c + b - 2c = 0 \quad \Rightarrow \quad b = 9c \tag{10}$$

From (6):

$$d = -2(-7c) - 9c = 5c \tag{11}$$

*Step 5: Final result*

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = c \begin{pmatrix} -7 \\ 9 \\ 1 \\ 5 \end{pmatrix}$$

Choosing $c = 1$, the plane equation is

$$-7x + 9y + z + 5 = 0 \tag{12}$$

Or equivalently,

$$\boxed{7x - 9y - z - 5 = 0} \tag{13}$$

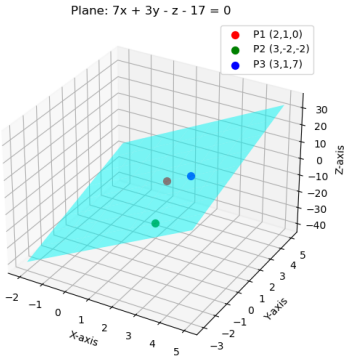


Fig. 1