5.4.23

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Question:

Using elementary transformations, find the inverse of the following matrix:

$$A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

Soultion: We want to find the inverse of the matrix

$$A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}.$$

Step 1: Assume the inverse matrix

Let

$$A^{-1} = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \tag{1}$$

By definition of inverse, we have

$$AA^{-1} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

Step 2: Multiply the matrices

$$\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 2x - 3z & 2y - 3w \\ -x + 2z & -y + 2w \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 2x - 3z & 2y - 3w \\ -x + 2z & -y + 2w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

From this multiplication, we get the following system of equations:

$$2x - 3z = 1 \tag{5}$$

$$2y - 3w = 0 \tag{6}$$

$$-x + 2z = 0 \tag{7}$$

$$-y + 2w = 1 \tag{8}$$

Step 3: Solve the equations

From equation (7):

$$-x + 2z = 0 \Rightarrow x = 2z \tag{9}$$

Substitute into equation (5):

$$2(2z) - 3z = 1 \Rightarrow 4z - 3z = 1 \Rightarrow z = 1 \tag{10}$$

Then,

$$x = 2z = 2 \tag{11}$$

From equation (8):

$$-y + 2w = 1 \Rightarrow y = 2w - 1 \tag{12}$$

Substitute into equation (6):

$$2(2w - 1) - 3w = 0 \Rightarrow 4w - 2 - 3w = 0 \Rightarrow w = 2 \tag{13}$$

Then,

$$y = 2w - 1 = 3 \tag{14}$$

Step 4: Write the inverse matrix

$$A^{-1} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \tag{15}$$

Graph presentation:

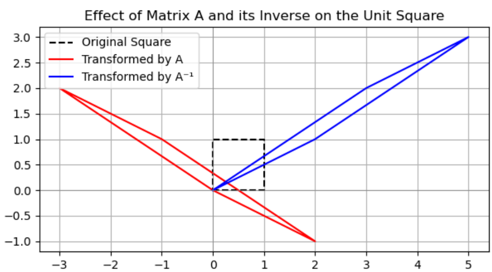

Fig. 1