4.13.33

EE25btech11028 - J.Navya sri

Question:

Find the locus of a variable point $\left(P\right) = \left((x, y)\right)$ whose distance from the point $\left(A\right) = \left((-2, 0)\right)$ is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$.

Solution:

Let

$$\left(x\right) = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad \left(a\right) = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \qquad \left(n\right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad c = \frac{9}{2}.$$

Distance condition (given):

$$\left\|\left(x\right) - \left(a\right)\right\| = \frac{2}{3}\left|\left(n\right)^T \left(x\right) + c\right|. \tag{1}$$

Square both sides:

$$(\left(x\right) - \left(a\right))^T(\left(x\right) - \left(a\right)) = \frac{4}{9}(\left(n\right)^T \left(x\right) + c)^2. \tag{2}$$

Evaluate each side in coordinates:

$$(x + 2)^2 + y^2 = \frac{4}{9}\left(x + \frac{9}{2}\right)^2 \tag{3}$$

$$x^2 + 4x + 4 + y^2 = \frac{4}{9}x^2 + 4x + 9 \tag{4}$$

(cancel $4x$ on both sides) $$x^2 + 4 + y^2 = \frac{4}{9}x^2 + 9 \tag{5}$$

Multiply both sides by 9:

$$9x^2 + 36 + 9y^2 = 4x^2 + 81$$

$$\Rightarrow 5x^2 + 9y^2 = 45.$$

Divide by 45 to get standard form:

$$\boxed{\frac{x^2}{9} + \frac{y^2}{5} = 1} \tag{6}$$

Thus the locus is an ellipse centered at the origin with semi-axes 3 (along x) and $\sqrt{5}$ (along y).

Graph presentation:

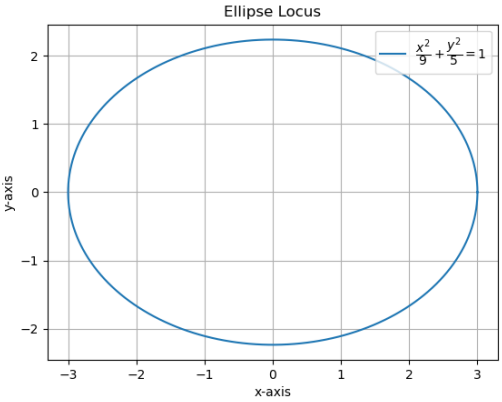

Fig. 1