EE25btech11028 - J.Navya sri

Question:

Find the equation of the plane passing through the intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

and

$$\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

and parallel to the X-axis. Hence, find the distance of the plane from the X-axis.

Solution:

Step1:Plane through Intersection

Let the equations of the given planes be:

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \tag{1}$$

$$\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 4 \tag{2}$$

Any plane passing through their intersection can be written as:

$$\left(\mathbf{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1\right)+\lambda\left(\mathbf{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})-4\right)=0\tag{3}$$

Expanding:

$$\mathbf{r} \cdot \left((\hat{i} + \hat{j} + \hat{k}) + \lambda (2\hat{i} + 3\hat{j} - \hat{k}) \right) = 1 + 4\lambda \tag{4}$$

The normal vector of the plane is:

$$\mathbf{N} = (1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}$$
(5)

Step2:Parallel to X-Axis

Since the plane is parallel to the X-axis, its normal N must be perpendicular to the X-axis direction \hat{i} :

(Coefficient of
$$\hat{i}$$
 in \mathbb{N}) = 0 \implies 1 + 2 λ = 0 \implies λ = $-\frac{1}{2}$ (6)

Substitute $\lambda = -\frac{1}{2}$:

$$\mathbf{N} = 0 \cdot \hat{i} + \left(1 + 3\left(-\frac{1}{2}\right)\right)\hat{j} + \left(1 - \left(-\frac{1}{2}\right)\right)\hat{k} = -\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}$$
 (7)

Equation of the plane (using the scalar form $\mathbf{r} \cdot \mathbf{N} = D$):

$$\mathbf{r} \cdot \left(-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right) = 1 + 4\left(-\frac{1}{2} \right) = -1 \tag{8}$$

$$-\frac{1}{2}y + \frac{3}{2}z = -1\tag{9}$$

$$-\frac{1}{2}y + \frac{3}{2}z + 1 = 0 \quad \Rightarrow \quad -y + 3z + 2 = 0 \tag{10}$$

Step3:Distance from X-Axis

The X-axis is the line y = 0, z = 0.

Distance from the plane to the X-axis (taking point (0,0,0)) is:

$$D = \frac{|-0+3\cdot 0+2|}{\sqrt{(-1)^2+3^2}} = \frac{2}{\sqrt{10}}$$
 (11)

Final Answers:

• Required plane: -y + 3z + 2 = 0• Distance from X-axis: $\frac{2}{\sqrt{10}}$

Graph presentation:

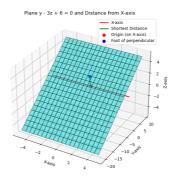


Fig. 1