4.13.33

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Question:

Find the locus of a variable point $\left(P\right) = \left((x, y)\right)$ whose distance from the point $\left(A\right) = \left((-2, 0)\right)$ is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$.

Solution:

Let

$$\begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad \begin{pmatrix} a \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} n \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad c = \frac{9}{2}.$$

Distance condition (given):

$$\left\| \begin{pmatrix} x \end{pmatrix} - \begin{pmatrix} a \end{pmatrix} \right\| = \frac{2}{3} \left| \begin{pmatrix} n \end{pmatrix}^T \begin{pmatrix} x \end{pmatrix} + c \right|. \tag{1}$$

Square both sides:

$$(\begin{pmatrix} x \end{pmatrix} - \begin{pmatrix} a \end{pmatrix})^T(\begin{pmatrix} x \end{pmatrix} - \begin{pmatrix} a \end{pmatrix}) = \frac{4}{9}(\begin{pmatrix} n \end{pmatrix}^T \begin{pmatrix} x \end{pmatrix} + c)^2. \tag{2}$$

Solution:

Evaluate each side in coordinates:

$$(x+2)^2 + y^2 = \frac{4}{9}\left(x + \frac{9}{2}\right)^2 \tag{3}$$

$$x^2 + 4x + 4 + y^2 = \frac{4}{9}x^2 + 4x + 9 \tag{4}$$

$$\text{(cancel } 4x \text{ on both sides)} \qquad x^2 + 4 + y^2 = \frac{4}{9}x^2 + 9 \tag{5}$$

Solution:

Multiply both sides by 9:

$$9x^2 + 36 + 9y^2 = 4x^2 + 81$$

$$\Rightarrow 5x^2 + 9y^2 = 45.$$

Divide by 45 to get standard form:

$$\boxed{\frac{x^2}{9} + \frac{y^2}{5} = 1} \tag{6}$$

Thus the locus is an ellipse centered at the origin with semi-axes 3 (along x) and $\sqrt{5}$ (along y).

Python Code

```python
import numpy as np
import matplotlib.pyplot as plt

# Define theta
theta = np.linspace(0, 2*np.pi, 400)

# Ellipse parameters
a = 3 # semi-major axis
b = np.sqrt(5) # semi-minor axis

x = a * np.cos(theta)
y = b * np.sin(theta)
```

Python Code

```
plt.plot(x, y, label=r'$\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$')

plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Ellipse Locus")
plt.grid(True)

# Move legend to top-right
plt.legend(loc="upper right")

plt.axis("equal")
plt.savefig("fig9.png")
plt.show()
```

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    FILE *fp = fopen("ellipse.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
```

C Code

```c
    double a = 3.0; // semi-major axis (x-direction)
    double b = sqrt(5.0); // semi-minor axis (y-direction)
    double theta;

    for (theta = 0; theta <= 2*M_PI; theta += 0.01) {
        double x = a * cos(theta);
        double y = b * sin(theta);
        fprintf(fp, "%lf %lf\n", x, y);
    }

    fclose(fp);
    printf("Data written to ellipse.dat\n");
    printf("Use: gnuplot -e \"plot 'ellipse.dat' with lines title
        'x^2/9 + y^2/5 = 1'\"\n");
    return 0;
}
```

Python and C code

```python

import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load compiled C library
lib = ctypes.CDLL("./libellipse.so")

# Define argument and return types
lib.ellipse_point.argtypes = [ctypes.c_double, ctypes.c_double,
                              ctypes.c_double,
                              ctypes.POINTER(ctypes.c_double),
                              ctypes.POINTER(ctypes.c_double)]

# Semi-axes
a = 3.0
b = np.sqrt(5.0)
```

Python and C Code

```
# Generate ellipse points
theta_vals = np.linspace(0, 2*np.pi, 400)
x_vals, y_vals = [], []

for theta in theta_vals:
    x = ctypes.c_double()
    y = ctypes.c_double()
    lib.ellipse_point(a, b, theta, ctypes.byref(x), ctypes.byref(
        y))
    x_vals.append(x.value)
    y_vals.append(y.value)
```

Python and C code

```
# Plot ellipse
plt.plot(x_vals, y_vals, label=r'$\dfrac{x^2}{9} + \dfrac{y^2}{5}
     = 1$')
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Ellipse Locus")
plt.grid(True)
plt.legend(loc="upper right")
plt.axis("equal")
plt.show()
```

Plot-Using Python