4.3.16

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Question:
Find the equation of the plane through the points

$$(2, 1, 0), \quad (3, -2, -2), \quad (3, 1, 7).$$

Solution: Let $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ be the given points and $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be a general point on the plane. The points are:

$$\mathbf{p}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{p}_2 = \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}, \quad \mathbf{p}_3 = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}.$$

Step 1: General Plane Equation

The equation of the plane is $\mathbf{n} \cdot \mathbf{x} + d = 0$, where $\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the normal vector.

$$ax + by + cz + d = 0 \tag{1}$$

Step 2: Substitution of Points

The points $\mathbf{p}_i$ must satisfy the plane equation:

$$2a + b + d = 0 \tag{2}$$

$$3a - 2b - 2c + d = 0 \tag{3}$$

$$3a + b + 7c + d = 0 \tag{4}$$

Step 3: Matrix Form

The system of linear equations can be written as $M\mathbf{v} = \mathbf{0}$, where $\mathbf{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$:

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 3 & -2 & -2 & 1 \\ 3 & 1 & 7 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Step 4: Solving the System

From (2):

$$d = -2a - b \tag{6}$$

Substitute (6) in (3):

$$3a - 2b - 2c + (-2a - b) = 0 \quad \Rightarrow \quad a - 3b - 2c = 0 \tag{7}$$

Substitute (6) in (4):

$$3a + b + 7c + (-2a - b) = 0 \quad \Rightarrow \quad a + 7c = 0 \tag{8}$$

From (8):

$$a = -7c \tag{9}$$

From (7), substitute (9):

$$(-7c) - 3b - 2c = 0 \quad \Rightarrow \quad -9c = 3b \quad \Rightarrow \quad b = -3c \tag{10}$$

From (6), substitute (9) and (10):

$$d = -2(-7c) - (-3c) = 14c + 3c \quad \Rightarrow \quad d = 17c \tag{11}$$

Step 5: Final Result The coefficient vector $\mathbf{v}$ is proportional to c:

$$\mathbf{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = c \begin{pmatrix} -7 \\ -3 \\ 1 \\ 17 \end{pmatrix}$$

Choosing $c = 1$, the normal vector is $\mathbf{n} = \begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix}$ and $d = 17$. The plane equation is:

$$-7x - 3y + z + 17 = 0 \tag{12}$$

Or equivalently, multiplying by -1:

$$\boxed{7x + 3y - z - 17 = 0} \tag{13}$$

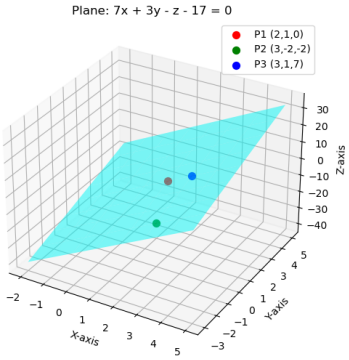

Fig. 1