4.3.16

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Question:

Find the equation of the plane through the points

$$(2, 1, 0), \quad (3, -2, -2), \quad (3, 1, 7).$$

Soultion:

Let $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ be the given points and $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be a general point on the plane. The points are:

$$\mathbf{p}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{p}_2 = \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}, \quad \mathbf{p}_3 = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}.$$

Soultion:

The equation of the plane is $\mathbf{n} \cdot \mathbf{x} + d = 0$, where $\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the normal vector.

$$ax + by + cz + d = 0 \tag{1}$$

The points $\mathbf{p}_i$ must satisfy the plane equation:

Soultion:

$$2a + b + d = 0 \tag{2}$$

$$3a - 2b - 2c + d = 0 \tag{3}$$

$$3a + b + 7c + d = 0 \tag{4}$$

The system of linear equations can be written as $M\mathbf{v} = \mathbf{0}$, where $\mathbf{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$:

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 3 & -2 & -2 & 1 \\ 3 & 1 & 7 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Soultion:

From (2):

$$d = -2a - b \tag{6}$$

Substitute (6) in (3):

$$3a - 2b - 2c + (-2a - b) = 0 \quad \Rightarrow \quad a - 3b - 2c = 0 \tag{7}$$

Substitute (6) in (4):

$$3a + b + 7c + (-2a - b) = 0 \quad \Rightarrow \quad a + 7c = 0 \tag{8}$$

Soultion:

From (8):

$$a = -7c \tag{9}$$

From (7), substitute (9):

$$(-7c) - 3b - 2c = 0 \;\Rightarrow\; -9c = 3b \;\Rightarrow\; b = -3c \tag{10}$$

From (6), substitute (9) and (10):

$$d = -2(-7c) - (-3c) = 14c + 3c \;\Rightarrow\; d = 17c \tag{11}$$

Soultion:

The coefficient vector $\mathbf{v}$ is proportional to c:

$$\mathbf{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = c \begin{pmatrix} -7 \\ -3 \\ 1 \\ 17 \end{pmatrix}$$

Choosing $c = 1$, the normal vector is $\mathbf{n} = \begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix}$ and $d = 17$. The plane equation is:

$$-7x - 3y + z + 17 = 0 \tag{12}$$

Or equivalently, multiplying by -1:

$$\boxed{7x + 3y - z - 17 = 0} \tag{13}$$

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Given points
P1 = np.array([2, 1, 0])
P2 = np.array([3, -2, -2])
P3 = np.array([3, 1, 7])
```

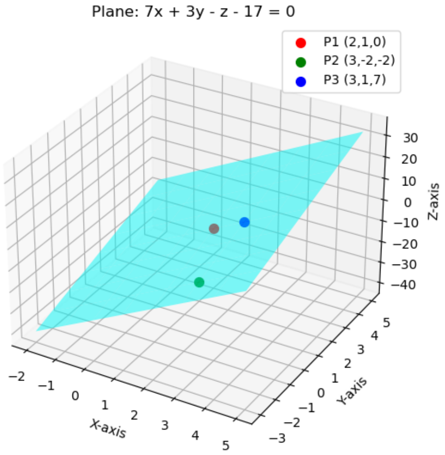
Python Code

```
# Create a meshgrid for x, y
xx, yy = np.meshgrid(range(-2, 6), range(-3, 6))

# Equation of plane: 7x + 3y - z - 17 = 0 => z = 7x + 3y - 17
zz = 7*xx + 3*yy - 17

# Plot
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')

# Plot the surface (plane)
ax.plot_surface(xx, yy, zz, alpha=0.5, color='cyan')

# Plot points
ax.scatter(*P1, color='r', s=50, label='P1 (2,1,0)')
ax.scatter(*P2, color='g', s=50, label='P2 (3,-2,-2)')
ax.scatter(*P3, color='b', s=50, label='P3 (3,1,7)')
```

Python Code

```
# Labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title("Plane: 7x + 3y - z - 17 = 0")

ax.legend()
plt.savefig("fig6.png")
plt.show()
```

Plot-Using by Python



C Code

```c
#include <stdio.h>

// Function to compute gcd
int gcd(int a, int b) {
    if (b == 0) return a > 0 ? a : -a;
    return gcd(b, a % b);
}

// Function to compute gcd of 4 numbers
int gcd4(int a, int b, int c, int d) {
    int g = gcd(a, b);
    g = gcd(g, c);
    g = gcd(g, d);
    return g;
}
```

C Code

```c
int main() {
    // Given three points
    int x1 = 2, y1 = 1, z1 = 0;
    int x2 = 3, y2 = -2, z2 = -2;
    int x3 = 3, y3 = 1, z3 = 7;
```

C Code

```c
// Direction vectors
    int v1x = x2 - x1, v1y = y2 - y1, v1z = z2 - z1;
    int v2x = x3 - x1, v2y = y3 - y1, v2z = z3 - z1;

    // Cross product  normal vector (a, b, c)
    int a = v1y * v2z - v1z * v2y;
    int b = v1z * v2x - v1x * v2z;
    int c = v1x * v2y - v1y * v2x;

    // Constant term d
    int d = -(a * x1 + b * y1 + c * z1);
```

C Code

```
// Simplify using gcd
    int g = gcd4(a, b, c, d);
    a /= g; b /= g; c /= g; d /= g;

    // Make leading coefficient positive
    if (a < 0) {
        a = -a; b = -b; c = -c; d = -d;
    }

    // Final output
    printf("The equation of plane is: %dx + %dy + %dz + %d = 0\n"
        , a, b, c, d);

    return 0;
}
```

Python and C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load the compiled C library
lib = ctypes.CDLL("./plane.so")

# Define the cross_product function signature
lib.cross_product.argtypes = [ctypes.POINTER(ctypes.c_double),
                              ctypes.POINTER(ctypes.c_double),
                              ctypes.POINTER(ctypes.c_double)]

# Points
P1 = np.array([2,1,0], dtype=np.double)
P2 = np.array([3,-2,-2], dtype=np.double)
P3 = np.array([3,1,7], dtype=np.double)
```

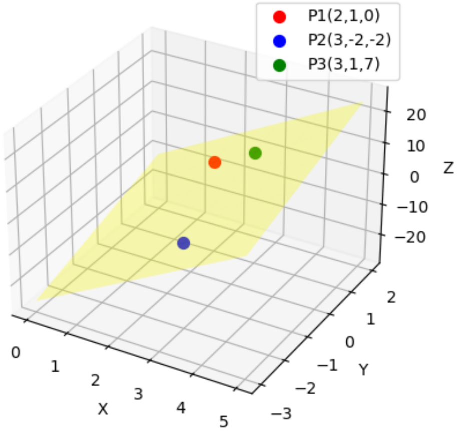
Python and C Code

```
# Direction vectors
v1 = P2 - P1
v2 = P3 - P1

# Prepare ctypes arrays
v1_c = (ctypes.c_double * 3)(*v1)
v2_c = (ctypes.c_double * 3)(*v2)
n_c = (ctypes.c_double * 3)()

# Call C function
lib.cross_product(v1_c, v2_c, n_c)

# Normal vector from C
n = np.array([n_c[0], n_c[1], n_c[2]])
print("Normal vector from C:", n)

# Equation of plane: n(X - P1) = 0  ax+by+cz+d=0
a, b, c = n
d = -(a*P1[0] + b*P1[1] + c*P1[2])
```

```python
print(f"Plane equation: {a}x + {b}y + {c}z + {d} = 0")

# ----- Plot -----
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot points
ax.scatter(*P1, color='red', s=50, label="P1(2,1,0)")
ax.scatter(*P2, color='blue', s=50, label="P2(3,-2,-2)")
ax.scatter(*P3, color='green', s=50, label="P3(3,1,7)")
```

Python and C Code

```
# Create grid for plane
xx, yy = np.meshgrid(range(0,6), range(-3,3))
zz = (-a*xx - b*yy - d)/c

ax.plot_surface(xx, yy, zz, alpha=0.3, color='yellow')

ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.legend()
plt.show()
```


P1(2,1,0)
P2(3,-2,-2)
P3(3,1,7)
20
10
0
−10
−20
Z
0
1
2
3
4
5
X
2
1
0
−1
−2
−3
Y