EE25btech11028 - J.Navya sri

Question:

Find the equation of the plane through the points

$$(2, 1, 0), (3, -2, -2), (3, 1, 7).$$

Solution: Let $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ be the given points and $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be a general point on the plane.

The points are:

$$\mathbf{p}_1 = \begin{pmatrix} 2\\1\\0 \end{pmatrix}, \quad \mathbf{p}_2 = \begin{pmatrix} 3\\-2\\-2 \end{pmatrix}, \quad \mathbf{p}_3 = \begin{pmatrix} 3\\1\\7 \end{pmatrix}.$$

Step 1: General Plane Equation

The equation of the plane is $\mathbf{n} \cdot \mathbf{x} + d = 0$, where $\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the normal vector.

$$ax + by + cz + d = 0 \tag{1}$$

1

Step 2: Substitution of Points

The points \mathbf{p}_i must satisfy the plane equation:

$$2a + b + d = 0 \tag{2}$$

$$3a - 2b - 2c + d = 0 (3)$$

$$3a + b + 7c + d = 0 (4)$$

Step 3: Matrix Form

The system of linear equations can be written as $M\mathbf{v} = \mathbf{0}$, where $\mathbf{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$:

$$\begin{pmatrix} 2 & 1 & 0 & 1 \\ 3 & -2 & -2 & 1 \\ 3 & 1 & 7 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
 (5)

From (2):

$$d = -2a - b \tag{6}$$

Substitute (6) in (3):

$$3a - 2b - 2c + (-2a - b) = 0 \implies a - 3b - 2c = 0$$
 (7)

Substitute (6) in (4):

$$3a + b + 7c + (-2a - b) = 0 \implies a + 7c = 0$$
 (8)

From (8):

$$a = -7c \tag{9}$$

From (7), substitute (9):

$$(-7c) - 3b - 2c = 0 \implies -9c = 3b \implies b = -3c$$
 (10)

From (6), substitute (9) and (10):

$$d = -2(-7c) - (-3c) = 14c + 3c \implies d = 17c \tag{11}$$

Step 5: Final Result The coefficient vector v is proportional to c:

$$\mathbf{v} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = c \begin{pmatrix} -7 \\ -3 \\ 1 \\ 17 \end{pmatrix}$$

Choosing c = 1, the normal vector is $\mathbf{n} = \begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix}$ and d = 17. The plane equation is:

$$-7x - 3y + z + 17 = 0 ag{12}$$

Or equivalently, multiplying by -1:

$$\boxed{7x + 3y - z - 17 = 0} \tag{13}$$

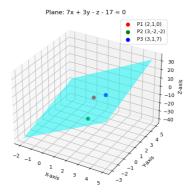


Fig. 1