

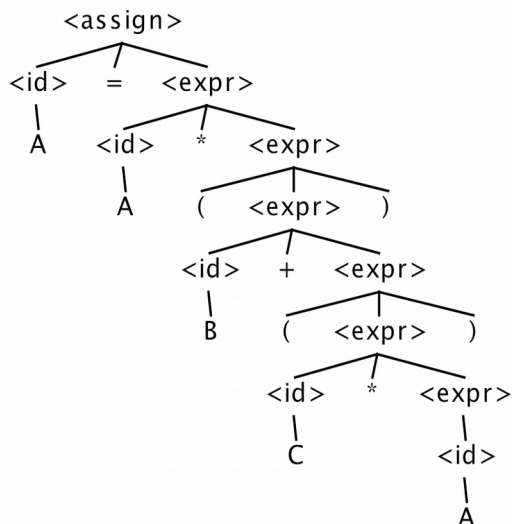
- Write a grammar for the language consisting of strings that have n copies of the letter a followed by the same number of copies of the letter b, where $n > 0$. For example, the strings ab, aaabbb, and aaaaabbbbb are in the language, but a, abb, ba, and aaabb are not in the language.

Solution: $\langle S \rangle \rightarrow ab \mid a \langle S \rangle b$

- Using the grammar below, show a parse tree and a leftmost derivation for each of the following statements:

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\langle \text{id} \rangle \rightarrow A \mid B \mid C$
 $\langle \text{expr} \rangle \rightarrow \langle \text{id} \rangle + \langle \text{expr} \rangle$
 $\quad \quad \quad \mid \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\quad \quad \quad \mid (\langle \text{expr} \rangle)$
 $\quad \quad \quad \mid \langle \text{id} \rangle$

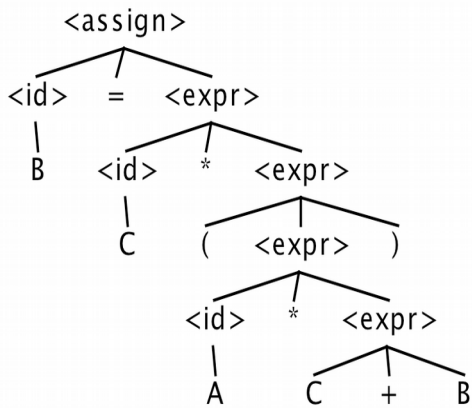
a. $A = A * (B + (C * A))$



Solution

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\rightarrow A = \langle \text{expr} \rangle$
 $\rightarrow A = \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\rightarrow A = A * \langle \text{expr} \rangle$
 $\rightarrow A = A * (\langle \text{expr} \rangle)$
 $\rightarrow A = A * (\langle \text{id} \rangle + \langle \text{expr} \rangle)$
 $\rightarrow A = A * (B + \langle \text{expr} \rangle)$
 $\rightarrow A = A * (B + (\langle \text{expr} \rangle))$
 $\rightarrow A = A * (B + (\langle \text{id} \rangle * \langle \text{expr} \rangle))$
 $\rightarrow A = A * (B + (C * \langle \text{expr} \rangle))$
 $\rightarrow A = A * (B + (C * \langle \text{id} \rangle))$
 $\rightarrow A = A * (B + (C * A))$

b. $B = C * (A * C + B)$



Solution

<assign> \rightarrow id = <expr>
 \rightarrow B = <expr>
 \rightarrow B = id * <expr>
 \rightarrow B = C * <expr>
 \rightarrow B = C * (<expr>)
 \rightarrow B = C * (id * <expr>)
 \rightarrow B = C * (A * <expr>)
 \rightarrow B = C * (A * id + <expr>)
 \rightarrow B = C * (A * C + <expr>)
 \rightarrow B = C * (A * C * id)
 \rightarrow B = C * (A * C + B)

3. Prove the following grammar is ambiguous:

<assign> \rightarrow <id> = <expr>
 <id> \rightarrow A | B | C
 <expr> \rightarrow <expr> + <expr>
 | <expr> * <expr>
 | (<expr>)
 | <id>

I can prove a grammar is ambiguous by creating two leftmost derivations for the same sentence. Consider the sentence, $A = B + C * A$. Below are two different leftmost derivations for the same sentence.

4. Consider the following grammar:

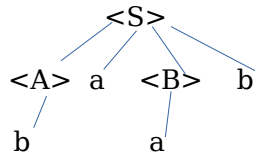
$$\begin{aligned} \langle S \rangle &\rightarrow \langle A \rangle a \langle B \rangle b \\ \langle A \rangle &\rightarrow \langle A \rangle b \mid b \\ \langle B \rangle &\rightarrow a \langle B \rangle \mid a \end{aligned}$$

Which of the following sentences are in the language generated by this grammar?

- a. baab
- b. bbbab
- c. bbaaaaaS
- d. bbaab

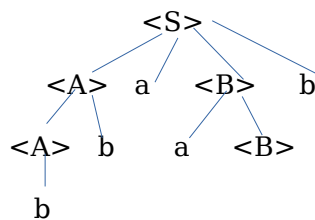
Solution:

a.



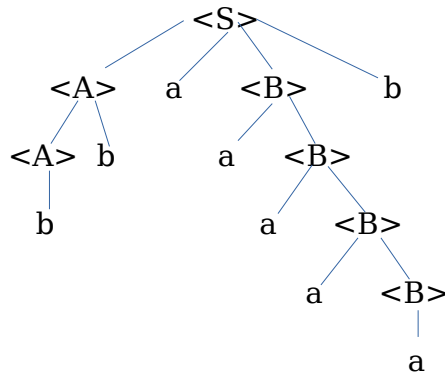
The sentence is generated.

b.



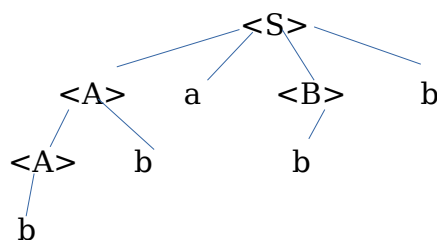
The sentence is not generated.

c.



The sentence is not generated.

d.



The sentence is generated.

5. Convert the following EBNF to BNF:

$\langle S \rangle \rightarrow \langle A \rangle \{ b \langle A \rangle \}$
 $\langle A \rangle \rightarrow a[b] \langle A \rangle$

Solution:

$S \rightarrow A \mid A B$
 $B \rightarrow b A \mid b A B$
 $A \rightarrow a A \mid a b A$