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CSE-105

ASSIGNMENT

Direct Proof:

Q-1: Directly prove that if m and n are odd integers then mn is also odd integer.

⇒ Here,

Proposition: If m and n are odd integers then mn is also odd integer.

p = m and n are odd integers

q = mn is odd integer.

The logical expression of the given statement is $p \to q$.

Proof:

Let,

$m = 2x + 1$

$n = 2y + 1$ [where x and y are integers]

So, $mn = (2x+1)(2y+1)$

$= 4xy + 2x + 2y + 1$

$= 2(2xy + x + y) + 1$

Since, mn can be written in this form, it is odd. Proved

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Q-2: Let m and n are integers. Directly prove that if m and n are perfect squares, then mn is also a perfect square.

⇒ Here,

Proposition: If m and n are perfect squares, then mn is also a perfect square.

p = m and n are perfect squares.

q = mn is perfect square.

The logical expression of the given statement is $p \rightarrow q$.

Proof:

Let,

$m = x^2$ [where x^2 and y^2 are

$n = y^2$ perfect squares]

So, $mn = x^2 y^2$

$= (xy)^2$

Since, mn can be written in this form, it is a perfect square. [Proved]

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Q-3: Directly prove that sum of two odd numbers is even.

⇒ Here,

Proposition: sum of two odd numbers is even.

Proof:

Let,

$a = 2p+1$ [where p and q are integers]

$b = 2q+1$

So, $a+b = 2p+1+2q+1$

$= 2p+2q+2$

$= 2(p+q+1)$

We know that, even numbers are divisible by 2.

Now, $\frac{a+b}{2} = \frac{2(p+q+1)}{2}$

$= p+q+1$

Since, sum of a, b is divisible by 2, $a+b$ is even.

[Proved]

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Q-4: Prove that if n is a multiple of 3, then n^2 is also a multiple of 3.

$\Rightarrow$ Here,

Proposition: If n is a multiple of 3, then n^2 is also a multiple of 3.

$p =$ n is a multiple of 3.
$q =$ n^2 is a multiple of 3.

The logical expression of the given statement is $p \rightarrow q$.

Proof:

Let,

$n = 3x$ [where x is an integer]

Then, $n^2 = (3x)^2$

$= 9x^2$

$= 3 \cdot 3x^2$

So, $\frac{n^2}{3} = \frac{3 \cdot 3x^2}{3}$

$= 3x^2$

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since n^2 is divisible by 3, it is a multiple of 3.

[Proved]

Q-5: Sum of an even integer and an odd integer is odd.

$\Rightarrow$ Here,

Proposition: Sum of an even integers and an odd integers is odd.

Proof:

Let x be an even integers and y be an odd integers.

$x = 2m$ [where m is an integers]

$y = 2m+1$

Now,

$$x+y = 2m+2m+1$$
$$= 4m+1$$
$$= 2\cdot(2m)+1$$

Now, we can see that the sum is in the form of the odd integers y. So, the sum of an even integers and an odd integers is odd.

[Proved]

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Indirect Proof:

Q-6: If $x=2$, then $3x-5 \neq 10$. Prove this statement true by indirect proof.

$\Rightarrow$ Here,

Proposition:

If $x=2$, then $3x-5 \neq 10$.

$p \Rightarrow x=2$; $q \Rightarrow 3x-5 \neq 10$

Logical expression $\Rightarrow p \rightarrow q$

Proof: Let's assume that the conclusion of this implication is false [$3x-5=10$].

$3x-5=10$ [negation of the conclusion]

$\Rightarrow 3x = 10+5$

$\Rightarrow 3x = 15$

$\Rightarrow x = 5$

So, $x \neq 2$ [$\neg q \rightarrow \neg p$]

We have shown that, if $3x-5=10$ then $x \neq 2$, which is the contrapositive of the implication. The contrapositive of the implication is true, so the implication itself is also true.

Proved

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Q-7: Let x be a real number. Prove that if $x^3-7x^2+x-7=0$, then $x=7$.

$\Rightarrow$ Here,

Proposition: If $x^3-7x^2+x-7=0$, then $x=7$.

Proof:

Let's assume that the conclusion of this implication is false.

$x \neq 7$

Now, $x^3-7x^2+x-7=0$

$\Rightarrow x^2(x-7)+1(x-7)=0$

$\Rightarrow (x-7)(x^2+1)=0$

$\Rightarrow x-7=0$ or $x^2+1=0$

$\Rightarrow x=7$ $\qquad \Rightarrow x^2=1$

$\therefore x=+i,-i$

But $x \neq 7$, So, $x=+i,-i$.

So, we have shown that if $x \neq 7$, then $x^3-7x^2+x-7=0$ has no real solutions.

So, the only real value which satisfies the equatition is $x=7$. Since the contraapositive of the implication is true, the implication itself is also true. Proved

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Proof By Induction:

Q-8: Prove that, $1^3 + 2^3 + 3^3 + \cdots + n^3 =$

$\Rightarrow$ For, $n = 0$, $S_0 = 0$

$$n = 1,\ S_1 = 1^3 = 1 = \frac{1^2 \cdot 2^2}{4}$$

$$n = 2,\ S_2 = 1^3 + 2^3 = 9 = \frac{2^2 \cdot 3^2}{4}$$

$$n = 3,\ S_3 = 1^3 + 2^3 + 3^3 = 36 = \frac{3^2 \cdot 4^2}{4}$$

$$\text{so},\ S_n = \frac{n^2 (n+1)^2}{4}$$

$$S_{n+1} = S_n + (n+1)^3$$

$$= \frac{n^2 (n+1)^2}{4} + (n+1)^3$$

$$= \frac{n^2 (n+1)^2}{4} + \frac{4(n+1)^3}{4}$$

$$= \frac{n^2 (n+1)^2 + 4(n+1)^3}{4}$$

$$= \frac{(n+1)^2 \, n^2 \cdot 4(n+1)}{4}$$

$$= \frac{(n+1)^2 (n^2 + 4n + 4)}{4}$$

$$= \frac{(n+1)^2 (n+2)^2}{4}$$

P.T.O.

So, the pattern is verified using proof by induction.

Q-9: Prove by induction that ~~the~~ $1^2+2^2+3^2+\dots+n = n(n+1)(2n+1)/6$ for every positive integer n.

$\Rightarrow$ For $n=0$, $S_n = 0$

$n=1$, $S_1 = 1^2 = \frac{1\cdot 2\cdot 3}{6}$

$n=2$, $S_2 = 1^2+2^2 = 5 = \frac{2\cdot 3\cdot 5}{6}$

$n=3$, $S_3 = 1^2+2^3+3^2 = 14 = \frac{3\cdot 4\cdot 7}{6}$

So, $S_n = \frac{n(n+1)(2n+1)}{6}$

$$S_{n+1} = S_n + (n+1)^2$$

$$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^2}{6}$$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}$$

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$$= \frac{(n+1)(2n^2+n+6n+6)}{6}$$

$$= \frac{(n+1)(2n^2+7n+6)}{6}$$

$$= \frac{(n+1)(2n^2+4n+3n+6)}{6}$$

$$= \frac{(n+1)\{2n(n+2)+3(n+2)\}}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

So, the pattern is verified using proof by induction.

P.T.O.

Q-10: Prove that $7^n - 1$ is a multiple of 6 for all N, i.e., $7^n - 1 = 6j$ for some integer j.

$\Rightarrow$ For $n=0$, $7^0 - 1 = 0$

$n=1$, $7^1 - 1 = 6 = 6 \cdot 1$

$n=2$, $7^2 - 1 = 48 = 6 \cdot 8$

$n=3$, $7^3 - 1 = 342 = 6 \cdot 57$

So, $7^n - 1 = 6j$ [where j is an integer]

$$7^{n+1} - 1 = 7^1 \cdot 7^n - 1$$
$$= 7 \cdot 7^n - 1$$
$$= 7(6j+1) - 1$$
$$= 7 \cdot 6j + 7 - 1$$
$$= 7 \cdot 6j + 6$$
$$= 6(7j+1)$$

Here $(7j+1)$ is an integer multiplied by 6. It ensures that $(7^{n+1} - 1)$ is indeed an multiple of 6. So, this is verified using proof by induction.

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Graph

Q1

Solutition ⇒

s	t	u	v	w	x	y	
0	∞	∞	∞	∞	∞	∞	s: { }
	∞	∞	1	4	2	3	s: {s}
	∞	2		4	2	3	s: {s, v}
	4			3	2	3	s: {s, v, u}
	4			3		3	s: {s, v, u, x}
	4					3	s: {s, v, u, x, w}
	4						s: {s, v, u, x, w, y}
	4						s: {s, v, u, x, w, y, t}

shortest path from s to others nodes:

s to t: s → y → t

s to u : s → v → u

s to v : s → v

s to w : s → v → u → w

s to x : s ~~x~~ → x

s to y : s → y

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Q-2

Solution ⇒

0	1	2	3	4	5	6	7	8	
0	∞	∞	∞	∞	∞	∞	∞	∞	s{ }
	4	∞	∞	∞	∞	∞	8	∞	s{0}
		12	∞	∞	∞	9	8	14	{0, 1}
		12	19	∞	11	9		14	{0, 1, 7}
		12	19	21	11			14	{0, 1, 7, 6}
		12	19	21				14	{0, 1, 7, 6, 5}
			19	21				14	{0, 1, 7, 6, 5, 2}
			19	21					{0, 1, 7, 6, 5, 2, 8}
				21					{01, 7, 6, 5, 2, 8, 3}
				21					{0, 1, 7, 6, 5, 2, 8, 3, 4}

Shortest path from 0 to other nodes :

0 to 1 : 0 → 1

0 to 2 : 0 → 1 → 2

0 to 3 : 0 → 1 → 2 → 3

0 to 4 : 0 → 7 → 6 → 5 → 4

0 to 5 : 0 → 7 → 6 → 5

0 to 6 : 0 → 7 → 6

0 to 7 : 0 → 7

0 to 8 : 0 → 1 → 2 → 8

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Tree :

Q - 1

(i) Solution ⇒

Inorder – 5, 15, 18, 20, 25, 30, 35, 40, 45, 50, 60

Preorder – 30, 20, 15, 5, 18, 25, 40, 35, 50, 45, 60

Postorder – 5, 18, 15, 25, 20, 35, 45, 60, 50, 40, 30

(ii) Solution ⇒

Inorder – DBAEGCHFI

Preorder – ABDCEGFHI

Postorder – DBGEHIFCA

(iii) Solution ⇒

Inorder – 10 20 30 100 150 200 300

Preorder – 100 20 10 30 200 150 300

Postorder – 10 30 20 150 300 200 100