Midterm Exam 1

1.) $s \to (p \lor \neg r) \land (\neg p \lor q) \land (r \lor \neg q)$

s	p	q	r	$(p \lor \neg r)$	$(\neg p \lor q)$	$(r \lor \neg q)$	$(p \lor \neg r) \land (\neg p \lor q) \land (r \lor \neg q)$	$s \to (p \lor \neg r) \land (\neg p \lor q) \land (r \lor \neg q)$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F	F
T	T	F	T	T	F	T	F	F
T	T	F	F	T	F	T	F	F
T	F	T	T	F	T	T	F	F
T	F	T	F	T	T	F	F	F
T	F	F	T	F	T	T	F	F
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F	T
F	T	F	T	T	F	T	F	T
F	T	F	F	T	F	T	F	T
F	F	T	T	F	T	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	T	F	T	T	F	T
F	F	F	F	T	T	T	T	T

2.) Use laws of logic to simplify $((\neg q \land p \land \neg r) \lor (q \land s \land \neg q) \lor (p \land \neg r \land q))$

$((\neg q \land p \land \neg r) \lor (q \land \neg q \land s) \lor (q \land p \land \neg r))$	Commutativity
$((\neg q \land p \land \neg r) \lor (F \land s) \lor (q \land p \land \neg r))$	Negation
$((\neg q \land (p \land \neg r)) \lor (F) \lor (q \land (p \land \neg r))$	Domination
$((\neg q \land (p \land \neg r)) \lor (q \land (p \land \neg r))$	Identity
$(((p \land \neg r) \land \neg q) \lor ((p \land \neg r) \land q)$	Commutativity
$\left((p \land \neg r) \land (\neg q \lor q)\right)$	Distributivity
$\left((p \land \neg r) \land (T)\right)$	Negation
$(p \land \neg r)$	Identity

3.a)

Premise 1) $w \rightarrow o$	s = Steak, w = Well done, o = Overcooked
Premise 2) $o \rightarrow a$	a = Fire alarm
Premise 3) $(b) \vee (a \wedge F)$	b = Batteries
Premise 4) $(b \rightarrow l)$	l = Ladder
Premise 5) $\neg l$	
Conclude $\neg w$	
$\neg t$	Premise 5
$(b \rightarrow l) \wedge \neg l$	Premise 4 + Premise 5
$\therefore \neg b$	Modus Tollens
$(F) \vee (a \wedge F)$	$\neg b$ + Premise 3
$(a \wedge F)$	Identity
$\therefore \neg a$	Domination
$(o \rightarrow a) \wedge \neg a$	Premise 2 + $\neg a$
$\therefore \neg o$	Modus Tollens
$(w \rightarrow o) \wedge \neg o$	Premise 1 + $\neg o$
$\therefore \neg w$	Modus Tollens

3.b) Because the premises don't take into account for every scenario and in real life q can be true while p is false. For example if someone enjoys their steak well done; then the steak would be well done and not considered overcooked even though that would violate $p \rightarrow q$ which cannot be $T \rightarrow F$. Or the fire alarm could go off despite the steak not being overcooked, perhaps something else burned or the fire alarm needs new batteries which would again violate the conditional proposition in a mathematical sense.

3.c)

Premise 1. I consider that if a steak is cooked well done then it has been overcooked

Premise 2. If the steak is overcooked and triggers the fire alarm the fire alarm will go off

Premise 4. When changing the fire alarm batteries only use the ladder that's in the room.

4.) Show $\left(\left((A \cup C) \cap (A \cup B)\right) - D\right) = \left((A - D) \cup \left((B \cap C) - D\right)\right)$ using universal generalization applied twice

Case 1: $\Big(\big((A \cup C) \cap (A \cup B)\big) - D\Big) = \Big((A - D) \cup \big((B \cap C) - D\big)\Big)$

$x \in \big((A \cup C) \cap (A \cup B)\big) - D$	Consider an element x
$x \in \big(A \cup (C \cap B)\big) - D$	Distributivity
$x \in (A) \cap (B \cap C) - D$	Commutativity
$x \in A \cup (B \cap C)\ \&\ x \notin D$	Relative Complacent
$x \in A\ \&\ x \notin D\ or\ x \in (B \cap C)\ \&\ x \notin D$	Definition of Union
Case 1.1 $x \in A\ \&\ x \notin D$	
$x \in A \cup S\ x \notin D$	Def of Union, S = any set
$x \in A \cup (B \cap C)\ x \notin D$	S = any set
$x \in ((A - D) \cup (B \cap C) - D)$	Relative Complacent
Case 1.2 $x \in (B \cap C)\ \&\ x \notin D$	
$x \in (B \cap C) \cup S\ \&\ x \notin D$	Def of Union, S = any set
$x \in A \cup (B \cap C)\ \&\ x \notin D$	S = any set, Commutativity
$x \in ((A - D) \cup (B \cap C) - D)$	Relative Complacent
$\therefore \Big(\big((A \cup C) \cap (A \cup B)\big) - D\Big) = \Big((A - D) \cup \big((B \cap C) - D\big)\Big)$	Universal Generalization

Case 2: $\Big((A - D) \cup \big((B \cap C) - D\big)\Big) = \Big(\big((A \cup C) \cap (A \cup B)\big) - D\Big)$

$x \in (A - D) \cup \big((B \cap C) - D\big)$	Consider an element x
$x \in A \cup (B \cap C) - D$	Definition of Union
$x \in A \cup (B \cap C)\ \&\ x \notin D$	Relative Complacent
$x \in A\ \&\ x \notin D\ or\ x \in (B \cap C)\ \&\ x \notin D$	Definition of Union
Case 2.1 $x \in A\ \&\ x \notin D$	
$x \in A \cup S\ x \notin D$	Def of Union, S = any set
$x \in A \cup (B \cap C)\ x \notin D$	S = any set
$x \in (A) \cap (C \cap B)\ x \notin D$	Commutativity
$x \in \big((A \cup C) \cap (A \cup B)\big)\ x \notin D$	Distributivity
$x \in \big((A \cup C) \cap (A \cup B)\big) - D$	Relative Complacent

Case 2.2 $x \in (B \cap C) \,\&\, x \notin D$

$x \in (B \cap C)$

$x \in (B \cap C) \cup S \,\&\, x \notin D$ — Def of Union, S = any set

$x \in A \cup (B \cap C) \,\&\, x \notin D$ — S = any set, Commutativity

$x \in (A) \cap (C \cap B)\; x \notin D$ — Commutativity

$x \in \big((A \cup C) \cap (A \cup B)\big)\; x \notin D$ — Distributivity

$x \in \big((A \cup C) \cap (A \cup B)\big) - D$ — Relative Complacent

$\therefore \Big((A - D) \cup \big((B \cap C) - D\big)\Big) = \Big(\big((A \cup C) \cap (A \cup B)\big) - D\Big)$ — Universal Generalization

5.) Disprove by counter-example $((A \cup B \cup C) - D \subseteq \Big(\big((A \cup C) \cap (A \cup B)\big) - D\Big)$

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	**D**	$(\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}) - \boldsymbol{D}$	$\big((\boldsymbol{A} \cup \boldsymbol{C}) \cap (\boldsymbol{A} \cup \boldsymbol{B})\big) - \boldsymbol{D}$	$((\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}) - \boldsymbol{D} \subseteq \Big(\big((\boldsymbol{A} \cup \boldsymbol{C}) \cap (\boldsymbol{A} \cup \boldsymbol{B})\big) - \boldsymbol{D}\Big)$
Y	Y	Y	Y	N	N	Y
Y	Y	Y	N	Y	Y	Y
Y	Y	N	Y	N	N	Y
Y	Y	N	N	Y	Y	Y
Y	N	Y	Y	N	N	Y
Y	N	Y	N	Y	Y	Y
Y	N	N	Y	N	N	Y
Y	N	N	N	Y	Y	Y
N	Y	Y	Y	N	N	Y
N	Y	Y	N	Y	Y	Y
N	Y	N	Y	N	N	Y
N	Y	N	N	Y	N	N
N	N	Y	Y	N	N	Y
N	N	Y	N	Y	N	N
N	N	N	Y	N	N	Y
N	N	N	N	N	N	Y