Problem 1: Division

1.)24 = quotient, 3 = remainder

2.)116 = quotient, 49 = remainder

3.)0 = quotient, 77 = remainder

4.)5 = quotient, 36 = remainder

5.)1 = quotient, 28 = remainder

Problem 2: GCD and LCM

1.) 123 and 67

$\frac{123}{67} = 1\ with\ 56\ remainder$

$\frac{67}{56} = 1\ with\ 11\ remainder$

$\frac{56}{11} = 5\ with\ 1\ remainder$

$\frac{11}{1} = 11\ evenly$

GCD = 1, LCM = $\frac{123*67}{1}$ = 8241

2.) 609 and 377

$\frac{609}{377} = 1\ with\ 232\ remainder$

$\frac{377}{232} = 1\ with\ 145\ remainder$

$\frac{232}{145} = 1\ with\ 87\ remainder$

$\frac{145}{87} = 1\ with\ 58\ remainder$

$\frac{87}{58} = 1\ with\ 29\ remainder$

$\frac{58}{29} = 2\ evenly$

GCD = 29, LCM = $\frac{609*377}{29}$ = 7917

3.) 135 and 198

$\frac{135}{198} = simplfies\ into\ \frac{15}{22}\ by\ dividing\ by\ 9$

GCD = 9, LCM = $\frac{135*198}{9}$ = 2970

4.) 923 and 7238

$\frac{923}{7238} = can't\ be\ simplified\ further\ therefore\ GCD = 1$

GCD = 1, LCM = $\frac{923*7238}{1}$ = 6680674

Problem 3: Modulo Rules

1.) ${}_5(7^{15})$

${}_5(7^3)^5$	Exponentiation
${}_5(343)^5$	Arithmetic
${}_5 3^5$	${}_5 3^8 so\ let\ x^8\ and\ f(343) = {}_5 f(3) \rightarrow 343^5 = {}_5 3^8$
${}_5 3 * {}_5 3^4$	Arithmetic
${}_5 3 * {}_5(3^2)^2$	Arithmetic
${}_5 3 * {}_5(9)^2 = {}_5 3 * {}_5 81$	Arithmetic
${}_5(3 * 1) = 3$	Arithmetic and Modulo

2.) ${}_7 11^{21}$

${}_7(11^3)^7$	Exponentiation
${}_7(1331)^7$	Arithmetic
${}_7(1)^7$	$1331 = {}_7 1\ \ so\ let\ x^7\ and\ f(1330) = {}_7 f(1) \rightarrow 1331^7 = {}_5 1^7$
${}_7 1$ = 1	Arithmetic and Modulo

3.)${}_{13} 2^{22}$

${}_{13}(2^2)^{11}$	Exponentiation
${}_{13}(4)^{11}$	Arithmetic
${}_{13}(4) * {}_{13}(4)^{10}$	Arithmetic
${}_{13} 4 * {}_{13}(4^2)^5$	Exponentiation
${}_{13}(4) * {}_{13}(16)^5$	Arithmetic

${}_{13}(4) * {}_{13}(3)^5$	13=${}_{13}$3 $so\ let\ x^5\ and\ f(13) = {}_{13}f(3) \rightarrow 16^5 = {}_{13}3^5$
${}_{13}(4) * ({}_{13}(3) * {}_{13}(3)^4)$	Arithmetic
${}_{13}(4) * ({}_{13}(3) * {}_{13}\left(3^2\right)^2)$	Exponentiation
${}_{13}(4) * ({}_{13}(3) * {}_{13}(9)^2)$	Arithmetic
${}_{13}(4) * ({}_{13}(3) * {}_{13}81)$	Arithmetic
${}_{13}4 * {}_{13}(243) = {}_{13}972$	Arithmetic
10	Modulo

4.)${}_{11}13^{30}$

${}_{11}(13^2)^{15}$	Exponentiation
${}_{11}(169)^{15}$	Arithmetic
${}_{11}4^{15}$	11=${}_{11}$4 $so\ let\ x^{15}\ and\ f(11) = {}_{11}f(4) \rightarrow 169^{15} = {}_{11}4^{15}$
${}_{11}(4^3)^5$	Exponentiation
${}_{11}(64)^5$	Arithmetic
${}_{11}(9)^5$	11=${}_{11}$9 $so\ let\ x^5\ and\ f(11) = {}_{11}f(9) \rightarrow 64^5 = {}_{11}9^5$
${}_{11}9^2 * {}_{11}9^2 * {}_{11}9$	Exponentiation
${}_{11}81 * {}_{11}81 * {}_{11}9$	Arithmetic
${}_{11}4 * {}_{11}4 * {}_{11}9 = {}_{11}144$	Arithmetic
${}_{11}144 = 1$	Modulo

5.) ${}_{17}3^{31}$

${}_{17}3 * {}_{17}(3^2)^{15}$	Exponentiation
${}_{17}3 * {}_{17}(9)^{15}$	Arithmetic
${}_{17}3 * {}_{17}(9^3)^5$	Exponentiation
${}_{17}3 * {}_{17}(729)^5$	Arithmetic
${}_{17}3 * {}_{17}(15)^5$	13=${}_{13}$15 $so\ let\ x^5\ and\ f(17) = {}_{17}f(15) \rightarrow 729^5 = {}_{17}15^5$

${}_{17}45 * {}_{17}(15^2)^2$	Arithmetic and Exponentiation
${}_{17}45 * {}_{17}(225)^2$	Arithmetic and Exponentiation
${}_{17}45 * {}_{17}(4)^2$	$17 = {}_{17}4$ $so\ let\ x^2\ and\ f(17) = {}_{17}f(4) \rightarrow 225^2 = {}_{17}4^2$
${}_{17}11 * {}_{17}16 = {}_{17}176$	Modulo and Arithmetic
6	Modulo

Problem 4: Divisibility

1.) Prove that if 3 divides an integer x with a remainder of 1, 9 divides x^3 with a remainder of 1

$3n = (x - 1)$	Divisibility Rules
$3n + 1 = x$	Algebra
$x^3 = (3n + 1)^3$	x^3
$x^3 = (27n^3 + 27n^2 + 9n) + 1$	Algebra
$x^3 = 9(3n^3 + 3n^2 + 1n) + 1$	Distribution
$x^3 = 9y + 1$	Divisible by 9 with a remainder of 1

2.) Prove that if 5 divides an integer n with a remainder of 4, 10 divides $2n^2 + 4n + 3$ with a remainder of 1

$2n^2 + 4n + 3 = 2(5x + 4)^2 + 4(5x + 4) + 3$	Substitution
$= (50x^2 + 100x + 51)$	Simplification
$= 50x^2 + 100x + 50 + 1$	Expand
$= 10(5x^2 + 10x + 5) + 1$	Distribution
$2n^2 + 4n + 3 = 10y + 1$	Divisible by 10 with a remainder of 1

3.) Prove that if 7 divides an integer n with a remainder of 4, 21 divides $3n^2$ with a remainder of 6

$3n^2 = 3(7x + 4)^2$	Substitution
$= 147x^2 + 168x + 48$	Simplification
$= (147x^2 + 168x + 42) + 6$	Expand
$= 7(21x^2 + 24x + 6) + 6$	Distribution

$3n^2 = 7y + 6$ Divisible by 7 with a remainder of 6

4.) Determine, with proof, all pairs of integers (x,y) which satisfy the equation $93411x + 2844y = 12345$

$a|b \text{ and } a|c \text{ then } a|(bx + cy) \text{for all integers } x \text{ and } y$

$93411x + 2844y = 12345$ Given

$\rightarrow 9(10379x + 316y) = 12345$ Distribution

$\rightarrow (10379x + 316y) = \frac{12345}{9}$ Distribution

$\rightarrow (10379x + 316y) \text{ is an integer but } \frac{12345}{9} \text{ is not}$