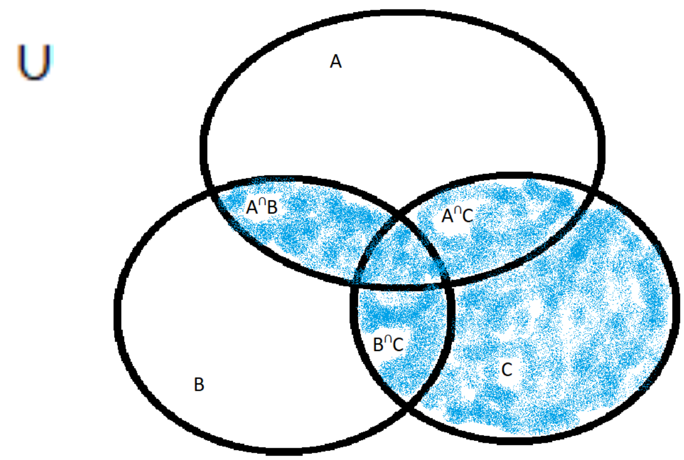
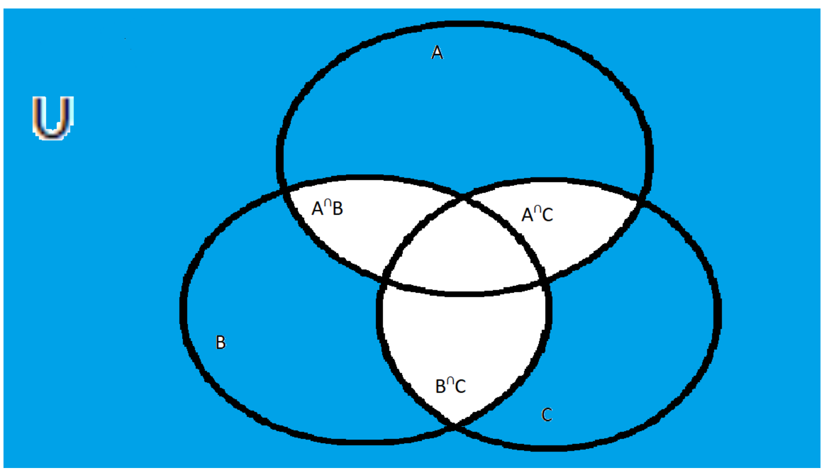
Problem 1: Venn Diagrams

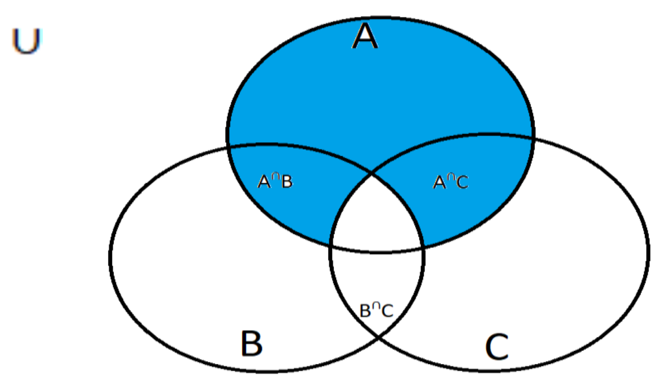
1.)$(A \cap B)$



2.)$(\neg A) \cup (\neg B) \cup (\neg C)$



3.)$A - (B \cap C)$



Problem 2: Proof of Laws

1.) prove absorption law $A \cup (A \cap B) = A$ by twice-applied universal generalization

Case 1: $A \cup (A \cap B) = A$

$x \in A \cup (A \cap B)$	Consider an element x
$x \in A\ or\ x \in (A \cap B)$	Definition of Union
Case 1.1: $x \in (A \cap B)$	
Case 1.1 $x \in A$	Definition of intersection
Case 1.2 $x \in A$	Definition of $\in$
Any case of $x \in A \cup (A \cap B), x \in A$	Universal Generalization
$\therefore A \cup (A \cap B) = A$	

Case 2: $A = A \cup (A \cap B)$

$x \in A$	Consider an element x
$x \in B\ or\ x \notin B$	Without loss of generality
Case2.1: $x \in B$	
Case 2.1: $x \in (A \cap B\ )$	Definition of Intersection
Case 2.2: $x \notin B$	
Case 2.2: $x \in (A \cup B)$	Definition of Union
Case 2.2: $x \in A$	Definition of Union
$\therefore A = A \cup (A \cap B)$	

2.) prove associative law $A \cap (B \cap C) = (A \cap B) \cap C$ by membership table

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$(\boldsymbol{B} \cap \boldsymbol{C})$	$(\boldsymbol{A} \cap \boldsymbol{B})$	$\boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$	$(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C}$	$\boldsymbol{A} \cap (\boldsymbol{B} \cap \boldsymbol{C}) \subseteq (\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C}$
Y	Y	Y	Y	Y	Y	Y	Y
Y	Y	N	N	Y	N	N	Y
Y	N	Y	N	N	N	N	Y
Y	N	N	N	N	N	N	Y
N	Y	Y	Y	N	N	N	Y
N	N	N	N	N	N	N	Y
N	N	Y	N	N	N	N	Y
N	Y	N	N	N	N	N	Y

Problem 3: Proof of Equalities

1.) Prove $(A \cup B) \subseteq (A \cup B \cup C)$

$x \in (A \cup B)$ Consider an element x

$x \in A \text{ or } x \in B$ Definition of Union

Case 1.1: $x \in A$

$x \in (A \cup B \cup C)$ Definition of Union

Case 1.2: $x \in B$

$x \in (A \cup B \cup C)$ Definition of Union

$\therefore (A \cup B) \subseteq (A \cup B \cup C)$ Universal generalization

2.) $(A \cap B \cap C) \subseteq (A \cap B)$

A	**B**	**C**	$(\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C})$	$(\boldsymbol{A} \cap \boldsymbol{B})$	$(\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C}) \subseteq (\boldsymbol{A} \cap \boldsymbol{B})$
Y	Y	Y	Y	Y	Y
Y	Y	N	N	Y	Y
Y	N	Y	N	N	Y
Y	N	N	N	N	Y
N	Y	Y	N	N	Y
N	Y	N	N	N	Y
N	N	Y	N	N	Y
N	N	N	N	N	Y

3.) $(A - C) \cap (C - B) = \emptyset$

A	**B**	**C**	$(\boldsymbol{A} - \boldsymbol{C})$	$(\boldsymbol{C} - \boldsymbol{B})$	$(\boldsymbol{A} - \boldsymbol{C}) \cap (\boldsymbol{C} - \boldsymbol{B})$	$(\boldsymbol{A} - \boldsymbol{C}) \cap (\boldsymbol{C} - \boldsymbol{B}) = \emptyset$
Y	Y	Y	N	N	N	Y
Y	Y	N	Y	N	N	Y
Y	N	Y	N	Y	N	Y
Y	N	N	Y	N	N	Y
N	Y	Y	N	N	N	Y
N	Y	N	N	N	N	Y
N	N	Y	N	Y	N	Y
N	N	N	N	N	N	Y

4.) $(B - A) \cup (C - A) = (B \cup C) - A$

$x \in (B - A) \cup (C - A) = (B \cup C) - A$ Consider an element x

$x \in B \;\&\; x \notin A \text{ or } x \in C \;\&\; x \notin A$ Definition of Union and Relative Complacent

$Case$ 1.1 $x \in B \;\&\; x \notin A$

$x \in (B \cup C) - A$ Definition of Union and relative complacent

Case 1.2 $x \in C \;\&\; x \notin A$

$x \in (B \cup C) - A$ Definition of Union and relative complacent

$\therefore (B - A) \cup (C - A) = (B \cup C) - A$ Universal Generalization

$x \in (B \cup C) - A = (B - A) \cup (C - A)$ Consider an element x

$x \in B - A$ $\boldsymbol{or}$ $x \in C - A$ Definition of Union

Case 2.1: $x \in B - A$

$x \in (B - A) \cup (S)$ Definition of Union, S = any set

$x \in (B - A) \cup (C - A)$ S = any set

Case 2.2 $x \in (C - A)$

$x \in (C - A) \cup (S)$ Definition of Union, S = any set

$x \in (C - A) \cup (B - A)$ S = any set

$\therefore (B \cup C) - A = (B - A) \cup (C - A)$ Universal Generalization

Problem 4: Set Equality Laws

1.) $\big((A \cap B) \cup (A \cap \neg B) \cup (\neg A \cap B)\big) = (A \cup B)$

$A \cap (B \cup \neg B) \cup (\neg A \cap B)$ Distributive

$A \cap U \cup (\neg A \cap B)$ Inverse

$A \cup (\neg A \cup B)$ Identity

$(A \cup \neg A) \cap (A \cup B)$ Distributive

$U \cap (A \cup B)$ Inverse

$(A \cup B)$ Identity

Problem 5: Disproof

1.) $(A - B) = (B - A)$ is not always true

A	**B**	$\boldsymbol{(A - B)}$	$\boldsymbol{(B - A)}$	$\boldsymbol{(A - C) \subseteq (C - B)}$
Y	Y	N	N	Y
Y	N	Y	N	Y
N	Y	N	Y	N
N	N	N	N	Y