Problem 1: Truth Tables

1.)

p	q	r	$(p \wedge q)$	$\neg r$	$(p \wedge q) \vee \neg r$
True	True	True	True	False	True
True	True	False	True	True	True
True	False	True	False	False	False
True	False	False	False	True	True
False	True	True	False	False	False
False	True	False	False	True	True
False	False	True	False	False	False
False	False	False	False	True	True

2.)

p	q	r	s	$\neg(p \vee q)$	$(r \oplus s)$	$\neg(p \vee q) \wedge (r \oplus s)$
T	T	T	T	F	F	F
T	T	T	F	F	T	F
T	T	F	T	F	T	F
T	T	F	F	F	F	F
T	F	T	T	F	F	F
T	F	T	F	F	T	F
T	F	F	T	F	T	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	T	F	F	T	F
F	T	F	T	F	T	F
F	T	F	F	F	F	F
F	F	T	T	T	F	F
F	F	T	F	T	T	T
F	F	F	T	T	T	T
F	F	F	F	T	F	F

3.)

q	r	s	p	$(q \oplus r)$	$(q \oplus r) \wedge s$	$[(q \oplus r) \wedge s] \rightarrow p$
T	T	T	T	F	F	T
T	T	T	F	F	F	T
T	T	F	T	F	F	T
T	T	F	F	F	F	T
T	F	T	T	T	T	T
T	F	T	F	T	T	F
T	F	F	T	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T

F	F	T	T	F	F	T
F	F	T	F	F	F	T
F	F	F	T	F	F	T
F	F	F	F	F	F	T

Problem 2: Boolean Algebraic Laws

1.) $[(p \to q) \vee (p \to r)] \leftrightarrow [p \to (q \vee r)]$

$[(\neg p \vee q) \vee (p \to r)]$	Implication
$[(\neg p \vee q) \vee (\neg p \vee r)]$	Implication
$[\neg p \vee (q \vee r)]$	Distributivity
$[p \to (q \vee r)]$	Implication

2.) $\neg[\neg(p \wedge q) \wedge (p \vee q)] \leftrightarrow [(p \to q) \wedge (q \to p)]$

$[\neg\neg(p \wedge q) \vee \neg(p \vee q)]$	De Morgan's
$[\neg\neg(p \wedge q) \vee (\neg p \wedge \neg q)]$	De Morgan's
$[(p \wedge q) \vee (\neg p \wedge \neg q)]$	Double Negation/Complement
$[m \vee (\neg p \wedge \neg q)]$	$m = (p \wedge q)$
$[(m \vee \neg p) \wedge (m \vee \neg q)]$	Distributivity
$[(\neg p \vee m) \wedge (\neg q \vee m)]$	Commutativity
$[\big(\neg p \vee (p \wedge q)\big) \wedge (\neg q \vee (p \wedge q))]$	$(p \wedge q) = m$
$[(\neg p \vee p) \wedge (\neg p \vee q) \wedge (\neg q \vee (p \wedge q))]$	Distributivity
$[(\neg p \vee p) \wedge (\neg p \vee q) \wedge (\neg q \vee p) \wedge (\neg q \vee q)]$	Distributivity
$[T \wedge (\neg p \vee q) \wedge (\neg q \vee p) \wedge (\neg q \vee q)]$	Negation\Inverse
$[T \wedge (\neg p \vee q) \wedge (\neg q \vee p) \wedge T]$	Negation\Inverse
$[T \wedge (p \to q) \wedge (\neg q \vee p) \wedge T]$	Implication
$[T \wedge (p \to q) \wedge (q \to p) \wedge T]$	Implication
$[(p \to q) \wedge (q \to p) \wedge T]$	Identity
$[(p \to q) \wedge (q \to p)]$	Identity

Problem 3: Disproof

$[(p \land q) \to r] \leftrightarrow [(p \to r) \land (q \to r)]$

$[(pT \land qF) \to rT] \leftrightarrow [(pT \to rF) \land (qF \to rT)]$ $(p = T)\ (q = F), (r = T)$

$[F \to rT] \leftrightarrow [(pT \to rF) \land (qF \to rT)]$ $(pT \land qF) = F$

$[T] \leftrightarrow [(pT \to rF) \land (qF \to rT)]$ $[F \to rT] = T$

$[T] \leftrightarrow [(T \land (qF \to rT))]$ $(pT \to rF) = T$

$[T] \leftrightarrow [T \land F]$ $(qF \to rT) = F$

$T \leftrightarrow F$ $[T \land F]$

$T \leftrightarrow F = False$

Problem 4: Inference – Verbal

Premise 1.) $\neg r \lor \neg f \to s \land d$

Premise 2.) $s \to t$

Premise 3.) $\neg t$

Conclude r

$\neg t$	Premise 3
$(s \to t) \land \neg t$	Premise 3 + Premise 2
$\neg s$	Modus Tollens
$\neg r \lor \neg f \to (F \land d)$	Premise 1 + $\neg s$
$\neg r \lor \neg f \to F$	Domination
$\neg(r \land f) \to F$	De Morgan's
$(r \land f)$	Contradiction
r	Conjunctive Simplification

Problem 5: Inference – Symbolic

Premise 1.) $(p \land t) \to (r \lor s)$

Premise 2.) $q \to (u \land t)$

Premise 3.) $u \to p$

Premise 4.) $\neg s$

Conclude that $q \to r$

$\neg s$	Premise 4
$(p \wedge t) \to (r \vee s) \wedge \neg s$	Premise 1+ Premise 4
$(p \wedge t) \to (s \vee r) \wedge \neg s$	Commutativity
$(p \wedge t) \to r$	Disjunctive Syllogism
$\neg(p \wedge t) \vee r$	Implication
$(\neg p \vee \neg t) \vee r$	De Morgan's
$\neg p \vee (\neg t \vee r)$	Associativity
$p \to (\neg t \vee r)$	Implication
$(u \to p) \wedge p \to (\neg t \vee r)$	Premise 3
$u \to (\neg t \vee r)$	Syllogism

$q \to (u \wedge t)$	Premise 2
$\neg q \vee (u \wedge t)$	Implication
$(\neg q \vee u) \wedge (\neg q \vee t)$	Distributivity
$(q \to u) \wedge (\neg q \vee t)$	Implication
$(q \to u) \wedge (q \to t)$	Implication
$q \to u$	Conjunctive Simplification
$(q \to u) \wedge (u \to (\neg t \vee r)) \to q \to (\neg t \vee r)$	$u \to (\neg t \vee r)$
$q \to (\neg t \vee r)$	Syllogism
$q \to (r \vee \neg t)$	Commutativity
$q \to r$	Identity