Homework Assignment 7

COT3100 – Spring 2021

1. **(21)** Let set $A = \{1,2,3,4\}$ and let R_1 and R_2 be binary relations on A. Specifically, let:

$$R_1 = \{(1,1), (1, 2), (2, 1), (2,2), (2, 4), (3, 4), (4, 2), (4, 3)\ (4,4)\}$$
$$R_2 = \{(1,2), (1, 3), (1, 4),(2,1), (2, 3), (4, 1), (4, 2)\}$$

Determine the following:

a) Whether R_1 is reflexive, irreflexive, symmetric, anti-symmetric and/or transitive.

Lacks [3,3] so is **not reflexive**, contains at least one $[x,x]$ therefore **not irreflexive**. Contains $[x,y]$ and $[y,x]$ for every point (1,2,3,4) therefore **is symmetric** and is **not anti-symmetric**. (3,4) is not transitive therefore **not transitive**.

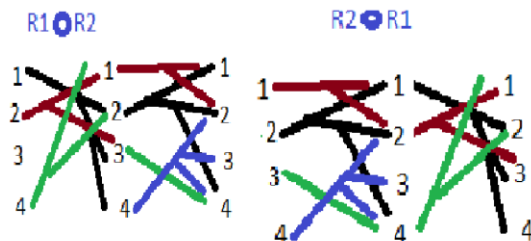b) Whether R_2 is reflexive, irreflexive, symmetric, anti-symmetric and/or transitive.

Lacks any $[x,x]$ so **is irreflexive, not reflexive**. Only two points, ($[1,2]\ and\ [2,1]$) are symmetric therefore is **not symmetric** and is **not anti-symmetric**. Lacks (4,3) as (4,1) relates to (1,3) but there is no (4,3) **not is transitive**.

c) $R_1 \circ R_2$.

$\{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (4,1), (4,2)\}$

d) $R_2 \circ R_1$.

$\{(1,2), (1,3), (1,4), (1,1), (2,2), (2,3), (2,4), (2,1), (3,1), (3,2), (4,1), (4,3), (4,2)$

R1○R2   R2○R1

e) $R_1 \cup R_2$.

$\{(1,1), (1, 2), (2, 1), (2,2), (2, 4), (3, 4), (4, 2), (4, 3), (4,4), (1, 3), (1, 4), (2, 3), (4, 1)\}$

f) $R_1 \cap R_2$.

$\{(1,2), (2,1), (4,2)\}$

g) The reflexive, symmetric and transitive closures of both R_1 and R_2. (14 pairs->r1 12 pairs ->r2)

2. **(10)** Let R be a binary relation over the positive integers defined as follows:

$$R = \{(a, b) \mid 2b < a < 3b\}$$

Determine whether R satisfies the following properties. Give brief justifications for your answers.

(i) reflexive

a can never be equal to b [$a\ != b$] because $2a$ can never be less than a [$2a < a$ doesn't work]. For example, $(1,1)$ doesn't work because $2 < 1 < 3$ is false. Therefore, it **isn't reflexive**

(ii) irreflexive

Since there can never be any reflexive pairs **it must be irreflexive**

(iii) symmetric

a can never be smaller than b [$b < a$ must be true for a possible combination] or the equation doesn't work and therefore there can never be a symmetric pair. For example, $(5,2)$ works but $(2,5)$ does not. Therefore, it **is not symmetric**

(iv) anti-symmetric

Because there cannot be any symmetric or reflexive pairs therefore **it must be anti-symmetric**

(v) transitive

$(4,10)$ & $(10,25)$ are both true however $(4,25)$ is not and therefore **is not transitive**

3. **(10)** Let $A = \{1,2, 3, 4, 5,6, 7, 8\}$. How many possible symmetric relations over A contain the ordered pairs $(2, 3)$, $(3, 2)$, $(4, 7)$, $(5, 5)$ and $(8, 7)$?

4. **(9)** Let $f(x) = \sqrt{e^x}$ and $g(x) = x^2$.

   a. (6) Determine $h_1(x) = f(g(x))$ and $h_2(x) = g(f(x))$.

   $$h_1(x) = \sqrt{e^{x^2}}$$
   $$h_2(x) = e^x$$

   b. (3) What are the largest possible domains for which h_1 and h_2 can be defined?

   $$h_1 = domain = (-\infty, \infty)$$
   $$h_2 = domain = (-\infty, \infty)$$