1.) Determine the matrix product in terms of n: $\begin{bmatrix} 2n+1 & 7 \\ -n+1 & 3 \end{bmatrix}\begin{bmatrix} 6 & n+1 \\ 3n-1 & 2 \end{bmatrix}$

$\begin{bmatrix} 33n-1 & 3n+1 \\ 3n+3 & 7 \end{bmatrix}$

2.) Using induction on n, prove for all positive integers n that: $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} F_{2n+1} & F_{2n} \\ F_{2n} & F_{2n-1} \end{bmatrix}$

Where F_n denotes the n^{th} Fibonacci number

Induction on n

$base\ case\ n = 1$

LHS $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^1 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$	Substitution
RHS $\begin{bmatrix} F_{2(1)+1} & F_{2(1)} \\ F_{2(1)} & F_{2(1)-1} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$	Fibonacci Sequence
Induction Hypothesis: $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^k = \begin{bmatrix} F_{2k+1} & F_{2k} \\ F_{2k} & F_{2k-1} \end{bmatrix}$	
$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^k * \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^1 = \begin{bmatrix} F_{2(k+1)+1} & F_{2(k+1)} \\ F_{2(k+1)} & F_{2(k+1)-1} \end{bmatrix}$	Exponents
$\begin{bmatrix} F_{2k+1} & F_{2k} \\ F_{2k} & F_{2k-1} \end{bmatrix} * \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} F_{2(k+1)+1} & F_{2(k+1)} \\ F_{2(k+1)} & F_{2(k+1)-1} \end{bmatrix}$	Induction Hypothesis
$\begin{bmatrix} 2^1 * F_{2k+1} + F_{2k} & F_{2k+1} + F_{2k} \\ 2^1 * F_{2k} + F_{2k-1} & F_{2k} + F_{2k-1} \end{bmatrix} = \begin{bmatrix} F_{2k+3} & F_{2k+2} \\ F_{2k+2} & F_{2k+1} \end{bmatrix}$	Arithmetic
$\begin{bmatrix} F_{2k+2} + F_{2k} & F_{2k+1} + F_{2k} \\ F_{2k+1} + F_{2k-1} & F_{2k+1} + F_{2k-1} \end{bmatrix} = \begin{bmatrix} F_{2k+3} & F_{2k+2} \\ F_{2k+2} & F_{2k+1} \end{bmatrix}$	Arithmetic
$\begin{bmatrix} F_{2k+3} & F_{2k+2} \\ F_{2k+2} & F_{2k+2} \end{bmatrix} = \begin{bmatrix} F_{2k+3} & F_{2k+2} \\ F_{2k+2} & F_{2k+1} \end{bmatrix}$	Arithmetic with Fibonacci
$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} F_{2n+1} & F_{2n} \\ F_{2n} & F_{2n-1} \end{bmatrix}$	Induction Principle

3.) Prove by induction on n, for all positive integers n: $\sum_{i=1}^{n}\big(i(i!)\big) = (n+1)! - 1$

Induction on n

$base\ case\ n = 1$

$\sum_{i=1}^{1}\big(1(1!)\big) = (1+1)! - 1$	Substitution

$1 = 1$ Arithmetic

Induction Hypothesis: $\sum_{i=1}^{k}\big(i(i!)\big) = (k+1)! - 1$

Induction Step: $\sum_{i=1}^{k+1}\big(i(i!)\big) = \big((k+1)+1\big)! - 1$

$\sum_{i=1}^{k+1}\big(i(i!)\big) = (k+2)! - 1$ Arithmetic

$\sum_{i=1}^{k}(i(i!)) + \big((k+1)(k+1)!\big) = (k+2)! - 1$ Definition of Summations

$\big((k+1)! - 1\big) + \big((k+1)(k+1)!\big) = (k+2)! - 1$ Induction Hypothesis

$(k+1)!\,\big((k+1)+1\big) - 1 = (k+2)! - 1$ Arithmetic

$(k+1)!\,(k+2) - 1 = (k+2)! - 1$ Arithmetic

$(k+2)! - 1 = (k+2)! - 1$ Definition of Factorial

$\therefore \sum_{i=1}^{n}\big(i(i!)\big) = (n+1)! - 1$ Induction Principle

4.) Prove by induction on n, for all positive integers n: $\sum_{i=1}^{n}\left(i(6^i)\right) = \frac{6(5n6^n - 6^n + 1)}{25}$

Induction on n

$base\ case\ n = 1$

$\sum_{i=1}^{1}\big(1(6^1)\big) = \frac{6\big(5(1)6^1 - 6^1 + 1\big)}{25}$ Substitution

$6 = 6$ Arithmetic

Induction Hypothesis: $\sum_{i=1}^{k}\left(i(6^i)\right) = \frac{6(5k6^k - 6^k + 1)}{25}$

Induction Step: $\sum_{i=1}^{k+1}\left(i(6^i)\right) = \frac{6(5(k+1)6^{k+1} - 6^{k+1} + 1)}{25}$

$\sum_{i=1}^{k+1}\left(i(6^i)\right) = \frac{6\big((5k+5)6^{k+1} - 6^{k+1} + 1\big)}{25}$ Arithmetic

$\sum_{i=1}^{k}\left(i(6^i)\right) + (k+1)(6^{k+1}) = \frac{6(5(k+1)6^{k+1} - 6^{k+1} + 1)}{25}$ Definition of Summation

$\frac{6(5k6^k - 6^k + 1)}{25} + (k+1)(6^{k+1}) = \frac{6(5(k+1)6^{k+1} - 6^{k+1} + 1)}{25}$ Induction Hypothesis

$\frac{6(5k6^k - 6^k + 1)}{25} + \frac{25(k+1)(6^{k+1})}{25} = \frac{6(5(k+1)6^{k+1} - 6^{k+1} + 1)}{25}$ Arithmetic

$\frac{6(5k6^k-6^k+1)+25(6^{k+1}k+6^{k+1})}{25} = \frac{6(5(k+1)6^{k+1}-6^{k+1}+1)}{25}$ Arithmetic

$\frac{6(5k6^k-6^k+1)+(25*6^{k+1}k+25*6^{k+1})}{25} = \frac{6(5(k+1)6^{k+1}-6^{k+1}+1)}{25}$ Arithmetic

$\frac{6(5(k+1)6^{k+1}-6^{k+1}+1)}{25} = \frac{6(5(k+1)6^{k+1}-6^{k+1}+1)}{25}$ Arithmetic

$\therefore \sum_{i=1}^{n}\left(i\left(6^i\right)\right) = \frac{6(5n6^n-6^n+1)}{25}$ Induction principle

5.) Prove by induction on n, for all positive integers n: $21|(4^{n+1}+5^{2n-1})$

Induction on n

$base\ case\ n = 1$

$21|(4^{1+1}+5^{2(1)-1})$ Substitution

$\frac{21}{21} = 1$ Arithmetic

Induction Hypothesis: $21|(4^{k+1}+5^{2k-1})$

Induction Step: $21|(4^{(k+1)+1}+5^{2(k+1)-1})$

$4^{k+2}+5^{2k+1}$ Arithmetic

$4(4^{k+1})+5^{2k+1}$ Exponents

$4(21x-5^{2k-1})+5^{2k+1}$ Induction Hypothesis

$4(21x-5^{2k-1})+25*5^{2k-1}$ Powers

$4*21x+21*5^{2k-1}$ Distribution

$21(4x+5^{2k-1})$ Distribution

$21(4x+5^{2k-1})$ is divisible by 21 Distribution

$\therefore 21|(4^{n+1}+5^{2n-1})$ By induction

6.) Let t_n be a sequence defined as follows:

a.) $t_0 = 5$, b.) $t_1 = 12$, c.) $t_n = 5t_{n-1} - 6t_{n-2}$ for $n \geq 2$

$base\ case\ n = 2$

$t_2 = 5t_1 - 6t_0$ Substitution

$30 = 5(12) - 6(5)$	Arithmetic
$30 = 30$	Arithmetic

$base\ case\ n = 3$

$t_3 = 5t_2 - 6t_1$	Substitution
$78 = 5(30) - 6(12)$	Arithmetic
$78 = 78$	Arithmetic

Inductive Hypothesis: $t_k = 5t_{k-1} - 6t_{k-2}$

Inductive Step: $t_{k+1} = 5t_{k+1-1} - 6t_{k+1-2}$

$t_k + t_{k-1} = 5t_k - 6t_{k-1}$	Definition of a Sequence
$5t_{k-1} - 6t_{k-2} + 5t_{k-2} - 6t_{k-3} = 5t_k - 6t_{k-1}$	Inductive Hypothesis
$5t_{k-1} + 5t_{k-2} - 6t_{k-2} - 6t_{k-3} = 5t_k - 6t_{k-1}$	Commutativity
$5t_k - 6t_{k-1} = 5t_k - 6t_{k-1}$	Definition of a Sequence
$\therefore t_0 = 5, t_1 = 12, t_n = 5t_{n-1} - 6t_{n-2}$ for $n \geq 2$	By Induction