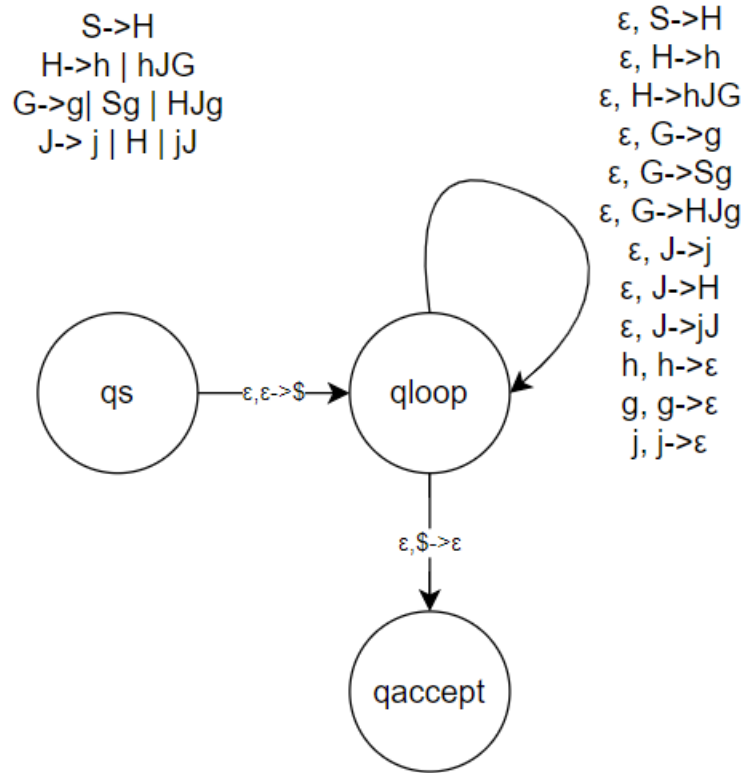


## Quiz 2

- (5) Convert  $G$  to a PDA using the method described in class

PDA



- (10) Convert  $G$  to Chomsky Normal Form

- $S \rightarrow H$
- $H \rightarrow h \mid hJG$
- $G \rightarrow g \mid Sg \mid HJg$
- $J \rightarrow h \mid H \mid jJ$

Add  $S$  and remove  $\epsilon$ :

$S_0 \rightarrow S \mid \epsilon$   
 $H \rightarrow h \mid hJG$   
 $G \rightarrow g \mid Sg \mid HJg$   
 $J \rightarrow h \mid H \mid jJ$

Remove single rewrites:

$S_0 \rightarrow h \mid hJG \mid \epsilon$   
 $H \rightarrow h \mid hJG$

$$G \rightarrow g|Sg|HJg$$

$$J \rightarrow h|h|hJG|jJ$$

Remove mixed/multiple terminals:

$$S_0 \rightarrow S|\epsilon$$

$$H \rightarrow h|H_0J_1$$

$$G \rightarrow g|SG_0|H_1G_0$$

$$J \rightarrow h|H|J_0J$$

$$H_0 \rightarrow h$$

$$H_1 \rightarrow HJ$$

$$G_0 \rightarrow g$$

$$J_0 \rightarrow j$$

$$J_1 \rightarrow JG$$

Remove long rewrites to get CNF:

$$S_0 \rightarrow S|\epsilon$$

$$H \rightarrow h|H_0JG$$

$$G \rightarrow g|SG_0|HJG_0$$

$$J \rightarrow h|H|J_0J$$

$$H_0 \rightarrow h$$

$$G_0 \rightarrow g$$

$$J_0 \rightarrow j$$

3. (10) Let  $FINITE_{TM} = \{ \langle M \rangle \mid M \text{ is a Turing machine, and } L(M) \text{ is finite} \}$ . Show that  $FINITE_{TM}$  is undecidable.

We know that all finite languages are regular, therefore if we prove that  $REGULAR_{TM} = \{ \langle M \rangle \mid M \text{ is a Turing machine, and } L(M) \text{ is regular} \}$  is undecidable, we know that  $FINITE_{TM}$  also has to be undecidable

Proof: Assume BWOC that  $REGULAR_{TM}$  is decidable, by definition of decidability let  $M_{REGULAR}$  is its decider

Then we build a  $M_{accept}$  where:

- We receive Turing machine  $M$  and string  $s$
- We mimic  $A_{TM}$
- We construct a new Turing machine  $M_{Internal}$  that,
  - Receives string  $x$
  - Accepts if  $x$  is in  $0^n 1^n$  for some  $n$
  - Otherwise simulates  $M$  on  $s$  and mimic if  $M$  accepts, rejects or fails to halt

Interrogate  $M_{Internal}$  for regularity using  $M_{REGULAR}$  and accept or reject in agreement. This causes a dichotomy and makes  $M_{Accept}$  decide  $A_{TM}$  and creates a contradiction, therefore  $FINITE_{TM}$  is undecidable