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May 2025

1 Executive summary

The bonus certificate offers a compelling investment vehicle, combining low-cost protection with high-growth potential, tailored to volatile markets. Its structure appeals to risk-averse investors seeking capital preservation while retaining upside exposure, making it a competitive alternative to conventional options-based products.

- **Price**: USD 23.9 (vs. stock price: USD 21.6)
- **Bonus (B)**:USD 21.5 (minimum guaranteed return unless barrier is breached in 15% cases)
- **Barrier Level (H)**: USD 5 (77% loss protection in 85% cases)
- **Maturity (T)**: 1 year

2 Methodology

2.1 Data

We fetch Option prices for Intel from yahoo finance (Yahoo! Inc 2025) on 2025-05-14. We clean the options with bid-ask spread ratio higher than 95% quantile. We also remove short-dated (less than 5 days to maturity) options. We also clean options with no or 0 bid/ask price. The stock price during the fetch was USD 21.6. We also remove most ITM options from the data leaving only those call options with strikes $K \geq 20.52$ and put options with strikes $K \leq 22.68$. The reason is that those options traders tend to trade OTM options while ITM options are usually held by investors. Hence, OTM options are often better priced and more liquid. We choose to leave some options close to ATM for better calibration around ATM.

We fetch yield for US treasuries from FRED (Federal Reserve Bank of St. Louis 2025) with maturities: 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y, 20Y, 30Y. We use the yields on 2025-05-14 for pricing.

Intel currently does not pay dividends. Hence, we assume that option traders do not price in the dividend yield until further announcements.

2.2 Options pricing model

We use Bates (Bates 1996) model for the distribution and dynamics of stock process. Bates model combines Heston stochastic volatility model (Heston 1993) for continuous component and Merton Jump-diffusion model (Merton 1976) for the jump process. Under Bates model stock process follows the following stochastic differential equations (SDEs) (Leoni 2025):

$$\begin{aligned} \tfrac{dS_t}{S_t} &= (r - q - \lambda\mu_j)\,dt + \sqrt{v_t}\,dW_t^S + J_t\,dN_t \\ dv_t &= \kappa(\theta - v_t)\,dt + \sigma_v\sqrt{v_t}\,dW_t^v \end{aligned} \tag{1}$$

where: r - risk-free rate; q - dividend yield; κ - mean-reversion speed of volatility; θ - long-term mean volatility; σ_v: Volatility of the variance (vol of vol); ρ - correlation between W_t^S and W_t^v; N_t - an independent Poisson process with frequency λ; $\log(1+J_t) \sim \text{Normal}\left(\log(1+\mu_J) - \frac{\sigma_J^2}{2}, \sigma_J^2\right)$.

We extend r to $r(t)$ in order to include the term structure of interest rate. The extension method is different for FFT algorithm and MC simulation, and we cover the specifics in the respective sections.

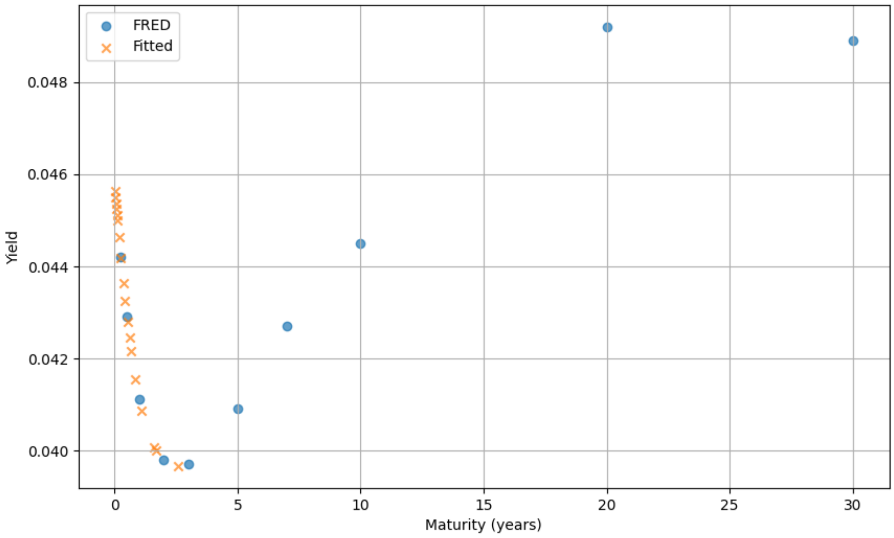

Figure 1: Interest rate fitted with cubic spline

2.3 Fast Fourier transform pricing algorithm

The algorithm uses Carr-Madan formula (Carr and Madan 1999)(Leoni 2025):

$$C(k,T) = e^{-\alpha k}\frac{1}{\pi}\int_0^{+\infty} e^{-\mathrm{i}vk}\varrho(v)\mathrm{d}v \tag{2}$$

where: $\varrho(v) = \dfrac{\exp(-rT)\phi(v-(\alpha+1)i,T)}{\alpha^2+\alpha-v^2+i(2\alpha+1)v}$; $\phi(\cdot)$ - a characteristic function of $\ln S_t$ under Bates.

The integral is approximated with Fast Fourier transform algorithm and Simpson's rule in the following form:

$$C(k,T) \approx \exp(-\alpha k)\frac{1}{\pi}\sum_{j=1}^{N}\exp(-\mathrm{i}v_j k)\varrho(v_j)\eta\left(\frac{3+(-1)^j-\delta_{j-1}}{3}\right), \qquad v_j = \eta(j-1) \tag{3}$$

where: $\lambda\eta = 2\pi/N$; The strike range $k_n = -b + \lambda(j-1)$, j as above, $\lambda = 2b/N$. We choose $N = 2^{14}$ and $\eta = 0.2$ to balance the strike grid resolution and approximation of the integral. We choose $\alpha = 1.5$ based on the lectures.

We use r, T pairs in the formula to model the term structure of the interest rate. The values are modelled using cubic spline (Press et al. 2007) as can be seen in the Figure 1. This approach assumes a constant risk-free rate in the period before maturity for each option with maturity T.

To calculate the put option payoff we use the put-call parity:

$$Put(K,T) = Call(K,T) - S_0e^{-qT} + Ke^{-rT} \tag{4}$$

where: K - strike; T - maturity.

2.4 Monte-Carlo simulation

We use Monte-Carlo simulation for stock paths. in order to price the exotic structure of the product. We use the same parameters for Bates as in the FFT pricing. We need to generate four random components: correlated Brownian motions (BMs), jump size, jump frequency - three independent standard normal distributions $N(0,1)$ and a Poisson distribution $Pois(\lambda dt)$ where dt is discrete time step. We use Ziggurat algorithm (Marsaglia and Tsang 2000) for standard normal distribution and Knuth algorithm (Knuth 1997) for Poisson distribution, both are implemented in numpy.

We relate standard normals to create correlated BMs as follows:

$$W^v = \rho W^S + \sqrt{1-\rho^2} N_2(0,1) \tag{5}$$

where: $W^S \sim N_1(0,1)$.

We use slightly different jump formula for generation in contrast to section 2.2. We generate the log jump size as follows:

$$J = \mu_j + \sigma_j N_3(0,1) \tag{6}$$

where J - log jump size. Hence, we need to change the drift compensation term from $\lambda\mu_j$ to $\lambda(e^{\mu_j+\frac{1}{2}\sigma_j^2}-1)$.

Stochastic volatility component is discretized with Milstein method and truncated to avoid negative volatility:

$$v_{t+1} = \max\left[v_t + \kappa(\theta - v_t)dt + \sigma_v\sqrt{v_t dt}W_t^v + \frac{1}{4}\sigma_v^2 dt(W_t^{v2} - 1), 0\right] \tag{7}$$

where: v_t - variance at discrete time step t with step size dt.

We also calculate r_t, which is the risk-free rate between maturities t-1 and t. We first extrapolate term structure for all maturities in the simulation $[0,T]$ with step size dt i.e. $0, dt, 2dt, 3dt, ..., T$. We fit a cubic spline similar, which is in fact the same as in 1 but with more fitted values. The we calculate drift term r(t) as follows:

$$r_t = \frac{r(t)t - r(t-dt)(t-dt)}{dt} \tag{8}$$

where: $r(t), r(t-dt)$ - interest rate for maturity t, t-dt. This is in fact equivalent to our approach in section 2.3 due to deterministic formula of interest rate.

The log return is then as follows:

$$\Delta X_t = [r_t - q - \lambda(e^{\mu_j+\frac{1}{2}\sigma_j^2} - 1) - \frac{1}{2}v_t]dt + \sqrt{v_t dt}W_t^S + Pois(\lambda dt)J_t \tag{9}$$

where: $Pois(\lambda dt)$ - 0 or 1 due to small dt; $\Delta X_t = X_{t+1} - X_t$, $X_t = \log S_t$.

The stock paths are then calculated as follows:

$$S_t = S_0 \exp\left(\sum_{i=1}^{t} \Delta X_i\right) \tag{10}$$

The paths are then used to calculate the price of exotic or vanilla options according to their payoff.

We calculate call payoff as follows:

$$\text{Payoff}_{call} = \max(S_T - K, 0) \tag{11}$$

We calculate put payoff as follows:

$$\text{Payoff}_{put} = \max(K - S_T, 0) \tag{12}$$

We calculate Down and Out Barrier Put (DOBP) payoff as follows:

$$\text{Payoff}_{BC} = \begin{cases} \max(K - S_T, 0), & \min(S_t) > H \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where: S_T - stock price at maturity T; K - strike price; H - barrier level.

2.5 Calibration

We calibrate the model on the FFT algorithm due to its computational efficiency.

We test several loss functions: Weighted Mean Squared error, Weighted Relative Squared Error, Weighted Normalized Absolute Percentage Error (WNAPE), Weighted Log Ratio Error. Weighted squared error is a bad choice because it implicitly gives more weight to options with higher prices hence longer maturity and closer to ATM. We want to fully control the weight to option prices with the weight term, hence we prefer relative error loss functions. We choose WNAPE because it shows better calibration performance.

The WNAPE loss function is constructed as follows:

$$L = \frac{1}{n}\sum_{i=1}^{n} w_i \frac{|y_{i,model} - y_{i,data}|}{|y_{i,model}| + |y_{i,data}| + \epsilon} \tag{14}$$

where: y - option prices; $\epsilon = 0.001$ - a small term which we use to ignore errors for prices close to 0.

We choose weights such that errors for options close to at-the-money in terms of strike and short dated options (small T) are emphasized. We use the following equation:

$$w_i = 10 \exp\left(-6\frac{|S_0 - K_i|}{S_0 + K_i} - T_i^2\right) \tag{15}$$

where: S_0 - the initial stock price; K - strike; T - maturity.

We log transform parameters that must be positive $(\kappa, \theta, \sigma_v, v_0, \sigma_J)$. We also give logical bounds to other parameters to avoid overfitting: κ: [0.01,10]; θ:[0.01,1]; σ_v: [0.01,2]; ρ: [-0.999, 0.999]; v_0: [0.01,1]; λ: [0,10]; μ_J: [-1,1]; σ_J: [0.01,0.5]. We use scipy implementation of L-BFGS-B algorithm to find the minimal loss.

2.6 Product structure

We structure a bonus certificate with the following payoff:

$$\text{Payoff}_{BC} = \begin{cases} \max(S_T, B), & \min(S_t) > H \\ S_T, & \text{otherwise} \end{cases} \tag{16}$$

where: B - bonus amount; H - barrier level.

We structure this payoff by buying a stock and buying DOBP with strike B and barrier level H.

For the optimal barrier level H, bonus B and maturity T. We decide to choose such H that gives protection to approximately 85% of possible losses. We check this level by approximating the mean frequency of knock out in the simulated data. This results in H at USD 5. For B, we choose the level that gives the most protection - USD 21.5. We choose maturity of 1 year as a usual investment horizon in protected products.

2.7 Delta

We calculate delta using the finite differences method as follows:

$$\Delta = \frac{f(S_0 + \epsilon) - f(S_0 - \epsilon)}{2\epsilon} \tag{17}$$

where: ϵ - small constant. We use 0.5% of S_0; S_0 - current stock price i.e. if we simulate delta this changes; $f(.)$ - derivative price.

We calculate the delta of DOBP and add it to the delta of the stock. Hence, we expect a positive delta. However, we sell the product, hence our delta is negative.

3 Results

3.1 Model fit

We show the results of calibration in the figure 2.

The final parameter values are as follows: κ: 0.0823; θ: 0.0354; σ_v: 0.1740; ρ: -0.999; v_0: 0.1356; λ: 1.5473; μ_J: 0.0615; σ_J: 0.2382.

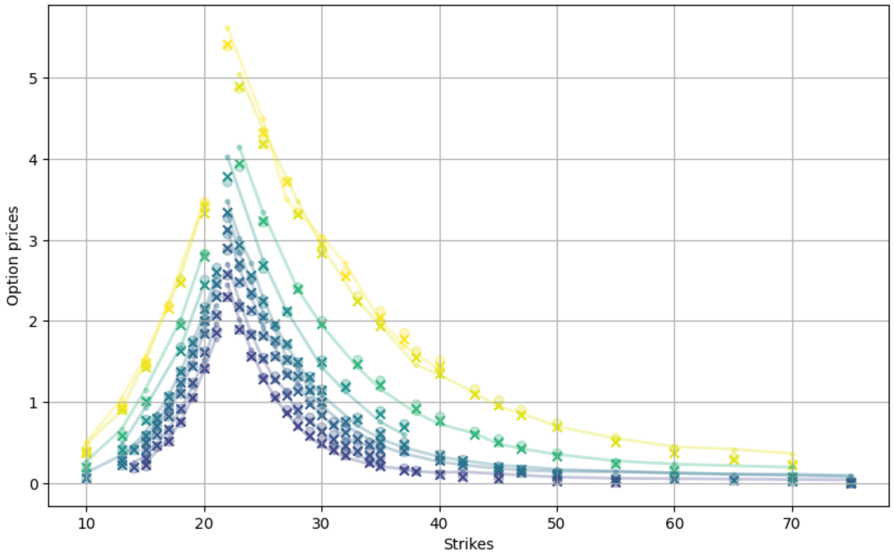

Figure 2: Model fit
The lines show option prices in the market data. X are values fitted with Bates model. Circles show prices generated with MC simulation and the same parameters.

Furthermore, volatility of stock paths in the Monte Carlo simulation is 46.68% which matches well the historical volatility of INTC.

3.2 Hedging strategy

We need to hedge our position in the product. We sell them BC that has positive delta, $\Delta_{stock} = 100\%$ and $\Delta_{DOBP} = -22.26\%$. Hence, our delta is $\Delta = -77.74\%$. Considering that we have notional of USD 1 000 000 in BC (notional=subtracted premium), we need to buy approximately 35 982 INTC stocks to become delta neutral.

We can continuously hedge our position afterwards.

3.3 Product attractiveness

The final payoff of the product is as follows:

$$\text{Payoff}_{BC} = \begin{cases} \max(S_T, 21.5), & \min(S_t) > 5 \\ S_T, & \text{otherwise} \end{cases} \tag{18}$$

The product payoff is similar to that of a principal protected note conditional on never breaching the barrier. This condition lets us offer partial protection with full participation in the underlying at a much cheaper price. With no premium the product price is USD 23.8359. We can round up the price to USD 23.9 and take the remainder as our profits.

The attractiveness of the product is in the following:

- The product covers for losses in 85% of cases up to 77% loss;
- The cost is minimal. Only USD 2.3 in addition to stock. When compared to buying puts that might cost USD 3.5 in addition to stock for the same maturity and strike;
- Given the volatility of 46.6% of INTC, the premium is covered very easily in either direction;
- Significantly increases potential return-to-risk ratios.

References

Bates, David S (1996). “Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options”. In: *The Review of Financial Studies* 9.1, pp. 69–107.

Carr, Peter and Dilip B Madan (1999). “Option valuation using the fast Fourier transform”. In: *Journal of Computational Finance* 2.4, pp. 61–73.

Federal Reserve Bank of St. Louis (2025). *FRED Economic Data.* US Treasury yield curve data accessed on 2025-05-17. URL: `https://fred.stlouisfed.org`.

Heston, Steven L (1993). “A closed-form solution for options with stochastic volatility with applications to bond and currency options”. In: *The Review of Financial Studies* 6.2, pp. 327–343.

Knuth, Donald E. (1997). *The Art of Computer Programming, Volume 2: Seminumerical Algorithms.* 3rd. Reading, Massachusetts: Addison-Wesley, pp. 137–138. ISBN: 0-201-89684-2.

Leoni, Peter (2025). *Financial Engineering Lectures.* Lecture notes.

Marsaglia, George and Wai Wan Tsang (2000). “The Ziggurat Method for Generating Random Variables”. In: *Journal of Statistical Software* 5.8, pp. 1–7. DOI: `10.18637/jss.v005.i08`.

Merton, Robert C (1976). “Option pricing when underlying stock returns are discontinuous”. In: *Journal of Financial Economics* 3.1-2, pp. 125–144.

Press, William H. et al. (2007). *Numerical Recipes: The Art of Scientific Computing.* 3rd. Covers cubic spline theory and implementation. Cambridge University Press. Chap. 3.3.

Yahoo! Inc (2025). *Yahoo Finance.* Intel (INTC) option chain data accessed on 2025-05-14. URL: `https://finance.yahoo.com`.