GATE SOLVED PAPER
Computer Science Engineering
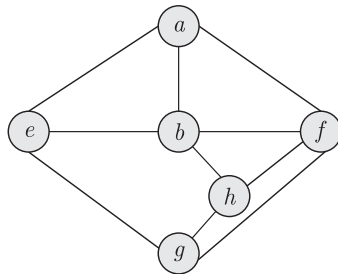**2003**

Q. 1   Consider the following three claims
1. $(n+k)^m = \Theta(n^m)$, where k and m are constants
2. $2^{n+1} = O(2^n)$
3. $2^{2^n+1} = O(2^n)$

Which of these claims are correct?
(A) 1 and 2   (B) 1 and 3

(C) 2 and 3   (D) 1, 2 and 3

Q. 2   Consider the following graph

Among the following sequences
1. *abeghf*   2. *abfehg*
3. *abfhge*   4. *agfhbe*

Which are depth first traversals of the above graph?
(A) 1, 2 and 4 only   (B) 1 and 4 only

(C) 2, 3 and 4 only   (D) 1, 3 and 4 only

Q. 3   The usual $\Theta(n^2)$ implementation of Insertion Sort to sort ab array uses linear search to identify the position where an element is to be inserted into the already sorted part of the array. If, instead, we use binary search to identify the position, the worst case running time will
(A) remain $\Theta(n^2)$ (B) become $\Theta(n(\log n)^2)$

(C) become $\Theta(n\log n)$ (D) become $\Theta(n)$

Q. 4   In a heap with n elements with the smallest element at the root, the 7th smallest element ban be found in time
(A) $\Theta(n\log n)$ (B) $\Theta(n)$

(C) $\Theta(\log n)$ (D) $\Theta(1)$

Common Data For Q. 5 and 6

Solve the problems and choose the correct answers.
In a permutation $a_1....a_n$ of n distinct integers, an inversion is a pair (a_l, a_j) such that $i < j$ and $a_i > a_j$.

Q. 5   If all permutation are equally likely, what is the expected number of inversions in a randomly chosen permutation of $1.....n$?
(A) $n(n-1)/2$
(B) $n(n-1)/4$
(C) $n(n+1)/4$
(D) $2n[\log_2 n]$

Q. 6   What would be the worst case time complexity of the insertion Sort algorithm, if the inputs are restricted to permutations of $1....n$ with at most n inversions?
(A) $\Theta(n^2)$ (B) $\Theta(n\log n)$
(C) $\Theta(n^{1.5})$ (D) $\Theta(n)$

Q. 7   The cube root of a natural number n is defined as the largest natural number m such that $m^3 \le n$. The complexity of computing the cube root of n (n is represented in binary notation) is
(A) $O(n)$ but not $O(n^{0.5})$
(B) $O(n^{0.5})$ but not $O(\log n)^k$) for any constant $k > 0$
(C) $O(\log n)^k$) for some constant $k > 0$, but not $O(\log\log n)^m$) for any constant $m > 0$
(D) $O(\log\log n)^k$) for some constant $k > 0.5$, but not $O(\log\log n)^{0.5}$)

Q. 8   Let $G = (V, E)$ be an undirected graph with a sub-graph $G_1 = (V_1, E_1)$, Weight are assigned to edges of G as follows
$$w(e) = \begin{cases} 0 \text{ if } e \in E, \\ 1 \text{ otherwise} \end{cases}$$

A single-source shortest path algorithm is executed on the weighted graph (V, E, w) with an arbitrary vertex v_1 of V_1 as the source. Which of the following can always be inferred from the path costs computed?
(A) The number of edges in the shortest paths from v_1 to all vertices of G
(B) G_1 is connected
(C) V_1 forms a clique in G
(D) G_1 is a tree

Q. 9   What is the weight of a minimum spanning tree of the following graph?
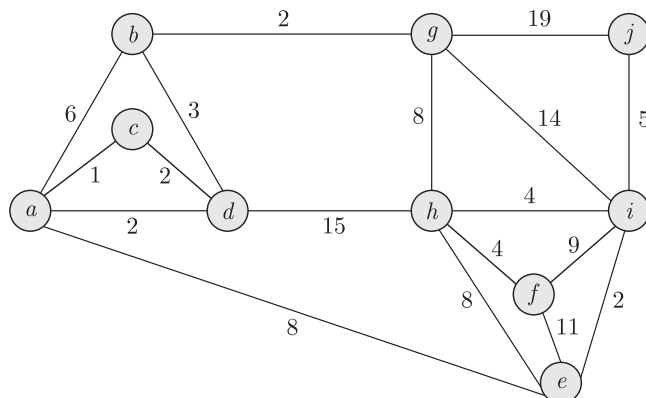
(A) 29   (B) 31
(C) 38   (D) 41

Q. 10   The following are the starting and ending times of cetivities A, B, C, D, E, F, G and H respectively in chronological order;"$a_s b_s a_a a_e d_s a_e e_s f_s b_e d_e g_s e_e f_e h_s g_e h_e$'Here, x_s denotes the starting time and x_e denotes the ending time of activity X. W need to schedule the activities in a set of rooms available to us. An activity can be scheduled in a room only if the room is reserved for the activity for its entire duration. What is the minimum number of rooms required?

(A) 3   (B) 4

(C) 5   (D) 6

Q. 11   Let $G = (V, E)$ be a direction graph with n vertices. A path from v_i to v_j in G is sequence of vertices $(v_i, v_{i1},....., v_j)$ such that $(v_k, v_{k+1}) \in E$ for all k in i through $j - 1$. A simple path is a path in which no vertex appears more than once.

Let A be an $n \times n$ array initialized as follow

$$A[j,k] = \begin{cases} 1 \text{ if } (j,k) \in E \\ 0 \text{ otherwise} \end{cases}$$

Consider the following algorithm

```
for i=1 to n
  for i = 1 to n
    for k = 1 to n
        A[j,k]=max (A[j,k](A[j,i]+ A[i,k]);
```

Which of the following statements is necessarily true for all j and k after terminal of the above algorithm?

(A) $A[j, k] \le n$

(B) If $A[j,j] \le n - 1$, then G has a Haniltonian cycle

(C) If there exists a path from j to k, $A[j, k]$ contains the longest path lens from j to k

(D) If there exists a path from j to k, every simple path from j to k contain most $A[j, k]$ edges

Q. 12   Which of the following suffices to convert an arbitrary CFG to an LL(1) grammar?

(A) Removing left recursion alone

(B) Factoring the grammar alone

(C) Removing left recursion and factoring the grammar

(D) None of this

Q. 13   Assume that the SLR parser for a grammar G has n_1 states and the LALR parser for G has n_2 states. The relationship between n_1 and n_2 is

(A) n_1 is necessarily less than n_2

(B) n_1 is necessarily equal to n_2

(C) n_1 is necessarily greater than n_2

(D) None of the above

Q. 14   In a bottom-up evaluation of a syntax directed definition, inherited attributes can

(A) always be evaluated

(B) be evaluated if the definition is L-attributed

(C) be evaluated only if the definition has synthesized attributes

(D) never be evaluated

Q. 15   Which of the following statements is FALSE?
(A) In statically typed language, each variable in a program has a fixed type

(B) In up-typed languages, values do not have any types

(C) In dynamically typed languages, variables have no types

(D) In all statically typed languages, each variable in a program is associated
with values of only a single type during the execution of the program

Q. 16   Consider the grammar shown below
$$S \to | \, EtSS \, | \, \alpha$$
$$S \to eS \, | \in$$
$$E \to b$$
In the predictive parse table. M, of this grammar, the entries $M[S',\varepsilon]$ and $M[S,\$]$
respectively are
(A) $\{s' \to eS\}$ and $\{S \to \varepsilon\}$ (B) $\{s' \to eS\}$ and $\{\}$

(C) $\{s' \to \varepsilon\}$ and $\{S \to \varepsilon\}$ (D) $\{s' \to eS, S \to \varepsilon\}$ and $\{S \to \varepsilon\}$

Q. 17   Consider the grammar shown below.
$$S \to C \, C$$
$$C \to eC \, | \, d$$
The grammar is
(A) LL (1)   (B) SLR (1) but not LL (1)

(C) LALR (1) but not SLR (1)   (D) LR (1) but not LALR (1)

Q. 18   Consider the translation scheme shown below
$S \to TR$
$R \to + \, T \, \{\text{print } ('+');\} \, R \, | \, \varepsilon$
$T \to \text{num } \{\text{print (num.val);}\}$
Here num is a token that represents an integer and num. val represents the
corresponding integer value. For an input string '9 + 5+ 2', this translation
scheme will print
(A) $9 + 5 + 2$ (B) $9\,5 + 2 +$

(C) $9\,5\,2 + +$ (D) $+ + 9\,5\,2$

Q. 19   Consider the syntax directed definition shown below
```
S→id:=E              {gen(id.place=E.place;);}
E→E₁+e₂             {t= newtemp ();
                     gen(t=E₁. place +E₂. place;);
                     E.place =t}
E→id                {E.place =id.place;}
```

Here, gen is a function that generates the output code, and newtemp is a function
that returns the name of a new temporary variable on every call. Assume that t_i
's are the temporary variable names generated by newtemp.
For the statement '$X := Y + Z$', the 3-address code sequence generated by this
definition is
(A) $X = Y + Z$ (B) $t_1 = Y + Z; X \, t_1$

(C) $t_1 = Y; t_2 = t_1 + Z; X = t_2$ (D) $t_1 = Y; t_2 = Z; t_3 + t_2; X = t_3$

Common Data For Q. 20 & 21

Solve the problems and choose the correct answers.
The following program fragment is written in a programming language that allows variables and does not allow nested declarations of functions.

```
global int i=100, j=5;
void P(x){
        int i=10
        print (x+10);
        i+200;
        j=20;
        print (x);
}
main () {P(i+j);}
```

Q. 20   If the programming language uses static scoping and call by need parameter passing mechanism, the values printed by the above program are
(A) 115, 220   (B) 25, 220
(C) 25, 15   (D) 115, 105

Q. 21   If the programming language uses dynamic scoping and call by name parameter passing mechanism, the values printed by the above program are
(A) 115, 220   (B) 25, 220
(C) 25, 15   (D) 115, 105

Q. 22   Consider the following class definitions in a hypothetical object oriented language that supports inheritance and uses dynamic binding. The language should not be assumed to be either Java or C++, thought the syntax is similar

```
Class      P {          Class Q subclass of P {
    void f(int i) {         void f (int i) {
        print (i);              print (2*i);
    }                       }
}                       }
```

Now consider the following program fragment:
P x = new $Q()$;
Q y = new $Q()$;
P z = new $Q()$;
$x.f(1); ((P) y). f(1); z.f(1);$
Here $((P) y)$ denotes a typecast of y to P. The output produced by executing the above program fragment will be
(A) 1 2 1   (B) 2 1 1
(C) 2 1 2   (D) 2 2 2

Q. 23   Which of the following is NOT an advantage of using shared, dynamically linked libraries as opposed to using statically linked libraries?
(A) Smaller sizes of executable

(B) Lesser overall page fault rate in the system

(C) Faster program startup

(D) Existing programs need not be re-linked to take advantage of newer versions of libraries

Q. 24   Which of the following assertions is false about the internet Protocol (IP) ?
(A) It is possible for a computer to have multiple IP addresses
(B) IP packets from the same source to the same destination can take different routes in the network
(C) IP ensures that a packet is farwarded if it is unable to reach its destination within a given number of hopes
(D) The packet source cannot set the route of an outgoing packets; the route is determined only by the routing tables in the routers on the way.

Q. 25   Which of the following functionalities must be implemented by a transport protocol over and above the network protocol ?
(A) Recovery from packet losses
(B) Detection of duplicate packets
(C) Packet delivery in the correct order
(D) End to end connectivity

Q. 26   The subnet mask for a particular network is 255.255.31.0 Which of the following pairs of IP addresses could belong to this network ?
(A) 172.57.88.62 and 172.56.87.233   (B) 10.35.28.2 and 10.35.29.4
(C) 191.203.31.87 and 191.234.31.88   (D) 128.8.129.43 and 128.8.161.55

Q. 27   A 2 km long brodcast LAN has 10^7 bps bandwidth and uses CSMA/CD. The signal travels along the wire at 2×10^8 m/s. What is the minimum packet size that can be used on this network ?
(A) 50 bytes   (B) 100 bytes
(C) 200 bytes   (D) None of the above

Q. 28   Host A is sending data to host B over a full duplex link. A and B are using the sliding window protocol for flow control. The send and receive window sizes are 5 packets each. Data packets (sent only from A to B) are all 1000 bytes long and the transmission time for such a packet is 50 μs. Acknowledgment packets (sent only from B to A), are very small and require negligible transmission time. The propagation delay over the link is 200 μs. What is the maximum achievable throughput in this communication ?
(A) 7.69×10^6 bps   (B) 11.11×10^6 bps
(C) 12.33×10^6 bps   (D) 15.00×10^6 bps

Q. 29   For a pipelined CPU with a single ALU, consider the following situations
1. The $j+1-st$ instruction uses the result of $j-th$ instruction as an operand
2. The execution of a conditional jump instruction
3. The $j-th$ and $j+1-st$ instructions require the ALU at the same time
Which of the above can cause a hazard?
(A) 1 and 2 only   (B) 2 and 3 only
(C) 3 only   (D) All the three

Q. 30   Consider an array multiplier for multiplying two n bit numbers. If each gate in the circuit has a unit delay, the total delay of the multiplier is
(A) $\Theta(1)$ (B) $\Theta(\log n)$
(C) $\Theta(n)$ (D) $\Theta(n^2)$

Q. 31   Consider the ALU shown below
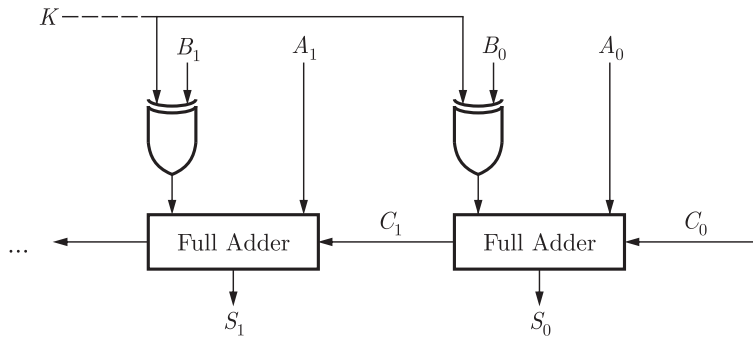
If the operands are in 2's complement representation, which of the following operations can be performed by suitably setting the control lines K and C_0 only (+ and - denote addition and subtraction respectively)?

(A) $A + B$, and $A - B$, but not $A + 1$

(B) $A + B$, and $A + 1$, but not $A - B$

(C) $A + B$, but not $A - B$, or $A + 1$

(D) $A + B$, and $A - B$, and $A + 1$

Common Data For Q. 32 and 33

Consider the following assembly language program for a hypothetical processor. A,B and C are 8 bit registers. The meanings of various instructions are shown as comments.

```
        MOV B, # 0      ; B←0
        MOV C, # 8      ; C←8
Z:      CMP C, # 0      ; compare C with 0
        JZX             ; jump to X if zero flag is set
        SUB C, # 1      ; C←C−1
        RRC A, # 1      ; right rotate A through carry
                        ; by one bit. Thus: if the
                        ; initial values of A and the
                        ; carry flag are a₇.......a₀ and c₀
                        ; respectively, their values
                        ; after the execution of this
                ; instruction will be c₀a₇.....a₁
                        ; and a₀ respectively.
        JCY             ;jump to Y if carry flag is set
        JMP Z           ; jump to Z
Y:      ADD B, # 1      ; B←B+1
        JMP Z           ; jump to Z
X:
```

Q. 32   If the initial value of register A is A_0, the value of register B after the program execution will be

(A) the number of 0 bits in A_0

(B) the number of 1 bits in A_0

(C) A_0

(D) 8

Q. 33   Which of the following instructions when inserted at location X will ensure that the value of register A after program execution is the same as its initial value?
(A) $RRC\ A, \#\ 1$
(B) NOP ;   no operation
(C) $LRC\ A, \#\ 1$;   left rotate A through carry flag by one bit
(D) $ADD\ A, \#\ 1$

Q. 34   Which of the following scenarios may lead to an irrecoverable error in a database system?
(A) A transaction writes a data item after it is read by an uncommitted transaction
(B) A transaction read a data item after it is read by an uncommitted transaction
(C) A transaction read a data item after it is written by an committed transaction
(D) A transaction read a data item after it is written by an uncommitted transaction

Q. 35   Consider the following SQL query
select distinct $a_1, a_2, \ldots\ldots a_n$ from $r_1, r_2, \ldots\ldots r_m$ where P
For an arbitrary predicate P, this query is equivalent to which of the following relational algebra expressions?

(A) $\displaystyle\prod_{a_1, a_2, \ldots a_n} \sigma_\rho (r_1 \times r_2 \times \ldots\ldots r_m)$ (B) $\displaystyle\prod_{a_1, a_2, \ldots a_n} \sigma_\rho (r_1 \bowtie r_2 \bowtie \ldots\ldots \bowtie r_m)$

(C) $\displaystyle\prod_{a_1, a_2, \ldots a_n} \sigma_\rho (r_1 \cup r_2 \cup \ldots\ldots \cup r_m)$ (D) $\displaystyle\prod_{a_1, a_2, \ldots a_n} \sigma_\rho (r_1 \cap r_2 \cup \ldots\ldots \cap r_m)$

Q. 36   Consider the following functional dependencise in a database:
Data_of_Birth→Age   Age→Eligibility
Name→Roll_number   Roll_number→Name
Course_number→Course_name   Course_number→Instructor
(Roll_number,Course_number)→Grade
The relation(Roll)number,Name,Date_of_brith,Age)is
(A) in second normal normal form but not in third normal form
(B) in third normal form but not in BCNF
(C) in BCNF
(D) in none of the above

Q. 37   Consider the set of relations shown below and the SQL query that follow:
Students:(Roll_number,Name,Date_of_birth)
Courses:(Course_number,Course_name,Instructor)
Grades:(Roll_number,Course_number,Grade)
select distrinct Name
from Students, Courses, Grades
Where Students,Roll_number=Grades. Toll_number
and Courses. Instructor=Korth
and Courses. Course_number=Grades. Course_number
and Grades.grade=A
Which of the following sets is computed by the above query?
(A) Names of students who have got an A grade in all courses taught by Korth
(B) Names of students who have got an A grade in all courses

(C) Name of students who have got an A grade in at least one of the courses taught by Korth

(D) None of the above

Q. 38   Consider three data items $D1, D2$ and $D3$ and the following execution schedule of transactions $T1$, $T2$ and $T3$. In the diagram, $R(D)$ and $W(D)$ denote the actions reading and writing the data item D respectively.

T1	T2	T3
	R(D3);	
	R(D2);	
	R(D2);	
		R(D2);
		R(D3);
R(D1);		
R(D1);		
		W(D2);
		W(D3);
	R(D1);	
R(D2);		
W(D2);		
	W(D1);	

(A) The schedule is serializable as T2; T3;T1;

(B) The schedule is serializable as T2; T1;T3;

(C) The schedule is serializable as T3; T2; T1;

(D) The schedule is not serializable

Q. 39   Assuming all numbers are in 2's complement representation, which of the following number is divisible by 11111011?

(A) 11100111   (B) 11100100

(C) 11010111   (D) 11011011

Q. 40   The following is a scheme for floating point number representation using 16 bits.

Bit Position   15   14... ... 9 8... 0

s	e	m

sign exponent Mantissa

Let s, c and m be the number represented in binary in the sign, exponent, and mantissa fields respectively. Then the flouting point number represented id

$\begin{cases} (-1)^2 (1 + m \times 2^{-9}) 2^{e-31}, & \text{if the exponent } 111111 \\ 0 & \text{otherwise} \end{cases}$

What is the maximum difference between two successive real numbers representable in this system?

(A) 2^{-40} (B) 2^{-9}

(C) 2^{22} (D) 2^{31}

Q. 41   A 1-input, 2-output synchronous sequential circuit behaves as follows.
Let z_k, n_k denote the number of 0's and 1's respectively in initial k bits of the input $(z_k + n_k = k)$. The circuit outputs 00 until one of the following conditions holds.
1. $n_k - n_k = 2$. In this case, the output at the k-th and all subsequency clock ticks is 10.
2. $n_k - z_k = 2$. In this case, the output at the k-th and all subsequent clock ticks is 01.
What in the minimum number of states required in the state transition graph of the above circuit?

(A) 5   (B) 6

(C) 7   (D) 8

Q. 42   The literal count of a boolean expression is the sum of the number of times each literal appears in the expression. For example, the literal count of $(xy + xz)$ is 4. What are the minimum possible literal counts of the product-of-sum and sum-of-product representations respectively of the function given by the following karnaugh map?
Here, ✗ denotes "don't care"

$xy \backslash^{zw}$	00	01	11	10
00	✗	1	0	1
01	0	1	✗	0
11	1	✗	✗	0
10	✗	0	0	✗

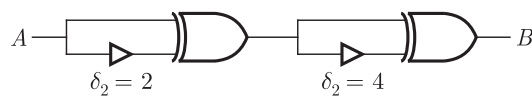(A) (11,9)   (B) (9,13)

(C) (9,10)   (D) (11,11)

Q. 43   Consider the following circuit composed of XOR gates and non-inverting buffers.

The non-inverting buffers have delays $\delta_1 = 2ns$ and $\delta_2 = 4ns$ as shown in the figure. both XOR gates and al wires have zero delay. Assume that all gate inputs, outputs and wires are stable at logic level 0. If the following waveform is applied at input. A, how many transition (s) (change of logic levels) occur (s) at B during the interval from 0 to 10 ns?
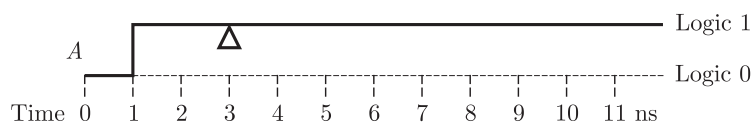
(A) 1   (B) 2

(C) 3   (D) 4

Q. 44   Let $P(E)$ denote the probability of the event E.
Given $P(A) = 1, P(B) = 1/2$, the values of $P(A/B)$ and $P(B/A)$ respectively are
(A) 1/4, 1/2   (B) 1/2, 1/4
(C) 1/2, 1   (D) 1, 1/2

Q. 45   Consider the set $\sum^*$ of all strings over the alphabet $\sum = \{0,1\}.\sum^*$ with the concatenation operator for strings
(A) does not form a group
(B) forms a non-commutative group
(C) does not have a right identity element
(D) forms a group if the empty string is removed from $\sum^*$

Q. 46   Let A be a sequence of 8 distinct integers sorted in ascending order.
How many distinct pairs of sequences. B and C are there such that (i) each is sorted in ascending order,
(ii) B has 5 and C 3 element, and
(iii) the result of merging B and C gives A?
(A) 2   (B) 30
(C) 56   (D) 256

Q. 47 n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is
(A) $\left(\dfrac{2n}{n}\right) * 2^n$ (B) 3^n

(C) $\dfrac{(2n)\,1}{2^n}$ (D) $\left(\dfrac{2n}{n}\right)$

Q. 48   Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between.
(A) k and n (B) $k-1$ and $k+1$
(C) $k-1$ and $n-1$ (D) $k+1$ and $n-k$

Q. 49   Let $(S, \leq)$ be a partial order with two minimal elements a and b, and a maximum element c. Let $P: S \to \{$True, False$\}$ be a predicate defined on S. Suppose that $P(a) = $ True, $P(b) = $False and $P(x) \Rightarrow p(y)$ for all $x, y \in S$ satisfying $x \leq y$, where $\Rightarrow$ stands for logical implication. Which of the following statements CANNOT be true?
(A) $P(x) = $True for all $x \in S$ such that $x \in$ such that $x \neq b$
(B) $P(x) = $False for all $x \in S$ such that $x \neq a$ and $x \neq c$
(C) $P(x) = $False for all $x \in S$ such that $b \leq x$ such that $x \neq c$
(D) $P(x) = $False for all $x \in S$ such that $a \leq x$ such that $b \leq x$

Q. 50   Which of the following is a valid first order formula?
(Here α and β are first order formulae with x as their only free variable)

(A) $(\forall x)[\alpha] \Rightarrow (\forall x)[\beta] \Rightarrow (\forall x)[\alpha \Rightarrow \beta]$ (B) $(\forall x)[\alpha] \Rightarrow [\exists x][\alpha \wedge \beta]$
(C) $(\forall x)[\alpha \vee \beta] \Rightarrow ((\exists x)[\alpha] \Rightarrow (\forall x)[\alpha]$ (D) $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow ((\forall x)[\alpha] \Rightarrow (\forall x)[\beta])$

Q. 51   Consider the following formula α and its two interpretations

I_1 and I_2 $\alpha : (\forall x)[P_x \Leftrightarrow (\forall y)[Q_{xy} \Leftrightarrow \neg Q_{yy}]] \Rightarrow (\forall x)[\neg P_x]$

I_1: Domain : the set of natural numbers
$P_x \equiv$ 'x is a prime number'
$Q_{xy} \equiv$ 'y divides x'
I_2; same as I_1 except that $P_x =$ 'x is a composite number.'
Which of the following statements is true?
(A) I_1 satisfies α, I_2 does not
(B) I_2 satisfies α, I_1 does not
(C) Neither I_2 nor I_2 satisfies α
(D) Both I_1 and I_2 satisfy α

Q. 52   Consider the following logic program P
$$A(x) \leftarrow B(x,y), C(y)$$
$$\leftarrow B(x,x)$$

Which of the following first order sentences is equivalent to P?
(A) $(\forall x)[\exists y][B(x,y) \wedge C(y)] \Rightarrow A(x)] \wedge \neg (\exists x)[B(xx)]$
(B) $(\forall x)[\forall y][B(x,y) \wedge C(y)] \Rightarrow A(x)] \wedge \neg (\exists x)[B(xx)]$
(C) $(\forall x)[\exists y][B(x,y) \wedge C(y)] \Rightarrow A(x)] \vee \neg (\exists x)[B(xx)]$
(D) $(\forall x)[\forall y][B(x,y) \wedge C(y)] \Rightarrow A(x)] \wedge \neg (\exists x)[B(xx)]$

Q. 53   The following resolution rule is used in logic programming:
Derive clause $(P \vee Q)$ from clauses $(P \vee R), (Q \vee \neg R)$
Which of the following statements related to this rule is FASLE?
(A) $(P \vee R) \wedge (Q \vee \neg R) \Rightarrow (P \vee Q)$ is logically valid
(B) $(P \vee Q) \Rightarrow (P \vee R) \wedge (Q \vee \neg R)$ is logically valid
(C) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable
(D) $(P \vee Q) \Rightarrow$ FALSE if and only if both P and Q are unclassifiable

Q. 54   A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$ respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by

(A) $f_1(t) + f_2(t)$

(B) $\int_0^t f_1(x) f_2(x)\, dx$

(C) $\int_0^t f_1(x) f_2(t-x)\, dx$

(D) $\max \{f_1(t), f_2(t)\}$

Q. 55   Let $: A \rightarrow B$ be injective (one-to-one) function. Define $g: 2^A \rightarrow 2^B$ as:
$g(C) = \{f(x) \; x \in C\}$, for all subsets C of A.
Define $h: 2^B \rightarrow 2^A$ as : $h(D) = \{x \mid x \in A, f(x) \in D\}$, for all subsets D of B.
Which of the following statements is always true?
(A) $g(h(D)) \subseteq D$
(B) $g(h(D)) \supseteq D$
(C) $g(h(D)) \cap D = \phi$
(D) $g(h(D)) \cap (B - D) = \phi$

Q. 56   Consider the set $\{a, b, c\}$ with binary operators $+$ and $\times$ defined as follows:

$+$	a	b	c	$\times$	a	b	c
a	b	a	c	a	a	b	c
b	a	b	c	b	b	c	a
c	a	c	b	c	c	c	b

For example, $a + c = c, c + a = a, c \times b = c$ and $b \times c = a$. Given the following set of equations;
$(a \times x) + (a \times y) = c$
$(b \times x) + (c \times y) = c$

The number of solution (s) $(i.e.,$ pair (s) (x, y) that satisfies the equations) is
(A) 0   (B) 1
(C) 2   (D) 3

Q. 57   Let $\Sigma = \{a,, b, c, d, e\}$ be an alphabet, We define an encoding scheme as follows $g(a) = 3, g(b) = 5, g(c) = 7, g(d) = 9, g(e) = 11$.
Let p_i denote the $i - th$ prime number$(p1 = 2)$.
For a non-empty string $s = a_1.....a_n$, where each $a_i \in \Sigma$, define $f(s) = \Pi^n_{i=1} p_i^{g(a_i)}$.
For a non-empty sequence $< s_1.....s_n >$ of strings from $\Sigma *$, define $h(< s_1.....s_n >) = \Pi^n_{i=1} p_i^{f(s_i).}$.
Which of the following numbers is the encoding, h of a non-empty sequence of strigs?
(A) $2^7 3^7 5^7$ (B) $2^8 3^8 5^8$
(C) $2^9 3^9 5^9$ (D) $2^{10} 3^{10} 5^{10}$

Q. 58 m identical balls are to be placed in n distinct bags. You are given that $m \geq kn$, where k is a natural number ≥ 1. In how many ways can the balls be placed in the bags if each bag must contain at least k balls?

(A) $\binom{m - k}{n - 1}$ (B) $\binom{m - kn + n - 1}{n - 1}$

(C) $\binom{m - 1}{n - k}$ (D) $\binom{m - kn + n + k - 2}{n - k}$

Q. 59   Consider the following recurrence relation
$T(n + 1) = T(n) + \lfloor \sqrt{n + 1} \rfloor$ for all $n \geq 1$
The value of $T(m^2)$ for $m \geq 1$ is
(A) $\frac{m}{6}(21m - 39) + 4$ (B) $\frac{m}{6}(4m^2 - 3m + 5)$

(C) $\frac{m}{2}(3m^{25} - 11m + 20) - 5$ (D) $\frac{m}{6}(5m^3 - 34m^2 + 137m - 104) + \frac{5}{6}$

Q. 60   How many perfect matching are there in a complete graph of 6 vertices?
(A) 15   (B) 24
(C) 30   (D) 60

Q. 61   A Graph $G = (V, E)$ satisfies $|E| \leq 3 |V| - 6$. The min-degree of G is defined as $\min_{v \in V} \{degree\ (v)\}$. Therefore, min-degree of G cannot be

(A) 3   (B) 4
(C) 5   (D) 6

Q. 62   Consider the following system of linear equations

$$\begin{bmatrix} 2 & 1 - 4 \\ 4 & 3 - 12 \\ 1 & 2 - 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$$
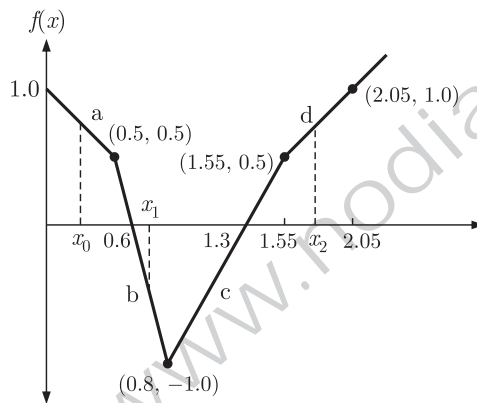
Notice that the second and the third columns of the coefficient matrix are linearly dependent. For how many values of α. does this system of equations have infinitely

many solutions?
(A) 0
(B) 1
(C) 2
(D) infinitely many

Q. 63   A piecewise linear function $f(x)$ is plotted using thick solid lines in the figure below (the plot is drawn to scale).

If we use the Newton-Raphson method to find the roots of $f(x) = 0$ using $x0, x1$ and $x2$ respectively as initial guesses, the roots obtained would be
(A) 1.3, 0.6 and 0.6 respectively
(B) 0.6, 0.6 and 1.3 respectively
(C) 1.3, 1.3 and 0.6 respectively
(D) 1.3, 0.6 and 1.3 respectively

Q. 64   Using a larger block size in a fixed block size file system leads to
(A) better disk throughput but poorer disk space utilization
(B) better disk throughput and better disk space utilization
(C) poorer disk throughput but better disk space utilization
(D) poorer disk throughput and poorer disk space utilization

Q. 65   In a system with 32 bit virtual addresses and 1 KB page size, use of one-level page tables for virtual to physical address translation is not practical because of
(A) the large amount of internal fragmentation
(B) the large amount of external fragmentation
(C) the large memory overhead in maintaining page tables
(D) the large computation overhead in the translation process

Q. 66   A uni-processor computer system only has two processes, both of which alternate 10 *ms* CPU bursts with 90 *ms* I/O bursts. Both the processes were created at nearly the same time. The I/O of both processes can proceed in parallel. Which of the following scheduling strategies will result in the least CPU utilizations (over a long period of time) for this system ?

(A) First come first served scheduling

(B) Shortest remaining time first scheduling

(C) Static priority scheduling with different priorities for the two processes

(D) Round robin scheduling with a time quantum of 5 *ms*.

Common Data For Q. 67 and 68

A processor uses 2-level page table fro virtual to physical address translation. Page table for both levels are stored in the main memory. Virtual and physical addresses are both 32 bits wide. The memory is byte addressable. For virtual to physical address translation, the 10 most significant bits of the virtual address are used as index into the first level page table while the next 10 bits are used as index into the second level page table. The 12 least significant bits of the virtual address are used as offset within the page. Assume that the page table entries in both levels of page tables are 4 a bytes wide. Further, the processor has a translation look aside buffer(TLB), with a hit rate of 96%. The TLB caches recently used virtual page numbers and the corresponding physical page numbers. The processor also has a physically addressed cache with a bit ratio of 90%. Main memory access time is 10 *ns*, cache access time is 1 *ns*, and {LB access time is also 1*ns*.

Q. 67   Assuming that no page faults occur, the average time taken to access a virtual address is approximately (to the nearest 0.5 *ns*)

(A) 1.5 *ns*

(B) 2 *ns*

(C) 3 *ns*

(D) 4 *ns*

Q. 68   Suppose a process has only the following pages in its virtual address space; two contiguous code pages starting at virtual address 0×0000000, two contiguous data pages starting at virtual address 0×00400000,and a stack page starting at virtual address $0 \times FFFFF000$. The amount of memory required for storing the page tables of this process is

(A) 8 KB

(B) 12 KB

(C) 16 KB

(D) 20 KB

Common Data For Q. 69 and 70

Suppose we want to synchronize two concurrent processes P and Q using binary semaphores S and T. The code for the processes P and Q is shown below.

```
Process P                              Process Q:
    while(1) {                             while(1) {
W:                                     Y:
    print '0';                             print '1'
    print '0';                             print '1'
X:                                     Z:
}                                      }
```

Synchronization statements can be inserted only at points W,X,Y and Z.

Q. 69 Which of the following will always lead to an output staring with '001100110011'?
(A) P(S) at W, V(S) at X, P(T) at Y, V(T) at Z, S and T inutuakkt 1
(B) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S initially 1, and T initially 0
(C) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 1
(D) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S initially 1, and T initially 0

Q. 70 Which of the following will ensure that the output string never contains a substring of the form 0.1" or 10"1 where n is odd?
(A) P(S) at W, V(S) at X, P(T) at Y, V(T) at Z, S and T initially 1
(B) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 1
(C) P(S) at W, V(S) at X, P(T) at Y, V(S) at Z, S initially 1
(D) V(S) at W, V(T) at X, P(T) at Y, P(S) at Z, S and T initially 1

Q. 71 Consider the following C function.
```
float f(float x, int y){
    float p, s; int i;
    for (s=1, p=1, i=1, i<y; i++)
    {
        p*=x/i;
        s+=p;
    }
    return s;
}
```
For large values of `y`, the return value of the function `f` best approximates
(A) x^y (B) e^x
(C) $In(1 + x)$ (D) x^x

Q. 72 Assume the following C variable declaration
```
int*A[10], B[10][10];
```
Of the following expressions
(1) A[2]   (2) A[2][3]
(3) B[1]   (4) B[2][3]
Which will not give compile-time errors if used as left hand sides of assignment statements in a C program ?
(A) 1, 2, and 4, only   (B) 2, 3, and 4, only
(C) 2 and 4 only   (D) 4 only

Q. 73   Let $T(n)$ be the number of different binary search trees on n distinct elements.
Then $T[n] = \sum_{k=1}^{n} T(k-1) T(x)$, where x is

(A) $n - k + 1$ (B) $n - k$

(C) $n - k - 1$ (D) $n - k - 2$

Q. 74   Suppose the numbers 7, 5, 1, 8, 3, 6, 0, 9, 4, 2 are inserted in that order into an
initially empty binary search tree. The binary search tree uses the usual ordering
on natural numbers. What is the inorder transversal sequence of the resultant
tree ?

(A) 7 5 1 0 3 2 4 6 8 9   (B) 0 2 4 3 1 6 5 9 8 7

(C) 0 1 2 3 4 5 6 7 8 9   (D) 9 8 6 4 2 3 0 1 5 7

Q. 75   A data structure is required for storing a set of integers such that each of the
following operations can be done is $(\log n)$ time, where n is the number of elements
in the set.

1. Delection of the smallest element.

2. Insertion of an element if it is not already present in the set.

Which of the following data structures can be used for this purpose ?

(A) A heap can be used but not a balanced binary search tree

(B) A balanced binary search tree can be used but not a heap

(C) Both balanced binary search tree and heap can be used

(D) Neither balanced binary search tree nor heap can be used

Q. 76   Let S be a stack of size $n \geq 1$. Starting with the empty stack, suppose we push the
first n natural numbers in sequence, and then perform n pop operations. Assume
that Push and Pop operation take X seconds each , and Y seconds elapse between
the end of the one such stack operation and the start of the next operation. For
$m \geq 1$, define the stack-life of mcs the time elapsed from the end or Push (m) to
the start of the pop operation that removes m from S. The average stack-life of
an element of this stack is

(A) $n(X + Y)$ (B) $3Y + 2X$

(C) $n(X + Y) - X$ (D) $Y + 2X$

Q. 77   Consider the following 2-3-4 tree (i.e., B-tree with a minimum degree of two) in
which each data item is a letter. The usual alphabetical ordering of letters is used
in constructing the tree
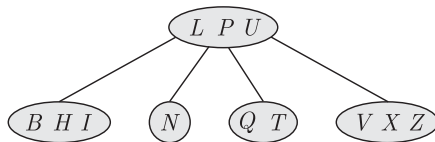
What is the result of inserting G in the above tree ?

(B)

(C)
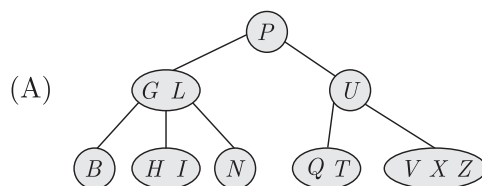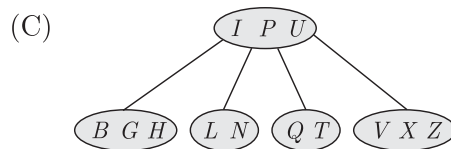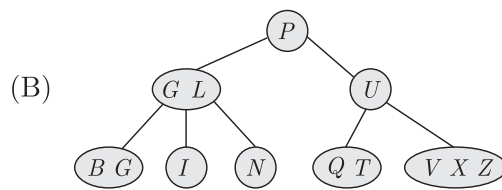
(D) None of the above

Q. 78 In the following C program fragment, `j, k, n` and `TwoLog_n` are integer variables, and `A` is an array of integers. The variable n is initialized to an integer ≥ 3, and `TwoLog_n` is initialized to the value of $2*\lfloor \log_2(n) \rfloor$

```
for (k=3;k<=n;k++)
        A[k]=0;
for (k=2;k<=TwoLog_n;k++)
        for (j=k+1;j<=n;j++)
            A[j]=A[j]||(j%k);
for(j=3;j<=n;j++)
        if (!A[j])printf("%d",j);
```

The set of number printed by this program fragment is

(A) $\{m \mid m \leq n, (\exists i)\,[m = i!]\}$

(B) $\{m \mid m \leq n, (\exists i)\,[m = i^2]\}$

(C) $\{m \mid m \leq n, m \text{ is prime}\}$

(D) $\{m \mid m \leq n, m \text{ is odd}\}$

Q. 79 Consider the C program shown below.

```
#include <stdio.h>
#define print(x)printf("%d",x)
int x;
void Q (int z){
    z+=x; print(z);
}
void p (int*y){
    int x=*y+2;
    Q(x);*y=x-1;
    print(x);
}
main (void){
    x=5;
    p(&x);
    print(x);
}
```

The output of this program is

(A) 12 7 6   (B) 22 12 11

(C) 14 6 6   (D) 7 6 6

Q. 80   Consider the function – defined below.

```
struct item {
      int data;
      struct item*next;
};
int f (struct item *p){
      return ((p==NULL)||(p->next==NULL)||
            ((p->data<=p->next->data)&&
                  f(p->next)));
}
```

For a given linked list p, the function f return 1 if and only if

(A) the list is empty or has exactly one element

(B) the elements in the list are sorted in non-decreasing order of data value

(C) the elements in the list are sorted in non-increasing order of data value

(D) not all elements in the list have the same data value

Q. 81   Ram and Shyam have been asked to show that a certain problem Π is NP-complete. Ram shows a polynomial time reduction from the 3-SAT problem to Π , and Shyam shows a polynomial time reduction from Π to 3-SAT. Which of the following can be inferred from these reduction?

(A) Π is NP-hard but not NP-complete

(B) Π is in NP, but is not NP-complete

(C) Π is NP-complete

(D) Π is neither Np-hard, nor in NP

Q. 82   Nobody knows yet if $P = NP$. Consider the language L defined as follows

$$L = \begin{cases} (0+1)^* & \text{if P = NP} \\ \phi & \text{othervise} \end{cases}$$

Which of the following statements is true?

(A) L is recursive

(B) L is recursively enumerable but not recu

(C) L is not recursively enumerable

(D) Whether L is recursive or not will be known after we find out if $P = NP$

Q. 83   The regular expression $0*(10)*$ denotes the same set as

(A) $(1*0)*1*$

(B) $0 + (0 + 10)*$

(C) $(0 + 1)*10(0 + 1)*$

(D) None of the above

Q. 84   If the strings of a language L can be effectively enumerated in lexicographic (i.e. alphabetic) order, which of the following statements is true?

(A) L is necessarily finite

(B) L is regular but not necessarily finite

(C) L is context free but not necessarily regular

(D) L is recursive but not necessarily context free

Q. 85 Consider the following deterministic finite state automaton M.
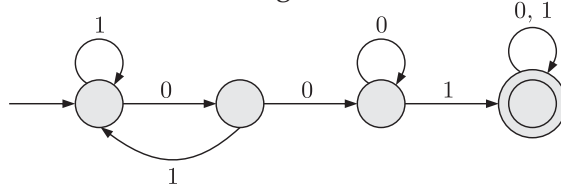
Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is

(A) 1   (B) 5

(C) 7   (D) 8

Q. 86 Let $G = (\{S\}, \{a, b\} R, S$ be a context free grammar where the rule set R is
$$S \to a\,S\,b \mid S\,S \mid \varepsilon$$

Which of the following statements is true?

(A) G is not ambiguous

(B) There exist $x, y, \in L(G)$ such that $xy \notin L(G)$

(C) There is a deterministic pushdown automaton that accepts $L(G)$

(D) We can find a deterministic finite state automaton that accepts $L(G)$

Q. 87 Consider two languages L_1 and L_2 each on the alphabet Σ. Let $f{:}\Sigma \to \Sigma$ be a polynomial time computable bijection such that $(\forall x[x \in L_1$ iff $f(x) \in L_2]$. Further, let f^1 be also polynomial time commutable.

Which of the following CANNOT be true?

(A) $L_1 \in P$ and L_2 finite

(B) $L_1 \in NP$ and $L_2 \in P$

(C) L_1 is undecidable and L_2 is decidable

(D) L_1 is recursively enumerable and L_2 is recursive

Q. 88 A single tape Turing Machine M has two states q^0 and q^1, of which q^0 is the starting state. The tape alphabet of M is {0,1,B} and its input alphabet is {0,1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table

	0	1	B
q^0	$q^{1,1,R}$	$Q^{1,1,R}$	Halt
q^1	$q^{1,1,R}$	$q^{0,1,L}$	$qH0, B, L$

The table is interpreted as illustrated below.

The entry $(q^{1,1,R})$ in row q^0 and column 1 signifies that if M is in state q^0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q^1.

Which of the following statements is true about M?

(A) M does not halt on any string in $(0 + 1)^+$

(B) M dies not halt on any string in $(00 + 1)^*$

(C) M halts on all string ending in a 0

(D) M halts on all string ending in a 1

Q. 89   Define languages L_0 and L_1 as follows
$L_0 = \{< M, w, 0 >| M \text{ halts on } w\}$

$L_0 = \{< M, w, 1 >| M \text{ does not halts on } w\}$

Here $< M, w, i >$ is a triplet, whose first component. M is an encoding of a Turing
Machine, second component, w, is a string, and third component, t, is a bit.
Let $L = L_0 \cup L_1$. Which of the following is true?
(A) L is recursively enumerable, but $\overline{L}$ is not

(B) $\overline{L}$ is recursively enumerable, but L is not

(C) Both L and $\overline{L}$ are recursive

(D) Neither L nor $\overline{L}$ is recursively enumerable

Q. 90   Consider the NFAM shown below.
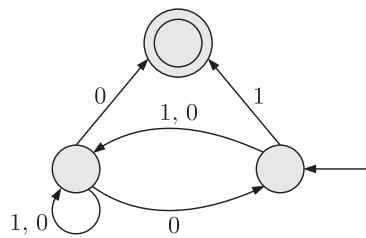
Let the language accepted by M be L. Let L_1 be the language accepted by the
$NFAM_1$, obtained by changing the accepting state of M to a non-accepting state
and by changing the non-accepting state of M to accepting states. Which of the
following statements is true?
(A) $L_1 = \{0, 1\}^* - L$ (B) $L_1 = \{0, 1\}^*$
(C) $L_1 \subseteq L$ (D) $L_1 = L$

****

ANSWER KEY

2003									
1	2	3	4	5	6	7	8	9	10
(A)	(D)	(C)	(C)	(B)	(B)	(C)	(B)	(B)	(B)
11	12	13	14	15	16	17	18	19	20
(D)	(C)	(B)	(C)	(A)	(A)	(D)	(D)	(C)	(D)
21	22	23	24	25	26	27	28	29	30
(A)	(D)	(D)	(A)	(D)	(C)	(D)	(B)	(D)	(D)
31	32	33	34	35	36	37	38	39	40
(D)	(A)	(A)	(D)	(A)	(A)	(C)	(C)	(A)	(C)
41	42	43	44	45	46	47	48	49	50
(C)	(A)	(C)	(D)	(D)	(D)	(A)	(D)	(C)	(D)
51	52	53	54	55	56	57	58	59	60
(A)	(C)	(B)	(C)	(A)	(D)	(B)	(B)	(B)	(A)
61	62	63	64	65	66	67	68	69	70
(C)	(B)	(B)	(D)	(A)	(A)	(B)	(B)	(A)	(C)
71	72	73	74	75	76	77	78	79	80
(C)	(D)	(A)	(B)	(B)	(D)	(C)	(B)	(A)	(D)
81	82	83	84	85	86	87	88	89	90
(C)	(D)	(D)	(A)	(A)	(D)	(C)	(D)	(A)	(C)