GATE SOLVED PAPER
Computer Science Engineering
Algorithms

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GATE SOLVED PAPER - CS

ALGORITHMS

YEAR 2001

Q. 1 Randomized quicksort is an extension of quicksort where the pivot is chosen randomly. What is the worst case complexity of sorting n numbers using randomized quicksort ?

(A) $0(n)$ (B) $0(n \log n)$

(C) $0(n^2)$ (D) $0(n!)$

Q. 2 Consider any array representation of an n element binary heap where the elements are stored from index 1 to index n of the array. For the element stored at index i of the array ($i \leq n$), the index of the parent is

(A) $i - 1$ (B) $\left\lfloor \frac{i}{2} \right\rfloor$

(C) $\left\lceil \frac{i}{2} \right\rceil$ (D) $\frac{(i+1)}{2}$

Q. 3 Let $f(n) = n^2 \log n$ and $g(n) = n(\log n)^{10}$ be two positive functions of n. Which of the following statements is correct ?

(A) $f(n) = 0(g(n))$ and $g(n) \neq 0(f(n))$ (B) $g(n) = 0(f(n))$ and $f(n) \neq 0(g(n))$

(C) $f(n) \neq 0(g(n))$ and $g(n) \neq 0(f(n))$ (D) $f(n) = 0(g(n))$ and $g(n) = 0(f(n))$

Q. 4 Consider the undirected unweighted graph G. Let a breadth-first traversal of G be done starting from a node r. Let $d(r,u)$ and $d(r,v)$ be the lengths of the shortest paths form r to u and v respectively in G. If u is visited before v during the breadth-first traversal, which of the following statements is correct ?

(A) $d(r,u) < d(r,v)$ (B) $d(r,u) > d(r,v)$

(C) $d(r,u) \leq d(r,v)$ (D) None of the above

Q. 5 How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{V_1, V_2, ...V_n\}$ of n vertices ?

(A) $\frac{n(n-1)}{2}$ (B) 2^n

(C) $n!$ (D) $2^{\frac{n(n-1)}{2}}$

Q. 6 What is the minimum number of stacks of size n required to implement a queue to size n ?

(A) One (B) Two

(C) Three (D) Four

YEAR 2002

Q. 7 The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 1, T(1) = 1$ is

(A) 2^k (B) $\frac{(2^{k+1} - 1)}{2}$

(C) $3^{\log_2^k}$ (D) $2^{\log_3^k}$

Q. 8 The minimum number of colours required to colour the vertices of a cycle with n nodes in such a way that no two adjacent nodes have same colour is.

(A) 2 (B) 3

(C) 4 (D) $n - 2\left[\frac{n}{2}\right] + 2$

Q. 9 In the worst case, the number of comparisons needed to search a single linked list of length n for a given element is

(A) $\log n$ (B) $\frac{n}{2}$

(C) $\log_2^n - 1$ (D) n

Q. 10 Maximum number of edges in a n-node undirected graph without self loops is

(A) n^2 (B) $\frac{n(n-1)}{2}$

(C) $(n-1)$ (D) $\frac{(n+1)(n)}{2}$

Q. 11 The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 3 children is :

(A) $\frac{n}{2}$ (B) $\frac{(n-1)}{3}$

(C) $\frac{(n-1)}{2}$ (D) $\frac{(2n+1)}{3}$

Q. 12 Consider the following algorithm for searching for a given number x in an unsorted array $A[1.....n]$ having n distinct values :

(1) Choose an i uniformly at random from $[1....n]$

(2) If $A[i] = x$ then stop else Goto 1;

Assuming that x is present A, What is the expected number of comparisons made by the algorithm before it terminates ?

(A) n (B) $n - 1$

(B) $2n$ (D) $\frac{n}{2}$

Q. 13 The running time of the following algorithm

Procedure $A(n)$

If $n <= 2$ return (1) else return $(A(\lceil \sqrt{n} \rceil))$;

Is best described by

(A) $0(n)$ (B) $0(\log n)$

(C) $0(\log\log n)$ (D) $0(1)$

Q. 14 A weight-balanced tree is a binary tree in which for each node, the number of nodes in the let sub tree is at least half and at most twice the number of nodes in the right sub tree. The maximum possible height (number of nodes on the path from the root to the furthest leaf) of such a tree on n nodes is best described by which of the following ?

(A) $\log_2 n$ (B) $\log_{\frac{4}{3}} n$

(C) $\log_3 n$ (D) $\log_{\frac{3}{2}} n$

Q. 15 To evaluate an expression without any embedded function calls

(A) One stack is enough (B) Two stack are needed

(C) As many stacks as the height of the expression tree are needed

(D) A Turning machine is needed in the general case

YEAR 2003 ONE MARK

Q. 16 Consider the following three claims

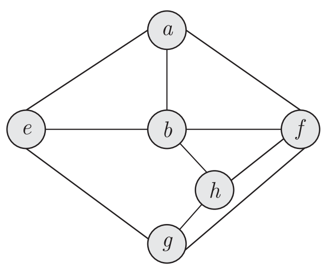
1. $(n+k)^m = \Theta(n^m)$, where k and m are constants
2. $2^{n+1} = O(2^n)$
3. $2^{2^n+1} = O(2^n)$

Which of these claims are correct?

(A) 1 and 2 (B) 1 and 3

(C) 2 and 3 (D) 1, 2 and 3

Q. 17 Consider the following graph



Among the following sequences

1. *abeghf* 2. *abfehg*

3. *abfhge* 4. *agfhbe*

Which are depth first traversals of the above graph?

(A) 1, 2 and 4 only (B) 1 and 4 only

(C) 2, 3 and 4 only (D) 1, 3 and 4 only

Q. 18 The usual $\Theta(n^2)$ implementation of Insertion Sort to sort ab array uses linear search to identify the position where an element is to be inserted into the already sorted part of the array. If, instead, we use binary search to identify the position, the worst case running time will

(A) remain $\Theta(n^2)$ (B) become $\Theta(n(\log n)^2)$

(C) become $\Theta(n \log n)$ (D) become $\Theta(n)$

Q. 19 In a heap with n elements with the smallest element at the root, the 7^{th} smallest element ban be found in time

(A) $\Theta(n \log n)$ (B) $\Theta(n)$

(C) $\Theta(\log n)$ (D) $\Theta(1)$

YEAR 2003 TWO MARKS

Common Data For Q. 20 & 21

Solve the problems and choose the correct answers.

In a permutation $a_1....a_n$ of n distinct integers, an inversion is a pair (a_1, a_j) such that $i < j$ and $a_i > a_j$.

Q. 20 If all permutation are equally likely, what is the expected number of inversions in a randomly chosen permutation of $1.....n$?

(A) $n(n-1)/2$ (B) $n(n-1)/4$

(C) $n(n+1)/4$ (D) $2n[\log_2 n]$

Q. 21 What would be the worst case time complexity of the insertion Sort algorithm, if the inputs are restricted to permutations of $1....n$ with at most n inversions?

(A) $\Theta(n^2)$ (B) $\Theta(n \log n)$

(C) $\Theta(n^{1.5})$ (D) $\Theta(n)$

Q. 22 The cube root of a natural number n is defined as the largest natural number m such that $m^3 \leq n$. The complexity of computing the cube root of n (n is represented in binary notation) is

(A) $O(n)$ but not $O(n^{0.5})$

(B) $O(n^{0.5})$ but not $O((\log n)^k)$ for any constant $k > 0$

(C) $O((\log n)^k)$ for some constant $k > 0$, but not $O((\log\log n)^m)$ for any constant $m > 0$

(D) $O((\log\log n)^k)$ for some constant $k > 0.5$, but not $O((\log\log n)^{0.5})$

Q. 23 Let $G = (V, E)$ be an undirected graph with a sub-graph $G_1 = (V_1, E_1)$, Weight are assigned to edges of G as follows

$$w(e) = \begin{cases} 0 \text{ if } e \in E, \\ 1 \text{ otherwise} \end{cases}$$

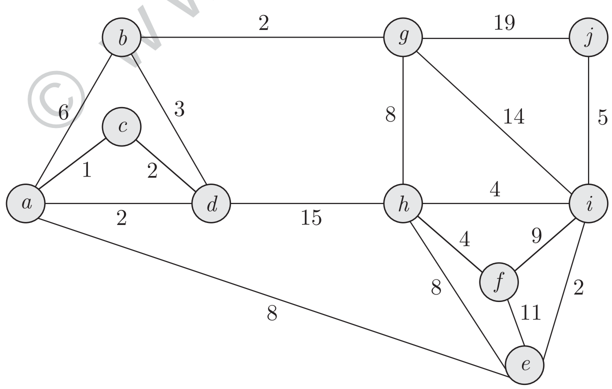
A single-source shortest path algorithm is executed on the weighted graph (V, E, w) with an arbitrary vertex v_1 of V_1 as the source. Which of the following can always be inferred from the path costs computed?

(A) The number of edges in the shortest paths from v_1 to all vertices of G

(B) G_1 is connected

(C) V_1 forms a clique in G (D) G_1 is a tree

Q. 24 What is the weight of a minimum spanning tree of the following graph?



(A) 29 (B) 31

(C) 38 (D) 41

Q. 25 The following are the starting and ending times of cetivities A, B, C, D, E, F, G and H respectively in chronological order;"$a_s b_s a_a a_e d_s a_e e_s f_s b_e d_e g_s e_e f_e h_s g_e h_e$'Here, x_s denotes the starting time and x_e denotes the ending time of activity X. W need to schedule the activities in a set of rooms available to us. An activity can be scheduled in a room only if the room is reserved for the activity for its entire duration. What is the minimum number of rooms required?

(A) 3 (B) 4

(C) 5 (D) 6

Q. 26 Let $G=(V,E)$ be a direction graph with n vertices. A path from v_i to v_j in G is sequence of vertices $(v_i, v_{i1}, ..., v_j)$ such that $(v_k, v_{k+1}) \in E$ for all k in i through $j-1$. A simple path is a path in which no vertex appears more than once.
Let A be an $n \times n$ array initialized as follow

$$A[j,k] = \begin{cases} 1 \text{ if (j,k)} \in \text{E} \\ 0 \text{ otherwise} \end{cases}$$

Consider the following algorithm

```
for i = 1 to n
  for i = 1 to n
     for k = 1 to n
          A[j,k]=max (A[j,k](A[j,i])+[i,k]);
```

Which of the following statements is necessarily true for all j and k after terminal of the above algorithm?

(A) $A[j,k] \le n$

(B) If $A[j,j] \le n-1$, then G has a Haniltonian cycle

(C) If there exists a path from j to $k, A[j,k]$ contains the longest path lens from j to k

(D) If there exists a path from j to k, every simple path from j to k contain most $A[j,k]$ edges

YEAR 2004 ONE MARK

Q. 27 Level order traversal of a rooted tree can be done by stating from the root and performing

(A) preorder traversal (B) inorder traversal

(C) depth first search (D) breadth first search

Q. 28 Given the following input(4322,1334,1471,9679,1989,6171,6173,4199) and the hash function x mod 10, which of the following statements are true?

1. 9679,1989,4199 hash to the same value

2. 1471,6171 hash to the same value

3. All element hashes to a different value

(A) 1 only (B) 2 only

(C) 1 and 2 only (D) 3 and 4 only

Q. 29 The tightest lower bound on the number of comparisons, in the worst case, for comparision-based sorting is of the order of

(A) n (B) n^2

(C) $n \log n$ (D) $n \log^2 n$

YEAR 2004 TWO MARKS

Q. 30 Consider the label sequences obtained by the following pairs of traversals on a labeled binary tree. Which of these pairs identify a tree uniquely?

1. preorder and postorderr 2. inorderr and postorder

3. preorder and inorder 4. level order and postorder

(A) 1 only (B) 2 and 3

(C) 3 only (D) 4 only

Q. 31 Two matrices M_1 and M_2 are to be stored in arrays A and B respectively. Each array can be stored either in row-major or column-major order in contiguous memory locations. The time complexity of an algorithm to compute $M_1 \times M_2 \times$ will be

(A) best if A is in row-major, and B is in column-major order

(B) best if both are in row-major order

(C) best if both are in column-major order

(D) independent of the storage scheme

Q. 32 Suppose each set is represented as a linked list with elements in arbitrary order. Which of the operations among union, intersection, membership, cardinality will be the slowest?

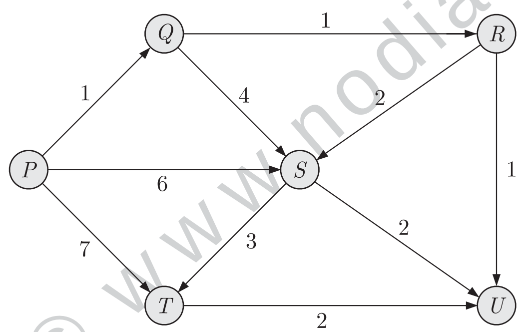
(A) union only

(B) intersection, membership

(C) membership, cardinality

(D) union, intersection

Q. 33 Suppose we run Dijkstra's single source shortest-path algorithm on the following edge-weighted directed graph with vertex P as as the source.



In what order do the nodes get included into the set of vertices ofr which the shortest path distances are finalized?

(A) P, Q, R, S, T, U

(B) P, Q, R, U, S, T

(C) P, Q, R, U, T, S

(D) P, Q, T, R, U, S

Q. 34 Let $A[1,....n]$ be an array storing a bit (1 or 0) at each location, and $f(m)$ is a function whose time complexity is $\theta(m)$. Consider the following program fragment

written in a C like language:

```
counter=0;
          for  (i=1;i<n;i++)
     {if       (A[i]==1)counter++
     else{f(counter); counter=0;}
     }
```

The complexity of this program fragment is

(A) $\Omega(n^2)$

(B) $\Omega(n \log n)$ and $O(n^2)$

(C) $\theta(n)$

(D) $o(n)$

Q. 35 The time complexity of the following C function is (assume $n > 0$)

```
int recursive(int n)     {
     if (n==1)
     return(1);
     else
     return(recursive(n-1)+recursive(n-1);
}
```

(A) $O(n)$ (B) $O(n \log n)$

(C) $O(n^2)$ (D) $O(2^n)$

Q. 36 The recurrence equation

$$T(1) = 1$$
$$T(n) = 2T(n-1) + n, n \leq 2$$

evaluates to

(A) $2^{n+1} - n - 2$ (B) $2^n - n$

(C) $2^{n+1} - 2n - 2$ (D) $2^n + n$

Q. 37 A program takes as input a balanced binary search tree with n leaf modes and computes the value of a function $g(x)$ for each node x. If the cost of computing $g(x)$ is min (number of leaf-nodes in lear-subtree of x, number of leaf-nodes in right-subtree of x) then the worst case time complexity of the program is

(A) $\theta(n)$ (B) $O(n \log n)$

(C) $O(n)^2$ (D) $O(2^n)$

YEAR 2005 ONE MARK

Q. 38 A undirected graph G has n nodes. Its adjacency matrix is given by by an $n \times n$ square matrix whose

1. diagonal elements are O's, and
2. non-diagonal elements are 1's. Which one of the following is TRUE?

(A) Graph G has no minimum spanning tree (MST)

(B) Graph G has a unique MST of cost in-1

(C) Graph G has multiple distinct MST's, each of cost n-1

(D) Graph G has multiple spanning trees of different costs

Q. 39 The time complexity of computing the transitive closure of a binary relation on a set of n elements is known to be

(A) $O(n)$ (B) $O(n \log n)$

(C) $O(n^{3/2})$ (D) $O(n^3)$

YEAR 2005 TWO MARKS

Q. 40 A priority-Queue is implemented as a Max-Heap, Initially, it has 5 elements. The level-order traversal of the heap is given below:

10, 8, 5, 3, 2

Two new elements '1' and '7' are inserted in the heap in that order. The level-order traversal of the heap after the insertion of the elements is

(A) 10, 8, 7, 5, 3, 2, 1 (B) 10, 8, 7, 2, 3, 1, 5

(C) 10, 8, 7, 1, 2, 3, 5 (D) 10, 8, 7, 3, 2, 1, 5

Q. 41 How many distinct binary search trees can be created out of 4 distinct keys?

(A) 5 (B) 14

(C) 24 (D) 35

Q. 42 In a complete k-ary, every internal node has exactly k children. The number of leaves in such a tree with n internal nodes is

(A) $n\ k$ (B) $(n-1)k+1$

(C) $n(k-1)+1$ (D) $n(k-1)$

Q. 43 Suppose $T(n)=2T(n/2)+n, T(0)=T(1)=1$

Which one of the following is FALSE?

(A) $T(n)=O(n^2)$ (B) $T(n)=\theta(n\log n)$

(C) $T(n)=\Omega(n^2)$ (D) $T(n)=O(n\log n)$

Q. 44 Let $G\ (V,E)$ an undirected graph with positive edge weights. Dijkstra's single source-shortes path algorithm can be implemented using the binary heap data structure with time complexity?

(A) $O(|V|^{2)}$ (B) $O(|E|)+|V|\log|V|)$

(C) $O(|V|\log|V|)$ (D) $O((|E|+|V|)\log|V|)$

Q. 45 Suppose there are $\log\ n$ sorted lists of $n/\log\ n$ elements each. The time complexity of producing a sorted list of all these elements is: (Hint: Use a heap data structure)

(A) $O(n\,\mathrm{loglog}n)$ (B) $\theta(n\log n)$

(C) $\Omega(n\log n)$ (D) $\Omega(n^{3/2})$

Common Data For Q. 46 & 47

Solve the problems and choose the correct answers.

Consider the following C-function:

```
double foo(int n){
     int i;
double sum;
if   (n==0) return 1.0;
     else {
     sum=0 0;
     for (i=0;<n;i+)
     sum += goo (i);
     return sum;
     }}
```

Q. 46 The space complexity of the above function is

(A) $O(1)$ (B) $O(n)$

(C) $O(n!)$ (D) $O(n^n)$

Q. 47 The space complexity of the above function is foo O and store the values of foo $(i), 0 <= i < n$, as and when they are computed. With this modification, the time complexity for function foo$O$ is significantly reduced. The space complexity of the modified function would be:

(A) $O(1)$ (B) $O(n)$

(C) $O(n^2)$ (D) $O(n!)$

Common Data For Q. 48 & 49

Solve the problems and choose the correct answers.

We are given 9 tasks $T_1, T_2, ... T_9$. The execution of each task requires one unit of time. We can execute one task at a time. T_i has a profit P_i and a deadline d_i profit P_i is earned if the task is completed before the end of the d_1^{th} unit of time.

Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
Profit	15	20	30	18	18	10	23	16	25
Deadline	7	2	5	3	4	5	2	7	3

Q. 48 Are all tasks completed in the schedule that gives maximum profit?

(A) All tasks are completed (B) T_1 and T_6 are left out

(C) T_1 and T_8 are left out (D) T_4 and T_6 are left out

Q. 49 What is the maximum profit earned?

(A) 147 (B) 165

(C) 167 (D) 175

YEAR 2006 ONE MARK

Q. 50 Consider the polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_2x^3$, where $a_i \neq 0, \forall i$. The minimum number of multiplications needed to evaluate p on an input x is

(A) 3 (B) 4

(C) 6 (D) 9

Q. 51 In a binary max heap containing n numbers, the smallest element can be found in time

(A) $\theta(n)$ (B) $\theta(\log n)$

(C) $\theta(\log\log n)$ (D) $\theta(1)$

Q. 52 Consider a weighted complete graph G on the vertex set $\{v_1, v_2, ... v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i - j|$. The weight of a minimum spanning tree of G is

(A) $n - 1$ (B) $2n - 2$

(C) $\binom{n}{2}$ (D)

Q. 53 To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, then data structure to be used is

(A) Queue (B) Stack

(C) Heap (D) B-Tree

Q. 54 A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. The roots is stored at $X[1]$. For a node stored at $X[1]$, the left child, if any, is stored in $X[2i]$ and the right child, if any, in $X[2i + 1]$. To be able to store any binary tree on n vertices, the minimum size of X should be

(A) $\log_2 n$ (B) n

(C) $2n + 1$ (D) $2n$

Q. 55 Which one the following in place sorting algorithms needs the minimum number of swaps?

(A) Quick-sort
(B) Insertion sort
(C) Selection sort
(D) Heap sort

Q. 56 Consider the following C-program fragment in which i, j, and n are integer variables.

$$\text{for } (i = n, j = 0; i > 0; i/2, j += i);$$

Let Val $(j) =$denote the value stored in the variable j after termination of the for loop. Which one of the following is true?

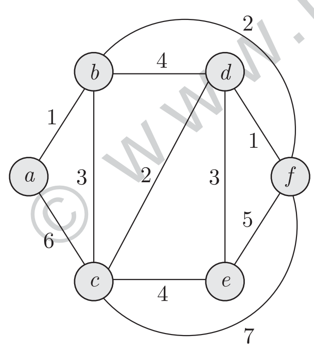
(A) $\text{val}(j) = \theta(\log n)$
(B) $val(j) = \theta(\sqrt{n})$
(C) $\text{val}(j) = \theta(n)$
(D) $\text{val}(j) = \theta(n \log n)$

Q. 57 An element in an array X is called a leader if it is grater than all elements to the right of it in X. The best algorithm to find all leaders in an array.

(A) Solves it in linear time using a left to right pass of the array
(B) Solves in linear time using a right to left pass
(C) Solves it is using divide and conquer in time $\theta(n \log n)$
(D) Solves it in time $\theta(n^2)$

YEAR 2006 TWO MARKS

Q. 58 Consider the following graph:



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm?

(A) $(a-b),(d-f),(b-f).(d-c),(d-e)$
(B) $(a-b),(d-f),(b-c),(b-f),(d-e)$
(C) $(d-f),(a-b),(d-c),(d-e),(d-e)$
(D) $(d-f),(a-b),(b-f),(d-e),(d-e)$

Q. 59 Let T be a depth first search tree in a undirected graph G Vertices u and v are leaves of this tree T. The degrees of both u and v in G are at least 2. Which one of the following statements is true?

(A) There must exist a vertex w adjacent to both u and v in G
(B) There must exist a vertex w whose removal disconnects u and v in G
(C) There must be exist a cycle in G containing u and v
(D) There must exist a cycle in G containing u and all its neighbours in G

Q. 60 A set X can be represented by an array $x[n]$ as follows

$$x[i] = \begin{cases} 1 & \text{if } i \in X \\ 0 & \text{otherwise} \end{cases}$$

Consider the following algorithm in which x,y and z are boolean arrays of size n ;

algorithm $zzz(x[], y[], z[])\{$
int i;
for $(i = 0; i < n; ++i)$
$z[i] = (x[i] \wedge \sim y[i]) \vee (\sim x[i] \wedge y[i])$
}

The set Z computed by the algorithm is

(A) $(X \cup Y)$
(B) $(X \cap Y)$
(C) $(X - Y) \cap (Y - X)$
(D) $(X - Y) \cup (Y - X)$

Q. 61 Consider the following is true?

$$T(n) = 2T([\sqrt{n}]) + 1, T(1) = 1$$

Which one of the following is true?

(A) $T(n) = \theta(\log\log n)$
(B) $T(n) = \theta(\log n)$
(C) $T(n) = \theta(\sqrt{n})$
(D) $T(n) = \theta(n)$

Q. 62 The median of n elements can be found in $O(n)$ time. Which one of the following is correct about the complexity of quick sort, in which remains is selected as pivot?

(A) $\theta(n)$
(B) $\theta(n \log n)$
(C) $\theta(n^2)$
(D) $\theta(n^3)$

Q. 63 Given two arrays of numbers $a_1 ....a_n$ and $b_1,.....b_n$ where each number is 0 or 1, the fastest algorithm to find the largest span (i,j) such that $a_{i+} + a_{i+1} + ... + a_j = b_i + b_{i+1} + ... + b_j$, or report that there is no such span,

(A) Takes $O(3^n)$ and $\Omega(2^n)$ time if hashing is permitted
(B) Takes $O(n^3)$ and $W(n^{2.5})$ time in the key comparison model
(C) Takes $\Theta(n)$ time and space
(D) Takes $O(\sqrt{n})$ time only if the sum of the $2n$ elements is an even number

Q. 64 Consider the following code written in a pass-by reference language like FORTAN and these statements about the code.

```
Subroutine swap (ix,iy)
          it=ix
L1 :      ix=iy
L2 :      iy=it
     end
     ia=3
     ib=8
     call swap (ia,ib+5)
     print*,ia,ib
     end
```

S1: The complier will generate code to allocate a temporary nameless cell, initialize it to 13, and pass the address of the cell to swap

S2: On execution the code will generate a runtime error on line 1.1

S3: On execution the code will generate a runtime error on line 1.2

S4: The program will print 13 and 8

S5: The program will print 13 and-2

Exactly the following set of statement (s) is correct:

(A) $S1$ and$S2$ (B) $S1$ and S4

(C) $S3$ (D) $S1$ amd S5

YEAR 2007 ONE MARK

Q. 65 The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes is a binary tree of height h is

(A) 2^h (B) $2^{h-1}-1$

(C)$2^{h+1}-1$ (D) 2^{h+1}

Q. 66 The maximum number of binary trees that can be formed with three unlabeled nodes is

(A) 1 (B) 5

(C) 4 (D) 3

Q. 67 Which of the following sorting algorithms has the lowest worst-case complexity?

(A) Merge sort (B) Bubble sort

(C) Quick sort (D) Selection sort

YEAR 2007 TWO MARKS

Q. 68 The inorder and preorder traversal of a binary tree are

dbeafcg and*abdecfg* respectively

The postorder traversal of the binary tree is

(A) *debfgca* (B) *edbgfca*

(C) *edbfgca* (D) *defgbca*

Q. 69 Consider the hash table of size seven, with starting index zero, and a has function $(3x+4)$ and 7. Assuming the has table is initially empty, which of the following is the contents of the table when the sequence 1,3,8,10 is inserted into the table using closed hashing?

Note that-denotes an empty location in the table

(A) 8,-,-,-,-,-,10 (B) 1,8,10,-,-,-,3

(C)1,-,-,-,-,-,,3 (D) 1,10,8,-,-,-,3

Q. 70 In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of time complexity, by

(A) Dijkstra's algorithm starting from S. (B) Warshall's algorithm

(C) performing a DFS starting from S (D) preforming a BFS starting from S

Q. 71 A complete n-ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If $L=41$, and $I=10$, what is the value of n

(A) 3 (B) 4

(C) 5 (D) 6

Q. 72 In the following C function, let $n \leq m$.

```
int gcd (n,m)
{
     if(n&m==0} return m;
     n=n&m;
     return gcd(m,n);
}
```

How many recursive calls are made by this function?

(A) $\Theta(\log_2 n)$

(B) $\Omega(n)$

(C) $\Theta(\log_2 \log_2 n)$

(D) $\Theta(\sqrt{n})$

Q. 73 What is the time complexity of the following recursive function :

```
     intDoSomething(int n)    {
     if       n<=2
          return 1;
     else
     return(DoSomething(floor(sqrt(n))
     )+   n);
          }
```

(A) $\Theta(n^2)$

(B) $\Theta(n \log_2 n)$

(C) $\Theta(\log_2 n)$

(D) $\Theta(\log_2 \log_2 n)$

Q. 74 Consider the process of inserting an element into a Max Heap, where the Max Heap is represented by an array. Suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of comparisons performed is

(A) $\theta(\log_2 n)$

(B) $\theta(\log_2 \log_2 n)$

(C) θn

(D) $\theta(n \log_2 n)$

Q. 75 Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w. Which of the following is FALSE ?

(A) There is a minimum spanning tree containing e.

(B) If e is not in a minimum spanning tree T,then in the cycle formed by adding e to T, all edges have the same weight.

(C) Every minimum spanning tree has an edge of weight w

(D) e is present in every minimum spanning tree

Q. 76 An array of n numbers is given, where n is an even number. The maximum as well as the minimum of these n numbers needs to be determined. Which of the following is TRUE about the number of comparisons needed?

(A) At least $2n$-c comparisons, for some constant c, are needed,

(B) At most $1.5n$-2 comparisons are needed.

(C) At least n,$\log_2 n$ comparisons are needed.

(D) None of the above

Q. 77 Consider the following C code segment:

```
int        IsPrime(n)
           {
                int i,n;
                for (i=2;i<=sqrt(n);i++)
                if (n&i==0)
                print(Not Prime\n"};
                          return0;)
                return 1;
           }
```

Let $T(n)$ denote the number of times the for loop is executed by the program on input n. Which of the following is TRUE?

(A) $T(n) = O(\sqrt{n})$ and $T(n) + \Omega(\sqrt{n})$

(B) $T(n) = O(\sqrt{n})$ and $T(n) + \Omega(1)$

(C) $T(n) = O(n)$ and $T(n) = \Omega(\sqrt{n})$

(D) None of these

Common Data For Q. 78 & 79

Solve the problems and choose the correct answers.

Suppose the letters a,b,c,d,e,f have probabilities $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32}$ respectively.

Q. 78 Which of the following is the Hoffman code for the letters a,b,c,d,e,f?

(A) 0, 10, 110, 1110, 11110, 11111

(B) 11, 10, 011, 010, 001, 000

(C) 11, 10, 01, 001, 0001, 0000

(D) 110, 100, 010, 000, 111

Q. 79 What is the average length of the correct answer to Q. ?

(A) 3

(B) 2.1875

(C) 2.25

(D) 1.781

YEAR 2008 ONE MARK

Q. 80 The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity.

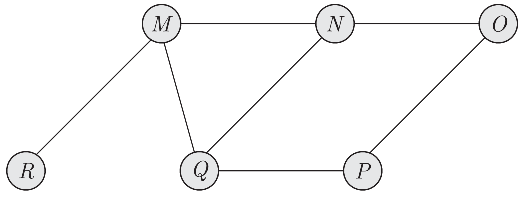
(A) $\Theta(n)$

(B) $\Theta(m)$

(C) $\Theta(m+n)$

(D) $\Theta(mn)$

Q. 81 The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



(A) $MNOPQR$

(B) $NQMPOR$

(C) $QMNPRO$

(D) $QMNPOR$

YEAR 2008 TWO MARKS

Q. 82 Consider the following function;

$$f(n) = 2^n$$
$$g(n) = n!$$
$$h(n) = n^{\log n}$$

Which of the following statements about the asymptotic behavior of $f(n)$. $g(n)$ and $h(n)$ is true?

(A) $f(n) = O(g(n)); g(n) = O(h(n))$

(B) $f(n) = \Omega(g(n)); g(n) = O(h(n))$

(C) $g(n) = O(f(n)); h(n) = O(f(n))$

(D) $h(n) = O(f(n)); g(n) = \Omega(f(n))$

Q. 83 The minimum number of comparison required to determine if an integer appears more than $n/2$ times in a sorted array of n integers is

(A) $\Theta(n)$ (B) $\Theta(\log n)$

(C) $\Theta(\log * n)$ (D) $\Theta(1)$

Q. 84 AB-tree of order 4 is built from scratch by 10 successive insertions. What is the maximum number of node splitting operations that may take place?

(A) 3 (B) 4

(C) 5 (D) 6

Q. 85 G is a graph on n vertices and $2n - 2$ edges. The edges of G can be partitioned into two edge-disjiont spanning trees. Which of the following is NOT true for G?

(A) For every subset of k vertices, the induced sub graph has a most $2k - 2$ edges.

(B) The minimum cut in G has a least two edges

(C) There are two edges-disjoint paths between every pair of vertices

(D) There are two vertex-disjoint paths between every pair of vertices.

Q. 86 Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into sub-lists each of which contains at least one-fifth of the elements. Let $T(n)$ be the number of comparisons required to sort n elements. Then

(A) $T(n) \le 2T(n/5) + n$ (B) $T(n) \le T(n/5) + T(4n/5) + n$

(C) $T(n) \le 2T(4n/5) + n$ (D) $T(n) \le 2T(n/2) + n$

Q. 87 The subset-sum problem is defined as follows: Given a set S of n positive integers and a positive integer W; determine whether there is aa subset of S whose elements sum to W.

An algorithm Q Solves this problem in $O(nW)$ time. Which of the following statements is false?

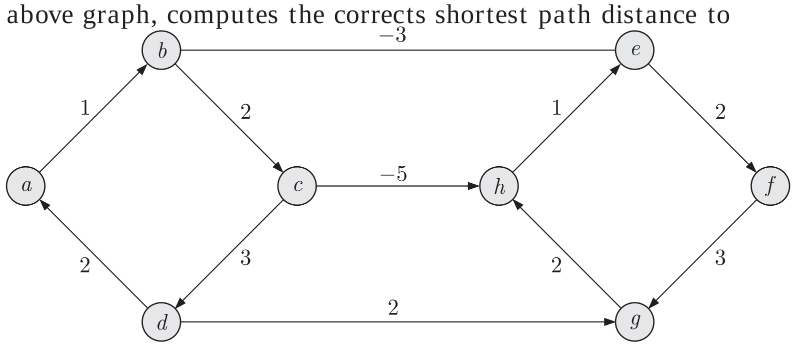
(A) Q sloves the subset-sum problem unpolynomial time when the input is encoded in unary

(B) Q solves the subset-sum problem is polynominal time when the input is encoded in binary

(C) The subset sum problem belongs to the class NP

(D) The subset sum problem in NP-hard

Q. 88 Dijkstra's single source shortest path algorithm when run from vertex a in the above graph, computes the corrects shortest path distance to



(A) only vertex a (B) only vertices a, e, f, g, h

(C) only vertices, a, b, c, d (D) all the vertices

Q. 89 You are given the postorder traversal, P of a binary search tree on the n elements 1,2,.....,n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

(A) $\Theta(\log n)$ (B) $\Theta(n)$

(C) $\Theta(n \log n)$

(D) none of the above, as the tree cannot be uniquely determined.

Q. 90 We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is

(A) $\Theta(\log n)$ (B) $\Theta(n)$

(C) $\Theta(n \log n)$ (D) $\Theta(n^2)$

Common Data For Q. 91 & 92:

Consider the following C functions:

```
int f1 (int n)
{
     If(n==0||n==1)
          return n;
     else
          return(2*f1(n-1)+3*f1(n-2));
}
int f2(int n)
{
     int i;
     int X[N], Y[N], Z[N];
     X[1]=1;Y[1]=2;Z[1]=3;
     for  (i=2;i<=n;i++){
         X[i]=Y[i-1]+Z[i-2];
         Y[i]=2*X[i];
         z[i]=3*X[i];
     }
     return X[n];
}
```

Q. 91 The running time of $f1(n)$ and $f2(n)$ are

(A) $\Theta(n)$ and $\Theta(n)$ (B) Θ(2") and $O(n)$

(C) $\Theta(n)$ and Θ(2") (D) Θ(2") and Θ(2")

Q. 92 $f1$ (8) $f2$ (8) return the values

(A) 1661 and 1640

(B) 59 and 59

(C) 1640 and 1640

(D) 1640 and 1661

Statement for Linked Q. 93 & 94:

The subset-sum problem is defined as follows. Given set of n positive integers, $S = \{a_1, a_2, a_3, ... a_n\}$ and a positive integer W is there a subset S whose elements sum of W? A dynamic program for solving this problem uses a 2-dimensiond Boolean array, X with n rows and W-1 columns $X[i,j], 1 \le i \le W$, is TRUE if and only if there is a subset of $\{a_1, a_2, ... a_i\}$ whose elements sum to j.

Q. 93 Which of the following is valid for $2 \le i \le n$ and $a_1 \le j \le W$?

(A) $X[i,j] = X[i-1,j] \vee X[i,j-a_i]$

(B) $X[i,j] = X[i-1,j] \vee X[i-1,j-a_i]$

(C) $X[i,j] = X[i-1,j] \wedge X[i,j-a_i]$

(D) $X[i,j] = X[i-1,j] \wedge X[i-1,j-a_i]$

Q. 94 Which entry of the array X, if TRUE, implies that there is a subset whose elements sum to W?

(A) $X[1, W]$ (B) $X[n, 0]$

(C) $X[n, W]$ (D) $X[n-1, n]$

Statement for Linked Q. 95 & 96:

Consider the following C program that attempts to locate an element x in an array Y[ ] using binary search. The program is erroneous.

```
f(int Y[10],int x){
     int i,j,k;
     i=0;j=9;
     do {
     k=(i+j)/2;
     if (Y[k]<x)i=k; else j=k;
} while ((Y[k]!=x)&&(i<j)),
if (Y[k]==x) print f(”x is in the array”);
     else print f(”x is not in the array”);
}
```

Q. 95 On which of the following contents of Y and x does the program fail?

(A) Y is [1 2 3 4 5 6 7 8 9 10] and $x < 10$

(B) Y is [1 3 5 7 9 11 13 15 17 19] and $x < 1$

(C) Y is [2 2 2 2 2 2 2 2 2 2] and $x > 2$

(D) Y is [2 4 6 8 10 12 14 16 18 20] and $2 < x < 20$ and x is even

Q. 96 The correction needed in the program to make it work properly is

(A) change line 6 to : if $(Y[k]) < x) i = k + 1;$ else j = k - 1;

(B) change line 6 to: if $(Y[k] < x) i = k - 1;$ else j = k + 1;

(C) change line 6 to: if $(Y[k] < x) i = k;$ else $j = k;$

(D) change line 7 to : } while $((Y[k] == x) \&\& (i < j));$

YEAR 2009 ONE MARK

Q. 97 What is the number of swaps required to sort n elements using selection sort, in the worst case ?

(A) $\theta(n)$ (B) $\theta(n \log n)$

(C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

Q. 98 Which of the following statement(s) is/are correct regarding Bellman-Ford shortest path algorithm ?

P. Always finds a negative weighted cycle, if one exists.

Q. Finds whether any negative weighted cycle is reachable from the source

(A) P only

(B) Q only

(C) Both P and Q

(D) Neither P nor Q

Q. 99 Let π_A be a problem that belongs to the class NP. Then which one of the following is TRUE ?

(A) There is no polynomial time algorithm for π_A

(B) If π_A can be solved deterministically in polynomial time, then $P = NP$

(C) If π_A is NP-hard, then it is NP-complete

(D) π_A may be undecidable

YEAR 2009 TWO MARKS

Q. 100 The running time of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} n & n \leq 3 \\ T\left(\frac{n}{3}\right) + cn & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm ?

(A) $\theta(n)$ (B) $\theta(n \log n)$

(C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

Q. 101 The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function $h(k) = k \bmod 10$ and linear probing. What is the resultant hash table ?

(A)

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

(B)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

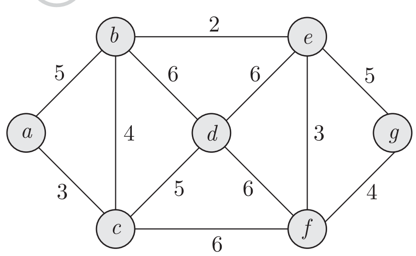
(C)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(D)

0	
1	
2	12, 2
3	13, 3, 23
4	
5	
6	
7	
8	
9	

Q. 102 Consider the following graph:



Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm ?

(A) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)

(B) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)

(C) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)

(D) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

Q. 103 In quick sort, for sorting n elements, the $(n/4^{th})$ smallest element is selected as pivot using an $O(n)$ time algorithm. What is the worst case time complexity of the quick sort ?

(A) $\theta(n)$ (B) $\theta(n \log n)$

(C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

Common Data For Q. 104 & 105 :

A sub-sequence of a given sequence is just the given sequence with some elements (possibly none of all) left out. We are given two sequence $X[m]$ and $Y[n]$ of length m and n, respectively, with indexes of X and Y starting from 0.

Q. 104 We wish to find the length of the longest common sub-sequence (LCS) of $x[m]$ and $Y[n]$ of lengths m and n, where an incomplete recursive definition for the function $l(i,j)$ to compute the length of the LCS of $X[m]$ and $Y[n]$ is given below :

$l(i,j)$ $=0$, if either $i = 0$ or $j = 0$

$=$expr1, if $i,\ j > 0$ and $x[i-1] = Y[j-1]$

$=$expr2, if $i,\ j > 0$ and $x[i-1] \neq Y[j-1]$

Which one of the following options is correct ?

(A) expr $1 \equiv l(i-1,j) + 1$

(B) expr $1 \equiv l(i,j-1)$

(C) expr $2 \equiv \max(l(i-1,j), l(i,j-1))$

(D) expr $2 \equiv \max(l(i-1,j-1), l(i,j))$

Q. 105 The values of $l(i,j)$ could be obtained by dynamic programming based on the correct recursive definition of $l(i,j)$ of the form given above, using an array $L[M,N]$, where $M = m+1$ and $N = n+1$, such that $L[i,j] = l(i,j)$.

Which one of the following statements would be true regarding the dynamic programming solution for the recursive definition of $l(i,j)$?

(A) All elements of L should be initialized to 0 for the values of $l(i,j)$ to be properly computed.

(B) The values of $l(i,j)$ may be computed in a row major order or column major order of $L[M,N]$

(C) The values of $l(i,j)$ cannot be computed in either row major order or column major order of $L[M,N]$

(D) $L[p,q]$ needs to be computed before $L[r,s]$ if either $p < r$ or $q < s$

YEAR 2010 ONE MARK

Q. 106 Two alternative package A and B are available for processing a database having 10^k records. Package A requires 0.0001 n^2 time units and package B requires 10 $n \log_{10} n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A ?

(A) 12 (B) 10

(C) 6 (D) 5

YEAR 2010 TWO MARKS

Q. 107 The weight of a sequence $a_0, a_1 ......, a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + ...a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements form the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, ...a_{n-1}$ and Y the maximum possible weight of a subsequence of $a_1, a_2, ..., a_{n-1}$. Then X is equal to

(A) max $(Y, a_0 + Y)$

(B) max $(Y, a_0 + Y/2)$

(C) max $(Y, a_0 + 2Y)$

(D) $a_0 + Y/2$

Common Data For Q. 108 & 109

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge $\{i, j\}$.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

Q. 108 What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T ?

(A) 7

(B) 8

(C) 9

(D) 10

Q. 109 What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges ?

(A) 7

(B) 8

(C) 9

(D) 10

YEAR 2011 ONE MARK

Q. 110 An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array $A[0:n-1]$ is given below.

Let L_i denote the length of the longest monotonically increasing sequence starting at index i in the array

Initialize $L_{n-1} = 1$

For all i such that $0 \le i \le n-2$

$$L_i = \begin{cases} 1 + L_{i+1} & \text{If} A[i] < A[i+1] \\ 1 & \text{Otherwise} \end{cases}$$

Finally the length of the longest monotonically increasing sequence is $\max(L_0, L_1, ..., L_{n+1})$. Which of the following statements is TRUE?

(A) The algorithm uses dynamic programming paradigm

(B) The algorithm has a linear complexity and uses branch and bound paradigm

(C) The algorithm has a non-linear polynomial complexity and uses branch and bound paradigm

(D) The algorithm uses divide and conquer paradigm

YEAR 2011 TWO MARKS

Q. 111 We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

(A) 0 (B) 1

(C) $n!$ (D) $\frac{1}{n+1}.^{2n}C_n$

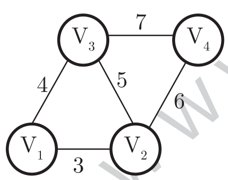
Q. 112 Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is $pqr + prs + pst$. If $p = 10$, $q = 100$, $r = 20$, $s = 5$ and $t = 80$, then the minimum number of scalar multiplications needed is

(A) 248000 (B) 44000

(C) 19000 (D) 25000

Statement for Linked Q. 113 and 114 :

An undirected graph $G(V,E)$ contains $n(n > 2)$ nodes named $v_1, v_2, ..., v_n$. Two nodes v_i, v_j are connected if an only if $0 < |i - j| \le 2$. Each edge (v_i, v_j) is assigned a weight $i + j$. A sample graph with $n = 4$ is shown below.



Q. 113 What will be the cost of the Minimum Spanning Tree (MST) of such a graph with n nodes?

(A) $\frac{1}{12}(11n^2 - 5n)$ (B) $n^2 - n + 1$

(C) $6n - 11$ (D) $2n + 1$

Q. 114 The length of the path from v_5 to v_6 in the MST of previous question with $n = 10$ is

(A) 11 (B) 25

(C) 31 (D) 41

YEAR 2012 ONE MARK

Q. 115 The Worst case running time to search for an element in a balanced binary search tree with $n2^n$ element is

(A) $\Theta(n \log n)$

(B) $\Theta(n2^n)$

(C) $\Theta(n)$

(D) $\Theta(\log n)$

Q. 116 Let $W(n)$ and $A(n)$ denote respectively, the worst case and average case running time of an algorithm executed on an input of zise n. Which of the following is ALWAYS TRUE?

(A) $A(n) = \Omega(W(n))$ (B) $A(n) = \Theta(W(n))$

(C) $A(n) = O(W(n))$ (D) $A(n) = o(W(n))$

YEAR 2012 TWO MARKS

Q. 117 Let G be a weighted graph with edge weights greater than one and G' be the graph constructed by squaring the weights of edge in G. Let T and T' be the minimum spanning trees of G and G' respectively, with total weights t and t'. Which of the following statements is TRUE?

(A) $T' = T$ with total weight $t' = t^2$ (B) $T' = T$ with total weight $t' < t^2$

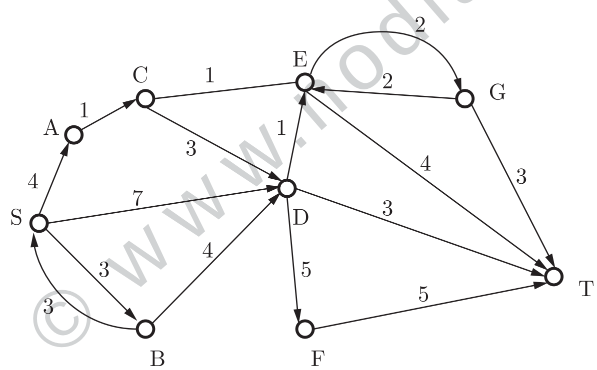
(C) $T' \neq T$ but total weight $t' = t^2$ (D) None of the above

Q. 118 A list of n strings, each of length n, is sorted into lexicographic order using the merge sort algorithm. The worst case running time of this computation is

(A) $O(n \log n)$ (B) $O(n^2 \log n)$

(C) $O(n^2 + \log n)$ (D) $O(n^2)$

Q. 119 Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



(A) SDT (B) SVDT

(C) SACDT (D) SACET

****

ANSWER KEY

Algorithms									
1	2	3	4	5	6	7	8	9	10
(A)	(D)	(A)	(C)	(B)	(A)	(C)	(B)	(B)	(B)
11	12	13	14	15	16	17	18	19	20
(D)	(D)	(C)	(C)	(B)	(D)	(D)	(B)	(C)	(D)
21	22	23	24	25	26	27	28	29	30
(A)	(A)	(C)	(D)	(D)	(B)	(C)	(B)	(B)	(A)
31	32	33	34	35	36	37	38	39	40
(C)	(B)	(D)	(A)	(A)	(A)	(B)	(C)	(B)	(D)
41	42	43	44	45	46	47	48	49	50
(C)	(C)	(D)	(A)	(D)	(A)	(C)	(C)	(A)	(C)
51	52	53	54	55	56	57	58	59	60
(B)	(A)	(A)	(B)	(D)	(C)	(A)	(D)	(A)	(D)
61	62	63	64	65	66	67	68	69	70
(B)	(C)	(A)	(D)	(C)	(C)	(D)	(B)	(A)	(D)
71	72	73	74	75	76	77	78	79	80
(B)	(D)	(C)	(B)	(B)	(B)	(C)	(B)	(C)	(C)
81	82	83	84	85	86	87	88	89	90
(A)	(A)	(B)	(C)	(A)	(C)	(D)	(B)	(C)	(B)
91	92	93	94	95	96	97	98	99	100
(C)	(*)	(D)	(B)	(A)	(B)	(C)	(B)	(C)	(C)
101	102	103	104	105	106	107	108	109	110
(C)	(D)	(B)	(D)	()	()	()	()	()	()
111	112	113	114	115	116	117	118	119	
()	()	()	()	()	()	()	()	()	