Ch- Vectors

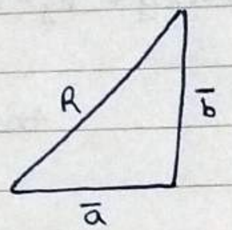
☆ Vectors

$AB = (\bar{b} - \bar{a})$

[$\bar{a}$ & $\bar{b}$ are position vectors]

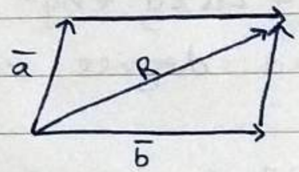
☆ Addition : Triangle Law

$R = \bar{a} + \bar{b}$ (3rd side of Δ)



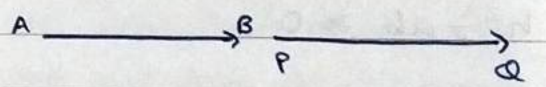
Parallelogram Law

$R = \bar{a} + \bar{b}$ [Diagonal of $\parallel^{gm}$]



☆ Collinear or parallel vector :-

$\overline{AB} = K \cdot \overline{PQ}$



$\overline{AB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$\overline{PQ} = 4\hat{i} + 6\hat{j} + 10\hat{k}$

$\therefore \frac{2}{4} = \frac{3}{6} = \frac{5}{10} = \frac{1}{2}$

$\overline{AB} = \cancel{2}\, 2\overline{PQ}$

☆ Distance of $P(x, y, z)$ from planes:

i) Distance of P from XY plane $= |z|$

ii) $YZ = |x|$

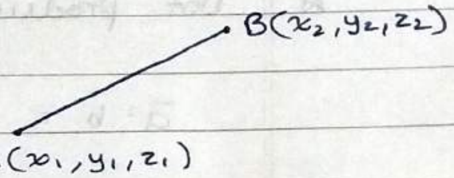
iii) $XZ = |y|$

☆ Distance of $P(x, y, z)$ from origin:

$$l(OP) = \sqrt{x^2 + y^2 + z^2}$$

☆ Distance between any two points in space

$$l(AB) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$



☆ Distance of point $P(x, y, z)$ from co-ordinates axes

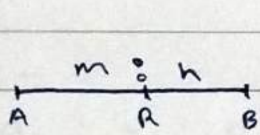
i) from x-axis $= \sqrt{y^2 + z^2}$

ii) from Y-axis $= \sqrt{x^2 + z^2}$

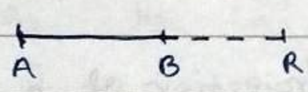
iii) from Z-axis $= \sqrt{x^2 + y^2}$

☆ Section Formula:-

Internal $\bar{r} = \dfrac{m\bar{b} + n\bar{a}}{m+n}$



External $\bar{r} = \dfrac{m\bar{b} - n\bar{a}}{m-n}$



☆ Midpoint formula

$$\frac{\bar{a}+\bar{b}}{2}$$

☆ Centroid formula

$$\Delta,\ \bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

$$||^{gm},\ \bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4}$$

☆ Dot product (Scalar)

$$\bar{a}\cdot\bar{b} = |\bar{a}||b|\cos\theta$$

$$\cos\theta = \frac{a\cdot b}{|a||b|}$$

☆ Cross product (Vector)

$$a\times b = |a||b|\sin\theta\cdot\hat{n}$$

$$|a\times b| = |a||b|\sin\theta$$

$$\sin\theta = \cancel{\frac{|a|\cdot|b|}{|a|}}\ \frac{|a\times b|}{|a||b|}$$

__Scalar__

$\bar{a}\perp\bar{b} = \theta = 90^\circ$

$\bar{a}\cdot\bar{b} = 0$

$\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$

$i\cdot i = j\cdot j = k\cdot k = 1$

$i\cdot j = j\cdot k = k\cdot i = 0$

__Vector__

$\bar{a} \parallel \bar{b} = \theta = 0$

$\bar{a}\times\bar{b} = 0$

$\bar{a}\times\bar{b} \neq \bar{b}\times\bar{a}$

$\bar{a}\times\bar{b} = -\bar{b}\times\bar{a}$

$i\times i = j\times j = k\times k = 0$

☆ Projection of $\bar{a}$ on $\bar{b}$ $= \dfrac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$

Projection of $\bar{b}$ on $\bar{a}$ $= \dfrac{\bar{b}\cdot\bar{a}}{|\bar{a}|}$

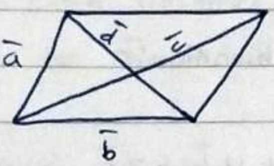
☆ Cross product

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

☆ Area of $\|^{gm}$

$= |\bar{a} \times \bar{b}|$

or $\frac{1}{2} |\bar{c} \times \bar{d}|$



☆ Area of triangle

$= \frac{1}{2} |\bar{AB} \times \bar{AC}|$

☆ Volume of parallelopiped $= [\bar{a}\ \bar{b}\ \bar{c}]$

☆ Volume of Tetrahedron $= \frac{1}{6} [\bar{a}\ \bar{b}\ \bar{c}]$

☆ If $\bar{a}, \bar{b}, \bar{c}$ are coplanar then $[\bar{a}\ \bar{b}\ \bar{c}] = 0$

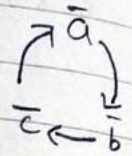
☆ Scalar Triple product (S.T.P) :-

$$\bar{a} \cdot |\bar{b} \times \bar{c}| = [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

☆ Vector triple product

$[\bar{a}\ \bar{b}\ \bar{c}] = [\bar{b}\ \bar{c}\ \bar{a}] = [\bar{c}\ \bar{a}\ \bar{b}]$

$[\bar{a}\ \bar{b}\ \bar{c}] = -[\bar{a}\ \bar{c}\ \bar{b}]$



$\bar{a} \times (\bar{b} \times \bar{c})$

clockwise = +ve
Anti-clockwise = -ve

☆ Linear Combination:-

$\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$

☆ Direction Angles $[\alpha, \beta, \gamma]$

Direction Cosines $[l, m, n]$

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

$l^2 + m^2 + n^2 = 1$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Direction ratios (a, b, c)

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda$$

$$l = \frac{\pm a}{\sqrt{a^2+b^2+c^2}},\quad m = \frac{\pm b}{\sqrt{a^2+b^2+c^2}},\quad n = \frac{\pm c}{\sqrt{a^2+b^2+c^2}}$$

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☆ Angle between two line if d.r.s is given

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \cdot \sqrt{a_2^2+b_2^2+c_2^2}}$$

☆ Unit vector:

$$\hat{a} = \frac{\bar{a}}{|a|}$$

☆ If $\bar{a}$, $\bar{b}$ are non-zero vectors and $\bar{a}\cdot\bar{b} = 0$ then $\bar{a}$ is orthogonal to $\bar{b}$.

☆ If $\bar{a} = K\cdot\bar{b}$ then $\bar{a}$ and $\bar{b}$ are parallel where K is scalar.

☆ The coordinates of the points which are at a distance of d units from the points P (x_1, y_1, z_1) are given by

$$(x_1 \pm ld \,,\, y_1 \pm md \,,\, z \pm nd)$$

☆ To find a unit vector perpendicular to $\bar{a}$ and $\bar{b}$

$$= \frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$$

☆ $a\cdot b = |a|\,|b| \cos\theta$

$|a\times b| = |a|\,|b| \sin\theta$

☆ To find direction ratios of vector perpendicular to $\bar{a}\times\bar{b}$ use $[\bar{a}\times\bar{b}]$ - Matrix Method.