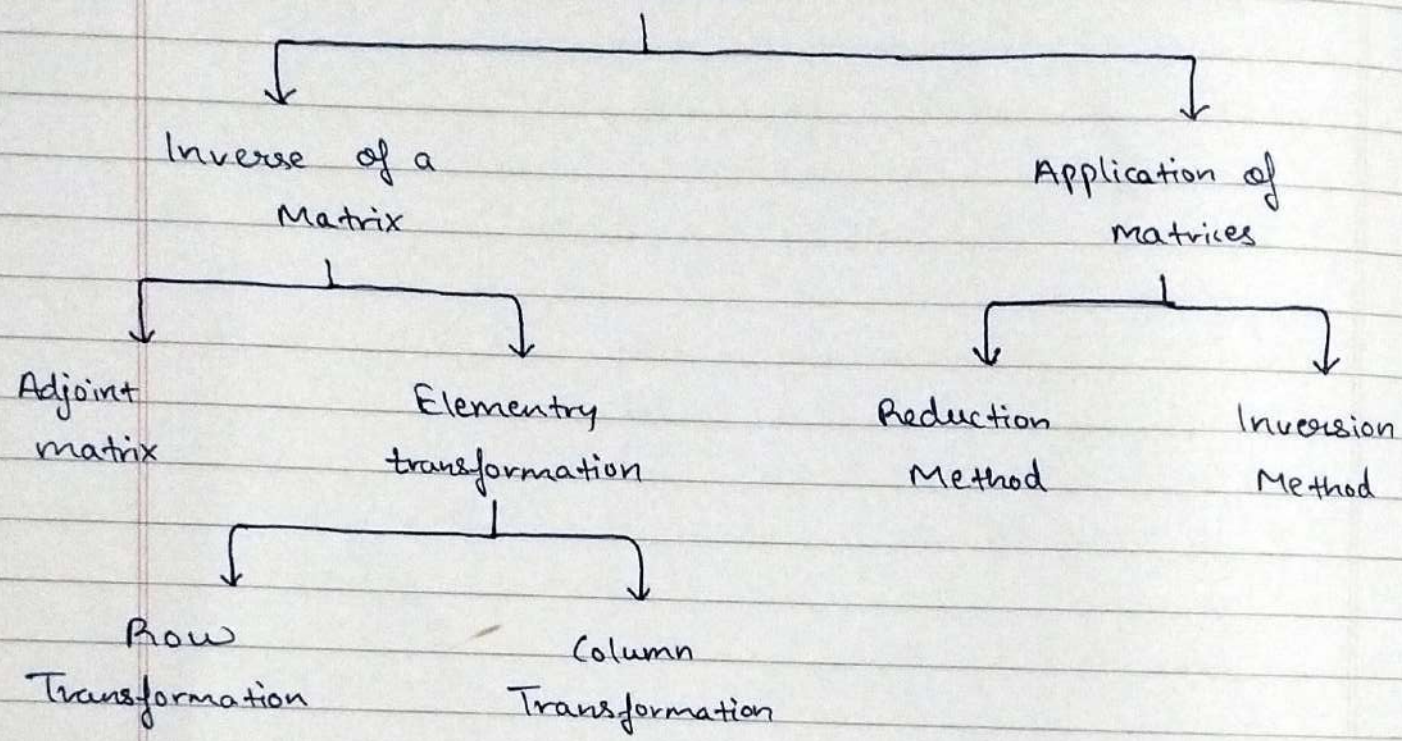
Ch - Matrices

Check whether Matrix is invertible or not in every sum.



☆ Inverse of non-singular or square matrix using Adjoint Method.

$$A^{-1} = \frac{1}{|A|} (adj\ A)$$

step 1:- Check whether Matrix A is invertible or not by calculating its value.

if $|A| = 0$ then matrix is not invertible.

Matrix should be $|A| \neq 0$

step 2:- Minors:-

Find minor of every element.

e.g:- for a_{12} :- Strike out 1st row and 2nd Column

Step 3:- Co-factors:-

Find co-factors of every element in the Matrix.

$$C_{ij} = (-1)^{i+j} \cdot M_{ij}$$

if $i+j$ = Even then no change.

if $i+j$ = odd then sign change of the element.

e.g:- $M_{12} = -3$

then $C_{12} = 3$

Step 5:- Adjoint Matrix:-

It is the transpose of co-factor Matrix.

e.g:-

$$\text{co-factor Matrix} = \begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & 3 \\ -2 & -2 & 2 \end{bmatrix}$$

$$\text{then } \text{adj}A = \begin{bmatrix} -4 & 3 & -2 \\ 2 & 0 & -2 \\ -2 & 3 & 2 \end{bmatrix}$$

Step 6:- Inverse of Matrix using:-

$$A^{-1} = \frac{1}{|A|} (\text{adj}A)$$

~~Elementary transformations~~

Row transformation for obtaining Inverse Matrix.

~~$AX = B^{-1}$~~

$$A A^{-1} = B$$

where B is Identity Matrix.

$$A A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Convert A into identity matrix using suitable row transformations and apply the changes to B aswell.

for Identity Matrix:-

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

1. Reduce a_{11} to 1
2. Using a_{11} make a_{21} and a_{31} 0.
3. ~~Using~~ Reduce a_{22} to 1.
4. Using a_{22} make a_{12} and a_{32} as 0.
5. Reduce a_{33} to 1
6. Using a_{33} make a_{13} and a_{23} as 0.

Column Transformations:-

$$AA^{-1} = B$$

where B = Identity Matrix

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Step (1): Reduce a_{11} to 1

(2) Using a_{11} make a_{12} and a_{13} as 0.

(3) Reduce a_{22} to 1.

(4) Using a_{22} make a_{21} and a_{23} as 0.

(5) Reduce a_{33} to 1.

(6) Using a_{33} make a_{31} and a_{32} as 0.

Reduction Method:-

i) $AX = B$

where, coefficient Matrix :- 'A'

Unknown Matrix :- 'X'

Constant Matrix :- 'B'

ii) Reduce the matrix A into an upper or lower triangular Matrix using Suitable Row Transformations only.

e.g:- $x + 3y + 2z = 6$

$3x - 2y + 5z = 5$

$2x - 3y + 6z = 7$

$$\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}}_{B}$$

★ Inversion Method :-

$AX = B$

$X = A^{-1} B$

∴ Find A^{-1} and then multiply both A^{-1} and B to get the value of unknown matrix 'X'.