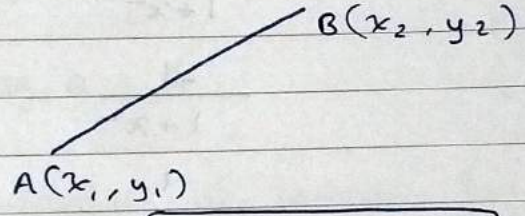
Ch- Pair of straight lines

① Slope of a line

$$\boxed{m = \tan\theta}$$

- If A, B, C are collinear points then
$$m_{AB} = m_{BC} = m_{AC}$$

- 

$$\boxed{m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}}$$

- If m_1 and m_2 are slopes of two intersecting lines then

$$\boxed{\tan\theta = \frac{m_1 - m_2}{1 - m_1 \cdot m_2}}$$

* Different forms of Eqⁿ :-

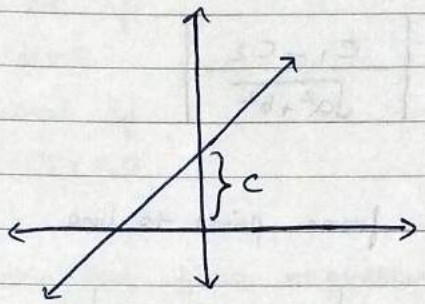
① Slope point form :-
$$(y - y_1) = m(x - x_1)$$

② Two point form :-

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

③ Slope intercept form:-

$y = mx + c$



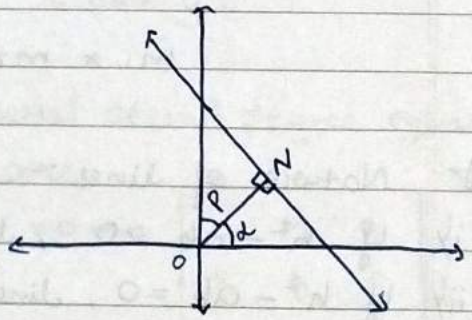
$y = mx$ if line passes through origin

④ Double intercept form:-

$$\frac{x}{a} = \frac{y}{b} = 1$$

⑤ Normal form

$x\cos\alpha + y\sin\alpha = p$



* General form

$ax + by + c = 0$

$m = \frac{-a}{b}$

* i) For parallel lines : $ax + by + c_1 = 0$

$ax + by + c_2 = 0$

ii) For perpendicular lines: $ax + by + c_1 = 0$

$bx + ay + c_2 = 0$

* Distance betⁿ parallel lines

$$d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

* Distance from point to line

$$PM = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

★ Sum and product of Slopes :-

$$m_1 + m_2 = \frac{-2h}{b}$$

$$m_1 \times m_2 = \frac{a}{b}$$

★ Nature of lines :-

i) If $h^2 - ab > 0$, lines are real and distinct

ii) If $h^2 - ab = 0$, lines are coincident or parallel

iii) If $h^2 - ab < 0$ then ~~lines~~ eqⁿ does not represent pair of lines

★ The Auxillary eqⁿ of pair of lines ~~$ax^2 + 2hyx$~~

$$ax^2 + 2hxy + by^2 = 0$$

$$\therefore \boxed{bm^2 + 2hm + a = 0}$$

★ Line represented by $ax^2 + 2hxy + by^2 = 0$
i) are perpendicular if
$$a + b = 0$$
ii) Parallel / coindent if
$$h^2 - ab = 0$$

★ Acute angle 'θ' between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is given by
$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$$

★ $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is called general second degree equation in x and y.

★ Conditions necessary for a general second degree equation to represent a pair of line are:
i) $abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \longrightarrow \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$
ii) $h^2 - ab \geq 0$

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines then
1) These lines are parallel to the lines represented by $ax^2 + 2hxy + by^2 = 0$
2) Acute $\&$ angle is given by $\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$
3) Condition for line to be 1^{er} is $a + b = 0$
4) Condition for line to be II^{el} is $h^2 - ab = 0$
5) Condition for lines to intersect each other, $h^2 - ab \geq 0$ and the coordinates of their point of intersection :-
$$\left[\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right]$$