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Rotational motion

Distinguish between centripetal and centrifugal force

Centripetal force	Centrifugal force
① It is real force	① It is not real force or imaginary force or pseudo force
(ii) It is responsible for circular motion	(ii) It is because of circular motion
(iii) It is acting along the radius and towards the centre	(iii) It is acting along the radius and away from the centre
(iv) It is acting in inertial frame of reference	(iv) It is acting in not inertial frame of reference

$\because f = ma$

$\therefore$ Centripetal force = mass × centripetal acceleration since $a_{cp} = r\omega^2 = \frac{v^2}{r} = v\omega$

$\therefore F_{cp} = mr\omega^2 = \frac{mv^2}{r} = mv\omega = F_{cf}$

$\Rightarrow$ Consider a vehicle of mass 'm' moving with linear speed 'v' on curved path of radius 'r'. If it's linear speed is doubled then centrifugal force increases by four times while centripetal force remains the same, because - - -.

$$\overset{\text{Depends}}{F_{cp}} = \frac{mv^2}{r} = \overset{\text{Depends}}{F_{cf}}$$

F_{cp}
- Gravitational force
- Electrostatic force
- Tension
- frictional force

$\because F_{cf} = \frac{mv^2}{r}$

$\therefore F \propto v^2$

$\therefore \frac{F_1}{F_2} = \left(\frac{v_1}{v_2}\right)^2$

$\frac{F_1}{F_2} = \left(\frac{v}{2v}\right)^2$

$\frac{F_1}{F_2} = \frac{1}{4}$

$\Rightarrow \boxed{F_2 = 4F_1}$

(3)

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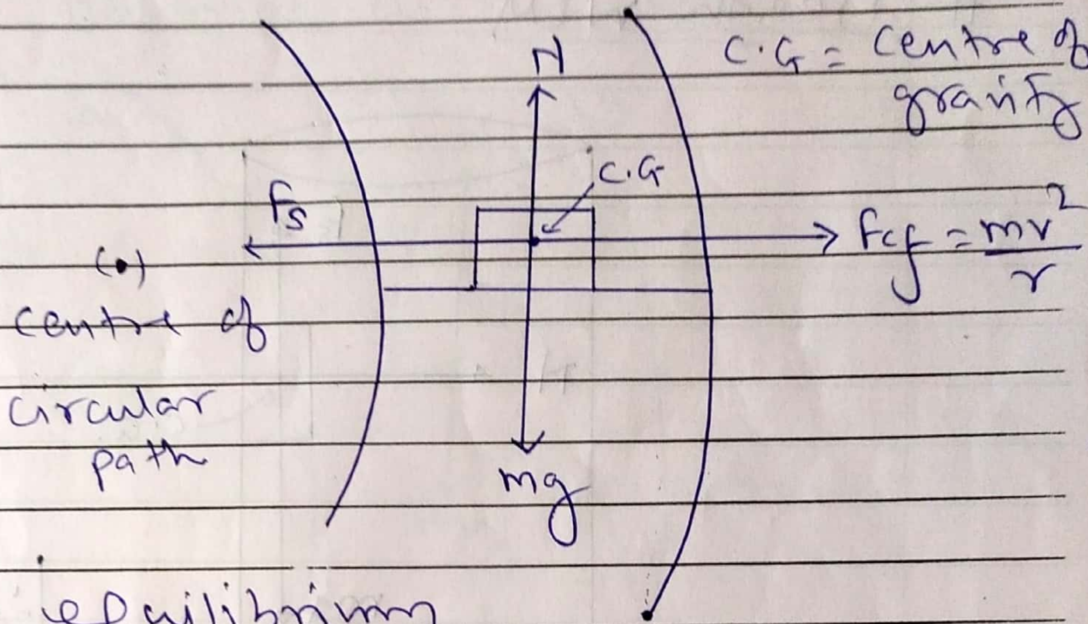
Expression for maximum speed of a vehicle on curved path:



In equilibrium

$$f_s = \frac{mv^2}{r} \quad \text{......(I)}$$

$$N = mg \quad \text{......(II)}$$

By definition of frictional force,

$$f_s = \mu N \quad \text{......(III)}$$

where μ is coefficient of force of friction

Now Substitute eqns (I) and (II) in eqn (III)

$$\frac{mv^2}{r} = \mu(mg)$$

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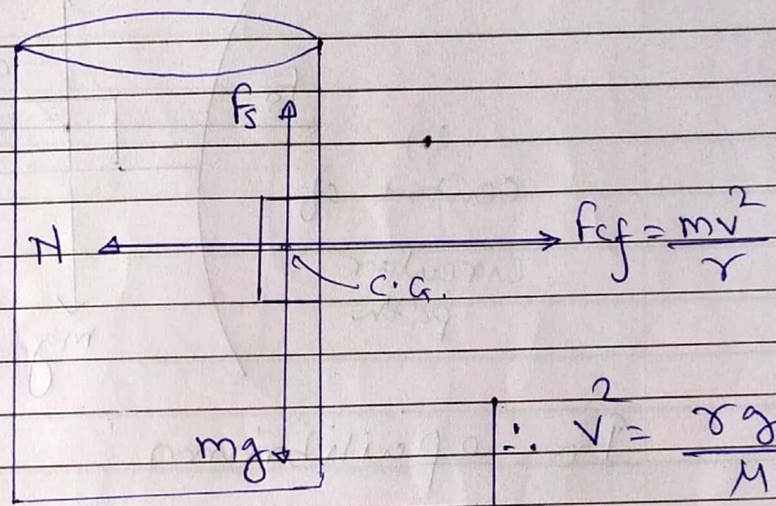
$$v^2 = \mu r g$$

$$\boxed{v = \sqrt{\mu r g}}$$

Hence maximum safe speed is independent of mass of the vehicle.

Death well



In equilibrium

$F_s = mg$ ---- (I)

$N = \frac{mv^2}{r}$ ---- (II)

By definition of force of friction

$F_s = \mu N$ ---- (III)

Now substitute eqn (I) and (II) in eqn (III)

$\therefore mg = \mu \left(\frac{mv^2}{r}\right)$

$\therefore v^2 = \frac{rg}{\mu}$

$$\therefore \boxed{v = \sqrt{\frac{rg}{\mu}}}$$

Since, $v = r\omega$

$\therefore r\omega = \sqrt{\frac{rg}{\mu}}$

$$\therefore \boxed{\omega = \sqrt{\frac{g}{\mu r}}}$$

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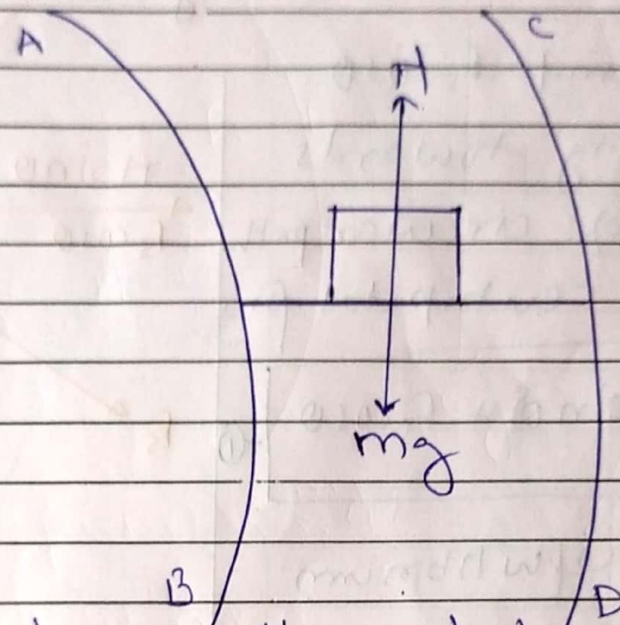
Banking of road

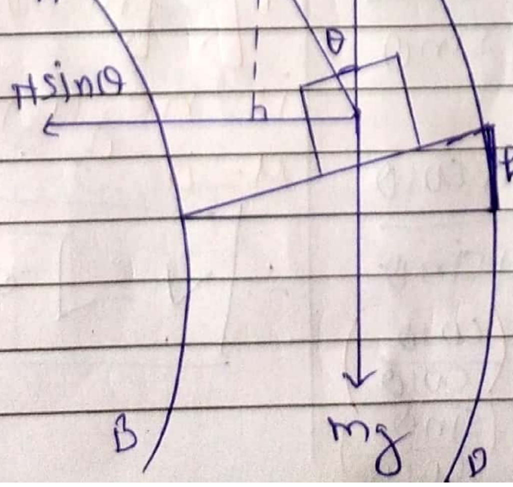
On normal curved path :-



→ Normal reaction balances weight of the vehicle.

→ centripetal force is provided by frictional force betn tyres of vehicle and road surface

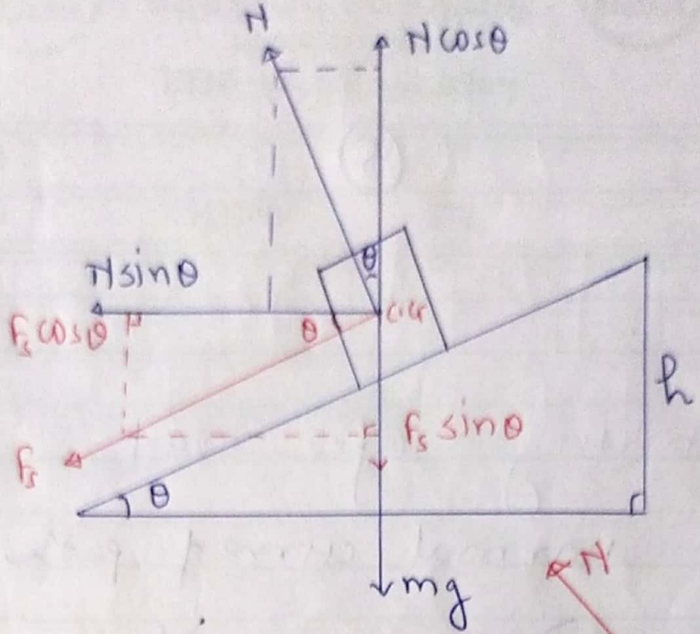
On curved banked path

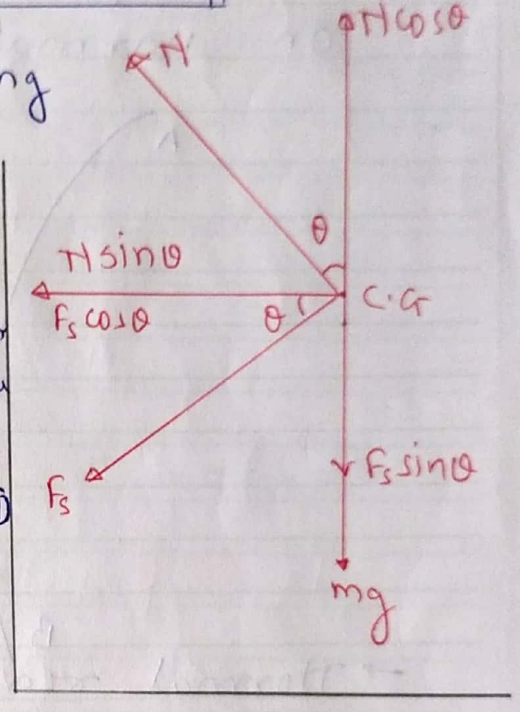


→ N cos θ balances weight of the vehicle

→ centripetal force is provided by friction and N sin θ.

Expression for maximum speed on banked path.





As $N\sin\theta$ and $f_s\cos\theta$ both are acting towards the centre of circular path it provides centripetal force

$$\therefore \boxed{\frac{mv^2}{r} = N\sin\theta + f_s\cos\theta} \quad \text{—(I)}$$

<u>In vertical equilibrium</u>

$$\therefore mg + f_s\sin\theta = N\cos\theta$$

$$\therefore \boxed{mg = N\cos\theta - f_s\sin\theta} \quad \text{—(II)}$$

Divide eqn (I) by eqn (II)

$$\therefore \frac{\left(\frac{mv^2}{r}\right)}{mg} = \frac{N\sin\theta + f_s\cos\theta}{N\cos\theta - f_s\sin\theta}$$

$$\therefore \frac{v^2}{rg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta}$$

$$\therefore \frac{v^2}{rg} = \frac{\left(\frac{\sin\theta}{\cos\theta}\right) + \mu\left(\frac{\cos\theta}{\cos\theta}\right)}{\left(\frac{\cos\theta}{\cos\theta}\right) - \mu\left(\frac{\sin\theta}{\cos\theta}\right)}$$

$$\frac{v^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\therefore \boxed{v = \sqrt{rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}}$$

If frictional force is negligible then $\mu = 0$

$$\therefore \boxed{v = \sqrt{rg\tan\theta}}$$

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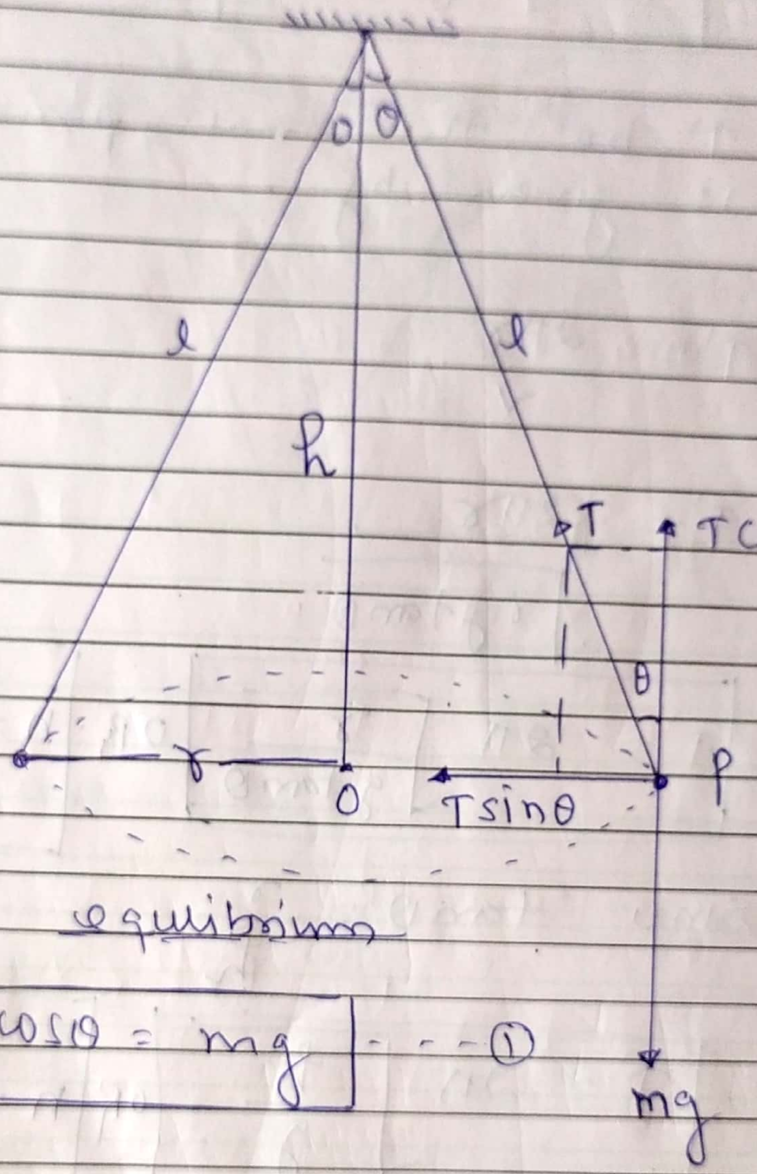
Conical pendulum



In equilibrium

$$\boxed{T\cos\theta = mg} \quad \text{---(i)}$$

As $T\sin\theta$ is acting towards centre it provides centripetal force

$$\therefore \boxed{T\sin\theta = \frac{mv^2}{r}} \quad \text{---(ii)}$$

Divide eqn (11) by eqn (1)

$$\therefore \frac{T\sin\theta}{T\cos\theta} = \frac{mv^2/r}{mg}$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$

$$\therefore \boxed{v = \sqrt{rg\tan\theta}}$$

Let T' be the time period then it is given by

$$T' = \frac{2\pi r}{v}$$

$$T' = \frac{2\pi r}{\sqrt{rg\tan\theta}}$$

$$\boxed{T' = 2\pi\sqrt{\frac{r}{g\tan\theta}}} \text{ OR } \boxed{n = \frac{1}{2\pi}\sqrt{\frac{g\tan\theta}{r}}}$$

Since $\tan\theta = \frac{r}{h}$

$$\therefore \boxed{T' = 2\pi\sqrt{\frac{h}{g}}} \text{ OR } \boxed{n = \frac{1}{2\pi}\sqrt{g/h}}$$

Since $h = l\cos\theta$

$$\therefore \boxed{T' = 2\pi\sqrt{\frac{l\cos\theta}{g}}} \text{ OR } \boxed{n = \frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}}}$$

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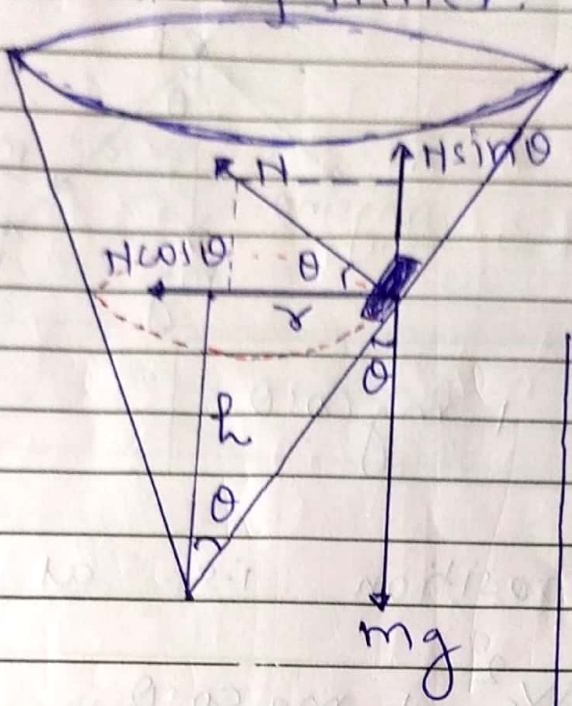
Conical funnel.



In equilibrium

$$\boxed{N \sin\theta = mg} \quad \text{---(I)}$$

$$\boxed{N \cos\theta = \frac{mv^2}{r}} \quad \text{---(II)}$$

Divide eqn (I) by eqn (II)

$$\frac{N \sin\theta}{N \cos\theta} = \frac{mg}{\left(\frac{mv^2}{r}\right)}$$

$$\tan\theta = \frac{rg}{v^2}$$

$\because \tan\theta = \frac{r}{h}$

$\therefore \frac{r}{h} = \frac{rg}{v^2}$

$\therefore \boxed{h = \frac{v^2}{g}}$

How we can find

Linear speed (v)

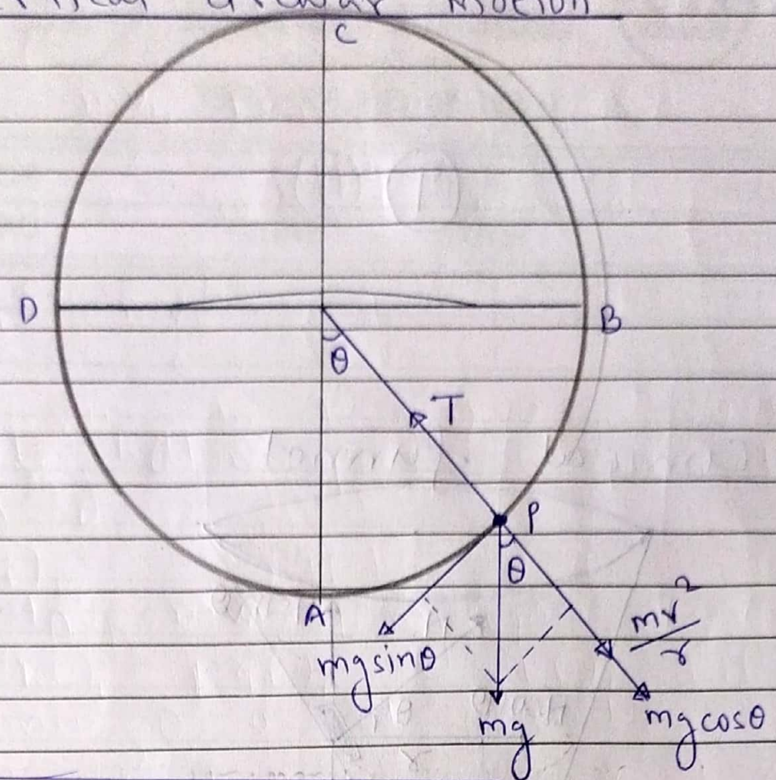
Angular speed (ω)

Time period (T)

frequency (n)

etc. ..

Vertical circular motion



$$\boxed{T_P = \frac{mv_P^2}{r} + mg\cos\theta}$$

At top position i.e. at point 'C'

$$\therefore T_C = \frac{mv_C^2}{r} + mg\cos\theta$$

Since $\theta = 180°$ and $\cos 180 = -1$

$$\therefore T_C = \frac{mv_C^2}{r} - mg$$

In order to complete vertical circular path, tension in the string should be greater than or equal to zero.

$$\therefore T_C \geq 0$$

$$\therefore \frac{mv_C^2}{r} - mg \geq 0$$

$$\therefore \frac{mv_C^2}{r} \geq mg$$

$$\therefore v_C \geq \sqrt{rg}$$

$$\therefore \boxed{v_C = \sqrt{rg}}$$

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At bottom position i.e. at point 'A'

$\because T_A = \frac{mv_A^2}{r} + mg\cos\theta$

Since $\theta = 0$ and $\cos\theta = 1$

$\therefore \boxed{T_A = \frac{mv_A^2}{r} + mg}$

According to law of conservation of energy.

Total energy at point A = Total energy at point C

$\therefore (T.E)_A = (T.E)_C$

$\therefore (KE)_A + (PE)_A = (KE)_C + (PE)_C$

$\therefore \frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$

$\therefore \frac{1}{2}mv_A^2 + mg(0) = \frac{1}{2}m(rg) + mg(2r)$

$\therefore \frac{1}{2}mv_A^2 = \frac{1}{2}mrg + 2mrg$

$\therefore \frac{1}{2}mv_A^2 = \frac{5}{2}mrg$

$\therefore \boxed{v_A = \sqrt{5rg}}$

At horizontal position i.e. at point 'B'

$T_B = \frac{mv_B^2}{r} + mg\cos\theta$

Since $\theta = 90$ and $\cos 90 = 0$

$\therefore \boxed{T_B = \frac{mv_B^2}{r}}$

According to law of conservation of energy

$\because (TE)_B = (TE)_C$

$\therefore (KE)_B + (PE)_B = (KE)_C + (PE)_C$

$\therefore \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}mv_C^2 + mgh_C$

$\therefore \frac{1}{2}mv_B^2 + mg(r) = \frac{1}{2}m(rg) + mg(2r)$

$\therefore \frac{1}{2}mv_B^2 + mrg = \frac{1}{2}mrg + 2mrg$

$\therefore \frac{1}{2}mv_B^2 = \frac{5}{2}mrg - mrg$

$\therefore \frac{1}{2}mv_B^2 = \frac{3}{2}mrg$

$\therefore \boxed{v_B = \sqrt{3rg}}$

Note

1) Total energy at any point in vertical circular motion is constant. i.e. $\frac{5}{2}mrg$

2) KE and PE changes place to place but total remains the same.

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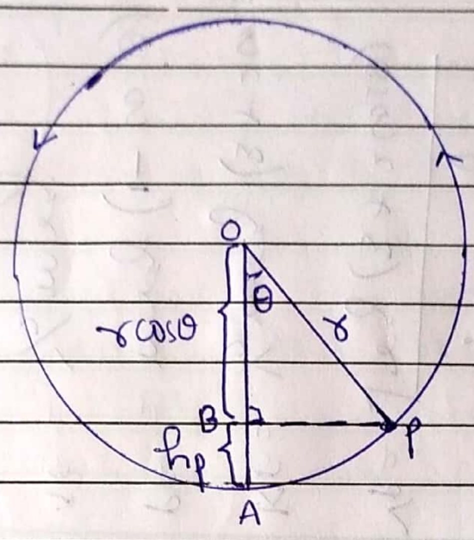
Expression for kinetic and potential energy in vertical circular motion.



∵ Potential energy at point 'P' is given by

$$\therefore PE = mg h_p$$

$$\therefore PE = mg[r - r\cos\theta]$$

$$\therefore \boxed{PE = mrg(1-\cos\theta)}$$

According to law of conservation of energy

$$(T.E)_P = (KE)_P + (PE)_P$$

$$\therefore \frac{5}{2}mrg = \frac{1}{2}mv_P^2 + mrg(1-\cos\theta)$$

$$\therefore \tfrac{1}{2} m v_p^2 = \tfrac{5}{2} m r g - \left(m r g (1 - \cos\theta) \right)$$

$$\therefore \tfrac{1}{2} m v_p^2 = \tfrac{5}{2} m r g - m r g + m r g \cos\theta$$

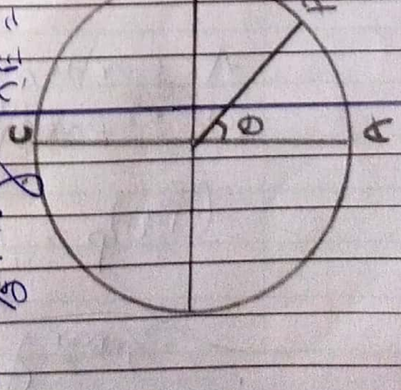
$$\therefore m v_p^2 = 5 m r g - 2 m r g + 2 m r g \cos\theta$$

$$\therefore v_p^2 = 3 r g + 2 r g \cos\theta$$

$$\therefore \boxed{v_p = \sqrt{r g (3 + 2\cos\theta)}}$$

Conclusion

At bottom position i.e. at point 'A'	At top position i.e. at point 'C'	At horizontal position i.e. at point 'B'	At any position i.e. at point 'P'
$T_A = \frac{m v_A^2}{r} + mg$	$T_C = \frac{m v_C^2}{r} - mg$	$T_B = \frac{m v_B^2}{r}$	$T_P = \frac{m v_P^2}{r} + mg\cos\theta$
$v_A = \sqrt{5rg}$	$v_C = \sqrt{rg}$	$v_B = \sqrt{3rg}$	$v_P = \sqrt{rg(3 + 2\cos\theta)}$
$KE = \frac{5}{2} mrg$	$KE = \frac{1}{2} mrg$	$KE = \frac{3}{2} mrg$	$KE = \frac{1}{2} mrg(3 + 2\cos\theta)$
$PE = 0$	$PE = 2mrg$	$PE = mrg$	$PE = mrg(1 - \cos\theta)$
$TE = \frac{5}{2} mrg$	$TE = \frac{5}{2} mrg$	$TE = \frac{5}{2} mrg$	$TE = \frac{5}{2} mrg$



$T_A - T_B = 6mg - 3mg = 3mg$

$T_B - T_C = 3mg - 0 = 3mg$

$T_A - T_C = 6mg - 0 = 6mg$

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* When all small masses of a body are performing circular motion in such a way that centres of all circular path line in the same line (axis of rotation) then motion of the body is said to be rotational motion.

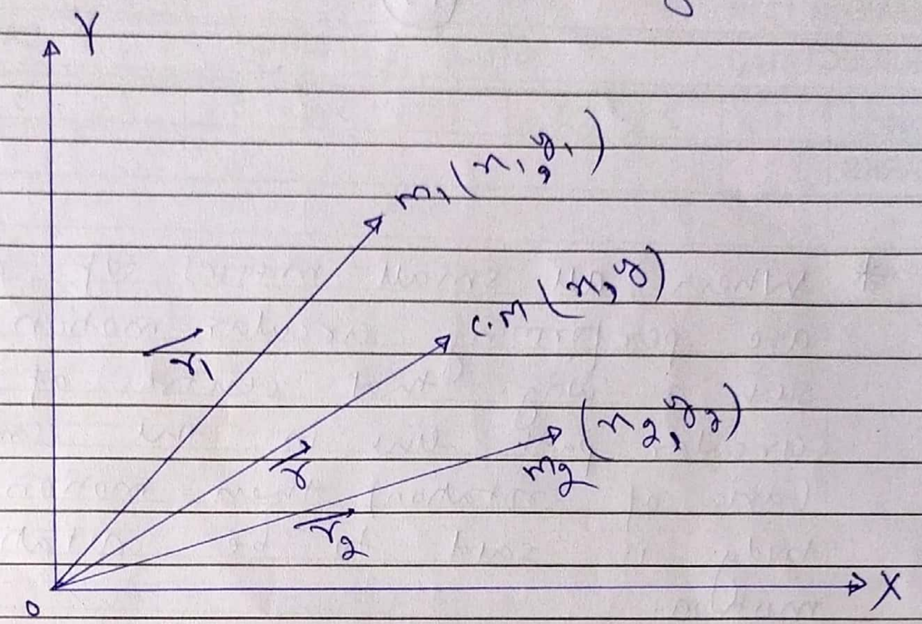
* When a body is rotating, few masses inside the body remains stationary. A line joining these all small masses is called axis of rotation.

* In rotational motion all small masses in a line moving with constant angular speed but different linear speed.

* Centre of mass :- Centre of mass of a body is defined as a point at which entire mass of body is supposed to be concentrated and at which newton's law of motion is applicable.

OR

* centre of mass of a body is a point with respect to which ~~entire~~ entire mass of the body is considered to be zero.



→ Position vector of centre of mass is given by

$$r = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

→ X - cordinate of centre of mass

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Similly $y = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$$z = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

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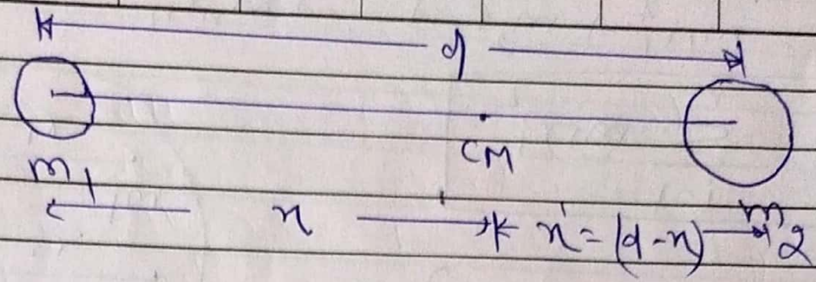


$\therefore m_1 x = m_2 (d - x)$

$\therefore m_1 x = m_2 d - m_2 x$

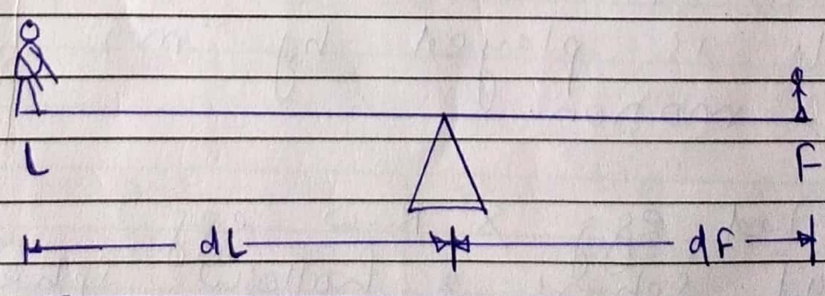
$\therefore m_1 x + m_2 x = m_2 d$

$\therefore x (m_1 + m_2) = m_2 d$

$$\therefore \boxed{x = \frac{m_2 d}{m_1 + m_2}}$$

Similarly $\boxed{x' = \frac{m_1 d}{m_1 + m_2}}$

II Principle of moments

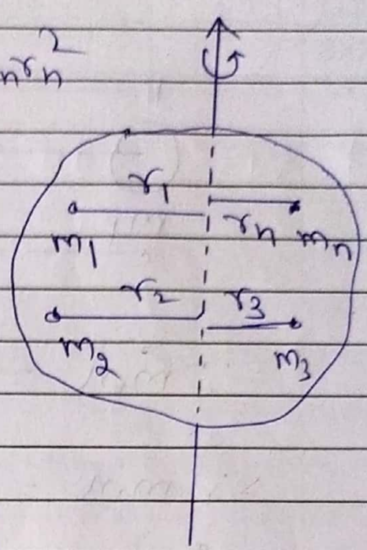


$$\boxed{L \times dL = F \times dF}$$

* **Moment of Inertia :-** Moment of inertia of a body about any axis of rotation is defined as sum of product of all small masses and square of distance between axis of rotation and them.

$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$



S.I unit - Kgm^2

C.G.S unit - gcm^2

Dimension - $[L^2 M^1 T^0]$

Significance of moment of inertia

Linear motion	Rotational motion
$F = ma$	$T = I\alpha$
$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
$p = mv$	$L = I\omega$

Hence whatever role played by mass in linear motion same role is played by M.I in rotational motion.

→ Boiled egg & Raw egg
→ Solid sphere & hollow sphere
→ Disc and Ring

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Radius of Gyration :- Radius of gyration of a body about an axis of rotation is defined as distance between axis of rotation and a point at which entire mass of the body is supposed to be concentrated so as moment of inertia of the body about given axis of rotation remains the same.

$\therefore I = mK^2$

$$\therefore \boxed{K = \sqrt{\frac{I}{M}}}$$

where K is radius of gyration.

S.I unit of K is metre. C.G.S unit is cm. Dimension - $[L^1 M^0 T^0]$

Significance of radius of gyration.

1) Radius of gyration indicates about mass distribution of the body.

11) If value of radius of gyration is more it means masses are widely distributed from axis of rotation.

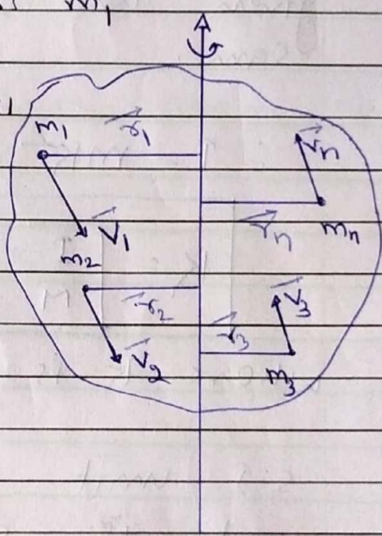
iii) If value of radius of gyration is less it means masses are closely distributed from axis of rotation.

Note :- Radius of gyration of a body about an axis of rotation is an average distance of all masses from axis of rotation.

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$$

Expression for kinetic energy of a rotating body

Consider a small mass m_1 moving with velocity v_1. Kinetic energy of mass m_1 is given by



$KE_1 = \frac{1}{2} m_1 v_1^2$

since $v_1 = r_1 w$

$\therefore KE_1 = \frac{1}{2} m_1 r_1^2 w^2$

similly $KE_2 = \frac{1}{2} m_2 r_2^2 w^2$

$KE_3 = \frac{1}{2} m_3 r_3^2 w^2$

$\vdots$

$KE_n = \frac{1}{2} m_n r_n^2 w^2$

Let KE be the total kinetic energy of rotating body then it is given by

$KE = KE_1 + KE_2 + KE_3 + \dots + KE_n$

$\therefore KE = \frac{1}{2} m_1 r_1^2 w^2 + \frac{1}{2} m_2 r_2^2 w^2 + \dots + \frac{1}{2} m_n r_n^2 w^2$

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$\therefore KE = \frac{1}{2}\left[m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2\right]\omega^2$

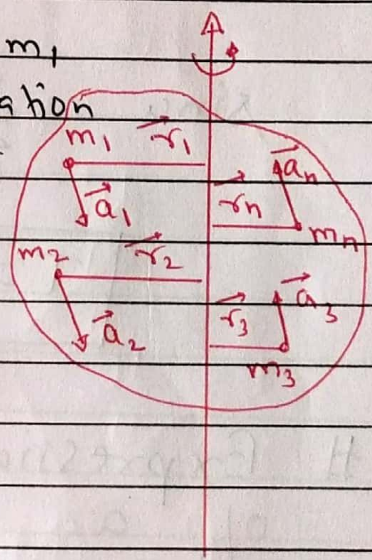
$\therefore KE = \frac{1}{2}\left[\sum_{i=1}^{n} m_ir_i^2\right]\omega^2$

But $\sum_{i=1}^{n} m_ir_i^2 = I$

$\therefore \boxed{KE = \frac{1}{2}I\omega^2}$

Expression for torque acting on a rotating body

Consider a small mass m_1 moving with an acceleration a_1. force acting on it is given by



$\therefore F_1 = m_1a_1$

Since $a_1 = r_1\alpha$

$\therefore F_1 = m_1r_1\alpha$

Let τ_1 be the torque acting on it then

$\therefore \tau_1 = r_1F_1$

$\therefore \tau_1 = r_1(m_1r_1\alpha)$

$\therefore \tau_1 = m_1r_1^2\alpha$

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similly $\tau_2 = m_2 r_2^2 \alpha$

$\tau_3 = m_3 r_3^2 \alpha$

⋮

$\tau_n = m_n r_n^2 \alpha$

Let τ be the total torque acting on rotating body then it is given by

$\therefore \tau = \tau_1 + \tau_2 + \tau_3 + \cdots + \tau_n$

$\therefore \tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \cdots + m_n r_n^2 \alpha$

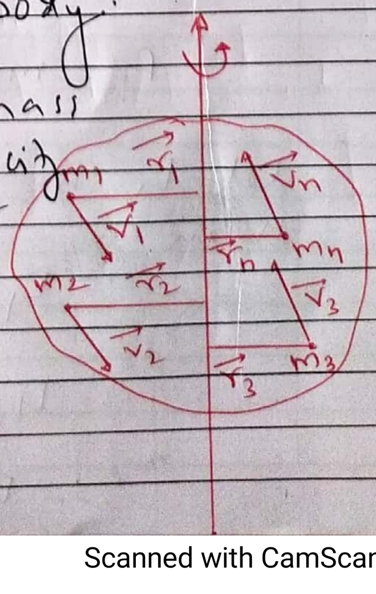
$\therefore \tau = [m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2] \alpha$

$\therefore \tau = \left[\sum_{i=1}^{n} m_i r_i^2\right] \alpha$

since $\sum_{i=1}^{n} m_i r_i^2 = I$

$\therefore \boxed{\tau = I\alpha}$

Expression for angular moment of a rotating body.



consider a small mass m_i moving with velocity v_i then it's linear momentum is given by

$p_1 = m_1 v_1$

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sinu $v_1 = r_1 \omega$

$\therefore \quad P_1 = m_1 (r_1 \omega)$

$\therefore \quad P_1 = m_1 r_1 \omega$

Let L_1 be the angular momentum of mass m_1

$\therefore \quad L_1 = r_1 P_1$

$\therefore \quad L_1 = r_1 (m_1 r_1 \omega)$

$\therefore \quad L_1 = m_1 r_1^2 \omega$

similly $L_2 = m_2 r_2^2 \omega$

$L_3 = m_3 r_3^2 \omega$

$\vdots$

$L_n = m_n r_n^2 \omega$

Let L be the total angular momentum of rotating body then it is given by

$\therefore L = L_1 + L_2 + L_3 + \cdots + L_n$

$\therefore L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + \dots + m_n r_n^2 \omega$

$\therefore L = [m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2]\omega$

$\therefore L = \left[\sum_{i=1}^{n} m_i r_i^2 \right] \omega$

Since $\sum_{i=1}^{n} m_i r_i^2 = I$

$\therefore \boxed{L = I\omega}$

Law of conservation of angular momentum.

Statement :- When external torque is acting then angular moment of body remains constant.

Note :- uv rule

$\frac{d(uv)}{dt} = \frac{du}{dt} \times v + u \times \frac{dv}{dt}$

Proof

$\because \vec{L} = \vec{r} \times \vec{p}$

Now diff above eqn w.r. to 't'

$\therefore \frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$

$\therefore \frac{d\vec{L}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times \vec{F}$

$\dots \left(\because \frac{d\vec{r}}{dt} = \vec{v}, \vec{p} = m\vec{v}, \frac{d\vec{p}}{dt} = \vec{F}\right)$

$\therefore \frac{d\vec{L}}{dt} = m(\vec{v} \times \vec{v}) + \vec{\tau} \dots (\because \vec{\tau} = \vec{r} \times \vec{F})$

$\therefore \frac{d\vec{L}}{dt} = \vec{\tau} \dots (\because \vec{v} \times \vec{v} = 0)$

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Since no external torque is acting,

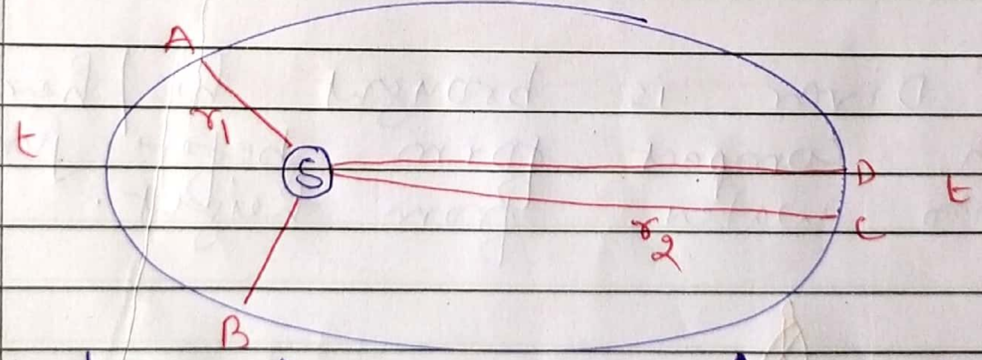
$\therefore \tau = 0$

$\therefore \boxed{\frac{d\vec{L}}{dt} = 0}$

$\Rightarrow \vec{L}$ is constant.

Application of law of conservation of angular moment.

① Kepler's second law

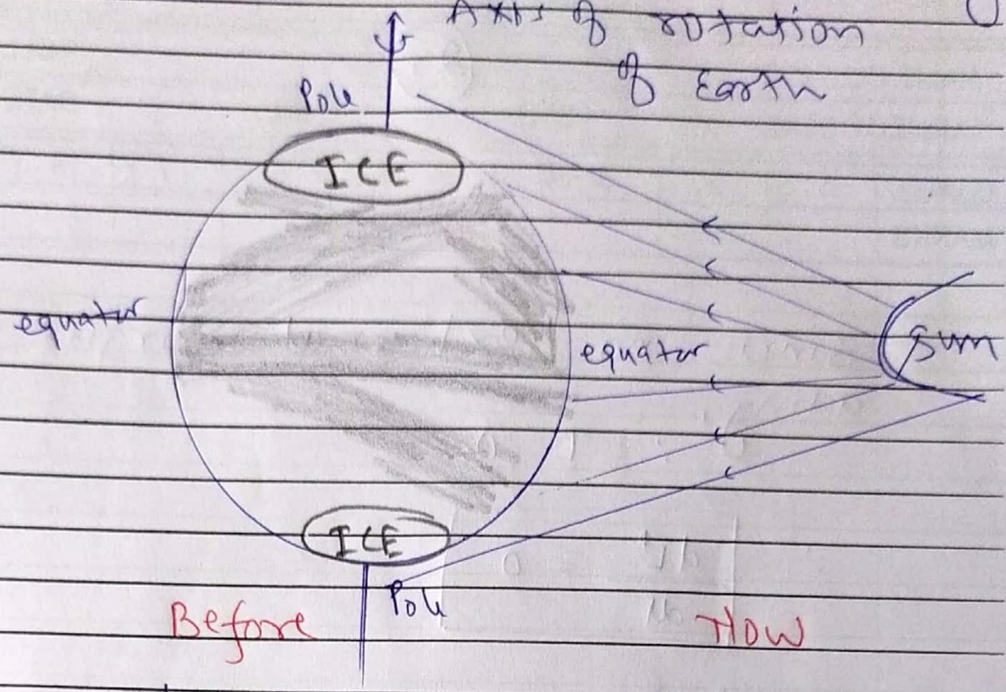


$L = \downarrow I_1 \omega_1 \uparrow = \uparrow I_2 \omega_2 \downarrow$

$(m r_1^2) \downarrow \omega_1 \uparrow = (m r_2^2) \uparrow \omega_2 \downarrow$

Hence satellite moves faster between AB compare to CD.

(ii) Due to global warming time period of Earth's rotational is increasing.

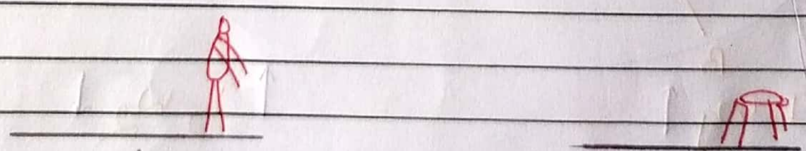


$$L = \downarrow I_1 \omega_1 \uparrow \quad = \quad \uparrow I_2 \omega_2 \downarrow$$

$$= (m r_1^2 \downarrow)\left(\frac{2\pi}{T_1}\right) \quad = \quad (m r_2^2 \uparrow)\left(\frac{2\pi}{T_2}\right)$$

$$\downarrow T_1 = \frac{2\pi}{\omega_1 \uparrow} = 24 \text{ hr.} \qquad T_2 = \frac{2\pi}{\omega_2 \downarrow} = \text{more than } 24 \text{ hr.}$$

(iii) A Diver is brought his/her body in compact form before jumping into water from height.



$$L = \uparrow I_1 \omega_1 \downarrow \quad = \quad \downarrow I_2 \omega_2 \uparrow$$

$$= (m r_1^2 \uparrow)(2\pi n_1)\downarrow \quad = \quad (m r_2^2 \downarrow)(2\pi n_2 \uparrow)$$

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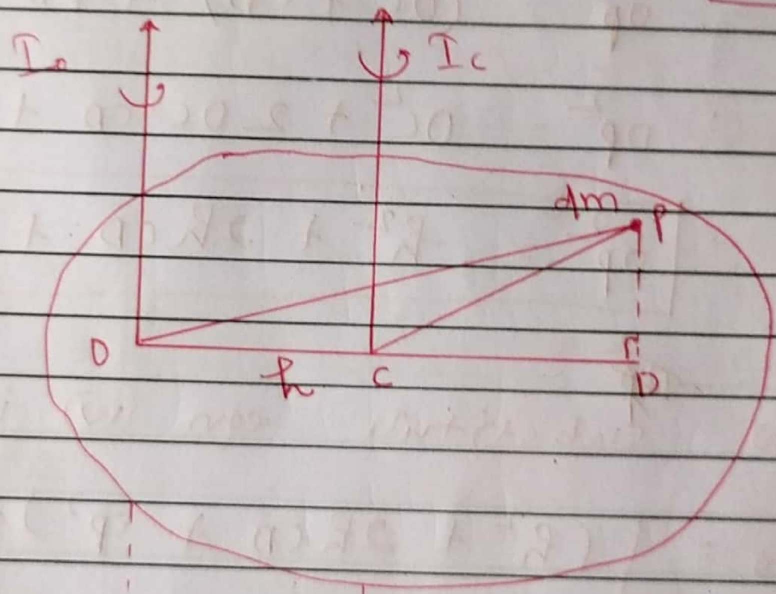
State and prove parallel axis theorem.

Statement :- M.I of parallel axis of a body is sum of M.I of a body about an axis of rotation which is passing through centre of mass and product mass of the body and square of distance betn parallel ~~axies~~ axes. i.e. $\boxed{I_0 = I_c + Mh^2}$



I_c - M.I of entire body about an axis of rotational which is passing through centre of mass

I_0 - M.I of entire body about parallel axis.

M - Mass of the rotating body

dm - small mass element

h - Distance between two parallel axes.

$\because I_c = \int CP^2 dm$ ...... (I)

$\therefore I_o = \int OP^2 dm$ ...... (II)

In ΔCDP, $\angle D = 90^\circ$

$\therefore \boxed{CP^2 = CD^2 + PD^2}$ ...... (III)

In ΔODP, $\angle D = 90^\circ$

$\therefore OP^2 = (OD)^2 + PD^2$

$\therefore OP^2 = (OC + CD)^2 + PD^2$

$\therefore OP^2 = OC^2 + 2 \cdot OC \cdot CD + \underline{CD^2 + PD^2}$

$\therefore \boxed{OP^2 = h^2 + 2hCD + CP^2}$ ... from eqn (III)

$\therefore \cancel{OP^2}$ ...... (IV)

Now substitute eqn (IV) in eqn (II)

$\therefore I_o = \int (h^2 + 2hCD + CP^2) dm$

$\therefore I_o = \int h^2 dm + \int 2hCD \, dm + \int CP^2 dm$

$\therefore I_o = Mh^2 + 0 + I_c$

$\therefore \boxed{I_o = I_c + Mh^2}$

Since $\int 2hCD \, dm = 0$ because

∵ [sum of moment of all small masses about centre of mass is zero].

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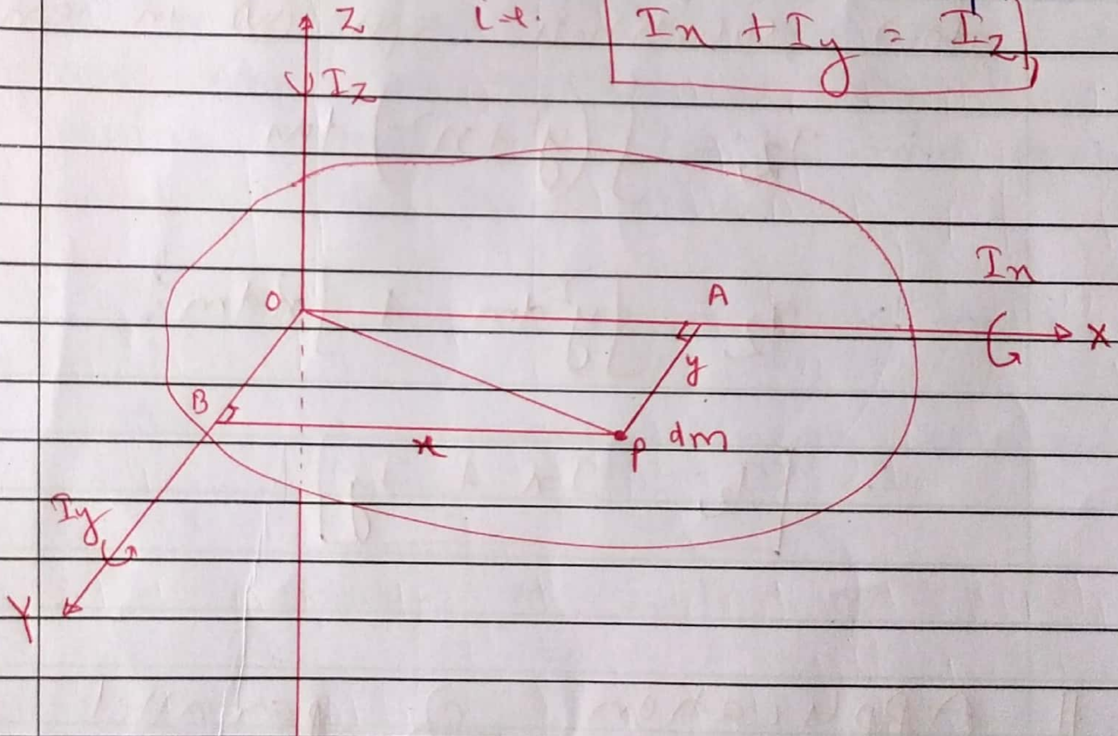
\# State and prove perpendicular axis theorem.

Statement :- Sum of moment of inertia of a body about two perpendular axes which are parallel with the plane is equal to M.I of the body about an axis of rotation which is passing through the point of intersection and perpendicular to the plane.

i.e. $\boxed{I_x + I_y = I_z}$



Let I_x be the M.I of the entire body about an axis of rotation along OX and it is given by

$$\boxed{I_x = \int y^2 dm} \quad \text{--- } (1)$$

let I_y be the M.I of the entire body about an axis of rotation along OY

$$\therefore \boxed{I_y = \int x^2 dm} \quad \dots \text{(ii)}$$

let I_z be the M.I of the entire body about an axis of rotation along OZ.

$$\therefore \boxed{I_z = \int OP^2 dm} \quad \dots \text{(iii)}$$

In ΔOAP $\angle A = 90°$

$$\therefore OP^2 = (AP)^2 + (OA)^2$$

$$\therefore \boxed{OP^2 = y^2 + x^2} \quad \dots \text{(iv)}$$

Now substitute eqn (iv) in eqn (iii)

$$\therefore I_z = \int (y^2 + x^2)\, dm$$

$$\therefore I_z = \int y^2 dm + \int x^2 dm$$

$$\therefore \boxed{I_z = I_x + I_y}$$

Application of parallel axis theorem and perpendicular axis theorem.

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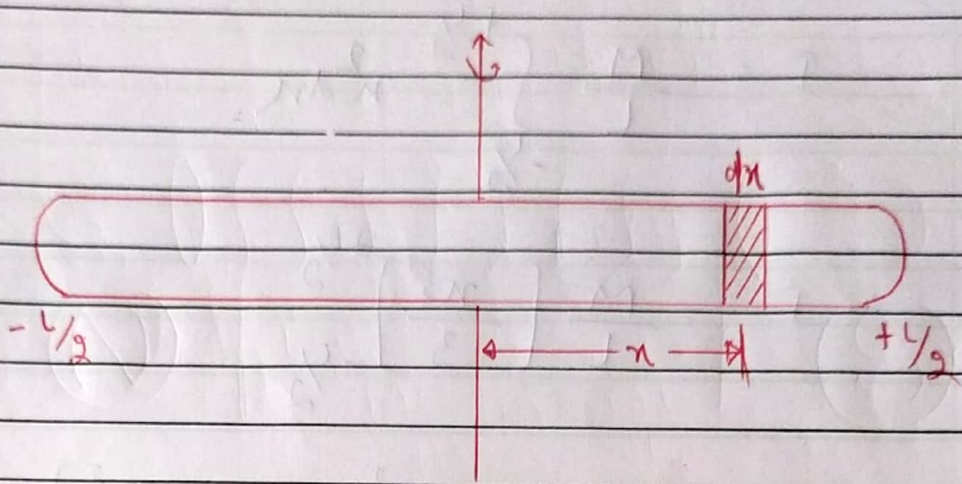
Moment of inertia of rod



consider a uniform rod of mass 'M' and length 'L' is rotating about an axis of rotational which is passing through the centre and perpendicular to the plane.

∵ Linear mass density of rod = $\frac{M}{L}$

∴ Mass of small length element 'dx' is given by

$$\boxed{dm = \frac{M}{L} dx}$$

Let dI be the M.I of small length element dx about given axis of rotation is given by

$$dI = x^2\left(\frac{M}{L} dx\right)$$

$\therefore$ Moment of inertia of entire rod about given axis of rotation is given by

$$\therefore \quad I = \int_{-L/2}^{+L/2} dI$$

$$\therefore \quad I = \int_{-L/2}^{+L/2} x^2 \left(\frac{M}{L} dx\right)$$

$$\therefore \quad I = \frac{M}{L} \int_{-L/2}^{+L/2} x^2 dx$$

$$\therefore \quad I = \frac{M}{L} \left[\frac{x^3}{3}\right]_{-L/2}^{+L/2}$$

$$\therefore \quad I = \frac{M}{L} \left[\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3}\right]$$

$$\therefore \quad I = \frac{M}{L} \left[\frac{L^3}{24} - \left(-\frac{L^3}{24}\right)\right]$$

$$\therefore \quad I = \frac{M}{L} \left[\frac{2L^3}{24}\right]$$

$$\therefore \quad \boxed{I = \frac{ML^2}{12}}$$

P.T.O.

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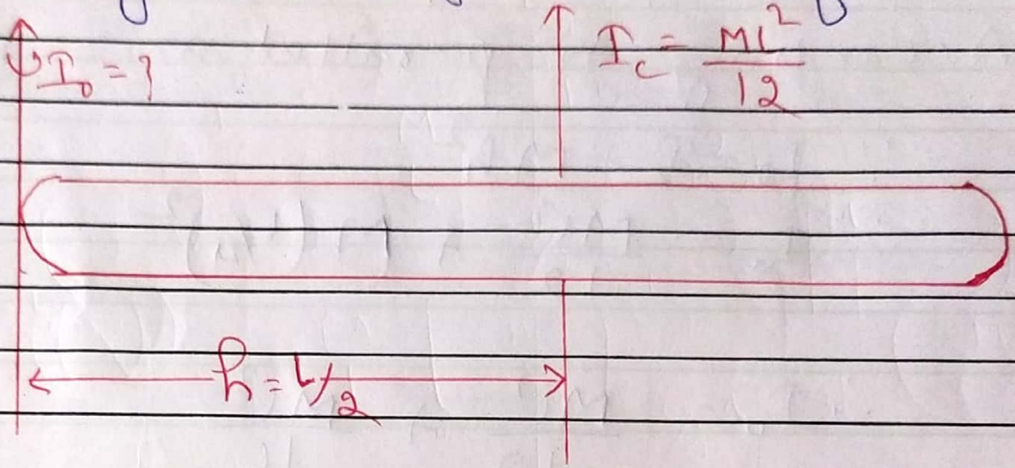
Case-1) M.I of uniform rod about an axis of rotation which is passing through one of the end.



According to parallel axis theorem

$\therefore I_0 = I_c + Mh^2$

$\therefore I_0 = \frac{ML^2}{12} + M\left(L/2\right)^2$

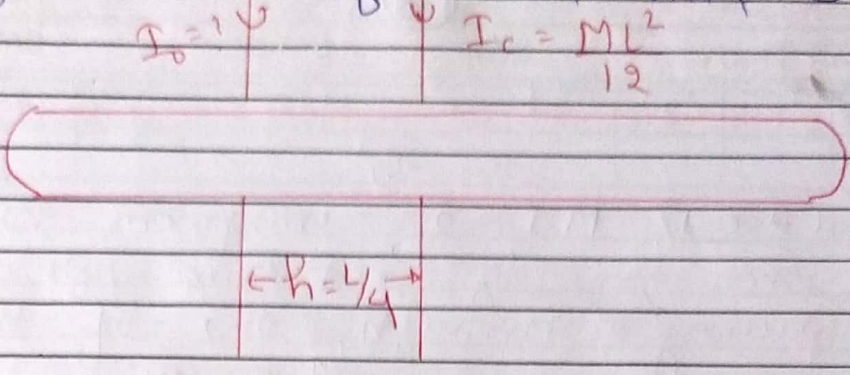
$\therefore I_0 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\therefore I_0 = \frac{ML^2 + 3ML^2}{12}$

$\therefore \boxed{I_0 = \frac{ML^2}{3}}$

P.T.O.

Case-2) M.I of rod about an axis of rotation which is passing through the mid point of one of end and centre



According to parallel axis theorem

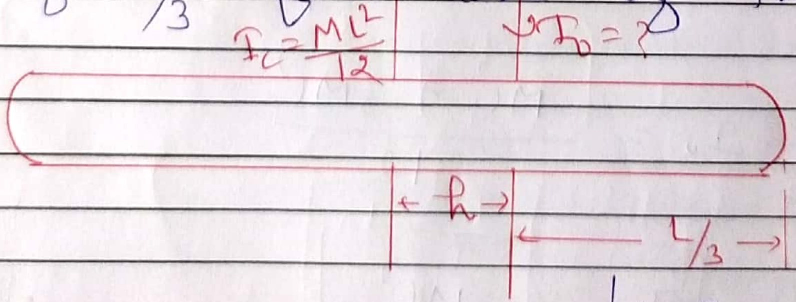
$\therefore I_0 = I_c + Mh^2$

$\therefore I_0 = \frac{ML^2}{12} + M(L/4)^2$

$\therefore I_0 = \frac{ML^2}{12} + \frac{ML^2}{16}$

$\therefore \boxed{I_0 = \frac{7}{48} ML^2}$

Case-3) M.I of rod about an axis of rotation which is passing through at a point at a distance of $L/3$ from one of the end.



$\therefore h = L/2 - L/3$

According to parallel axis theorem

$I_0 = I_c + Mh^2$

$\therefore I_0 = \frac{ML^2}{12} + M(L/6)^2$

$I_0 = \frac{ML^2}{12} + \frac{ML^2}{36}$

$\boxed{I_0 = \frac{ML^2}{9}}$

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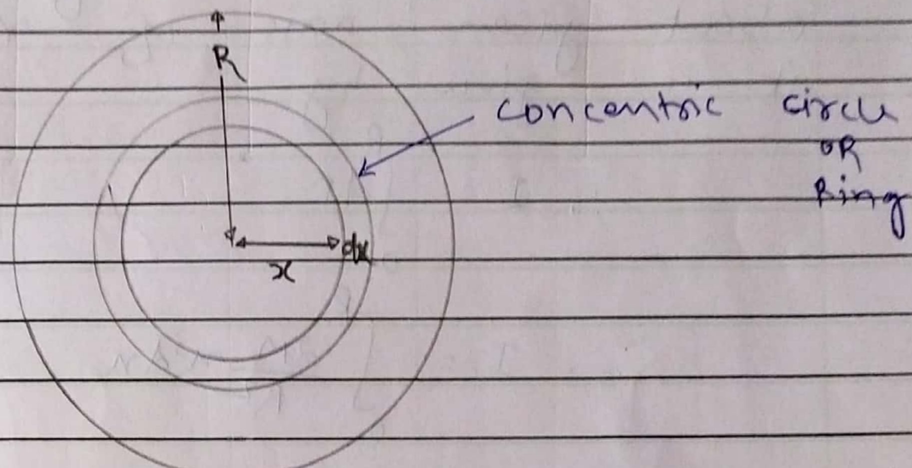
Expression for M.I of uniform disc about an axis of rotation which is passing through the centre and perpendicular to the plane



Consider an uniform disc of mass 'M' and radius 'R'.

Since mass per unit area $= \frac{M}{\pi R^2}$

Since area of concentric circle $= l \times b$
$= 2\pi x \cdot dx$

Since dm be the mass of concentric ring then it is given by

$$dm = \left(\frac{M}{\pi R^2}\right)(2\pi x\, dx)$$

$$\therefore dm = \frac{2M}{R^2} x\, dx$$

Let dI be the M.I of concentric ring about given axis of rotation then it is given by

$$\therefore dI = x^2 dm$$

$$\therefore dI = x^2 \left(\frac{2M}{R^2}\right) x\,dx$$

$$\therefore dI = \frac{2M}{R^2} x^3 dx$$

Let I be the M.I of entire disc about given axis of rotation then it is given by

$$\therefore I = \int_0^R dI$$

$$\therefore I = \int_0^R \frac{2M}{R^2} x^3 dx$$

$$\therefore I = \frac{2M}{R^2} \int_0^R x^3 dx$$

$$\therefore I = \frac{2M}{R^2} \left[\frac{x^4}{4}\right]_0^R$$

$$\therefore I = \frac{2M}{R^2} \left(\frac{R^4}{4} - \frac{0}{4}\right)$$

$$\therefore I = \frac{2M}{R^2} \left(\frac{R^4}{4}\right)$$

$$\therefore \boxed{I = \frac{MR^2}{2}}$$

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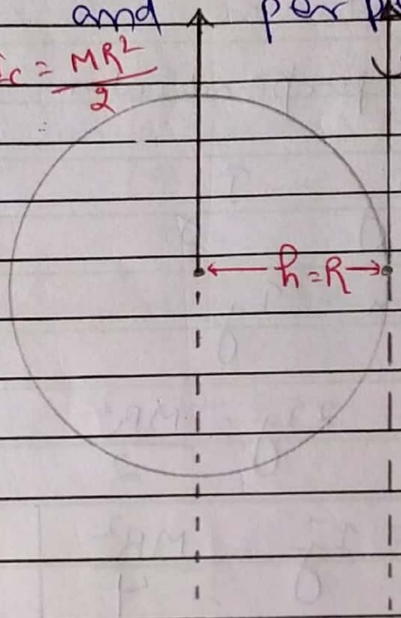
Case 1) M.I of uniform disc about an axis of rotation passing through tangent and perpendicular to the plane.



According to parallel axis theorem

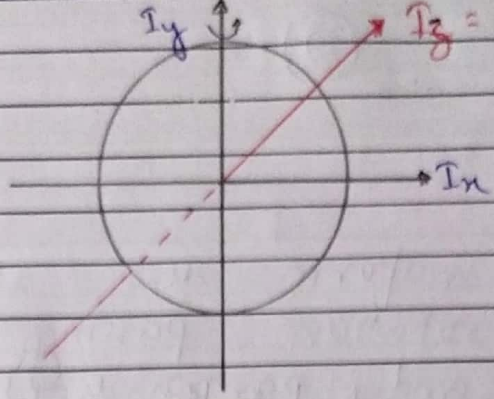
$$\therefore I_0 = I_c + Mh^2$$

$$\therefore I_0 = \frac{MR^2}{2} + M(R^2)$$

$$\therefore I_0 = \frac{MR^2}{2} + MR^2$$

$$\therefore \boxed{I_0 = \frac{3}{2} MR^2}$$

Case.2) M.I of uniform disc about an axis of rotation which is passing through centre and parallel with plane.



According to perpendicular axis theorem.

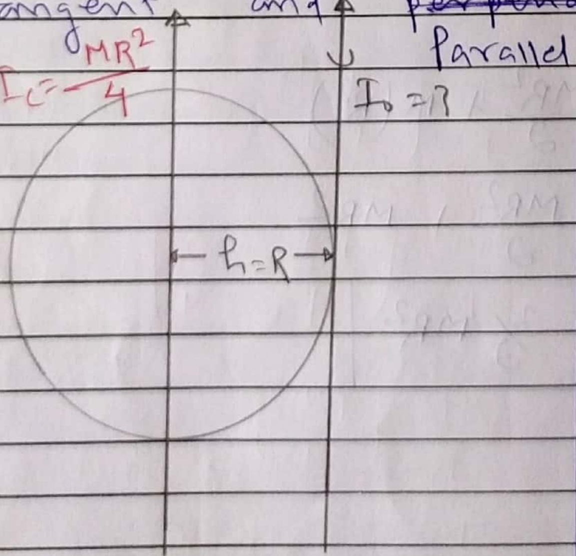
$\because I_x + I_y = I_z$

Since $I_x = I_y$

$\therefore 2I_x = 2I_y = \frac{MR^2}{2}$

$\therefore \boxed{I_x = I_y = \frac{MR^2}{4}}$

Case.3) M.I of uniform disc about an axis of rotation which is passing through tangent and ~~perpendicular to~~ the plane
Parallel with



According to parallel axis theorem

$\because I_0 = I_c + Mh^2$

$\therefore I_0 = \frac{MR^2}{4} + M(R^2)$

$\therefore I_0 = \frac{MR^2}{4} + MR^2$

$\therefore \boxed{I_0 = \frac{5}{4} MR^2}$

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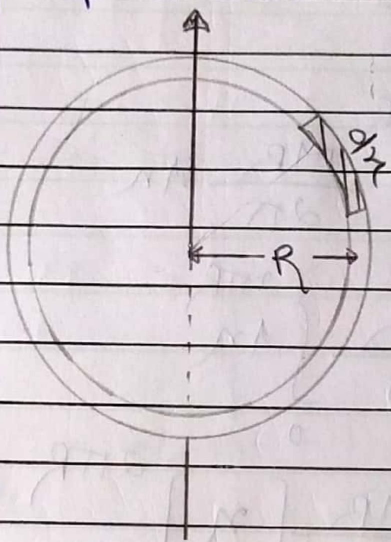
Moment of inertia of uniform Ring about an axis of rotation which is passing through centre and perpendicular to the plane.



Consider an uniform Ring of mass 'M' and radius R. which is rotating about an axis of rotation which is passing through centre and perpendicular to plane.

∴ Mass per unit length of Ring = $\frac{M}{2\pi R}$

let 'dm' be the mass of small element 'dx' and it is given by

$$dm = \left(\frac{M}{2\pi R}\right) dx$$

let dI be the M.I of small length element dx about given axis of rotation and it is given by

$$\because dI = R^2 \, dm$$

$$\therefore dI = R^2 \left(\frac{M}{2\pi R} dx \right)$$

$$\therefore dI = \frac{MR}{2\pi} dx$$

Let I be the M.I of entire ring about given axis of rotation then it is given by

$$\therefore I = \int_0^{2\pi R} dI$$

$$\therefore I = \int_0^{2\pi R} \frac{MR}{2\pi} dx$$

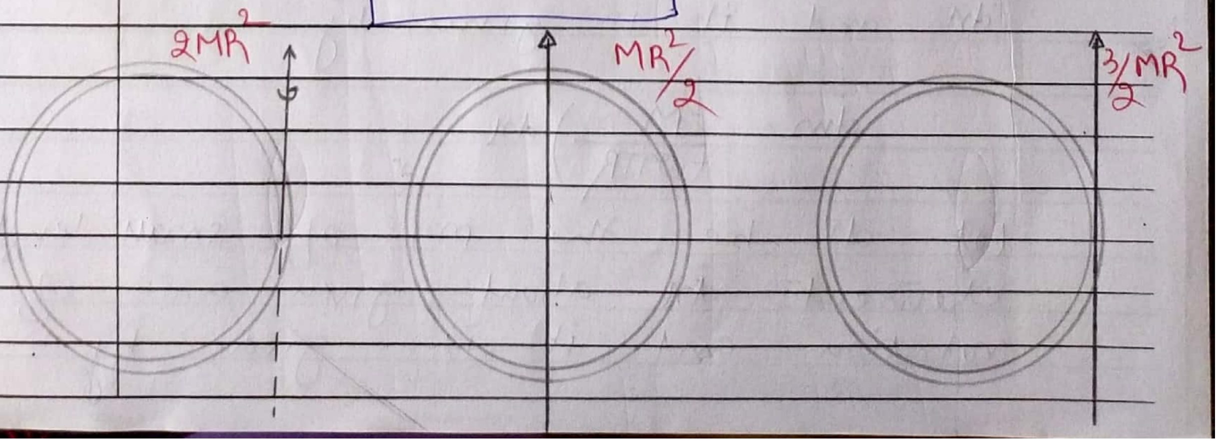
$$\therefore I = \frac{MR}{2\pi} \int_0^{2\pi R} dx$$

$$\therefore I = \frac{MR}{2\pi} \left[x \right]_0^{2\pi R}$$

$$\therefore I = \frac{MR}{2\pi} \left[2\pi R - 0 \right]$$

$$\therefore I = \frac{MR}{2\pi} (2\pi R)$$

$$\therefore \boxed{I = MR^2}$$



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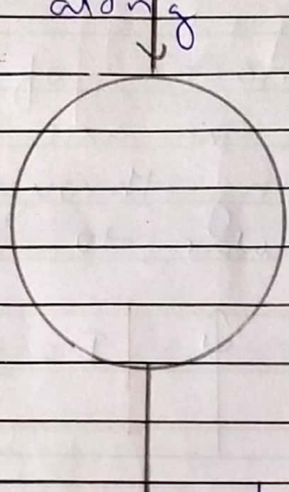
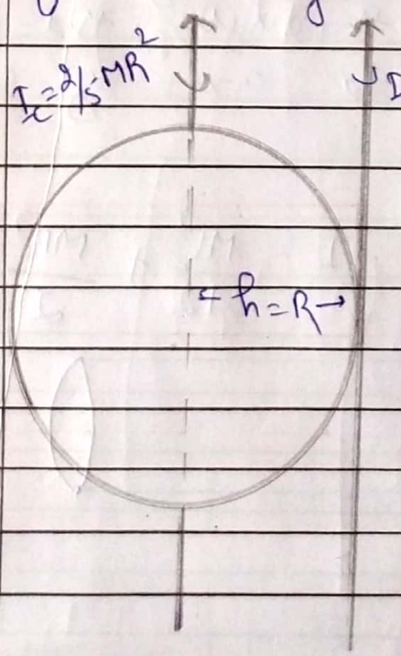
Moment of inertia of uniform solid sphere about an axis of rotation along one of the diameter



$$\boxed{I = 2/5\, MR^2}$$

Where M is mass of solid sphere and R is radius of solid sphere

Moment of inertia of solid sphere about an axis of rotation along one of the tangent.



According to parallel axis theorem

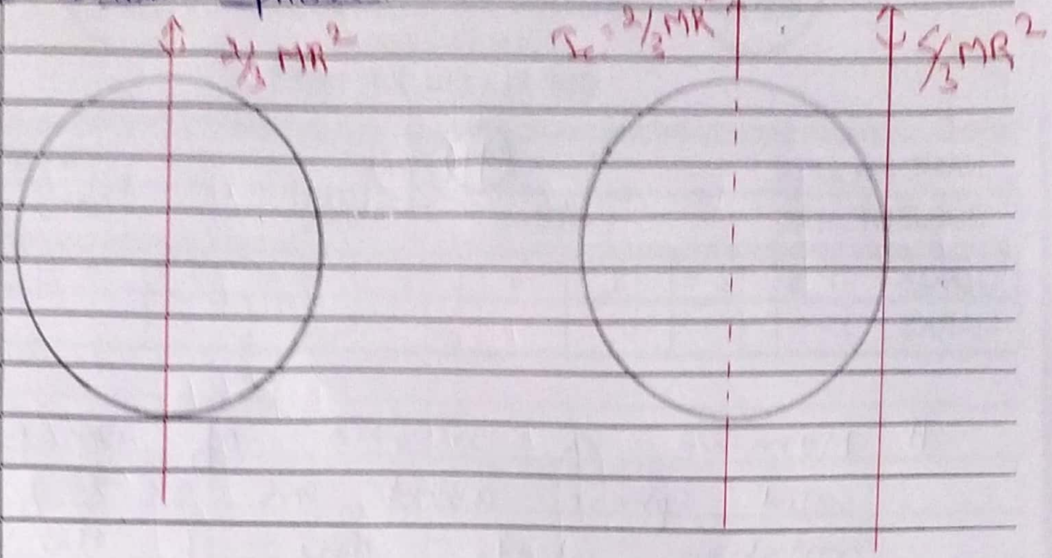
$\therefore I_o = I_c + Mh^2$

$\therefore I_o = 2/5\, MR^2 + M(R)^2$

$\therefore I_o = 2/5\, MR^2 + MR^2$

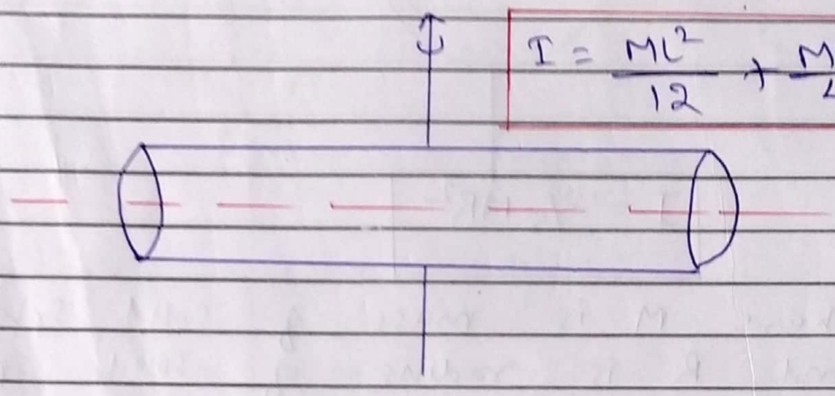
$\therefore \boxed{I_o = 7/5\, MR^2}$

Hollow Sphere :-



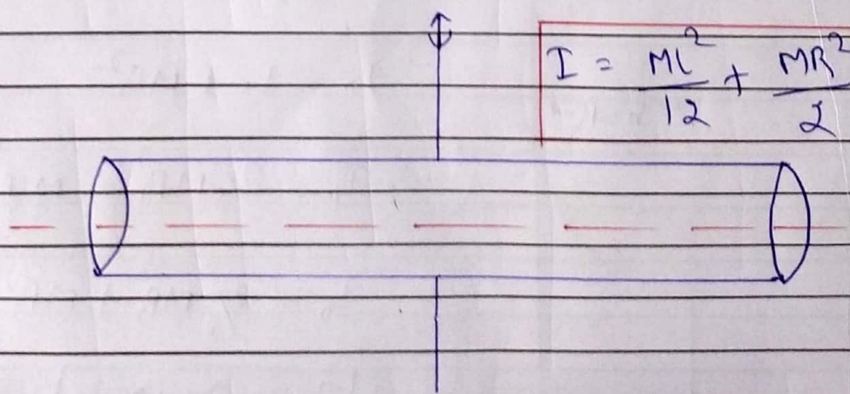
Moment of inertia of uniform solid cylinder about an axis of rotation which is passing through the centre and perpendicular to axis of cylinder.

$$\boxed{I = \frac{ML^2}{12} + \frac{MR^2}{4}}$$



Moment of inertia of uniform hollow cylinder about an axis of rotation which is passing through the centre and perpendicular to axis of cylinder

$$\boxed{I = \frac{ML^2}{12} + \frac{MR^2}{2}}$$



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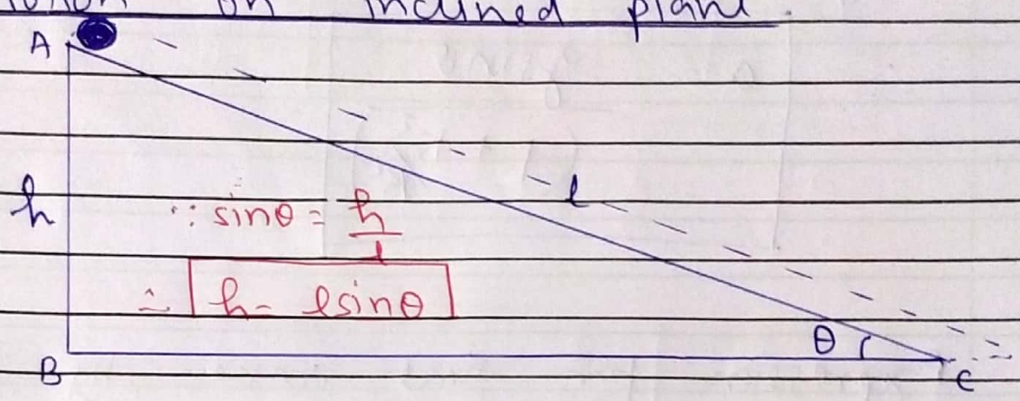
Motion on inclined plane



$\because \sin\theta = \frac{h}{l}$

$\therefore \boxed{h = l\sin\theta}$

Case.1) When body is rolling

$\because$ PE = Linear KE + Rotational KE

$\therefore mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\therefore mgh = \frac{1}{2}mv^2 + \frac{1}{2}(MK^2)\left(\frac{v^2}{R^2}\right)$

$\therefore mgh = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$

$$\therefore \boxed{v = \sqrt{\frac{2gh}{\left(1 + \frac{K^2}{R^2}\right)}}}$$

$$\therefore \boxed{v = \sqrt{\frac{2gl\sin\theta}{\left(1 + K^2/R^2\right)}}}$$

Expression for acceleration

$$v^2 = u^2 + 2as$$

Here, $u = 0\, m/s$, $s = l$ & $v = \sqrt{\dfrac{2gl\sin\theta}{(1 + K^2/R^2)}}$

$$\therefore \frac{2gl\sin\theta}{(1 + K^2/R^2)} = 2al$$

$$\therefore \boxed{a = \frac{g\sin\theta}{(1 + K^2/R^2)}}$$

Expression for time taken by body to reach at bottom.

$$\therefore s = ut + \tfrac{1}{2}at^2$$

Here, $s = l$, $u = 0\, m/s$, $a = \dfrac{g\sin\theta}{(1 + K^2/R^2)}$

$$\therefore l = 0 + \tfrac{1}{2}\left(\frac{g\sin\theta}{1 + K^2/R^2}\right) \cdot t^2$$

$$\therefore t^2 = \frac{2l}{g\sin\theta}\left(1 + K^2/R^2\right)$$

$$\therefore \boxed{t = \sqrt{\frac{2l}{g\sin\theta}\left(1 + K^2/R^2\right)}}$$

→ Case-2) When body is sliding

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$$\boxed{v = \sqrt{2gh} = \sqrt{2gl\sin\theta}}$$

$$\boxed{a = g\sin\theta}$$

$$\boxed{t = \sqrt{\frac{2l}{g\sin\theta}}}$$

Total Kinetic energy of rolling bodies.

<u>for Disc or solid cylinder</u>

$\because$ T.KE = Linear KE + Rotational KE

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$

$$\boxed{T.KE = \frac{3}{4}mv^2}$$

for Ring or hollow cylinder

(2) $T.KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$\boxed{T.KE = mv^2}$

(3) for uniform solid sphere

$T.KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2$

$\boxed{T.KE = \frac{7}{10}mv^2}$

4) for uniform hollow sphere

$T.KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{3}mv^2$

$\boxed{T.KE = \frac{5}{6}mv^2}$

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List of formulas

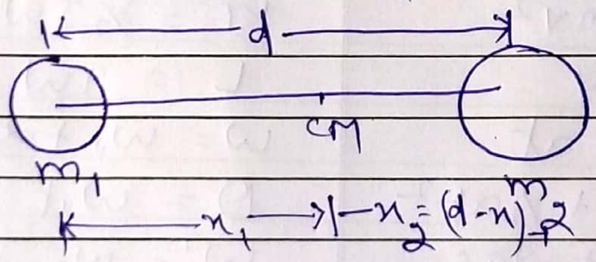
1) Centre of mass

$$r = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$z = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$



$$x_1 = \frac{m_2 d}{m_1 + m_2}$$

$$x_2 = \frac{m_1 d}{m_1 + m_2}$$

2) Principle of moments

$$\boxed{F_1 \times dF = L \times dL}$$

3) Moment of inertia

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2} \dots \text{(for discrete mass dist}^n\text{)}$$

$$I = \int x^2 dm \dots \text{(for continuous mass distribution)}$$

4) Radius of Gyration (K)

$$I = MK^2$$

$$K = \sqrt{\frac{I}{M}}$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$$

5)

Linear motion	Rotational motion
$F = ma$	$\tau = I\alpha$
$KE = \frac{1}{2} m v^2$	$KE = \frac{1}{2} I \omega^2$
$P = mv$	$L = I\omega$
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2} a t^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$W = FS$	$W = \tau\theta$
$P = FV$	$P = \tau\omega$
$F = \frac{dp}{dt}$	$\tau = \frac{dL}{dt}$
$V = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

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6) According to law of conservation of angular moment, when $I = 0$ then

$$L = I_1\omega_1 = I_2\omega_2$$

7) Parallel axis theorem

$$I_0 = I_c + Mh^2$$

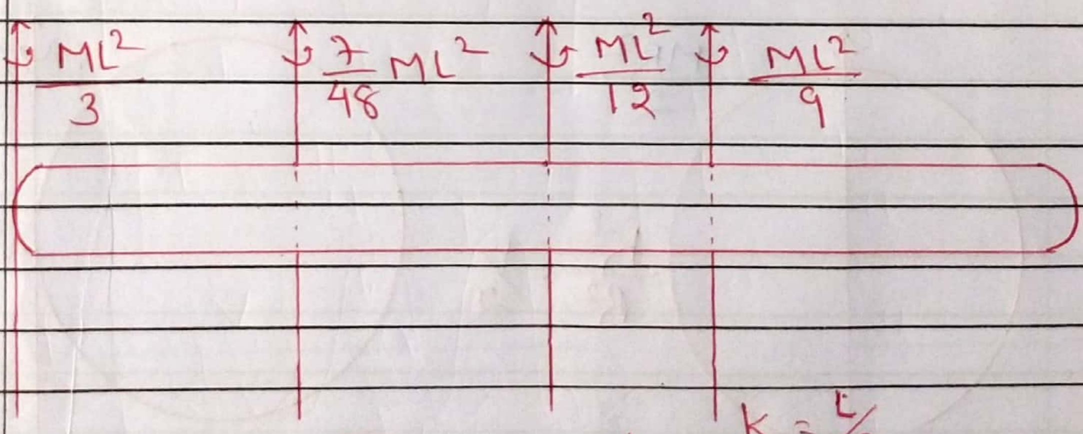
$$MK_0^2 = MK_c^2 + Mh^2$$

$$K_0^2 = K_c^2 + h^2$$

8) Perpendicular axis theorem

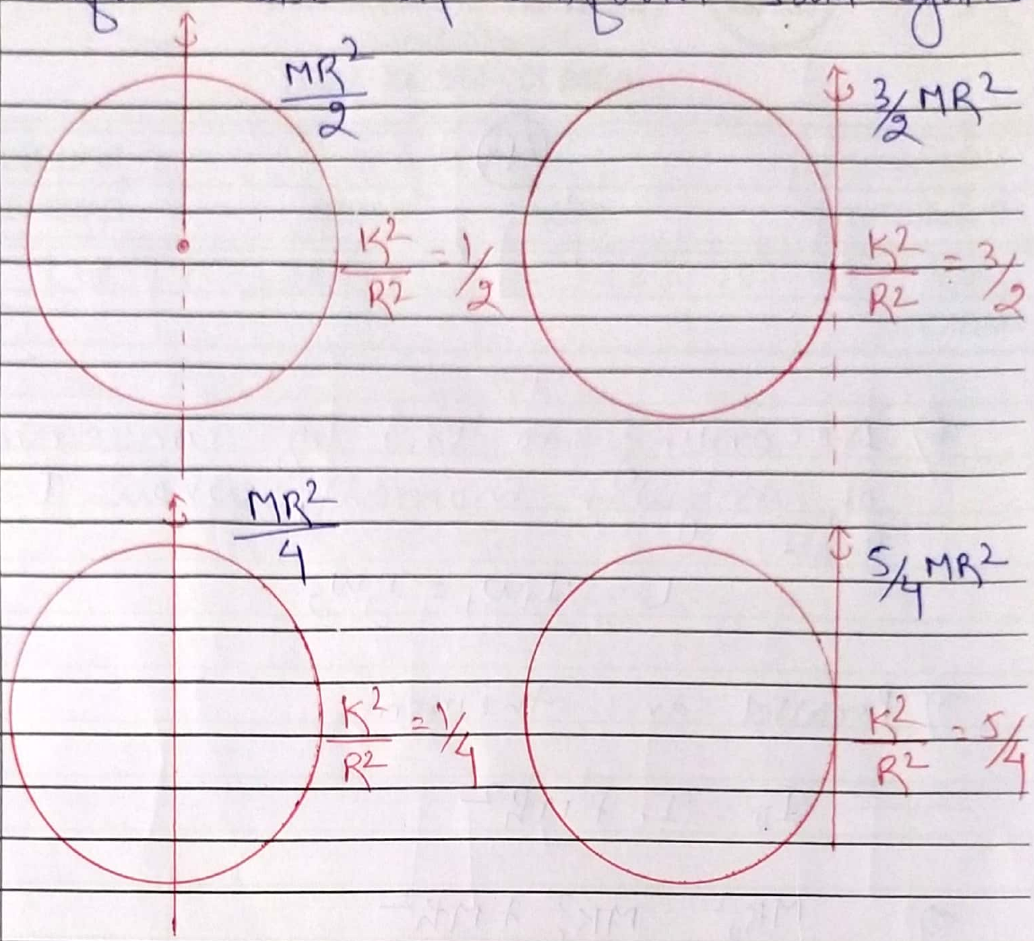
$$I_x + I_y = I_z$$

10) <u>Uniform rod</u>

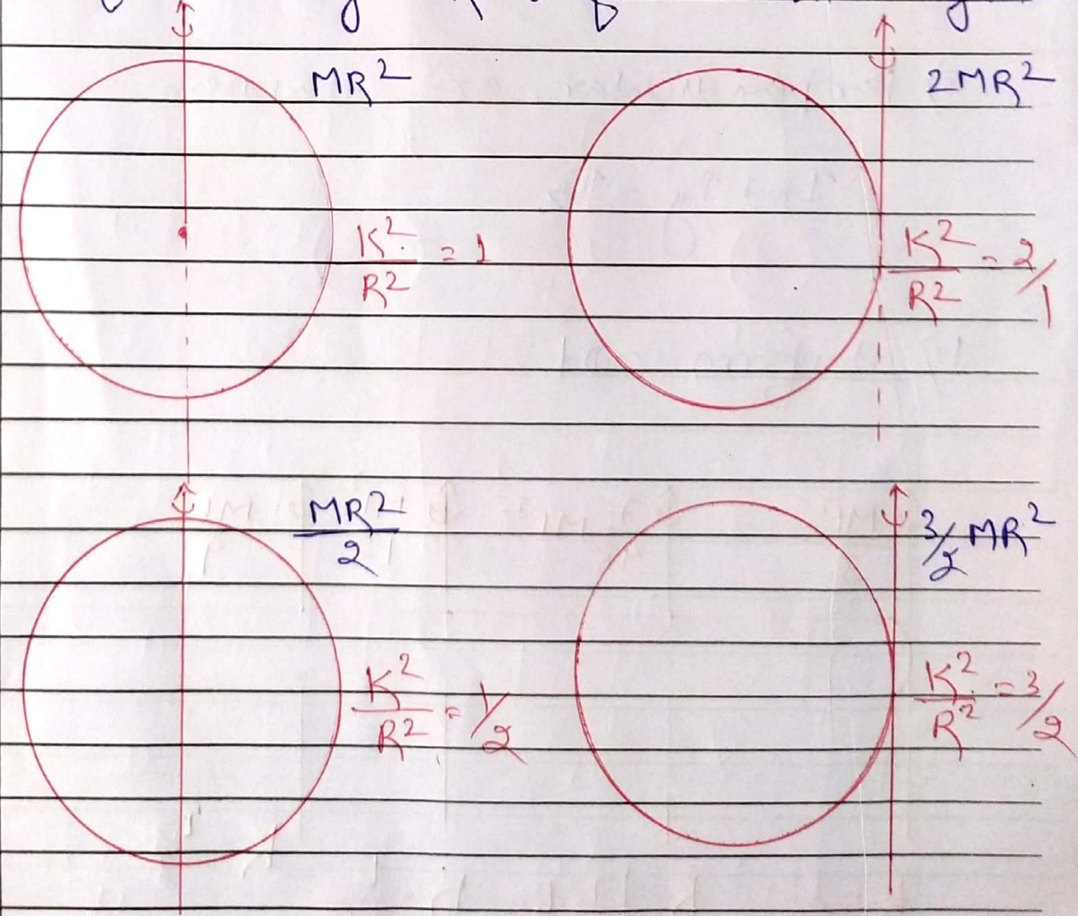


$K = \frac{L}{\sqrt{3}}$ $K = \sqrt{\frac{7}{48}}L$ $K = \frac{L}{2\sqrt{3}}$ $K = L/3$

Uniform disc OR uniform solid cylinder



Uniform Ring OR uniform hollow cylinder



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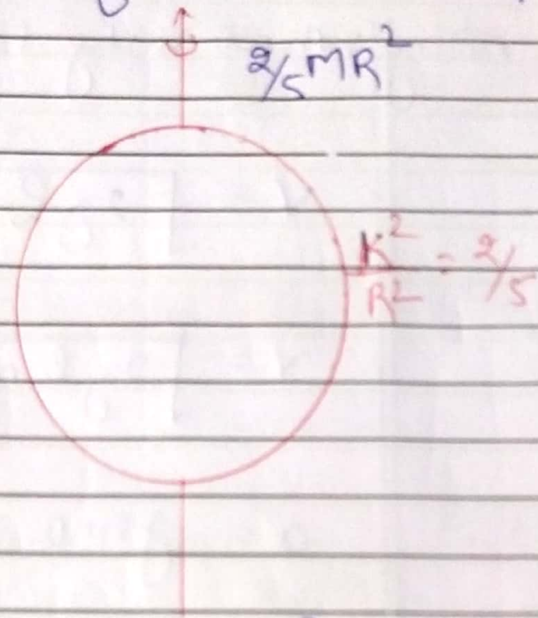
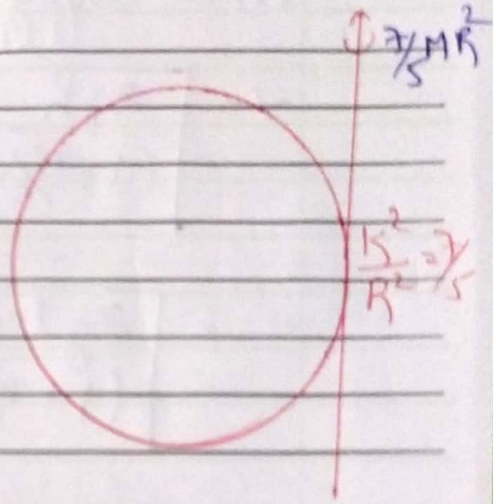
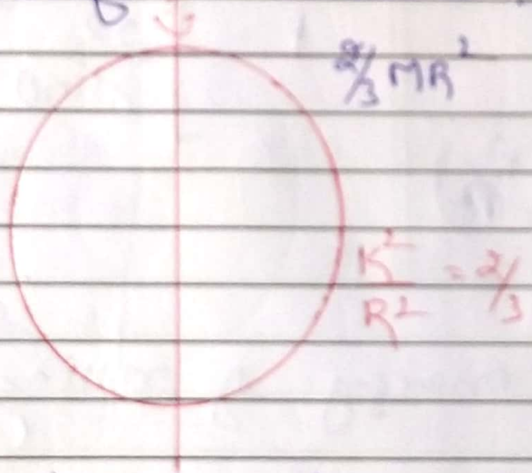
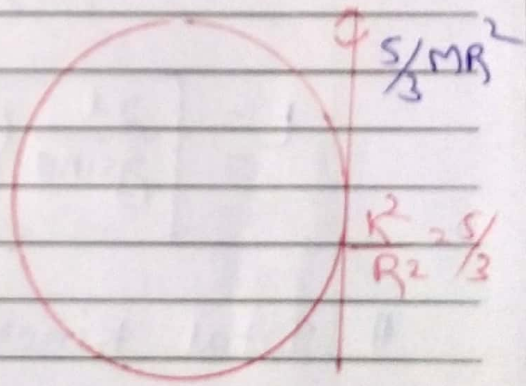
Uniform solid sphere





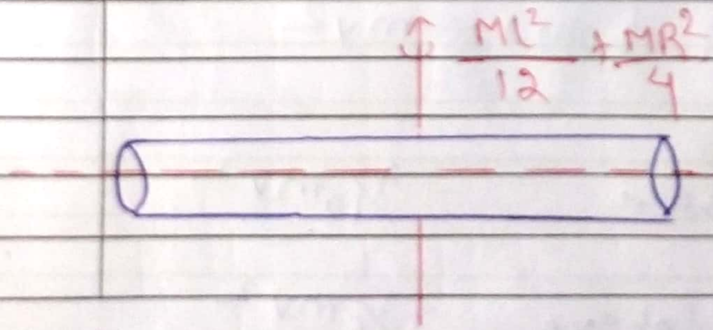
Uniform hollow sphere

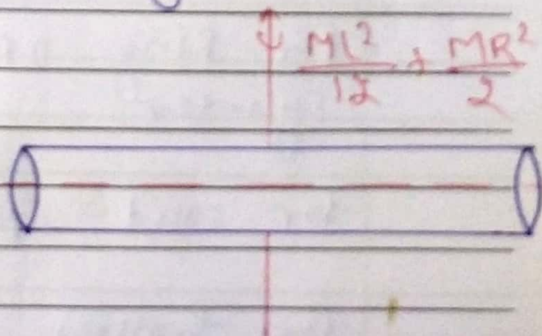




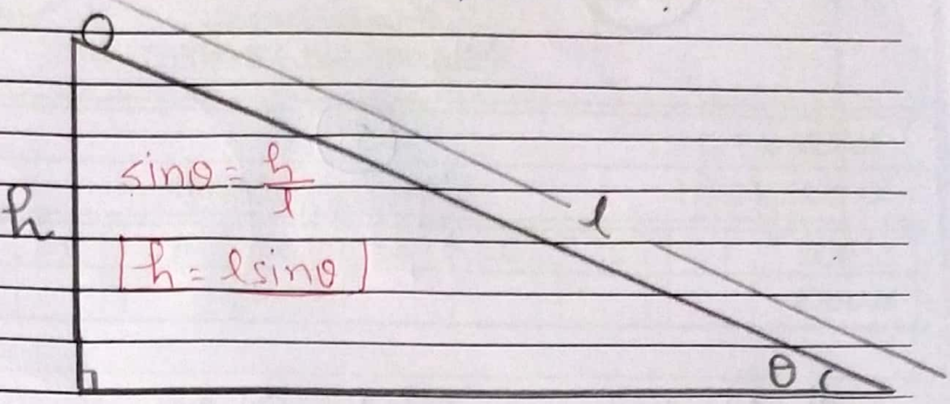
Uniform solid cylinder

Uniform hollow cylinder





Motion on inclined plane



$\sin\theta = \frac{h}{l}$

$\boxed{h = l\sin\theta}$

When body is rolling	When body is sliding
$v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}$	$v = \sqrt{2gh}$
$= \sqrt{\frac{2gl\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}}$	$v = \sqrt{2gl\sin\theta}$
$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$	$a = g\sin\theta$
$t = \sqrt{\frac{2l}{g\sin\theta}\left(1+\frac{K^2}{R^2}\right)}$	$t = \sqrt{\frac{2l}{g\sin\theta}}$

Total kinetic energy of rolling bodies

for disc OR solid cylinder - $\frac{3}{4}mv^2$

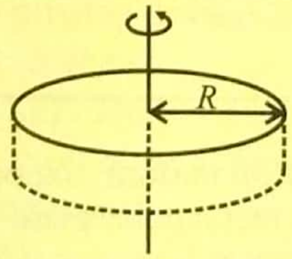
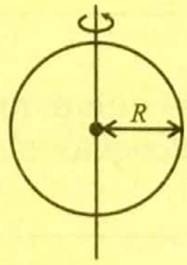
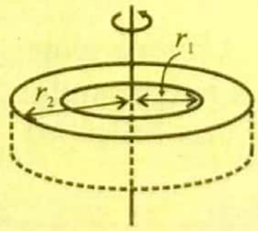
for Ring OR hollow cylinder - mv^2

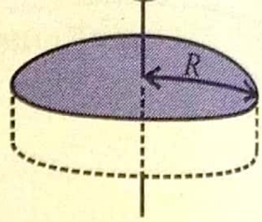
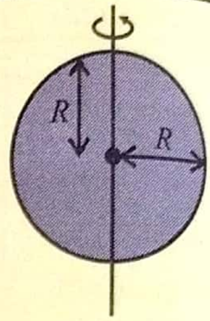
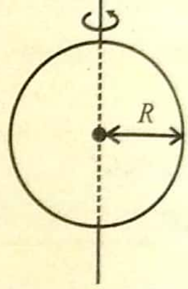
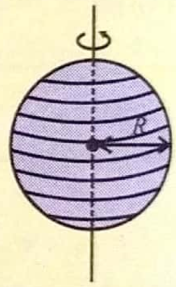
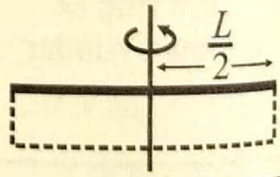
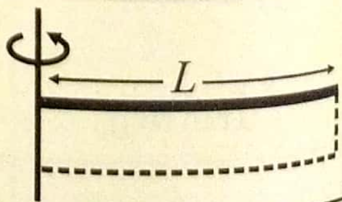
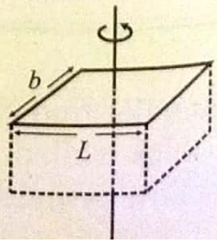
for solid sphere - $\frac{7}{10}mv^2$

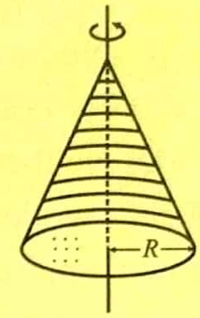
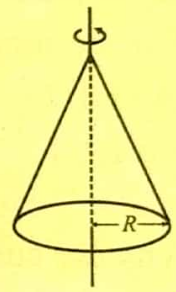
for hollow sphere - $\frac{5}{6}mv^2$

Translational motion		Rotational motion		
Quantity	Symbol/ expression	Quantity	Symbol/ expression	Inter-relation, if possible
Linear displacement	$\vec{s}$	Angular displacement	$\vec{\theta}$	$\vec{s} = \vec{\theta} \times \vec{r}$
Linear velocity	$\vec{v} = \frac{d\vec{s}}{dt}$	Angular velocity	$\vec{\omega} = \frac{d\vec{\theta}}{dt}$	$\vec{v} = \vec{\omega} \times \vec{r}$
Linear acceleration	$\vec{a} = \frac{d\vec{v}}{dt}$	Angular acceleration	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$	$\vec{\alpha} = \vec{a} \times \vec{r}$
Inertia or mass	m	Rotational inertia or moment of inertia	I	$I = \int r^2 dm = \sum m_i r_i^2$
Linear momentum	$\vec{p} = m\vec{v}$	Angular momentum	$\vec{L} = I\vec{\omega}$	$\vec{L} = \vec{r} \times \vec{p}$
Force	$\vec{f} = \frac{d\vec{p}}{dt}$	Torque	$\vec{\tau} = \frac{d\vec{L}}{dt}$	$\vec{\tau} = \vec{r} \times \vec{f}$
Work	$W = \vec{f} \cdot \vec{s}$	Work	$W = \vec{\tau} \cdot \vec{\theta}$	------
Power	$P = \frac{dW}{dt} = \vec{f} \cdot \vec{v}$	Power	$P = \frac{dW}{dt} = \vec{\tau} \cdot \vec{\omega}$	------

Table 3: Expressions for moment of inertias for some symmetric objects:

Object	Axis	Expression of moment of inertia	Figure
Thin ring or hollow cylinder	Central	$I = MR^2$	
Thin ring	Diameter	$I = \frac{1}{2} MR^2$	
Annular ring or thick walled hollow cylinder	Central	$I = \frac{1}{2} M\left(r_2^2 + r_1^2\right)$	

Uniform disc or solid cylinder	Central	$I = \frac{1}{2} MR^2$	
Uniform disc	Diameter	$I = \frac{1}{4} MR^2$	
Thin walled hollow sphere	Central	$I = \frac{2}{3} MR^2$	
Solid sphere	Central	$I = \frac{2}{5} MR^2$	
Uniform symmetric spherical shell	Central	$I = \frac{2}{5} M \frac{\left(r_2^5 - r_1^5\right)}{\left(r_2^3 - r_1^3\right)}$	
Thin uniform rod or rectangular plate	Perpendicular to length and passing through centre	$I = \frac{1}{12} ML^2$	
Thin uniform rod or rectangular plate	Perpendicular to length and about one end	$I = \frac{1}{3} MR^2$	
Uniform plate or rectangular parallelepiped	Central	$I = \frac{1}{12} M(L^2 + b^2)$	

Uniform solid right circular cone	Central	$I = \frac{3}{10} MR^2$	
Uniform hollow right circular cone	Central	$I = \frac{1}{2} MR^2$	

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MARKS									

Numericals:

(12) 17) $R = 0.5\,m$

$n_1 = 0$ rps

$n_2 = 2$ rps

$t = 10$ sec

to find

no of revolution = ?

Time taken by wheel for first revolution = ?

Time taken by wheel for last revolution = ?

formula

$\alpha = \dfrac{\omega_2 - \omega_1}{t}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

no of revolution $= \dfrac{\theta}{2\pi}$

Calculation

$\because \alpha = \dfrac{2\pi n_2 - 2\pi n_1}{t}$

$= \dfrac{2\pi(2-0)}{10}$

$\boxed{\alpha = 2\pi/5 \text{ rad/sec}^2}$

$\because \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\therefore \theta = 0 + \frac{1}{2} \times 2\pi/5 \times (10)^2$

$\therefore \boxed{\theta = 20\pi}$

$\because$ no of revolution $= \dfrac{\theta}{2\pi}$

$= \dfrac{20\pi}{2\pi}$

$= 10$ revolution

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(ii) $\therefore \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

for one revolution $\theta = 2\pi$.

$\therefore 2\pi = 0 + \frac{1}{2} \times \frac{2\pi}{5} \times t^2$

$\therefore t^2 = 10$

$\therefore t = \sqrt{10} = 3.16$ sec

(iii) since time taken by wheel for nine revolution

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\therefore 18\pi = 0 + \frac{1}{2} \times \frac{2\pi}{5} \times t^2$

$\therefore \boxed{t = 9.486 \text{ sec}}$

$\therefore$ time taken by wheel for last revolution

$= 10 - 9.486$

$= 0.5131$ sec

(13) 2) Data

$\mu = 0.5$

$R = 8 \times 10^{-2}$ m

$r_1 = \pi \times 10^{-2}$ m

$r_2 = 8 \times 10^{-2}$ m

$g = \pi^2$ m/s^2

To find frequency at which coin will slipping off?

formula

frictional force = centripetal force

Calculation

$\therefore \mu m g = m r_1 \omega_1^2$

$\therefore \omega_1^2 = \frac{\mu g}{r_1} = \frac{0.5 \times \pi^2}{\pi \times 10^{-2}}$

$\therefore 4\pi^2 n_1^2 = 0.5\pi \times 10^2$

$\therefore n_1^2 = \frac{0.5 \times \pi \times 10^2}{4\pi^2}$

$\therefore \boxed{n_1 \approx 2 \text{ r.p.s}}$

Since, $F = mr\omega^2$

$\therefore r\omega^2 =$ const

$\therefore \frac{r_1}{r_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 = \left(\frac{n_2}{n_1}\right)^2$

$\therefore \frac{\pi \times 10^{-2}}{8 \times 10^{-2}} = \left(\frac{n_2}{n_1}\right)^2$

$\therefore \frac{n_2}{n_1} = \sqrt{\pi/8} = 0.6264$

$\therefore \boxed{n_2 = 0.6264\, n_1}$

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14 3) Data

$r = 72\ m$

$V = 216\ km/hr$

$= \frac{216 \times 5}{18}\ m/s$

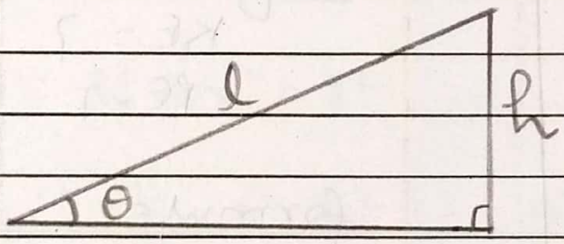
$= 60\ m/s$

$l = 10\ m$

to find

$\theta = ?$

$h = ?$



formula

$\tan\theta = \frac{v^2}{rg}$

$h = l\sin\theta$

Calculation

$\tan\theta = \frac{(60)^2}{72 \times 10}$

$\therefore \tan\theta = \frac{3600}{72 \times 10}$

$\therefore \tan\theta = 5$

$\therefore \theta = \tan^{-1}(5) = (78.69)^\circ$

$\therefore \sin(78.69)^\circ = 0.9805$

$\therefore h = l\sin\theta$

$= 10 \times 0.9805$

$\boxed{h = 9.805\ m}$

4) Data

15 $r = 72\ m$

$\theta = (78.69)^\circ$

$g = 9.8\ m/s^2$

to find

$V_{min} = ?$

$V_{max} = ?$

formula

$V_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta}\right)}$

$V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$

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Calculation

$$V_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu}{1+\mu\tan\theta}\right)}$$

$$= \sqrt{72 \times 9.8\left(\frac{5 - 0.8}{1+(0.8\times5)}\right)}$$

$$= \sqrt{604.8}\ m/s$$

$$= \sqrt{604.8} \times \frac{18}{5}\ km/hr$$

$$= 24.59 \times 18/5$$

$$\boxed{V_{min} = 88.53\ Km/hr}$$

(M) As angle of banking is more than $45°$ there is no upper limit.

(16) 5) Data

$r = 6.05\ m$

$\mu = 0.5$

$m = 50\ Kg$

to find

$V_{min} = ?$

$f_s = ?$

formula

$$V = \sqrt{\frac{rg}{\mu}}$$

$$f_s = \mu mg$$

Calculation

$$V = \sqrt{\frac{6.05\times10}{0.5}}$$

$$= \sqrt{118.56}$$

$$\boxed{V_{min} = 10.88 \approx 11\ m/s}$$

$\because f_s = \mu mg$

$= 0.5 \times 50 \times 10$

$= 250\ N$

6) Data

(15)

$l = 20cm = 0.2\ m$

$m = 100g = 0.1\ Kg$

$n = 75\ rpm$

$= \frac{75}{60}\ r.p.s.$

$\cos\theta = 0.8$

to find

$KE = ?$

$PE = ?$

formula

$KE = \frac{1}{2}mv^2$

$PE = mgh$

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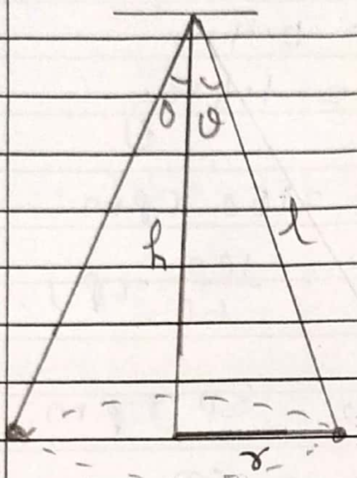
Calculation



$\because \cos\theta = \frac{h}{l}$

$\therefore h = l\cos\theta$

$= 0.2 \times 0.8$

$= 0.16\ m$

$\because h^2 + r^2 = l^2$

$\therefore r^2 = l^2 - h^2$

$\therefore r^2 = 0.04 - 0.0256$

$\therefore r^2 = 0.0144$

$\therefore \boxed{r = 0.12\ m}$

$\because v = r\omega$

$= 0.12 \times 2 \times \pi \times \frac{75}{60}$

$\boxed{v = 0.942\ m/s}$

$KE = \frac{1}{2} m v^2$

$= \frac{1}{2} \times 0.1 \times (0.942)^2$

$\boxed{KE = 0.044\ J}$

$\because$ since height of the bob from mean position

$= l - h$

$= 0.2 - 0.16$

$= 0.04\ m$

$\because PE = mgh$

$= 0.1 \times 10 \times 0.04$

$\boxed{PE = 0.04\ J}$

8) Data (Ring → Disc)

(19) $m = 1\ kg$

$I_R = 1\ kgm^2$

to find

$I_D = ?$

formula

$I_R = \frac{MR^2}{2}$ (Ring)

$I_D = \frac{MR^2}{2}$ (Disc)

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Calculation

$\because I_R = \frac{MR^2}{2}$

$\therefore 1 = \frac{1 \times R^2}{2}$

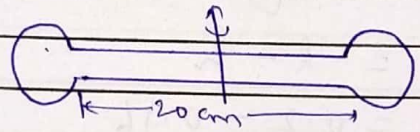
$\therefore \boxed{R^2 = 2}$

$\because I_D = \frac{MR^2}{2}$

$= \frac{1 \times 2}{2}$

$\boxed{I_D = 1 \ \text{kgm}^2}$

20 9) Data



$m_R = 60 \ g$

$L_R = 20 \ cm$

$m_s = 50 \ g$

$R_s = 10 \ cm$

to find

$I = ?$

formula

$I = \frac{M_R L_R^2}{12} + 2 m_s r^2$

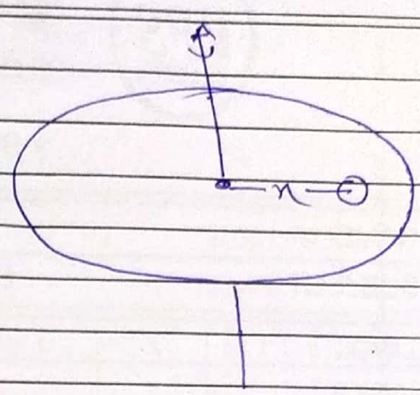
Calculation

$I = \frac{60 \times (20)^2}{12} + 2 \times 50 \times (20)^2$

$= 2000 + 40000$

$= 42000 \ g cm^2$

10) 21



$m_s = 10 \ Kg$

$R_s = 0.4 \ m$

$m_c = 1.6 \ Kg$

$n_1 = 100 \ r.p.m$

$= \frac{100}{60} \ r.p.s$

$n_2 = 80 \ r.p.m$

$= \frac{80}{60} \ r.p.s$

to find

$x = ?$

formula

$L = I_1 \omega_1 = I_2 \omega_2$

Calculation

$\because I_1 (2\pi n_1) = I_2 (2\pi n_2)$

$\therefore \frac{MR^2}{2} \times n_1 = \left[m_c x^2 + \frac{m_s R_s^2}{2} \right] n_2$

$\therefore \frac{10 \times 0.16 \times 100}{2 \times 60}$

$= \left[1.6 \times x^2 + \frac{10 \times 0.16}{2} \right] \frac{80}{60}$

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$\therefore \frac{80}{60} = [1.6x^2 + 0.8]\, 80/60$

$\therefore 1.6x^2 + 0.8 = 1$

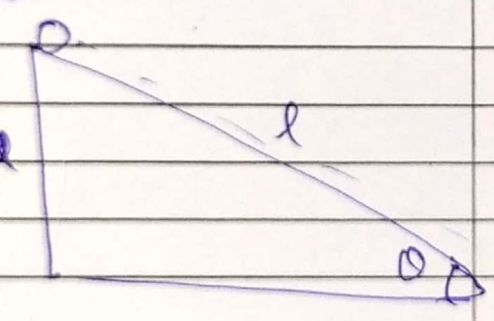
$\therefore 1.6x^2 = 0.2$

$\therefore x^2 = 1/8$

$\therefore x = \frac{1}{2\sqrt{2}}$

$\therefore \boxed{x = 0.35\ m}$

22 11) Data



$h = 3/5\, l$

$v = \sqrt{10}\ m/s$

$h = \frac{3l}{5}$

$\cancel{l} = 5/3$

To find
shape of the body =)

formula

$v = \sqrt{\frac{2gl\sin\theta}{(1+K^2/R^2)}}$

Calculation

$v^2 = \frac{2gl\sin\theta}{(1+K^2/R^2)}$

$\therefore 10 = \frac{2\times 10\times 5/3 \times (h/l)}{(1+K^2/R^2)}$

$\therefore \left(1+\frac{K^2}{R^2}\right) = 2\times 5/3 \times \left(\frac{3/5\, l}{l}\right)$

$\therefore \left(1+\frac{K^2}{R^2}\right) = 2$

$\therefore \frac{K^2}{R^2} = 2-1 = 1$

$\therefore \boxed{\frac{K^2}{R^2} = 1}$

Hence body is Ring.