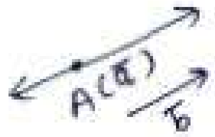
Line and Plane

1. Equation of a line passing through a given point and parellel to given vector.

   1. Vector Form:



$$\bar{r} = \bar{a} + \lambda \bar{b}$$

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\bar{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\bar{b} = a\hat{i} + y\hat{j} + z\hat{k}$$

   2. Cartesian form.

   Point $A(x_1, y_1, z_1)$

   drs of parellel line is a, b, c.

$$\therefore \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

or

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

2. Equation of a Line passing through two given points.

   1. Vector form:

$$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$$

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\bar{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\bar{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

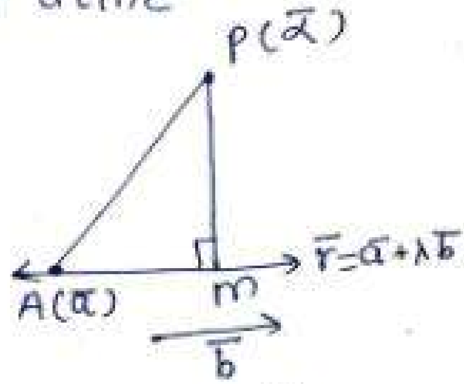
2. Cartesian form:

$A = (x_1, y_1, z_1)$

$B = (x_2, y_2, z_2)$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Distance of Point from a line



$$PM = \sqrt{|\bar{\alpha} - \bar{a}|^2 - \left[\frac{(\bar{\alpha} - \bar{a}) \cdot \bar{b}}{|\bar{b}|}\right]^2}$$

Where,

Eqn of Line is

$\bar{r} = \bar{a} + \lambda \bar{b}$

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Skew Lines

Neither Parellel nor intersecting

Shortest Distance $= PQ$.

$$\therefore PQ = \left| \frac{(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2)}{|\bar{b}_1 \times \bar{b}_2|} \right|$$

Let, $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ &

$\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_1}{c_2}$

are two skew lines.

$\therefore$ Shortest Distance

$$= \left| \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-b_1a_2)^2+(b_1c_2-c_1b_2)^2+(a_1c_2-c_1a_2)^2}} \right|$$

Lines are intersecting if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & a_2 & c_2 \end{vmatrix} = 0$$

Note: If the lines are intersecting then the shortest distance between them is zero.

Distance between Parellel lines

$$\bar{r} = \bar{a}_1 + \lambda \bar{b} \quad \& \quad \bar{r} = \bar{a}_2 + \lambda \bar{b}$$

$$\therefore \quad d = \left| \frac{(\bar{a}_2 - \bar{a}_1) \times \bar{b}}{|\bar{b}|} \right|$$

$\longleftrightarrow \bar{r} = a_1 + \lambda \bar{b}$

$\longleftrightarrow \bar{r} = \bar{a}_2 + \lambda \bar{b}$

PLANE

1. Equation of plane.

Normal form.

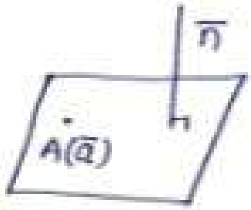
$$\bar{r} \cdot \hat{n} = p$$

$\hat{n}$ = unit Vector along the Normal

p = length of perpendicular from origin to plane

2. Eqn of Plane passing through a given point and perpendicular to a given vector

$$\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$$



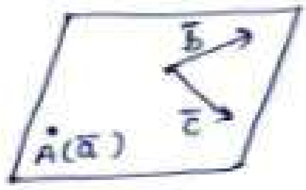
$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\bar{n} = a\hat{i} + b\hat{j} + z\hat{k}$$
$$\bar{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

Cartesian eqn of Plane is

$$ax + by + cz + d = 0.$$

3. Eqn. of Plane passing through apoint and parellel to two vectors.

$$\bar{n} = \bar{b} \times \bar{c}$$



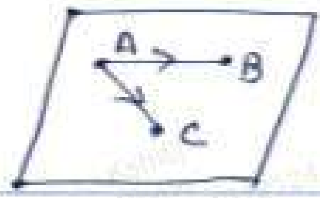
$$\bar{r} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c})$$

Parametric form

$$\bar{r} = \bar{a} + \lambda \bar{b} + \mu \bar{c}.$$

4. Eq. of Plane passing through 3 Non-collinear points.

$$\bar{r} \cdot (\overline{AB} \times \overline{AC}) = \bar{a} \cdot (\overline{AB} \times \overline{AC})$$



5. Intercept form

If a, b, c are the intercepts made by the plane on coordinate axes. x, y, z resp.

$\therefore$ Eqn of Plane is. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

6. Equation of plane passing through the intersection of two planes.

$\bar{r}\cdot\bar{n}_1 = p_1$ & $\bar{r}\cdot\bar{n}_2 = p_2$

$\therefore$ Eqn of plane is

$$\bar{r}\cdot(\bar{n}_1 + \lambda\bar{n}_2) = p_1 + \lambda p_2$$

Cartesian form:

$$\left.\begin{array}{l} a_1x + b_1y + c_1z + d_1 = 0 \\ a_2x + b_2y + c_2z + d_2 = 0 \end{array}\right\} \text{Eqn of planes}$$

$\therefore$ Eqn of plane is

$$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$$

λ : Parameter.

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Angle between two planes

1. Eqn. of planes are $\bar{r}\cdot\bar{n}_1 = p_1$
$\bar{r}\cdot\bar{n}_2 = p_2$

$$\therefore \cos\theta = \left|\frac{\bar{n}_1\cdot\bar{n}_2}{|\bar{n}_1||\bar{n}_2|}\right|$$

θ : an angle between planes.

2. Eqn of planes are
$a_1x + b_1y + c_1z + d_1 = 0$
$a_2x + b_2y + c_2z + d_2 = 0$

$$\therefore \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Note:

* Two lines are Perpendicular

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

* Two lines are parellel.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Angle between Line & Plane

Line: $\bar{r} = \bar{a} + \lambda \bar{b}$

Plane: $\bar{r} \cdot \bar{n} = p$

$\boxed{\text{Dineshsir}}$

$$\therefore \sin\theta = \left| \frac{\bar{b} \cdot \bar{n}}{|\bar{b}||\bar{n}|} \right|$$

Coplanarity of Two Lines

$\bar{r} = \bar{a}_1 + \lambda \bar{b}_1$ & $\bar{r} = \bar{a}_2 + \mu \bar{b}_2$

Above lines are coplaner if and only if $(\bar{a}_1 - \bar{a}_2) \cdot (\bar{b}_1 \times \bar{b}_2) = 0$

Eqn of Plane containing above lines is

$\bar{r} \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2)$ or

$\bar{r} \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_2 \cdot (\bar{b}_1 \times \bar{b}_2)$

Cartesian form

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \quad \&$$

$$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}.$$

Above eqns are coplaner if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

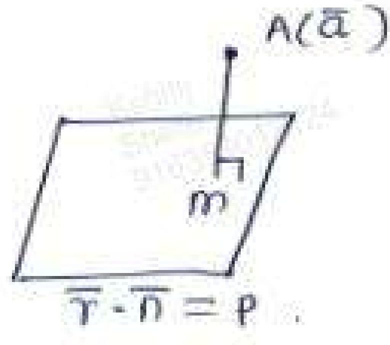
Eqn of plane containing above Lines is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0 \quad \text{or}$$

$$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Distance of Point from a plane

$$AM=\left|\frac{(\bar{a}\cdot\bar{n})-p}{|\bar{n}|}\right|$$



Length of perpendicular drawn from point (x_1, y_1, z_1) to the plane $ax+by+cz+d=0$ is

$$\left| \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}} \right|$$

Distance between two parellel planes $ax+by+cz+d_1=0$ & $ax+by+cz+d_2=0$ is

$$\left| \frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}} \right|$$