

# 1 Physics!

## Magnetic Field Due to Electric current

The phenomena of producing magnetic field by passing electric current through a conductor is called magnetic field due to electric current.

Q. What is Lorentz Force?

When a charged particle moves through a region of electric field ( $\vec{E}$ ) and magnetic field ( $\vec{B}$ ) then the net force experienced by charged particle is called as Lorentz force.

$$F_L = \vec{F}_e + \vec{F}_m$$

$$F_L = q\vec{E} + q(\vec{V} \times \vec{B})$$

$$F_L = q \cdot \vec{E} + (\vec{V} \times \vec{B})$$

Q. Define cyclotron formula.

- (i) In a magnetic field a charged particle typically undergoes circular motion.
  - (ii) centripetal force necessary for circular motion is the magnetic force on the particles.
- $\therefore$  Centripetal Force = Magnetic Force.

$$\frac{mv^2}{R} = qvB$$

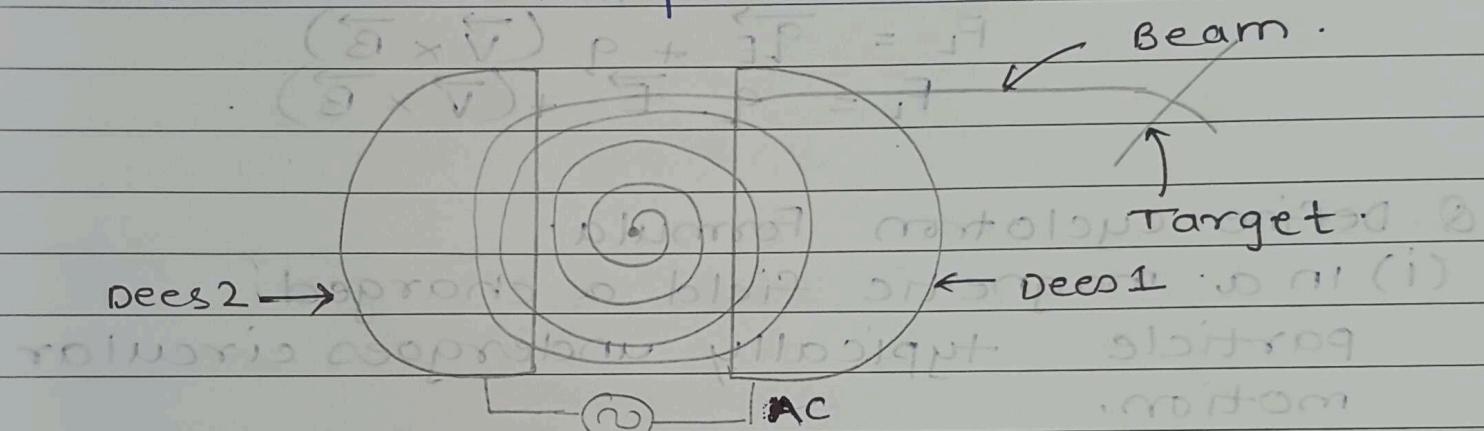
This is known as cyclotron formula.

$$\frac{mv}{R} = qB$$

$$mv = qBR$$

Q. Explain working principle, draw the construction and working of cyclotron. Also derive the formula for velocity of charge, time period, frequency, kinetic energy.

- Definition: cyclotron is a charged particle accelerator used for accelerating charged particle to higher energies.
- Working principle: in cyclotron, both electric and magnetic field are used. Magnetic Field puts ions (particles) into circular path and electric field accelerates the particles.



→ Using cyclotron formula,

$$mv = qBR$$

$$v = \frac{qBR}{m}$$

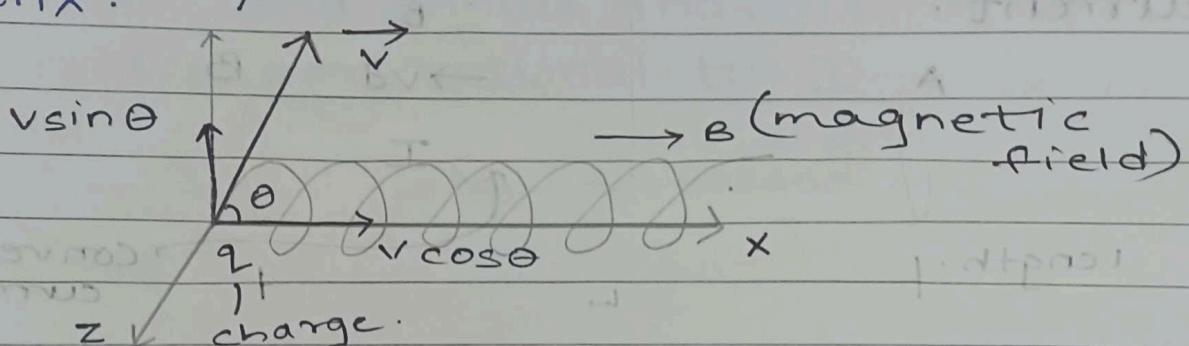
$$\text{But, } T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi R}{\frac{qBR}{m}} = \frac{2\pi m}{qB}$$

$$\text{Frequency of revolution} = f = \frac{1}{T} = \frac{1}{\frac{2\pi m}{qB}} = \frac{qB}{2\pi m}$$

$$\text{Kinetic Energy: } \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m}$$

K.E directly proportional to square of Radius

Q. Helical Motion: draw and explain the path of charged particle become helix.



→ The above fig. shows helical motion of charged particles when it moves with velocity ( $\vec{v}$ ).

→ This velocity is resolved into two components.

(I) Parallel to magnetic field.

(II) Perpendicular to magnetic field.

→ Due to this, it obtains helical path,

$$F_m = \vec{v} \times \vec{B}$$

$$F_m = VB \sin \theta$$

(I)  $v$  parallel to  $B$ .  $\theta = 0^\circ$   $\sin 0^\circ = 0$   
 $\therefore F_m = 0$

(II)  $v$  perpendicular to  $B$ .  $\theta = 90^\circ$   $\sin 90^\circ = 1$

$$\therefore \text{at speed } F_m = VB$$

$$\theta \text{ is abt } v \quad I = \frac{q}{m} t$$

$$bv$$

$$\theta \text{ is abt } I = \theta \text{ is abt } \frac{q}{m} t$$

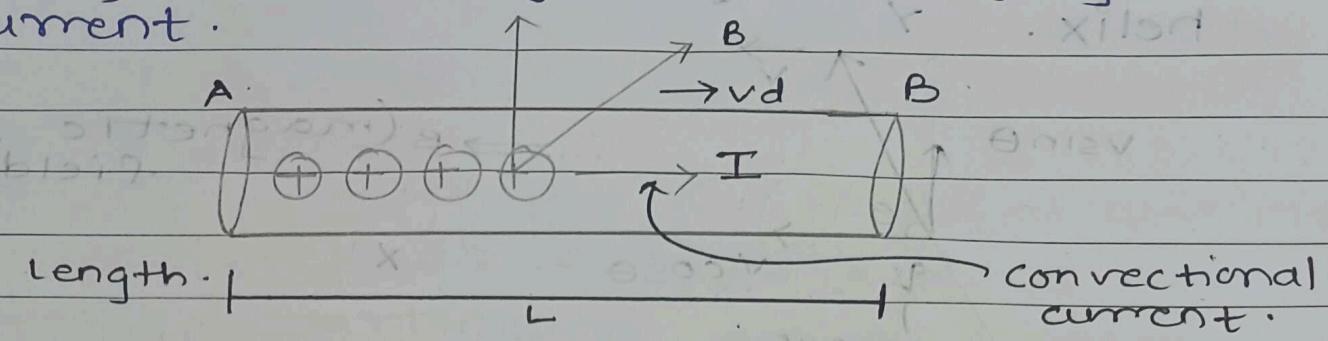
$$\therefore \text{curv. tripathi rot } - (\vec{B} \times \vec{v}) I = \frac{q}{m} t$$

$$\text{from abt of abt } - (\vec{B} \times \vec{v}) I = \frac{q}{m} t$$

$$(\vec{B} \times \vec{v}) I = \frac{q}{m} tb$$

$$(\vec{B} \times \vec{v}) I^2 = \frac{q}{m} tb^2$$

Q. Derive the relation for magnetic force acting on a straight wire carrying current.



→ Consider a straight wire of length ( $L$ ) in which magnetic field is applied.

→ Let 'I' be the current flowing through it by velocity ( $v_d$ ).

$$I = \frac{q}{t}$$

$$q = It \quad \text{(i)}$$

$$But, v_d = \frac{L}{t} \quad \text{(ii)}$$

$$\therefore t = \frac{L}{v_d} \quad \text{(iii)}$$

$$q = I \times \frac{L}{v_d} \quad \text{(iv)}$$

The magnetic force on the charge is -

$$\vec{F}_m = q (\vec{v}_d \times \vec{B})$$

$$\vec{F}_m = I \frac{L}{v_d} v_d B \sin \theta$$

$$\vec{F}_m = I L B \sin \theta = B I L \sin \theta$$

In vector form,

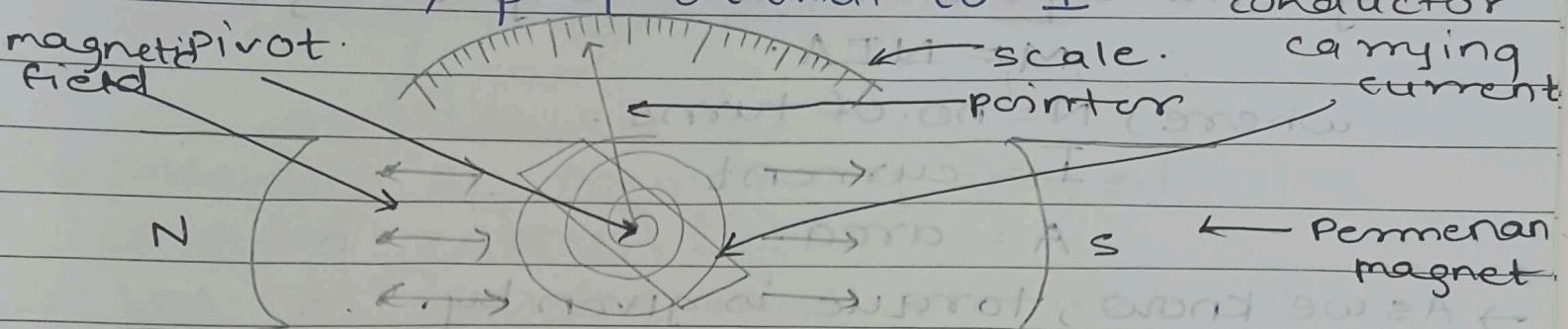
$$\vec{F}_m = I (\vec{L} \times \vec{B}) \quad \text{— For straight wire.}$$

For an arbitrary shape (we have to take small length.)

$$d\vec{F}_m = I (\vec{dL} \times \vec{B})$$

$$\int d\vec{F}_m = S I (\vec{dL} \times \vec{B})$$

Q. Draw a neat labelled diagram of moving suspended coil galvanometer. State its principle and shows that its deflection is directly proportional to  $I$ .



→ When a current carrying coil is suspended in a uniform magnetic field, it experiences torque and maximum due to which coil rotates and maximum flux passes through coil.

Proof.

(1) The coil of mcg rotates due to torque acting on it which is called as deflecting torque  
Formula:  $\tau_d = NIAB$  — (I).

(II) The restoring torque ( $\tau_r$ ) due to spring is given by

$$\tau_r \propto \phi$$

$$\tau_r = K\phi \quad \text{--- (II)}$$

$\phi$  = deflection,  $K$  = torsional constant.

$$\tau_d = \tau_r$$

$$NIAB = K\phi$$

$$I = \frac{K\phi}{nAB}$$

$K, N, A, B$  are constant.

$$I = \left[ \frac{K}{nAB} \right] \phi$$

$$I \propto \phi$$

Hence proved.

Q. What is magnetic dipole moment (m)?  
 Derive an expression for potential energy (U) and torque ( $\tau$ ).  
 → magnetic dipole moment is given by,

$$\mathbf{M} = NIA$$

where,  $N$  = no. of turns.

$I$  = current

$A$  = area.

→ As we know, torque is given by,

$$\tau = NIBA \sin \theta$$

→ We know that, magnetic potential energy is given by  $U = -mB \cos \theta$

$$\text{case I: if } \theta = 0^\circ \text{ then } U = -mB \sin 0^\circ = 0$$

This is minimum potential energy.

case II: If  $\theta = 180^\circ$  then.

$$U = -mB \cos 180^\circ = mB$$

$$U = mB$$

This is maximum potential energy.

Q. state and explain Biot-savart law.

- consider an arbitrary shaped wires carrying current ( $I$ ).
- Let ' $dL$ ' be the small element of the wire.
- we have to find magnetic field ' $dB$ ' at point  $P$  which is

(I) Directly proportional to small element.  
 $dB \propto dL$  — (I)

(II) Directly proportional to ' $I$ '.  
 $dB \propto I$  — (II)

(III) Inversely proportional to square of distance

$$dB \propto 1/r^2 \quad \text{— (III)}$$

→ From (I) (II) and (III) and (IV) ( $dB \propto \sin\theta$ )  $dB$  is directly proportional to

$$\frac{IdL \sin\theta}{r^2}$$

$$\therefore dB \propto \frac{IdL \sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{IdL \sin\theta}{r^2}$$

$\mu_0$  = permeability of free space.

$$4\pi \times 10^{-7}$$

$$\therefore \frac{\mu_0}{4\pi} \times 10^{-7}$$

$$\therefore d\theta = 10^{-7} \frac{Id \sin\theta}{r^2}$$

## Application 1.

Q. derive an expression for magnetic field at centre due to current in a circular arc of wire.

(i) consider a circular arc of wire carrying current  $I$ .

(ii) The circular arc has an angle ( $\theta$ ) from centre.

(iii) Let 'dL' be the small element.

→ Using Biot - Savart Law at point O,

$$dB = \frac{\mu_0}{4\pi} I dL \sin \theta \quad (i)$$

$$\theta = 90^\circ \text{ or } \sin 90^\circ = 1 \quad (ii)$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dL}{r^2} \quad (iii)$$

Total magnetic field at point O is:

$$B = \oint dB$$

$$B = \frac{\mu_0}{4\pi} \int \frac{I dL}{r^2} \quad (ii) \text{ (i) } \rightarrow \int d\theta$$

$$\text{But } dL = r \times d\theta$$

$$B = \int \frac{\mu_0}{4\pi} \frac{I \times r \times d\theta}{r^2}$$

$$B = \int \frac{\mu_0}{4\pi} \frac{I d\theta}{r}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} \int d\theta$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} \times \theta$$

For circular wire,

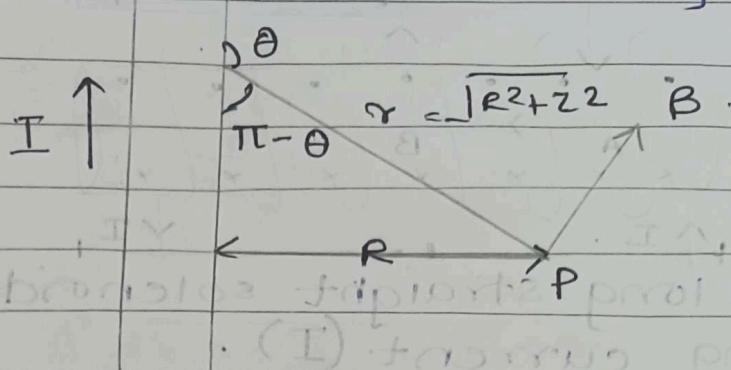
$$\theta = 2\pi$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} \times 2\pi$$

$$B = \frac{\mu_0 I}{2r}$$

## Application 2.

Q. Derive an expression for magnetic field at a point which is perpendicular distance from infinite long straight wire.



- consider an infinite long straight wire carrying current  $I$ .
- Let  $P$  be the point at distance  $R$  from the wire.

From the figure,

$$r^2 = L^2 + R^2$$

$$r = \sqrt{L^2 + R^2}$$

$$\text{But } \sin(\pi - \theta) = \frac{R}{\sqrt{R^2 + L^2}} = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{R}{\sqrt{R^2 + L^2}} \cdot \sin(180 - \theta)$$

Magnetic field  $dB$  at point  $P$  is

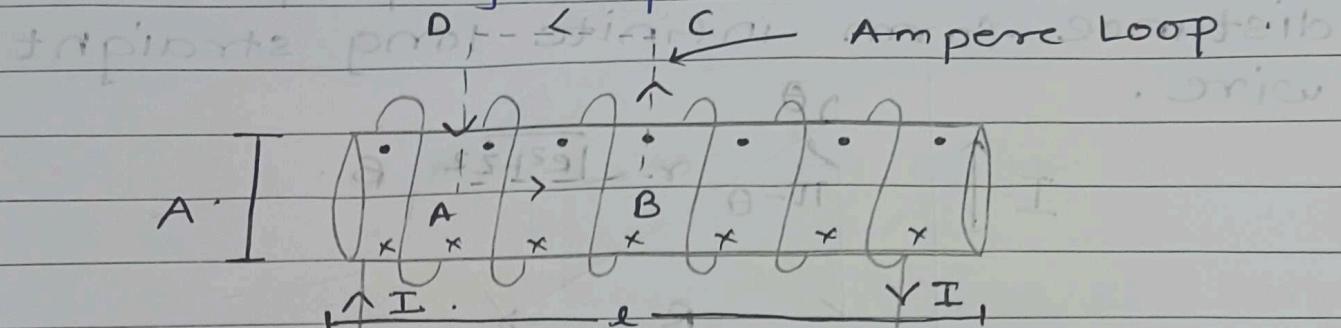
$$dB = \frac{\mu_0 I dL \sin \theta}{4\pi r^2} \quad (\text{Biot-Savart Law})$$

Q. State and explain Ampere's circuit law.

→ The line integral of magnetic induction ( $\vec{B}$ ) around any closed path in the free space ( $\mu_0$ ) is equal to times the total current flowing through area bounded by the closed path.

Mathematically:  $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

Q. Derive an expression for magnetic field along the axis due to long, straight, no solenoid using ampere's Law.



→ consider a long straight solenoid of length (L) carrying current (I).

→ The (•) sign shows the current is coming out form is & the plain and (x) shows current is going into the plane.

→ let 'n' be the no. of turns per unit length

$$n = \frac{N}{L}$$

→ Acc. to ampere circuit law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 N T$$

But  $\frac{N}{L} = n$

$$n \times L = N$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 n L I \quad (1)$$

(LHS) ~~is~~ ~~is~~ (RHS) ~~is~~ ~~is~~

case I: LHS =  $\oint \vec{B} \cdot d\vec{l}$

For rectangular loop ABCD.

$$\oint \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \cdot d\vec{l} + \int_B^C \vec{B} \cdot d\vec{l} + \int_C^D \vec{B} \cdot d\vec{l} + \int_D^A \vec{B} \cdot d\vec{l} \quad (2)$$

Integrate each part of loop in (2) ~~is~~ ~~is~~

be careful while apportion primary terms

$I_{loop} = 2b \cdot s \cdot \varphi$

I) For path AB,

$$\oint \vec{B} \cdot d\vec{l} = \int_A^B B dl \cos \theta$$

$\theta = 0^\circ$

[ $\vec{B} \parallel \text{path AB}$ ]

$$\begin{aligned} &= \int_A^B B dl \\ &= B \int_A^B dl \\ &= BL. \quad \text{--- (A)} \end{aligned}$$

II) For path BC,

$$\oint_B^C \vec{B} \cdot d\vec{l} = \int_B^C B dl \cos \theta$$

$\theta = 90^\circ$

$$\cos 90^\circ = 0$$

$$\oint_B^C \vec{B} \cdot d\vec{l} = 0 \text{ --- (B)}$$

(III) similarly path AD = 0,  $\oint_D^A \vec{B} \cdot d\vec{l} = 0 \text{ --- (C)}$ .

(IV) outside the solenoid the mF is very weak as compared to inside  
 $\therefore$  it is consider to be zero.

$$\oint \vec{B} \cdot d\vec{l} = 0 \text{ --- (D)}$$

put (A) (B) (C) and (D) in eq (2).

$$\oint \vec{B} \cdot d\vec{l} = BL + 0 + 0 + 0$$

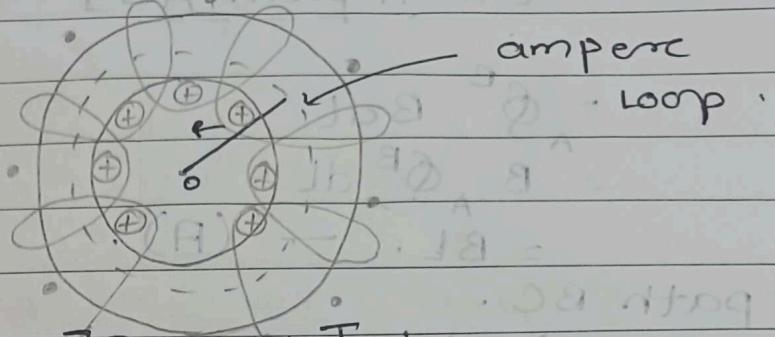
$$\oint \vec{B} \cdot d\vec{l} = BL \quad \text{--- (3)}$$

from eq (3) and (1).

$$BK = \mu_0 n K I$$

$$B = \underline{\underline{\mu_0 n I}}$$

Q. What is Toroid? Using ampere's Law derive an expression for magnetic induction at a point along the axis of toroid.



→ When solenoid is bent to form a circular loop like structure then it is called Toroid.

→ Consider an ampere loop of radius 'R' along the axis of Toroid carrying current 'I' with total no. of turns 'N'.

→ Acc. to ampere law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 NI$$

$$\oint B dl \cos \theta = \mu_0 NI$$

$$\theta = 0^\circ \cos 0^\circ = 1$$

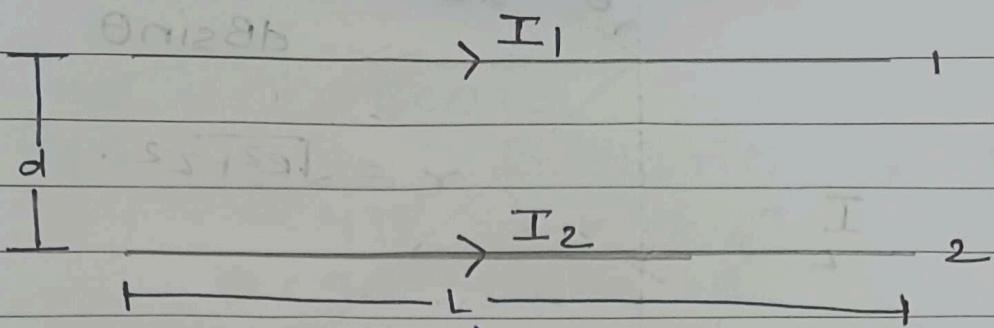
$$\oint B dl = \mu_0 NI$$

$$\text{But } \oint dl = 2\pi R + 2\pi R = 4\pi R$$

$$B \times 2\pi R = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi R}$$

Q. Derive an expression for force between two long current carrying wires separated by small distance 'd'.



→ Consider two wire carrying current  $I_1$  and  $I_2$  in the same direction separated by distand 'd'.

→ The current  $I_1$  in the first wire produces magnetic field  $B$  on 2nd wire. That is  $B = \frac{\mu_0 I_1}{2\pi d}$

→ Now due to current  $I_2$  and magnetic field  $\vec{B}$  form 1<sup>st</sup> wire produces a force which acts on 2nd wire.

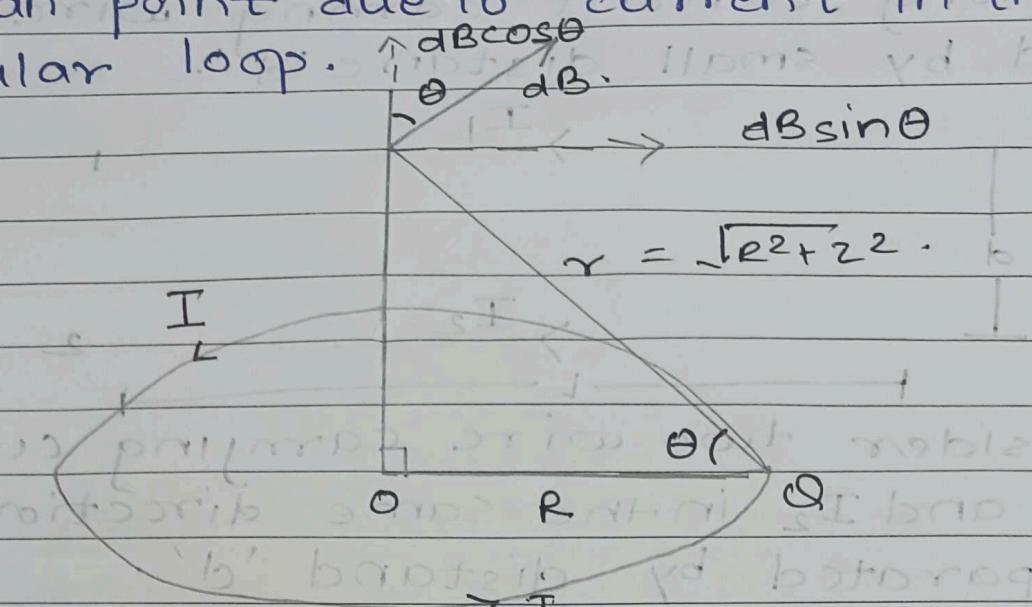
$$F = B I_2 L$$

$$(F = \frac{\mu_0 I_1 I_2 L}{2\pi d} \text{ (It is attractive force)})$$

Force per unit length of wire is.

$$\cdot (2) \quad \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Q. Derive an expression for magnetic field at a point due to current in the circular loop.



→ consider a circular loop of wire carrying current 'I'. Let P be the point at distance 'r'.

From the figure, . . .

$$\text{Given } r^2 = R^2 + z^2 \quad \text{Hypotenuse of triangle} \\ r = \sqrt{R^2 + z^2} \quad \text{Hyp}$$

$$\cos \theta = \frac{R}{r} = \frac{\text{Adj}}{\text{Hyp}}$$

$$\cos \theta = \frac{R}{\sqrt{R^2 + z^2}} \quad (1)$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dL \sin \theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dL \sin \theta}{R^2 + z^2} \quad (2)$$

Now,  $dB$  is resolved.

$dB \sin \theta$  = Horizontal

$dB \cos \theta$  = vertical.

Q. The component  $dB \sin\theta$  will cancel out due to symmetry. Therefore net magnetic field is.

$$B_{\text{net}} = \int dL B \cos\theta$$

$$B_{\text{net}} = \int \frac{\mu_0}{4\pi} \frac{I dL \sin\theta}{(R^2 + z^2)} \times \frac{R}{\sqrt{R^2 + z^2}}$$

$$\theta = 90^\circ \quad \sin 90^\circ = 1$$

$$B_{\text{net}} = \int \frac{\mu_0}{4\pi} \frac{I dL}{(R^2 + z^2)} \times \frac{R}{\sqrt{R^2 + z^2}}$$

$$\begin{aligned} B_{\text{net}} &= \frac{\mu_0}{4\pi} \frac{I}{R^2 + z^2} \times \frac{R}{\sqrt{R^2 + z^2}} \int dL \\ &= \frac{\mu_0}{4\pi} \frac{I}{(R^2 + z^2)} + \frac{R}{(\sqrt{R^2 + z^2})^{1/2}} \int dL. \\ &= \frac{\mu_0 I R}{4\pi (R^2 + z^2)^{3/2}} \int dL. \end{aligned}$$

$$\text{But } \int dL = 2\pi R.$$

$$B_{\text{net}} = \frac{\mu_0 I R}{4\pi (R^2 + z^2)^{3/2}} \times \frac{2\pi R}{2}$$

$$B_{\text{net}} = \frac{\mu_0 I R}{2(R^2 + z^2)^{3/2}}.$$