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Ch - Trigonometric Functions

★ Principle Solution

A Solution of Trigonometric equation is called principle solution only if ~~angle lies~~ $0 \leq \theta \leq 2\pi$.

e.g:- $\sqrt{3}\ \text{cosec}\ x - 2 = 0$

$$\text{cosec}\ x = \frac{2}{\sqrt{3}}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\sin x = \sin(\pi/3)$$

and $\sin \pi/3 = \sin(\pi - \pi/3)$

$= \sin \frac{2\pi}{3}$

$\therefore$ Principle Solution is

$0 \leq \pi/3 \leq 2\pi$ and $0 \leq \frac{2\pi}{3} \leq 2\pi$

★ General Solution

(1) i) If $\sin x = 0$ then $x = n\pi$

ii) If $\cos x = 0$ then $x = (2n+1)\pi/2$

iii) If $\tan x = 0$ then $x = n\pi$

(2) i) If $\sin x = \sin\alpha$ then $x = n\pi + (-1)^n \alpha$

ii) If $\cos x = \cos\alpha$ then $x = 2n\pi \pm \alpha$

iii) If $\tan x = \tan\alpha$ then $x = n\pi + \alpha$

(3) i) If $\sin^2 x = \sin^2\alpha$

ii) If $\cos^2 x = \cos^2\alpha$

iii) If $\tan^2 x = \tan^2\alpha$

$\Big\}\ x = n\pi \pm \alpha$

$\Big\}\ n \in Z$

~~if equation~~

★ If equation is in the form $a\sin x + b\cos x = c$ then divide the eq^n throughout by $\sqrt{a^2+b^2}$ and solve.

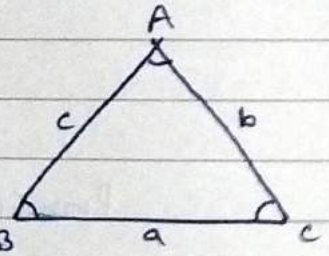
★ Solution of Triangle

1) Polar and cartesian coordinates

$$\left.\begin{aligned} y &= r\sin\theta \\ x &= r\cos\theta \end{aligned}\right\} \text{ Cartesian coordinates}$$

$$\left.\begin{aligned} r &= \sqrt{x^2+y^2} \\ \tan\theta &= \frac{y}{x} \end{aligned}\right\} \text{ Polar coordinates}$$

★ If A, B, C are the angles of triangle a, b, c are the sides opposite to there angles then:-



① Sine rule:-

i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = K$

ii) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = K$

iii) $\frac{a}{b} = \frac{\sin A}{\sin C}, \quad \frac{b}{c} = \frac{\sin B}{\sin C}$

② Cosine rule :-

i) $\cos A = \frac{b^2+c^2-a^2}{2bc}$

ii) $\cos B = \frac{a^2+c^2-b^2}{2ac}$

iii) $\cos C = \frac{a^2+b^2-c^2}{2ab}$

③ Projection rule :-

i) $a = b\cos C + c\cos B$

ii) $b = a\cos C + c\cos A$

iii) $c = a\cos B + b\cos A$

★ Half angle formulae

In $\triangle ABC$,

$a+b+c = 2s$

① i) $\sin A/2 = \sqrt{\frac{(s-b)(s-c)}{bc}}$

ii) $\cos A/2 = \sqrt{\frac{s(s-a)}{bc}}$

iii) $\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

② i) $\sin B/2 = \sqrt{\frac{(s-a)(s-c)}{ac}}$

ii) $\cos B/2 = \sqrt{\frac{s(s-b)}{ac}}$

iii) $\tan B/2 = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

③ i) $\sin C/2 = \sqrt{\frac{(s-a)(s-b)}{ab}}$

ii) $\cos C/2 = \sqrt{\frac{s(s-c)}{ab}}$

iii) $\tan C/2 = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

④ $\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

★ Inverse Trigonometric function:-

Functions	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	R	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cot^{-1} x$	R	$[0, \pi]$
$y = \sec^{-1} x$	$R-(-1,1)$	$[0, \pi] - \{\pi/2\}$
$y = \text{cosec}^{-1} x$	$R-(-1,1)$	$[-\pi/2, \pi/2] - \{0\}$

★ Property ①

i) $\sin^{-1}\left(\frac{1}{x}\right) = \text{Cosec}^{-1} x$

$\text{Cosec}^{-1}\left(\frac{1}{x}\right) = \sin^{-1} x$

2) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$

$\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1} x$

3) $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$

$\cot^{-1}\left(\frac{1}{x}\right) = \tan^{-1} x$

★ Property (2)

1) $\sin^{-1}(-x) = -\sin^{-1} x$

2) $\tan^{-1}(-x) = -\tan^{-1} x$

3) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

★ Property (3)

1) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

2) $\sec^{-1}(-x) = \pi - \sec^{-1} x$

3) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

★ Property (4)

1) $\sin^{-1} x + \cos^{-1} x = \pi/2$

2) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \pi/2$

3) $\tan^{-1} x + \cot^{-1} x = \pi/2$

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★ Property (5)

1) $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

2) $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

★ Property (6)

$$2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

★ Property (7)

1) $\sin^{-1}(\sin x) = x$

$$\text{——} \quad \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$