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Superposition of Waves

→ Wave motion is a mode of transfer of energy from one point to another point without rare migration of the particles.

→ Wave:
~~Dis~~ An oscillatory disturbance propogating in the medium is called wave.

Waves

Mechanical Wave	Electromagnetic Wave
→ A wave which requires material medium for its propogation is called mechanical wave.	→ A wave which does not require material medium for its propogation is called electromagnetic wave.

→ Progressive Wave.
A wave which is propogating in a medium without changing ~~its form~~ its called progressive waves.

→ Properties of progressive waves:

1. Each particle in a medium executes the same type of vibration. Particles vibrate about their mean positions performing simple harmonic motion.

2. All vibrating particles of the medium have the same amplitude, period and frequency.

3. The phase (i.e state of vibration of a particle), changes from one particle to another.

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4. No particle remains permanently at rest. Each particle comes to rest momentarily while at the extreme positions of vibration.

5. The particles attain maximum velocity when they pass through their mean positions.

6. During the propogation of wave, energy is transferred along the wave. There is no transfer of matter.

7. The wave propogates through the medium with a certain velocity. This velocity depends upon properties of the medium.

8. Progressive waves are of two types- transverse & longtitudinal waves.

9. In a transverse wave, vibrations of particles are perpendicular to the direction of propogation of wave, and produce crests & troughs in their medium of travel. In longitudinal waves, vibrations of particles produce compressions and rarefactions along the direction of propogation of the wave.

10. Both, the transvese as well as longitudinal, mechanical waves can propogate through solids, but only longitudinal waves can propogate through fluids.

11. For propogation of a wave, medium should have elasticity and inertia properties.

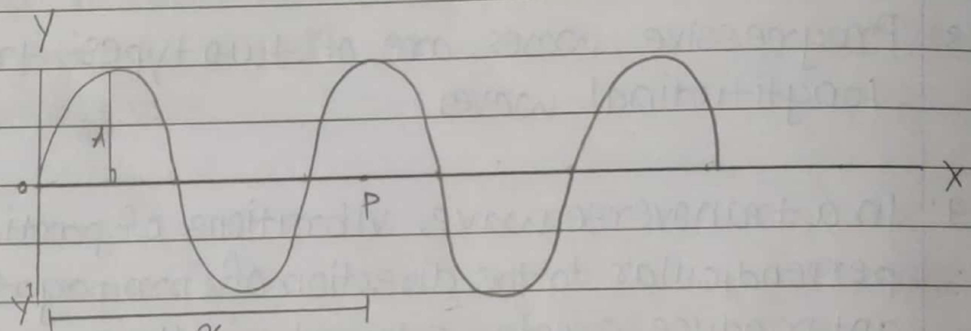
12. In simple harmonic progressive wave, there are two types of motion:

i. Particle's motion, which is oscillatory motion and its velocity is given by $\boxed{V_p = \omega\sqrt{A^2 - x^2}}$

ii. Wave motion, which is linear motion and its velocity is given by $\boxed{V_w = n\lambda}$

13. There is a constant phase difference between any two points in the medium.

⇒ Expression for equation of simple harmonic progressive wave propogating along positive direction of x-axis & find its different form.



Consider a simple harmonic progressive wave propogating along positive direction of x-axis

Let A be the amplitude, then at any time instant 't', displacement of particle 'o' is given by,

$$y = A \sin \omega t \qquad \text{— I}$$

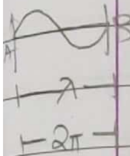
where, A is amplitude and ω is angular velocity of SHM.

Consider another particle of the medium 'P', which is at a distant of 'x' from point 'O'. Displacement of particle 'P' at same time instant 't' is given by.

$$y = A\sin(\omega t - \phi) \qquad \text{— II}$$

where, ϕ is phase difference between particles O & P.

Since, phase difference of 2π correspondance to path difference of λ, Hence, phase difference ϕ is given by,



$$\phi = \frac{2\pi x}{\lambda} \qquad \text{— III}$$

$2\pi - \lambda$

$\phi = x$

$\phi = \frac{2\pi x}{\lambda}$

Now substitute eq^n III in II.

$$\therefore y = A\sin(\omega t - 2\pi x/\lambda) \qquad \text{— IV}$$

Since, $\omega = \frac{2\pi}{T}$

$$\therefore y = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda}\right)$$

$$\boxed{y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

since, $n = \frac{1}{T}$

$$y = A\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$$

$\because v = n\lambda \Rightarrow n = \frac{v}{\lambda}$

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$$\therefore y = A \sin 2\pi \left(\frac{v}{\lambda}t - \frac{x}{\lambda}\right)$$

$$y = A \sin \frac{2\pi}{\lambda}(vt - x)$$

Since, $\frac{2\pi}{\lambda} = k$, called wave constant,

$$\therefore y = A \sin(\omega t - kx) \quad \text{— from IV}$$

If wave is propogating along −ve, direction of x-axis.

$$\therefore y = A \sin(\omega t + kx)$$

In general,

$$y = A \sin(\omega t \mp kx)$$

→ Superposition Law:

When two or more waves arrive at a point simultaneously then each wave produces its own displacement which is independent of others ~~a~~ resultant displacement is given by vector sum of all the waves.

Q What is beat? what is waxing? and what is wanning?

→ Beat is resultant of superposition of two sound waves, identical in all respects except their frequencies, propogating on the same path in the same direction.

A point at which intensity of sound energy is maximum is called as waxing.

A point at which intensity of sound energy is minimum is called as waning.

→ What is beat? find expression for its frequency. OR show that beat frequency is differency b/w frequency of two sound waves.

Consider two sound waves which are identical in all respects except their frequencies propogating on the same path in the same direction. Let A be the amplitude, n_1 & n_2 be the frequencies of two sound waves then displacement produced by these two waves on a particle of a medium are given by,

$x_1 = A \sin 2\pi n_1 t$

$x_2 = A \sin 2\pi n_2 t$

Let x be the resultant displacement then it is given by,

$x = x_1 + x_2$

$x = A \sin 2\pi n_1 t + A \sin 2\pi n_2 t$

$x = A[\sin 2\pi n_1 t + \sin 2\pi n_2 t]$

$x = 2A \sin 2\pi \left(\frac{n_1 + n_2}{2}\right) t \cos 2\pi \left(\frac{n_1 + n_2}{2}\right) t$

$x = 2A \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t \sin 2\pi \left(\frac{n_1 + n_2}{2}\right) t$

$x = R \sin 2\pi n t$.

where, $R = 2A \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t$ called resultant amplitude $n = \left(\frac{n_1 + n_2}{2}\right)$

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Waxing:
A point at which intensity of sound energy is maximum is called waxing.
for waxing:
$R = \pm 2A$

$$2A \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t = \pm 2A$$

$$\therefore \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t = \pm 1$$

$$2\pi \left(\frac{n_1 - n_2}{2}\right) t = \cos^{-1}(\pm 1)$$

$$2\pi \left(\frac{n_1 - n_2}{2}\right) t = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi \ldots$$

$$t_1 = 0$$

$$t_2 = \frac{1}{n_1 - n_2}$$

$$t_3 = \frac{2}{n_1 - n_2}$$

$$t_4 = \frac{3}{n_1 - n_2}$$

$$t_5 = \frac{4}{n_1 - n_2}$$

Time period of waxing,

$$= t_2 - t_1 = \frac{1}{n_1 - n_2} - 0 = \frac{1}{n_1 - n_2}$$

$$= t_3 - t_2 = \frac{2}{n_1 - n_2} - \frac{1}{n_1 - n_2} = \frac{1}{n_1 - n_2}$$

$$= t_4 - t_3 = \frac{3}{n_1 - n_2} - \frac{2}{n_1 - n_2} = \frac{1}{n_1 - n_2}$$

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~~Beat~~ frequency is given by,
Beat

$$n = \frac{1}{T}$$

$$n = \frac{1}{\left(\frac{1}{n_1 - n_2}\right)}$$

$$\boxed{n = n_1 - n_2}$$

~~for~~ waning:
A point at which intensity of sound energy is mininum is called waning.
for waning:

$$R = 0$$

$$2A \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t = 0$$

$$\cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t = 0$$

$$2\pi \left(\frac{n_1 - n_2}{2}\right) t = \cos^{-1}(0)$$

$$2\pi \left(\frac{n_1 - n_2}{2}\right) t = \frac{\pi}{2}, \frac{3\pi}{2}, \cancel{\frac{4\pi}{2}}, \frac{5\pi}{2}, \cancel{\frac{6\pi}{2}}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$t_1 = \frac{1}{2(n_1 - n_2)}$$

$$t_2 = \frac{3}{2(n_1 - n_2)}$$

$$t_3 = \frac{5}{2(n_1 - n_2)}$$

$$t_4 = \frac{7}{2(n_1 - n_2)}$$

$$t_5 = \frac{9}{2(n_1 - n_2)}$$

Time period of waning.

$$= t_2 - t_1 = \frac{3}{2(n_1-n_2)} - \frac{1}{2(n_1-n_2)} = \frac{1}{n_1-n_2}$$

$$= t_3 - t_2 = \frac{5}{2(n_1-n_2)} - \frac{3}{2(n_1-n_2)} = \frac{1}{n_1-n_2}$$

$$= t_4 - t_2 = \frac{7}{2(n_1-n_2)} - \frac{5}{2(n_1-n_2)} = \frac{1}{n_1-n_2}$$

$\therefore$ Beat frequency is given by,

$$n = \frac{1}{T}$$

$$= \frac{1}{\frac{1}{(n_1-n_2)}}$$

$$n = n_1 - n_2$$

Hence, beat frequency is same for waxing & waning.

Ent: $\Rightarrow$ Reflection of transverse wave from denser medium

1. wave velocity is reversed.
2. Particle velocity is reversed.
3. There is change in phase of π^c or 180°.
4. Crest - trough,
   ~~thro~~ trough - crest.

⇒ Reflection of transverse wave from rarer medium

i. Wave velocity is reversed.
ii. Particle velocity is not reversed
iii. There is no change in phase.
iv. crest - crest
trough - trough

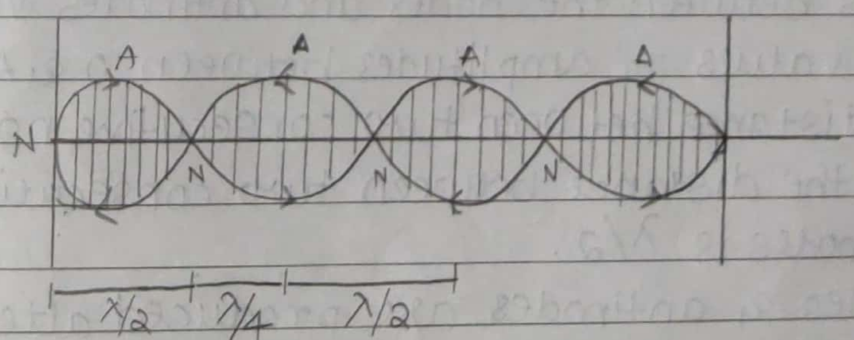
⇒ Reflection of longitudinal wave from denser medium.

1. Wave velocity is reversed.
2. Particle velocity is reversed.
3. There is chage in phase by 180° or πc.
4. comp - ~~no~~ comp
rare f - rare -f

⇒ Reflection of longitudinal wave from rarer medium.

1. wave velocity is reversed.
2. Particle velocity is not reversed.
3. There is no change is phase.
4. Comp - rare. f.
raref - comp.

What is stationary wave? Explain its properties.



→ Properties.

1. Stionary waves are produced due to superposition of two identical waves (either transverse or longitudinal waves) traveling throug a medium along the same path in oppôsite directions.

2. If two identical transverse progressive wave superimpose or interfere, the resultant wave is a transverse stationary wave.

- When a transverse stationary wave is produced on a string, some points on the string are motionless. The points which do not move are called nodes.
- There are some points on the string which oscillate with greatest amplitude (say A). They are called antinodes.
- Points between the nodes and antinodes vibrate with values of amplitudes between 0 & A.

3. If two identical longitudinal progessive waves superimose or interfere, the resultant wave is a longitudinal stationary wave.

- The points, at which the amplitude of the particles of the medium is minimum (zero), are called nodes.
- The points at which the amplitude of the particles of the medium is maximum (say A) are called antinodes.
- Points between the nodes and antinodes vibrate with values of amplitudes between 0 & A.

4. The distance between two consecutive node is $\lambda/2$, and the distance between two consecutive antinodes is $\lambda/2$.

5. Nodes & antinodes are produced alternately. The distance between a node and an adjacent antinode is $\frac{\lambda}{4}$.

6. The amplitude of vibration varies periodically in space. All points vibrate with the same frequency.
7. Though all the particles (except those at the nodes) possess energy, there is no propagation of energy. The wave is localized ~~it~~ and its velocity is zero. Therefore, we call it a stationary wave.
8. All the particles between adjacent nodes (i.e, in one loop) vibrate in phase. There is no progressive change of phase from one particle to another particle. All the particles in the ~~small~~ same loop are in the same phase phase ~~of~~ of oscillation which reverses for the adjacent loop.
9. In transverse stationary wave node is node and antinode is antinode. In longitudinal stationary wave node is pressure antinode and antinode is pressure node.

→ Explain analytical analysis of stationary wave.

Consider, two simple harmonic progressive waves which are identical in all respects, propogationg on the same path in opposite direction.
Let A, λ & T be the amplitude, wavelength & timeperiod, then displacements of two simple harmonic progressive waves at any time instant 't' are given by,

$$y_1 = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

$$y_2 = A\sin\left(\frac{2\pi t}{T} + \frac{2\pi x}{\lambda}\right)$$

Let y be the resultant displacement then it is given by,

$$y = y_1 + y_2$$

$$y = A\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) + A\sin\left(\frac{2\pi t}{T} + \frac{2\pi x}{\lambda}\right)$$

$$y = A\left[\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) + \sin\left(\frac{2\pi t}{T} + \frac{2\pi x}{\lambda}\right)\right]$$

$$y = 2A\sin\left(\frac{2\pi t}{T}\right)\cos\left(-\frac{2\pi x}{\lambda}\right)$$

$$y = 2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi t}{T}\right) \qquad - \cos(-\theta) = \cos\theta.$$

$$y = R\sin\omega t$$

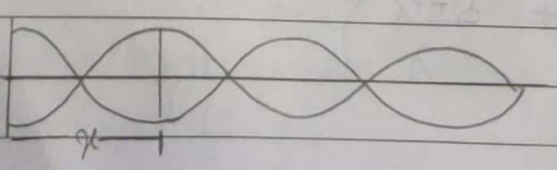
It is expression for resultant displacement where,
$R = 2A\cos\left(\frac{2\pi x}{\lambda}\right)$ called resultant amplitude.

As above expression is displacement of a particle performing SHM, hence all particles at a medium are performing SHM with the same time period as that of component wave.

As there is no 'x' term in the resultant expression hence, there is no wave motion and it is called Stationary wave.

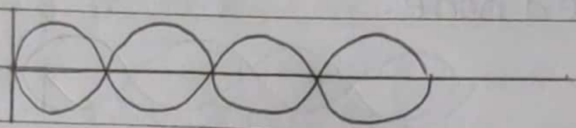
NOTE

$R = 2A\cos\left(\frac{2\pi x}{\lambda}\right)$ Distance from antinode is given or nothing is given.



$$R = 2A\sin\left(\frac{2\pi t}{T}\right)$$

distance from node is given



→ Antinode.

A point at which, amplitude of oscillation is maximum is called antinode.

for antinode.

$R = \pm 2A$

$$\therefore 2A\cos\left(\frac{2\pi x}{\lambda}\right) = \pm 2A$$

$$\cos\left(\frac{2\pi x}{\lambda}\right) = \pm 1$$

$$\therefore \frac{2\pi x}{\lambda} = \cos^{-1}(\pm 1)$$

$$\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, 3\pi, 4\pi.$$

$x_1 = 0$

$x_2 = \lambda/2$

$x_3 = \lambda$

$x_4 = 3\lambda/2$

$x_5 = 2\lambda$

$\therefore$ Distance b/w two consecutive antinode.

$= x_2 - x_1 = \lambda/2 - 0 = \lambda/2$

$= x_3 - x_2 = \lambda - \lambda/2 = \lambda/2$

$= x_4 - x_3 = 3\lambda/2 - \lambda = \lambda/2$

Node :
A point at which, amplitude of oscillation is minimum, is called node.

for node,

$R = 0$

$2A\cos\left(\frac{2\pi x}{\lambda}\right) = 0$

$\cos\left(\frac{2\pi x}{\lambda}\right) = 0$

$\frac{2\pi x}{\lambda} = \cos^{-1} 0$

$\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$

$x_1' = \frac{\lambda}{4}$

$x_2' = \frac{3\lambda}{4}$

$x_3' = \frac{5\lambda}{4}$

$x_4' = \frac{7\lambda}{4}$

$\therefore$ Distance, b/w two consecutive node.

$x_2' - x_1' = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$

$x_3' - x_2' = \frac{5\lambda}{4} - \frac{3\lambda}{4} = \frac{\lambda}{2}$

$x_4' - x_3' = \frac{7\lambda}{4} - \frac{5\lambda}{4} = \frac{\lambda}{2}$

Distance b/w two consecutive node & antinode.

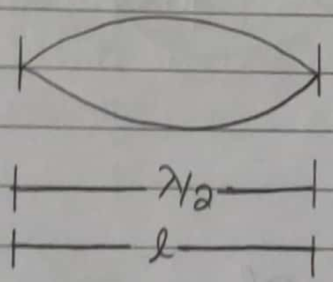
$= x_1' - x_1 = \frac{\lambda}{4} - 0 = \frac{\lambda}{4}$

$= x_2' - x_2 = \frac{3\lambda}{4} - \frac{\lambda}{2} = \frac{\lambda}{4}$

$= x_3' - x_3 = \frac{5\lambda}{4} - \lambda = \frac{\lambda}{4}$

→ Vibration in string

Boundry condition:
at fixed end node forms and at free end antinode forms



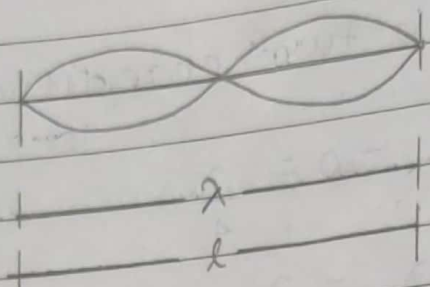
$\therefore \lambda/2 = l$

$\lambda_1 = 2l$

$n_1 = \frac{v}{\lambda_1}$

$n_1 = \frac{v}{2l}$

It is fundamental frequency and first harmonic.



$\lambda_2 = l$

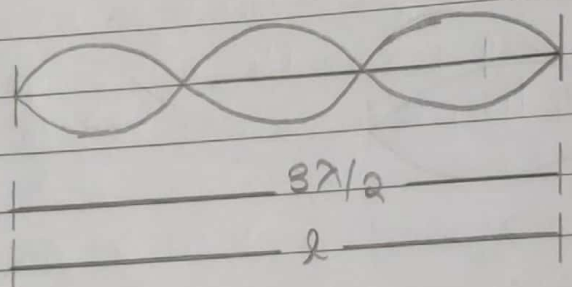
$n_2 = \frac{v}{\lambda_2}$

$n_2 = \frac{v}{l}$

$n_2 = \frac{2v}{2l}$

$n_2 = 2n_1$

It is second harmonic and first overtone.



$3\lambda/2 = l$ $\qquad \lambda_3 = \frac{2l}{3}$

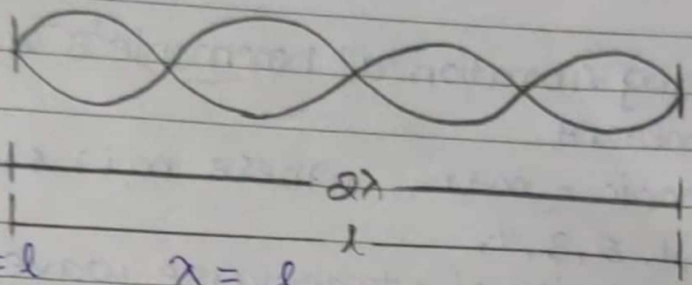
$n_3 = \frac{v}{\lambda_3}$

$n_3 = \frac{v}{\left(\frac{2l}{3}\right)}$

$n_3 = \frac{3v}{2l}$

$n_3 = 3n_1$

It is third harmonic and second overtone.



$2\lambda = l \qquad \lambda = \frac{l}{2}$

$\lambda_4 = \frac{l}{2}$

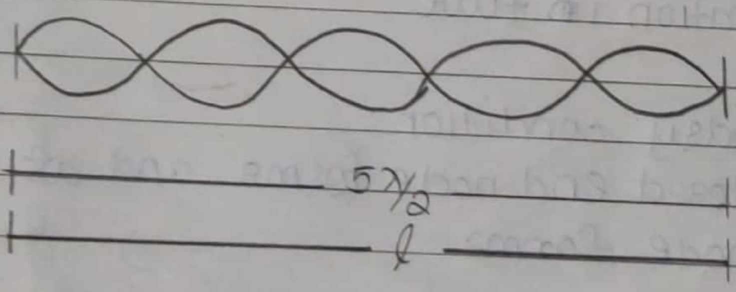
$n_4 = \frac{V}{\lambda_4}$ $\therefore$ It is fourth harmonic & third overtone.

$= \frac{V}{\left(\frac{l}{2}\right)}$

$n_4 = \frac{2V}{l}$

$n_4 = \frac{4V}{2l}$

$n_4 = 4n_1$



$5\lambda/2 = l$

$\lambda_5 = \frac{2l}{5}$ $\qquad \therefore n_5 = 5n_1$

$n_5 = \frac{V}{\lambda_5}$

$= \frac{V}{\left(\frac{2l}{5}\right)}$

$n_5 = \frac{5V}{2l}$

i. In string vibration all harmonic's & all overtones are present.

ii. Harmonic = m+1, where m is overtone and m = 0, 1, 2, 3, 4, ...

iii. Since, velocity of a transverse wave is a medium is given by $V = \sqrt{\frac{T}{m}}$,

$\therefore n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ T = tension is the medium, m = linear mass density.

iv. As per given condition, least frequency which is possible is called fundamental frequency.

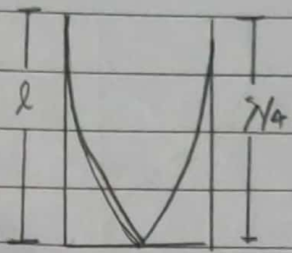
v. Harmonic's & overtones's are an integral multiple's of a fundamental frequency.

vi. Harmonic's are all integral multiples while overtones are higher integral mutiples.

→ Vibration in tube

Boundry condition:
At closed end node forms and at open end antinode forms.

Case i: Tube whose one end is open. (Resonance tube)



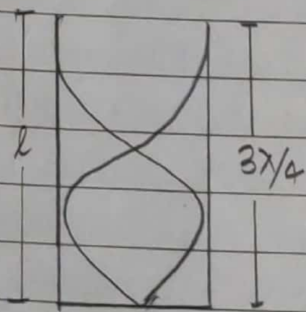
$$\frac{\lambda}{4} = l$$

$$\lambda_1 = 4l$$

$$n_1 = \frac{V}{\lambda_1}$$

$$n_1 = \frac{V}{4l}$$

It is fundamental frequency & first harmonic.



$$\therefore \frac{3\lambda}{4} = l$$

$$\therefore \lambda_2 = \frac{4l}{3}$$

$$n_2 = \frac{V}{\lambda_2}$$

$$= \frac{V}{\left(\frac{4l}{3}\right)}$$

$$n_2 = \frac{3V}{4l}$$

$$\therefore n_2 = 3n_1$$

∴ It is third harmonic & first overtone.