Project: Exotic Derivatives Pricing and Sensitivity Analysis using Monte Carlo Simulations and Finite Difference Methods

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Report Structure:

- **The report is organized into five distinct sections, as outlined below:**

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- **1.2) Numerical Procedures used in this project**
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Task Overview:

The project revolves around using Monte Carlo simulations to price two types of exotic options: Asian options and Lookback options.

The objective is to employ Monte Carlo methods, particularly with the Euler-Maruyama scheme for simulating stock prices, to compute the expected payoff of these options under risk-neutral valuation and Options Sensitivity analysis with Greeks calculated by Finite Difference Methods parameter perturbation approach.

In financial markets, exotic options offer more complex features compared to standard options, which often leads to more specific hedging needs or investment strategies by traders.

This project focuses on two types of exotic options:

1) Asian Options:

- These options have a payoff that depends on the average price of the underlying asset over a specified period. Unlike standard options, where the payoff is determined by the price of the underlying asset at a single point in time (at expiration).

- Asian options reduce the risk of price manipulation and smooth out volatility because the payoff is affected by the average price. This makes Asian options particularly useful in markets where the price of the underlying asset is susceptible to significant fluctuations or temporary spikes.

2) Lookback Options:

- These options provide a payoff based on the maximum or minimum price of the underlying asset during the life of the option. This feature allows the holder to "look back" over time to determine the payoff, which eliminates the risk of making poor timing decisions.

- A lookback option with a payoff linked to the maximum price benefits the holder by allowing them to buy (call option) or sell (put option) the asset at its most favorable price.

Section-1) Outline of Finance Problems and Numerical Methods.

1.1) Problems in valuing exotic options and role of Monte Carlo Methods

The valuation of Asian and Lookback options presents unique challenges due to their path-dependent nature and the complexity of their payoff structures. These challenges are not typically present in standard options (like European or American options), which makes the exotic options particularly difficult to handle using traditional analytical methods.

1) Problems in Valuing Asian Options:

- **i) Averaging Feature:** Asian options depend on the average price of the underlying asset over a certain period, rather than its price at maturity. This averaging can be arithmetic or geometric, complicating the payoff calculation. Traditional models, like the Black-Scholes model, which are designed for options depending on the terminal price, cannot directly be applied.
- **ii) Volatility Impact:** The averaging process dampens the impact of volatility on the option's value, making it less intuitive to assess how changes in market volatility affect the option. Traditional pricing methods often struggle to accurately account for this reduced sensitivity to volatility.
- **iii) Lack of Closed-Form Solution:** Except for some special cases, Asian options do not have closed-form solutions, necessitating numerical methods for their valuation.

2) Problems in Valuing Lookback Options:

- **i) Path Dependency:** The payoff of a Lookback option depends on the maximum or minimum price of the underlying asset over its life. This makes the option highly path-dependent, requiring knowledge of the entire price path, not just the terminal price.
- **ii) Extreme Value Sensitivity:** Lookback options are sensitive to the extreme values (maximum or minimum) that the asset price reaches during the option's life. Predicting these extreme values is challenging, especially in volatile markets.

- **iii) Computational Complexity:** The need to track the high or low watermark of asset prices throughout the option's life adds computational complexity to the valuation process.

Role of Monte Carlo Methods:

Monte Carlo simulations address these challenges effectively by leveraging their inherent flexibility and power to simulate complex, non-linear, and path-dependent financial instruments.

1. Handling Path Dependency:

- **Monte Carlo for Asian Options:** By simulating multiple possible paths for the underlying asset's price, Monte Carlo methods can directly compute the average price for each path, which is essential for determining the Asian option's payoff. This allows for an accurate valuation that fully accounts for the averaging feature across diverse market conditions.

- **Monte Carlo for Lookback Options:** For Lookback options, Monte Carlo simulations track and record the maximum or minimum price reached in each simulated price path. This makes it possible to evaluate the payoff based on the extreme values, directly addressing the path dependency issue.

2. Flexibility in Model Specification:

- **Adapting to Market Conditions:** Monte Carlo simulations can easily incorporate changes in market conditions, like shifts in volatility or interest rates, by adjusting the parameters used in the simulations. This adaptability is crucial when dealing with options where such parameters significantly impact the valuation.

3. Estimation of Expected Payoff:

- **Risk-Neutral Valuation:** Using Monte Carlo simulations under the risk-neutral measure, where it's assumed all investors are risk-neutral and expect to earn at the risk-free rate, allows for the appropriate discounting of expected payoffs. This is essential for complying with financial theory and ensuring fair valuation.

4. Addressing the Absence of Analytical Solutions:

- **Numerical Estimates:** Where closed-form solutions are lacking, Monte Carlo provides a numerical estimate of the option price by averaging the discounted payoffs across all simulated paths. This method is broadly applicable and can be adapted to virtually any type of option by modifying the payoff function and simulation parameters.

5. Computational Efficiency:

- **Improving with Technology:** While Monte Carlo simulations are computationally intensive, advances in computing power, parallel processing, and optimization algorithms have significantly reduced these challenges, making it a practical choice for real-time and complex scenario analyses.

1.2) Numerical Procedure Used in this Project:

1.2.1) Monte Carlo Methods with Euler - Maruyama and Risk Neutral Valuation:

i) Monte Carlo Simulation:

- Monte Carlo simulation is a flexible and powerful computational technique used in finance to model and analyze complex systems where analytic solutions might be unattainable. The technique is particularly useful for pricing exotic options, assessing risk, and simulating various financial scenarios.

Purpose:

- The primary purpose of using Monte Carlo simulation in option pricing is to estimate the expected payoff of options where closed-form solutions are not feasible due to the path-dependent nature of the option or complex payoffs.
- It allows practitioners to model the uncertainty inherent in financial markets by simulating thousands or even millions of possible future outcomes for the underlying asset's price, under the assumptions of risk neutrality.

Framework:

Monte Carlo Simulation Steps for Option Pricing:

1. **Simulating Multiple Stock Price Paths:** Use the Euler-Maruyama scheme to approximate the evolution of stock prices over time.

2. **Computing Values from Each Path:** Depending on the type of option, compute the average or extreme values (maximum/minimum) from each simulated path.

3. **Calculating the Option's Payoff:** Use the appropriate formulas to calculate the payoff for each path.

4. **Averaging Payoffs:** Average these individual payoffs across all paths to estimate the expected payoff.

5. **Discounting to Present Value:** Discount the expected payoff to its present

value using the risk-free rate. The formula for the discount factor is:

$$e^{-r(T-t)}$$

where (T) is the maturity of the option and (t) is the current time.

Detailed Monte Carlo Framework:

1) Stochastic Process Modeling:

The underlying asset prices are typically modeled using a stochastic differential equation (SDE) like the Geometric Brownian Motion (GBM). GBM is used to represent the continuous evolution of stock prices in the market, incorporating drift (mean return) and volatility (standard deviation) along with a stochastic term:

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t$$

In risk-neutral pricing, μ (the expected return) is replaced by r (the risk-free rate), modifying the equation to:

$$dS_t = rS_t\, dt + \sigma S_t\, dW_t$$

- where S_t is the stock price, r is the risk-free rate, σ is the volatility, and dW_t represents the increments of a Wiener process (Brownian motion).

2) Euler-Maruyama Discretization:

To simulate paths of the stock price under the GBM model using Monte Carlo methods, the continuous-time SDE must be discretized into a form suitable for numerical computation. The Euler-Maruyama method provides a straightforward means to achieve this:

$$S_{t+\delta t} = S_t + rS_t\delta t + \sigma S_t\sqrt{\delta t}Z_t$$

Where:

- δt is a small time increment,
- Z_t is a random draw from a standard normal distribution (N(0,1)).

Transformation to an Exponential Form:

For numerical stability and to ensure that stock prices do not become negative (a necessary condition in financial modeling), the GBM model is often rewritten in an exponential form:

$$S_{t+\delta t} = S_t \exp\left(\left(r - \frac{1}{2}\sigma^2\right)\delta t + \sigma\sqrt{\delta t}Z_t\right)$$

Here:

- $\left(r - \frac{1}{2}\sigma^2\right)\delta t$ adjusts the drift term to account for the continuous compounding of returns,
- $\sigma\sqrt{\delta t}Z_t$ represents the random fluctuations in the stock price, scaled by the volatility and the square root of the time increment.

The exponential form ensures that the simulated stock prices are always positive and closely follow the log-normal distribution implied by the GBM model.

3) Formula for Option Valuation:

Risk-Neutral Valuation:

The price of the option is calculated using the formula:

$$V(S,t) = e^{-r(T-t)}\mathbb{E}^Q[\text{Payoff}(S_T)]$$

- This formula states that the value of the option at any time t before expiry is the present value of the expected payoff under the risk-neutral measure Q, discounted at the risk-free rate r. This approach assumes that the expected return of the underlying asset is the risk-free rate.

4) Payoff Calculations:

- **Asian Option Payoff:**
  - Payoff for an Asian Call Option: The payoff is the excess of the average price of the underlying asset over the strike price, if positive. Mathematically, it's expressed as:

$$\text{Payoff} = \max(0, \bar{S} - K)$$

- Payoff for an Asian Call Option: The payoff is the excess of the average price of the underlying asset over the strike price, if positive. Mathematically, it's expressed as:

$$\text{Payoff} = \max(0, K - \bar{S})$$

- where $\bar{S}$ is the average of simulated asset prices.

- **Lookback Option Payoff:**

  - Payoff for a Lookback Call Option: This option gives the holder the right to buy the stock at its minimum price during the life of the option. Thus, the payoff is:

$$\text{Payoff} = \max(0, S_{\max} - K)$$

- Payoff for a Lookback Put Option: This option gives the holder the right to sell the stock at its maximum price during the life of the option. The payoff is:

$$\text{Payoff} = \max(0, K - S_{\min})$$

- where $S_{\max}$ and $S_{\min}$ are the maximum and minimum of the simulated asset prices, respectively.

5) Averaging and Discounting:

- The final step involves averaging the payoffs across all simulated paths to estimate the expected payoff under the risk-neutral probability measure. This average is then discounted back to the present value using the exponential discount factor:

$$e^{-r(T-t)}$$

- where (T) is the time to maturity and (t) is the current time.

1.2.2) Finite Difference Method with Parameter Perturbation for Option Sensitivity (Greeks):

- Finite differences are employed to approximate how changes in the underlying parameters (like the stock price, volatility, etc.) affect the option price. This involves slightly modifying the parameters and recalculating the option prices to observe the changes.

Finite Difference Method in Option Pricing Without Explicit Grid:

- In the scenario of option pricing, particularly when paired with Monte Carlo simulation, FDM does not use a spatial grid across the domain of the underlying asset or time as seen in typical PDE problems. Instead, it employs a perturbation approach where the grid concept is abstracted to only a few specific points of interest—namely the parameters we wish to perturb (like stock price, volatility, or time).

Simulating Paths:

- We simulate thousands of potential future paths for the underlying asset's price using the Monte Carlo method. This method uses the assumption of geometric Brownian motion, as typically modeled in financial markets, and incorporates random variations through a normal distribution, mimicking real-world uncertainty.

Parameter Perturbation with FDM:

- After simulating the stock price paths, we perturb the parameters of interest slightly up and down. For each perturbed parameter, new Monte Carlo simulations are run to calculate the resulting option prices.
- For Delta, we change the initial stock price slightly and observe the difference in option pricing. For Vega, we adjust the volatility and note the changes in pricing.
- Similarly, for Theta, we alter the time to expiration. This approach allows us to measure the sensitivity of the option price to each parameter, providing insights into how each factor influences the option's value in real market conditions.

Greeks calculations:

1. Delta (Δ):

- **Definition:** Measures the sensitivity of the option price to changes in the price of the underlying asset. It is the first derivative of the option price with respect to the stock price.

- **Formula:**

$$\Delta \approx \frac{V(S+\Delta S) - V(S-\Delta S)}{2\Delta S}$$

- **Calculation:** We compute the option price at slightly higher $(S+\Delta S)$ and lower $(S-\Delta S)$ stock prices. The change in the option price divided by the change in the stock price gives the Delta.

2. Gamma (Γ):

- **Definition:** Measures the rate of change of Delta with respect to changes in the underlying asset's price. It is the second derivative of the option price with respect to the stock price.

- **Formula:**

$$\Gamma \approx \frac{V(S+\Delta S) - 2V(S) + V(S-\Delta S)}{(\Delta S)^2}$$

- **Calculation:** Gamma uses the same prices as Delta but focuses on the curvature of the option price's response to changes in the stock price.

3. Vega (ν):

- **Definition:** Measures the sensitivity of the option price to changes in the volatility of the underlying asset.

- **Formula:**

$$\nu \approx \frac{V(\sigma + \Delta\sigma) - V(\sigma)}{\Delta\sigma}$$

- **Calculation:** We calculate the option price for slightly increased volatility $(\sigma + \Delta\sigma)$ and use the original price to find the rate of change in price with respect to volatility.

4. Theta (Θ):

- **Definition:** Measures the sensitivity of the option price to the passage of time, essentially the "time decay" of the option.
- **Formula:**

$$\Theta \approx \frac{V(T + \Delta T) - V(T)}{\Delta T}$$

- **Calculation:** This is calculated by incrementing the time to expiration and observing how the option price decreases, given that all other factors remain constant.

Section-2) Results - Tables (Contains Code - Outputs) (Detailed Comparison and Explanation in Step-3))

2.1) Option pricing for Asian Options:

2.1.1) Asian Option pricing with Initial Parameter Sample and Distribution of Payoffs

- **For Initial Sample: Today's stock price** So **:** 100, **Strike E:** 100, **Time to Expiry:** 1, **volatility:** 0.20, **risk-free rate:** 0.05.

In [164…

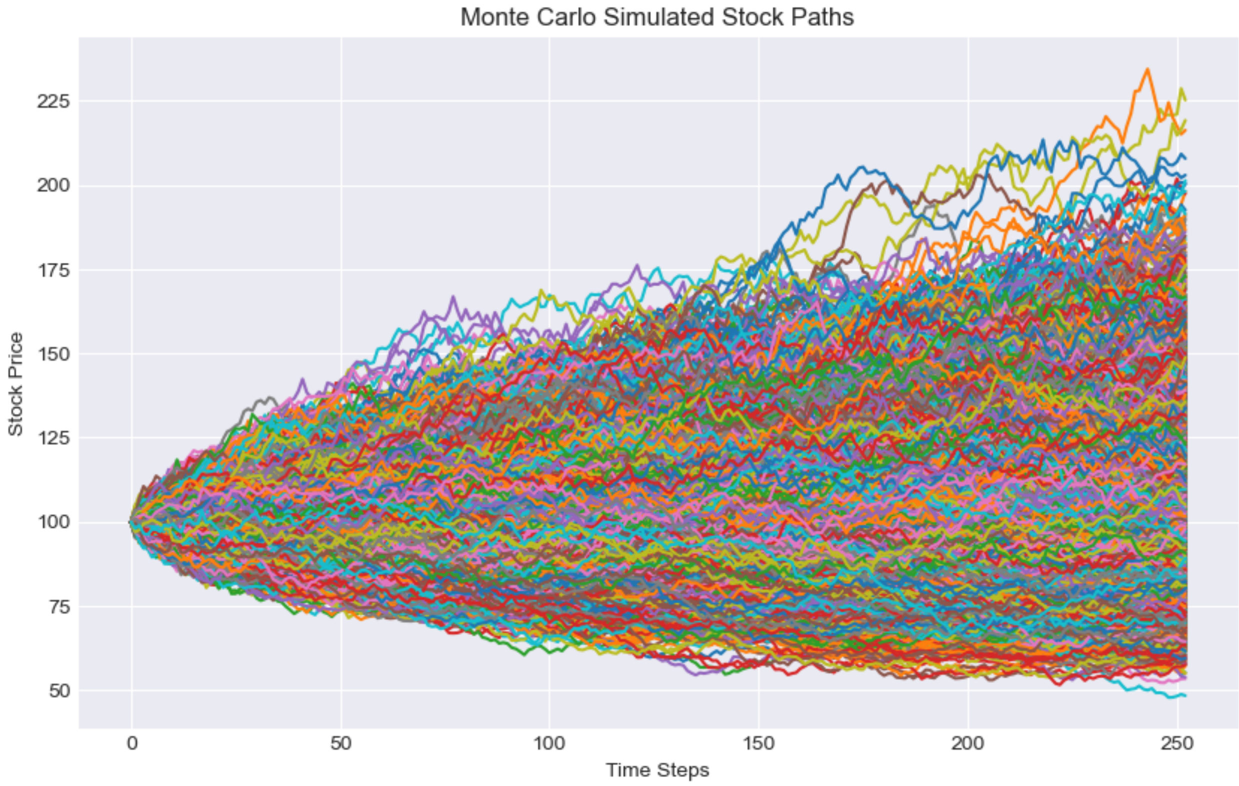
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from IPython.display import display
```

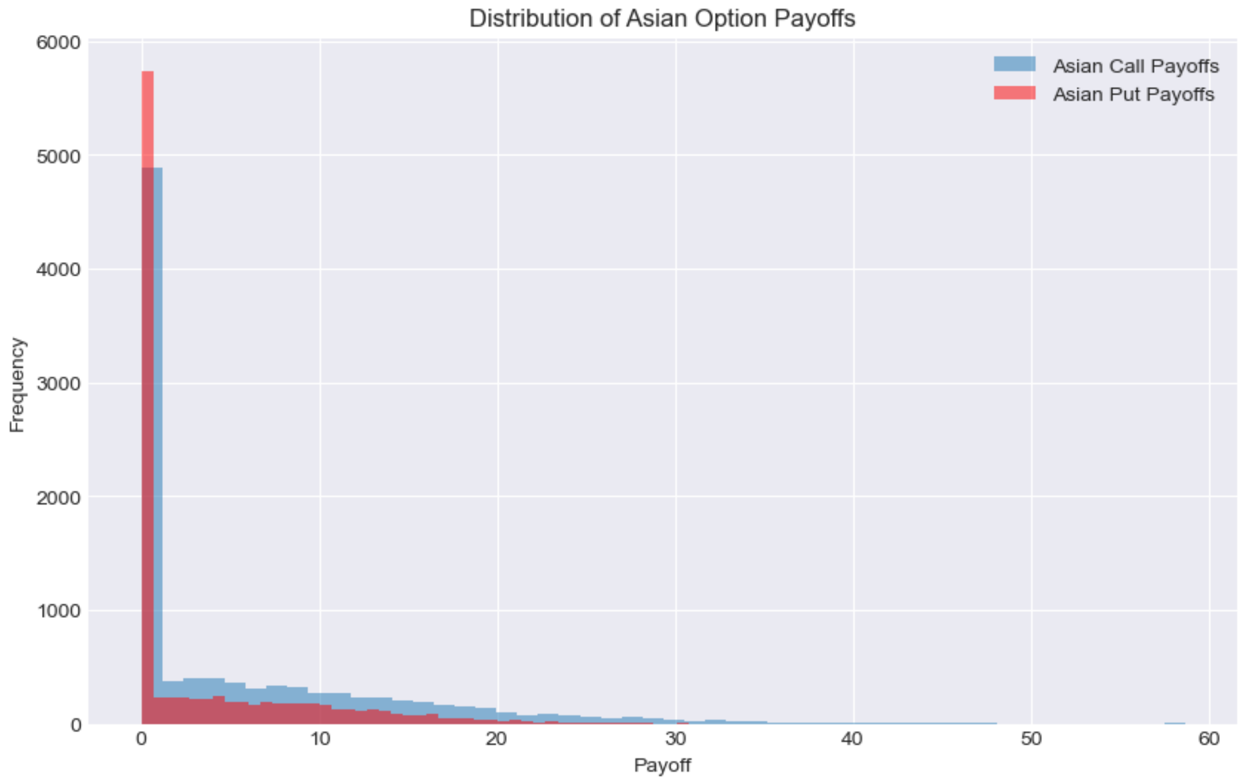
In [165…

```
# Function to simulate stock prices using the Euler-Maruyama method
def monte_carlo_simulated_stock_prices(S0, r, sigma, T, dt, nsim):
    nsteps = int(T / dt)
    S = np.zeros((nsteps + 1, nsim))
    S[0] = S0
    for t in range(1, nsteps + 1):
        Z = np.random.standard_normal(nsim)
        S[t] = S[t-1] * np.exp((r - 0.5 * sigma**2) * dt + sigma * np.sqr
```

```
    return S

# Function to calculate Asian option prices
def asian_option_prices(S, K, r, T):
    avg_prices = np.mean(S, axis=0)
    asian_call_payoffs = np.maximum(avg_prices - K, 0)
    asian_put_payoffs = np.maximum(K - avg_prices, 0)
    asian_call_price = np.exp(-r * T) * np.mean(asian_call_payoffs)
    asian_put_price = np.exp(-r * T) * np.mean(asian_put_payoffs)
    return asian_call_price, asian_put_price, asian_call_payoffs, asian_p

# Parameters
S0 = 100
K = 100
T = 1
r = 0.05
sigma = 0.20
nsim = 10000
dt = T / 252

# Monte Carlo Simulation of stock prices
prices = monte_carlo_simulated_stock_prices(S0, r, sigma, T, dt, nsim)

# Calculate option prices
asian_call_price, asian_put_price, asian_call_payoffs, asian_put_payoffs

# Print the prices
print(f"For Initial Sample Parameters:")
print(f"Asian Call Option Price: {asian_call_price:.4f}")
print(f"Asian Put Option Price: {asian_put_price:.4f}")

# Visualize the simulated stock paths
plt.figure(figsize=(10, 6))
plt.plot(prices)
plt.title('Monte Carlo Simulated Stock Paths')
plt.xlabel('Time Steps')
plt.ylabel('Stock Price')
plt.show()

# Visualization of the Payoff Distribution
plt.figure(figsize=(10, 6))
plt.hist(asian_call_payoffs, bins=50, alpha=0.5, label='Asian Call Payoff
plt.hist(asian_put_payoffs, bins=50, alpha=0.5, label='Asian Put Payoffs'
plt.title('Distribution of Asian Option Payoffs')
plt.xlabel('Payoff')
plt.ylabel('Frequency')
plt.legend()
plt.show()
```

```
For Initial Sample Parameters:
Asian Call Option Price: 5.7176
Asian Put Option Price: 3.4057
```





2.1.2) Varying the Parameters to generate table for Asian Option prices

- **Parameter Variations** :
    - **sigmas = [0.15, 0.20, 0.25]**
    - **interest_rates = [0.03, 0.05, 0.07]**

- **strike_prices = [90, 100, 110]**
- **times_to_expiration = [0.5, 1, 1.5]**

In [166…

```
def asian_option_prices_and_greeks(S, K, r, sigma, T, dt):
    avg_prices = np.mean(S, axis=0)
    call_payoffs = np.maximum(avg_prices - K, 0)
    put_payoffs = np.maximum(K - avg_prices, 0)

    call_price = np.exp(-r * T) * np.mean(call_payoffs)
    put_price = np.exp(-r * T) * np.mean(put_payoffs)

    # Sensitivity calculations using finite differences
    dS = 0.01 * S0
    prices_up = S * (1 + dS)
    prices_down = S * (1 - dS)

    avg_prices_up = np.mean(prices_up, axis=0)
    avg_prices_down = np.mean(prices_down, axis=0)

    # Delta
    call_price_up = np.exp(-r * T) * np.mean(np.maximum(avg_prices_up - K
    call_price_down = np.exp(-r * T) * np.mean(np.maximum(avg_prices_down
    put_price_up = np.exp(-r * T) * np.mean(np.maximum(K - avg_prices_up,
    put_price_down = np.exp(-r * T) * np.mean(np.maximum(K - avg_prices_d

    call_delta = (call_price_up - call_price_down) / (2 * dS * S0)
    put_delta = (put_price_up - put_price_down) / (2 * dS * S0)

    # Gamma
    call_gamma = (call_price_up - 2 * call_price + call_price_down) / (dS
    put_gamma = (put_price_up - 2 * put_price + put_price_down) / (dS * S

    # Vega
    sigma_up = sigma + 0.01
    S_sigma_up = monte_carlo_simulated_stock_prices(S0, r, sigma_up, T, d
    avg_prices_sigma_up = np.mean(S_sigma_up, axis=0)
    call_vega_price = np.exp(-r * T) * np.mean(np.maximum(avg_prices_sigm
    put_vega_price = np.exp(-r * T) * np.mean(np.maximum(K - avg_prices_s

    call_vega = (call_vega_price - call_price) / 0.01
    put_vega = (put_vega_price - put_price) / 0.01

    # Theta
    T_up = T + 1/252
    S_T_up = monte_carlo_simulated_stock_prices(S0, r, sigma, T_up, dt, n
    avg_prices_T_up = np.mean(S_T_up, axis=0)
    call_theta_price = np.exp(-r * T_up) * np.mean(np.maximum(avg_prices_
    put_theta_price = np.exp(-r * T_up) * np.mean(np.maximum(K - avg_pric

    call_theta = (call_theta_price - call_price) / (-1/252)
    put_theta = (put_theta_price - put_price) / (-1/252)

    return {
```

```
        'call_price': call_price,
        'put_price': put_price,
        'call_delta': call_delta,
        'put_delta': put_delta,
        'call_gamma': call_gamma,
        'put_gamma': put_gamma,
        'call_vega': call_vega,
        'put_vega': put_vega,
        'call_theta': call_theta,
        'put_theta': put_theta
    }

# Parameters
S0 = 100
T = 1
nsim = 10000
dt = 1 / 252

# Parameter variations
sigmas = [0.15, 0.20, 0.25]
interest_rates = [0.03, 0.05, 0.07]
strike_prices = [90, 100, 110]
times_to_expiration = [0.5, 1, 1.5]

# Results dataframe
results = []

# Vary each parameter and compute option prices and Greeks
for sigma in sigmas:
    for rate in interest_rates:
        for strike in strike_prices:
            for time in times_to_expiration:
                prices = monte_carlo_simulated_stock_prices(S0, rate, sig
                result = asian_option_prices_and_greeks(prices, strike, r
                results.append({
                    'Sigma': sigma,
                    'Rate': rate,
                    'Strike': strike,
                    'Time to Expiration': time,
                    'Call Price': result['call_price'],
                    'Put Price': result['put_price'],
                    'Call Delta': result['call_delta'],
                    'Put Delta': result['put_delta'],
                    'Call Gamma': result['call_gamma'],
                    'Put Gamma': result['put_gamma'],
                    'Call Vega': result['call_vega'],
                    'Put Vega': result['put_vega'],
                    'Call Theta': result['call_theta'],
                    'Put Theta': result['put_theta']
                })

# Convert results to DataFrame
df_asian_options = pd.DataFrame(results)
print("Asian Call - Put Option Price and Greeks Table:")
```

```
df_asian_options
```

```
Asian Call - Put Option Price and Greeks Table:
```

Out[166…

	Sigma	Rate	Strike	Time to Expiration	Call Price	Put Price	Call Delta	Put Delta	
0	0.15	0.03	90	0.5	10.666483	0.063497	0.549330	-0.443300	0.
1	0.15	0.03	90	1.0	11.516082	0.284562	0.549016	-0.436700	0
2	0.15	0.03	90	1.5	12.022739	0.554349	0.544883	-0.430199	0.
3	0.15	0.03	100	0.5	2.738964	2.057210	0.499374	-0.492556	0.
4	0.15	0.03	100	1.0	4.101629	2.712016	0.499119	-0.485223	0
...	...	...	...	...	...	...	...	...	
76	0.25	0.07	100	1.0	7.315580	3.863468	0.500718	-0.466197	0
77	0.25	0.07	100	1.5	9.363304	4.350483	0.500290	-0.450162	0
78	0.25	0.07	110	0.5	1.428315	9.312899	0.452237	-0.531083	0
79	0.25	0.07	110	1.0	3.292877	9.230701	0.453438	-0.512817	0
80	0.25	0.07	110	1.5	5.081170	9.256273	0.453432	-0.495174	0.

81 rows × 14 columns

Table Explanation for Asian Options:

Option Price Sensitivity to Parameters:

- **Impact of Volatility (Sigma):**
  - **Call and Put Prices:** Increasing volatility generally leads to higher option prices for both calls and puts. This is because higher volatility increases the probability of the option ending in the money.
  - **Example:** Comparing Sigma=0.15 to Sigma=0.25 shows an increase in call prices across similar strike prices and time to expiration.
- **Impact of Strike Price (Strike):**
  - **Call Prices:** Higher strike prices generally lead to lower call prices since the option is less likely to be in the money.
  - **Put Prices:** Conversely, higher strike prices result in higher put prices.
  - **Example:** For a fixed Sigma=0.15 and Rate=0.03, increasing the strike price from 90 to 110 decreases the call prices and increases the put prices.
- **Impact of Time to Expiration (Time to Expiration):**
  - **Both Calls and Puts:** Options with longer times to expiration are generally more expensive due to the greater uncertainty over a longer period.
  - **Example:** Extending the time to expiration from 0.5 to 1.5 years increases

both call and put prices across all strikes and volatilities.

- **Impact of Risk-Free Rate (Rate):**
  - **Rate Increases:** Generally leads to an increase in call prices and a decrease in put prices, as the present value of the strike price payment (at expiration) is discounted more.
  - **Example:** Increasing the rate from 0.03 to 0.07 tends to increase call prices and decrease put prices.

Sensitivity Analysis with Greeks:

- **Delta (Call Delta and Put Delta):**
  - Reflects the sensitivity of option prices to changes in the underlying asset's price.
  - **Observation:** Call delta remains relatively stable across different times to expiration and strike prices but decreases slightly as the strike price increases.
- **Gamma (Call Gamma and Put Gamma):**
  - Indicates the rate of change of delta. A higher gamma suggests greater sensitivity of delta to movements in the underlying asset's price.
  - **Observation:** Gamma tends to decrease slightly with higher strike prices and longer expiration times, indicating less sensitivity as the option moves further in- or out-of-the-money.
- **Vega (Call Vega and Put Vega):**
  - Measures sensitivity to volatility. An increase in vega indicates a higher impact of volatility changes on the option price.
  - **Observation:** Vega generally increases with an increase in time to expiration, reflecting higher sensitivity to volatility as the option's maturity increases.
- **Theta (Call Theta and Put Theta):**
  - Theta represents the sensitivity of the option price to the passage of time, typically indicating the loss in value as the option approaches expiration.
  - **Observation:** Theta is generally negative, indicating that the value of options decreases as expiration approaches. However, its absolute value can vary significantly based on other parameters.

2.2) Option pricing for Lookback options

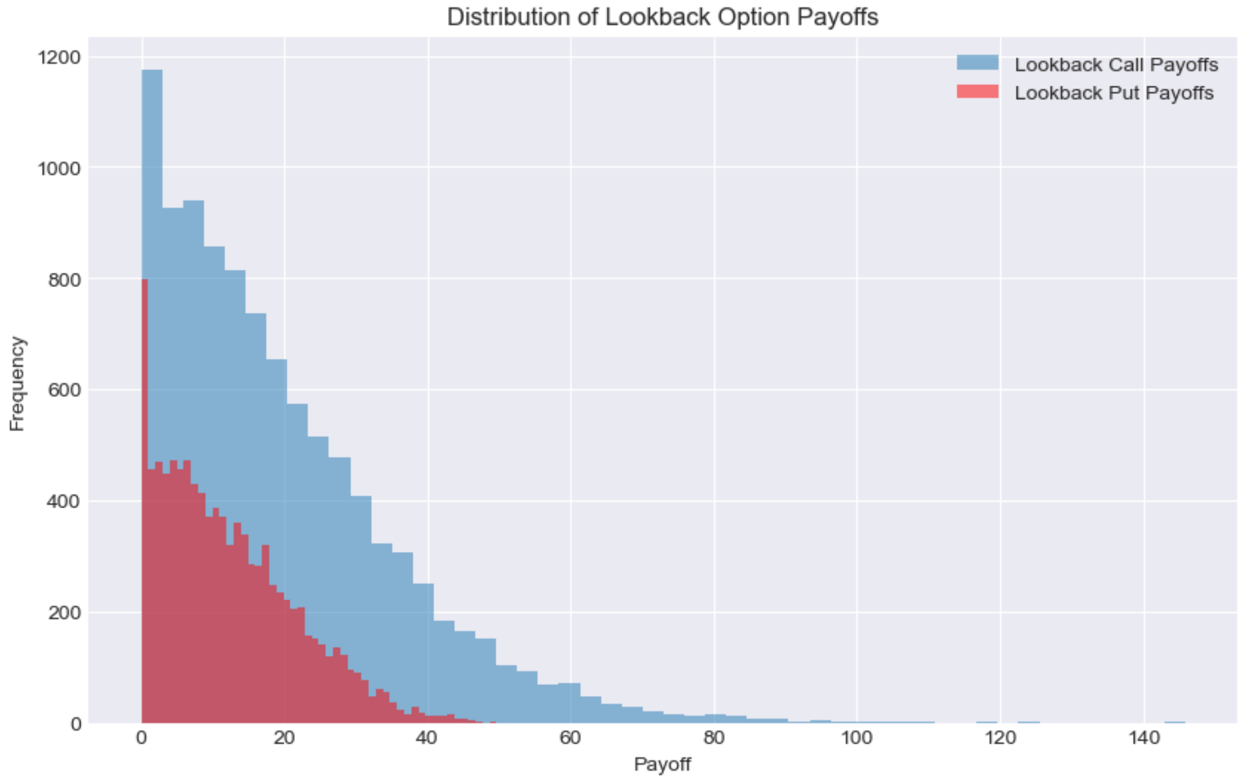
2.2.1) Lookback Option pricing with Initial Parameter Sample and Distribution of Payoffs

- **For Initial Sample: Today's stock price So :** 100, **Strike E:** 100, **Time to Expiry:** 1, **volatility:** 0.20, **risk-free rate:** 0.05.

In [109…

```
# Function to calculate Lookback option prices and payoffs
def lookback_option_prices(S, K, r, T):
    max_prices = np.max(S, axis=0)
    min_prices = np.min(S, axis=0)

    lookback_call_payoffs = np.maximum(max_prices - K, 0)
    lookback_put_payoffs = np.maximum(K - min_prices, 0)

    lookback_call_price = np.exp(-r * T) * np.mean(lookback_call_payoffs)
    lookback_put_price = np.exp(-r * T) * np.mean(lookback_put_payoffs)

    return lookback_call_price, lookback_put_price, lookback_call_payoffs

# Initial sample parameters
S0 = 100
K = 100
T = 1
r = 0.05
sigma = 0.20
nsim = 10000
dt = T / 252

# Simulate stock prices
prices = monte_carlo_simulated_stock_prices(S0, r, sigma, T, dt, nsim)

# Calculate Lookback option prices
lookback_call_price, lookback_put_price, lookback_call_payoffs, lookback_

# Print the Lookback option prices
print(f"For Initial Sample Parameters:")
print(f"Lookback Call Option Price: {lookback_call_price:.4f}")
print(f"Lookback Put Option Price: {lookback_put_price:.4f}")

# Visualization of the Payoff Distribution
plt.figure(figsize=(10, 6))
plt.hist(lookback_call_payoffs, bins=50, alpha=0.5, label='Lookback Call
plt.hist(lookback_put_payoffs, bins=50, alpha=0.5, label='Lookback Put Pa
plt.title('Distribution of Lookback Option Payoffs')
plt.xlabel('Payoff')
plt.ylabel('Frequency')
plt.legend()
plt.show()
```

```
For Initial Sample Parameters:
Lookback Call Option Price: 18.5429
Lookback Put Option Price: 11.6466
```



2.2.2) Varying the Parameters to generate table for Lookback Option prices:

- **Parameter Variations** :
  - **sigmas = [0.15, 0.20, 0.25]**
  - **interest_rates = [0.03, 0.05, 0.07]**
  - **strike_prices = [90, 100, 110]**
  - **times_to_expiration = [0.5, 1, 1.5]**

In [57]:
```
def lookback_option_prices_and_greeks(S, K, r, sigma, T, dt):
    max_prices = np.max(S, axis=0)
    min_prices = np.min(S, axis=0)

    lookback_call_payoffs = np.maximum(max_prices - K, 0)
    lookback_put_payoffs = np.maximum(K - min_prices, 0)

    lookback_call_price = np.exp(-r * T) * np.mean(lookback_call_payoffs)
    lookback_put_price = np.exp(-r * T) * np.mean(lookback_put_payoffs)

    # Delta and Gamma using finite differences
    dS = 0.01 * S0
    prices_up = S * (1 + dS)
    prices_down = S * (1 - dS)

    max_prices_up = np.max(prices_up, axis=0)
    max_prices_down = np.max(prices_down, axis=0)
    min_prices_up = np.min(prices_up, axis=0)
    min_prices_down = np.min(prices_down, axis=0)
```

```
    call_price_up = np.exp(-r * T) * np.mean(np.maximum(max_prices_up - K
    call_price_down = np.exp(-r * T) * np.mean(np.maximum(max_prices_down
    put_price_up = np.exp(-r * T) * np.mean(np.maximum(K - min_prices_up,
    put_price_down = np.exp(-r * T) * np.mean(np.maximum(K - min_prices_d

    call_delta = (call_price_up - call_price_down) / (2 * dS * S0)
    put_delta = (put_price_up - put_price_down) / (2 * dS * S0)

    call_gamma = (call_price_up - 2 * lookback_call_price + call_price_do
    put_gamma = (put_price_up - 2 * lookback_put_price + put_price_down)

    # Vega and Theta
    sigma_up = sigma + 0.01
    S_sigma_up = monte_carlo_simulated_stock_prices(S0, r, sigma_up, T, d
    max_prices_sigma_up = np.max(S_sigma_up, axis=0)
    min_prices_sigma_up = np.min(S_sigma_up, axis=0)

    call_vega_price = np.exp(-r * T) * np.mean(np.maximum(max_prices_sigm
    put_vega_price = np.exp(-r * T) * np.mean(np.maximum(K - min_prices_s

    call_vega = (call_vega_price - lookback_call_price) / 0.01
    put_vega = (put_vega_price - lookback_put_price) / 0.01

    T_up = T + 1/252
    S_T_up = monte_carlo_simulated_stock_prices(S0, r, sigma, T_up, dt, n
    max_prices_T_up = np.max(S_T_up, axis=0)
    min_prices_T_up = np.min(S_T_up, axis=0)

    call_theta_price = np.exp(-r * T_up) * np.mean(np.maximum(max_prices_
    put_theta_price = np.exp(-r * T_up) * np.mean(np.maximum(K - min_pric

    call_theta = (call_theta_price - lookback_call_price) / (-1/252)
    put_theta = (put_theta_price - lookback_put_price) / (-1/252)

    return {
        'call_price': lookback_call_price,
        'put_price': lookback_put_price,
        'call_delta': call_delta,
        'put_delta': put_delta,
        'call_gamma': call_gamma,
        'put_gamma': put_gamma,
        'call_vega': call_vega,
        'put_vega': put_vega,
        'call_theta': call_theta,
        'put_theta': put_theta
    }

# Parameters
S0 = 100
T = 1
nsim = 10000
dt = 1 / 252
```

```
# Parameter variations
sigmas = [0.15, 0.20, 0.25]
interest_rates = [0.03, 0.05, 0.07]
strike_prices = [90, 100, 110]
times_to_expiration = [0.5, 1, 1.5]

# Results dataframe
results = []

# Vary each parameter and compute option prices and Greeks
for sigma in sigmas:
    for rate in interest_rates:
        for strike in strike_prices:
            for time in times_to_expiration:
                prices = monte_carlo_simulated_stock_prices(S0, rate, sig
                result = lookback_option_prices_and_greeks(prices, strike
                results.append({
                    'Sigma': sigma,
                    'Rate': rate,
                    'Strike': strike,
                    'Time to Expiration': time,
                    'Call Price': result['call_price'],
                    'Put Price': result['put_price'],
                    'Call Delta': result['call_delta'],
                    'Put Delta': result['put_delta'],
                    'Call Gamma': result['call_gamma'],
                    'Put Gamma': result['put_gamma'],
                    'Call Vega': result['call_vega'],
                    'Put Vega': result['put_vega'],
                    'Call Theta': result['call_theta'],
                    'Put Theta': result['put_theta']
                })

# Convert results to DataFrame
df_lookback_options = pd.DataFrame(results)
print("Lookback Call - Put Option Price and Greeks Table:")
df_lookback_options
```

```
Lookback Call - Put Option Price and Greeks Table:
```

Out[57]:

	Sigma	Rate	Strike	Time to Expiration	Call Price	Put Price	Call Delta	Put Delta
0	0.15	0.03	90	0.5	18.809247	1.225197	0.631393	-0.443300
1	0.15	0.03	90	1.0	22.859492	2.831900	0.665295	-0.436700
2	0.15	0.03	90	1.5	26.425259	4.112463	0.694451	-0.430199
3	0.15	0.03	100	0.5	8.883546	6.978448	0.581391	-0.492556
4	0.15	0.03	100	1.0	13.102848	9.546772	0.616251	-0.485223
...	...	...	...	...	...	...	...	...
76	0.25	0.07	100	1.0	23.498923	13.853505	0.701186	-0.466112
77	0.25	0.07	100	1.5	29.935436	15.902367	0.749517	-0.449692
78	0.25	0.07	110	0.5	8.085411	20.415261	0.589467	-0.531073
79	0.25	0.07	110	1.0	15.517643	23.203519	0.654066	-0.512457
80	0.25	0.07	110	1.5	22.388620	24.707304	0.708289	-0.493491

81 rows × 14 columns

Table Explanation for Lookback Options:

Option Sensitivity to Parameters:

- **Impact of Volatility (Sigma):**
  - **Higher Volatility:** Generally increases the prices of both Lookback call and put options. This is due to the increased likelihood that the underlying will reach new extremes, which directly benefits the payoff structure of Lookback options.
  - **Example:** At Sigma=0.15 and Rate=0.03 for a Strike of 90 and Time to Expiration of 0.5, the Call Price is 18.67; increasing Sigma to 0.25 raises the Call Price to 24.63.
- **Impact of Strike Price (Strike):**
  - **Higher Strike Prices:**
    - **Call Options:** Higher strikes generally lead to lower prices for Lookback calls because the maximum value of the underlying might not exceed the higher strike as easily.
    - **Put Options:** In contrast, Lookback puts become more valuable as the strike price increases since the minimum value of the underlying asset is more likely to be below the strike, enhancing the payoff.
  - **Example:** At Sigma=0.15 and Rate=0.03, increasing the Strike from 90 to 110

decreases the Call Price from 18.67 to 2.59 for a Time to Expiration of 0.5.

- **Impact of Time to Expiration (Time to Expiration):**
  - **Longer Expirations:** Lead to higher prices for both calls and puts due to the increased chance of the underlying asset reaching more extreme values over a longer period.
  - **Example:** For a Strike of 90 and Sigma=0.15, extending Time to Expiration from 0.5 to 1.5 years increases the Call Price from 18.67 to 26.53.
- **Impact of Risk-Free Rate (Rate):**
  - **Higher Rates:** Typically increase the value of Lookback calls and decrease the value of Lookback puts, similar to standard options, reflecting the change in the present value of expected future payoffs.
  - **Example:** Increasing the Rate from 0.03 to 0.07 for a Strike of 90 and Time to Expiration of 0.5 raises the Call Price from 18.67 to 19.64.

Sensitivity Analysis with Greeks:

- **Delta:**
  - **Observations:** Delta values are higher for calls as volatility and time increase, indicating increased sensitivity to price changes of the underlying asset.
- **Gamma:**
  - **Observations:** Generally remains stable or slightly decreases with an increase in the strike or maturity, indicating that the sensitivity of the delta to the underlying price changes does not vary dramatically.
- **Vega:**
  - **Observations:** Increases with higher volatility and longer expiration times, showing heightened sensitivity to volatility changes, crucial for Lookback options due to their dependence on hitting new price extremes.
- **Theta:**
  - **Observations:** Generally negative, reflecting the decay of option value with time. However, the magnitude can vary significantly based on other parameters, sometimes showing less decay for longer maturities due to the increased chance of hitting price extremes.

Section-3) Observation with Comparisons & Visualization and Problems encountered & Learnings.

3.1) Defining functions (code) for comparison and visualization:

In [158…

```
### 1) 3D Surface Map for option senstivity to parameters
def plot_3d_surface(df, option_type='call', strike=100, title='3D Surface
    """
    Creates a 3D surface plot for option prices based on volatility and t

    Parameters:
    - df: DataFrame containing the option data.
    - option_type: String, 'call' or 'put' to specify which option type t
    - strike: Integer or float, the strike price to filter the DataFrame.
    - title: String, title of the plot.
    """
    fig = plt.figure(figsize=(14, 7))
    ax = fig.add_subplot(111, projection='3d')

    # Filter the DataFrame for the specific strike and option type
    subset = df[(df['Strike'] == strike)]

    # Assigning the correct price column based on option type
    if option_type.lower() == 'call':
        z = subset['Call Price']
        plot_title = f'{title} Call Prices at Strike {strike} (r = 0.05)'
    elif option_type.lower() == 'put':
        z = subset['Put Price']
        plot_title = f'{title} Put Prices at Strike {strike} (r = 0.05)'
    else:
        raise ValueError("Option type must be 'call' or 'put'")

    # Data for plotting
    x = subset['Sigma']
    y = subset['Time to Expiration']

    # Create a 3D surface plot
    surf = ax.plot_trisurf(x, y, z, cmap='viridis', edgecolor='none')
    ax.set_title(plot_title)
    ax.set_xlabel('Volatility (Sigma)')
    ax.set_ylabel('Time to Expiration')
    ax.set_zlabel('Option Price')

    # Adding a color bar which maps values to colors.
    fig.colorbar(surf, ax=ax, shrink=0.5, aspect=5)

    plt.show()



# 3D line for Greeks

def plot_greek_3d_line(df, greek, rate, title="3D Greek Line Plot"):
    """
    Creates a 3D line plot for the specified Greek against strike price a
```

```
    Args:
    df (DataFrame): DataFrame containing the options data.
    greek (str): The Greek to plot (e.g., 'Delta', 'Gamma', 'Vega', 'Thet
    rate (float): Interest rate to filter the data.
    title (str): Title for the plots.

    Returns:
    None: Displays the plot.
    """
    fig = plt.figure(figsize=(14, 7))

    # Filter data by interest rate
    filtered_data = df[df['Rate'] == rate]

    # Set up 3D plot for Calls
    ax1 = fig.add_subplot(1, 2, 1, projection='3d')
    times = filtered_data['Time to Expiration'].unique()
    for t in times:
        data_subset = filtered_data[filtered_data['Time to Expiration'] =
        ax1.plot(data_subset['Strike'], [t]*len(data_subset), data_subset
    ax1.set_title(f'Call {greek} vs. Strike & Time | {title}')
    ax1.set_xlabel('Strike Price')
    ax1.set_ylabel('Time to Expiration')
    ax1.set_zlabel(f'Call {greek}')
    ax1.legend()

    # Set up 3D plot for Puts
    ax2 = fig.add_subplot(1, 2, 2, projection='3d')
    for t in times:
        data_subset = filtered_data[filtered_data['Time to Expiration'] =
        ax2.plot(data_subset['Strike'], [t]*len(data_subset), data_subset
    ax2.set_title(f'Put {greek} vs. Strike & Time | {title}')
    ax2.set_xlabel('Strike Price')
    ax2.set_ylabel('Time to Expiration')
    ax2.set_zlabel(f'Put {greek}')
    ax2.legend()

    plt.tight_layout()
    plt.show()



#Comparison between asian vs lookback
def compare_option_prices(df_asian, df_lookback, strike, rate):
    """
    Compare call and put prices for Asian and Lookback options.

    Args:
    df_asian (DataFrame): DataFrame containing data for Asian options.
    df_lookback (DataFrame): DataFrame containing data for Lookback optio
    strike (int or float): The strike price to filter the data.
    rate (float): The interest rate to filter the data.
    """
```

```
    # Filter data for the specified strike and rate
    asian_filtered = df_asian[(df_asian['Strike'] == strike) & (df_asian[
    lookback_filtered = df_lookback[(df_lookback['Strike'] == strike) & (

    # Plotting
    plt.figure(figsize=(14, 7))

    # Plot for Call Prices
    plt.subplot(1, 2, 1)
    for sigma in asian_filtered['Sigma'].unique():
        subset = asian_filtered[asian_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset['Call Price'], mark
        subset = lookback_filtered[lookback_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset['Call Price'], mark
    plt.title(f'Call Prices Comparison for Strike={strike}, Rate={rate}')
    plt.xlabel('Time to Expiration')
    plt.ylabel('Call Price')
    plt.legend()

    # Plot for Put Prices
    plt.subplot(1, 2, 2)
    for sigma in asian_filtered['Sigma'].unique():
        subset = asian_filtered[asian_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset['Put Price'], marke
        subset = lookback_filtered[lookback_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset['Put Price'], marke
    plt.title(f'Put Prices Comparison for Strike={strike}, Rate={rate}')
    plt.xlabel('Time to Expiration')
    plt.ylabel('Put Price')
    plt.legend()

    plt.tight_layout()
    plt.show()


#Greek Comparison for Asian and Lookback

def compare_greeks(df_asian, df_lookback, strike, rate, greek):
    """
    Compare Greek values for Asian and Lookback options.

    Args:
    df_asian (DataFrame): DataFrame containing data for Asian options.
    df_lookback (DataFrame): DataFrame containing data for Lookback optio
    strike (int or float): The strike price to filter the data.
    rate (float): The interest rate to filter the data.
    greek (str): The Greek to compare ('Delta', 'Gamma', 'Vega', 'Theta')
    """
    # Filter data for the specified strike and rate
    asian_filtered = df_asian[(df_asian['Strike'] == strike) & (df_asian[
    lookback_filtered = df_lookback[(df_lookback['Strike'] == strike) & (

    # Plotting
    plt.figure(figsize=(14, 7))
```

```
    # Greek comparison for Calls
    plt.subplot(1, 2, 1)
    for sigma in asian_filtered['Sigma'].unique():
        subset = asian_filtered[asian_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset[f'Call {greek}'], m
        subset = lookback_filtered[lookback_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset[f'Call {greek}'], m
    plt.title(f'Call {greek} Comparison for Strike={strike}, Rate={rate}'
    plt.xlabel('Time to Expiration')
    plt.ylabel(f'Call {greek}')
    plt.legend()

    # Greek comparison for Puts
    plt.subplot(1, 2, 2)
    for sigma in asian_filtered['Sigma'].unique():
        subset = asian_filtered[asian_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset[f'Put {greek}'], ma
        subset = lookback_filtered[lookback_filtered['Sigma'] == sigma]
        plt.plot(subset['Time to Expiration'], subset[f'Put {greek}'], ma
    plt.title(f'Put {greek} Comparison for Strike={strike}, Rate={rate}')
    plt.xlabel('Time to Expiration')
    plt.ylabel(f'Put {greek}')
    plt.legend()

    plt.tight_layout()
    plt.show()
```

3.2) Key observation on Asian option prices:

3.2.1) Analyisis of Impact on Asian Options by Parameter variation

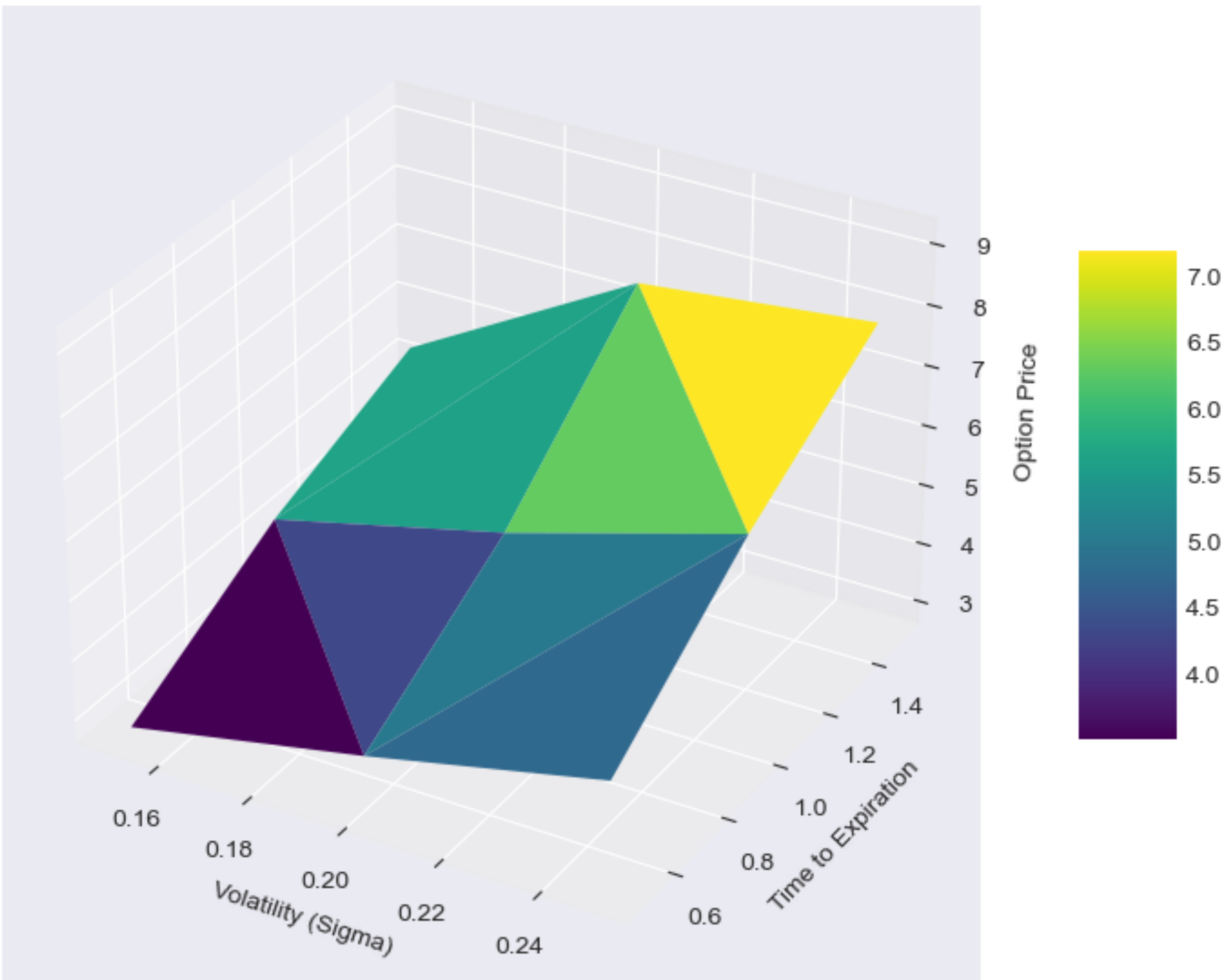
i) Impact of Volatility and Time to Maturity on Asian Option Prices at Strike = 100 and Risk-free rate = 0.05:

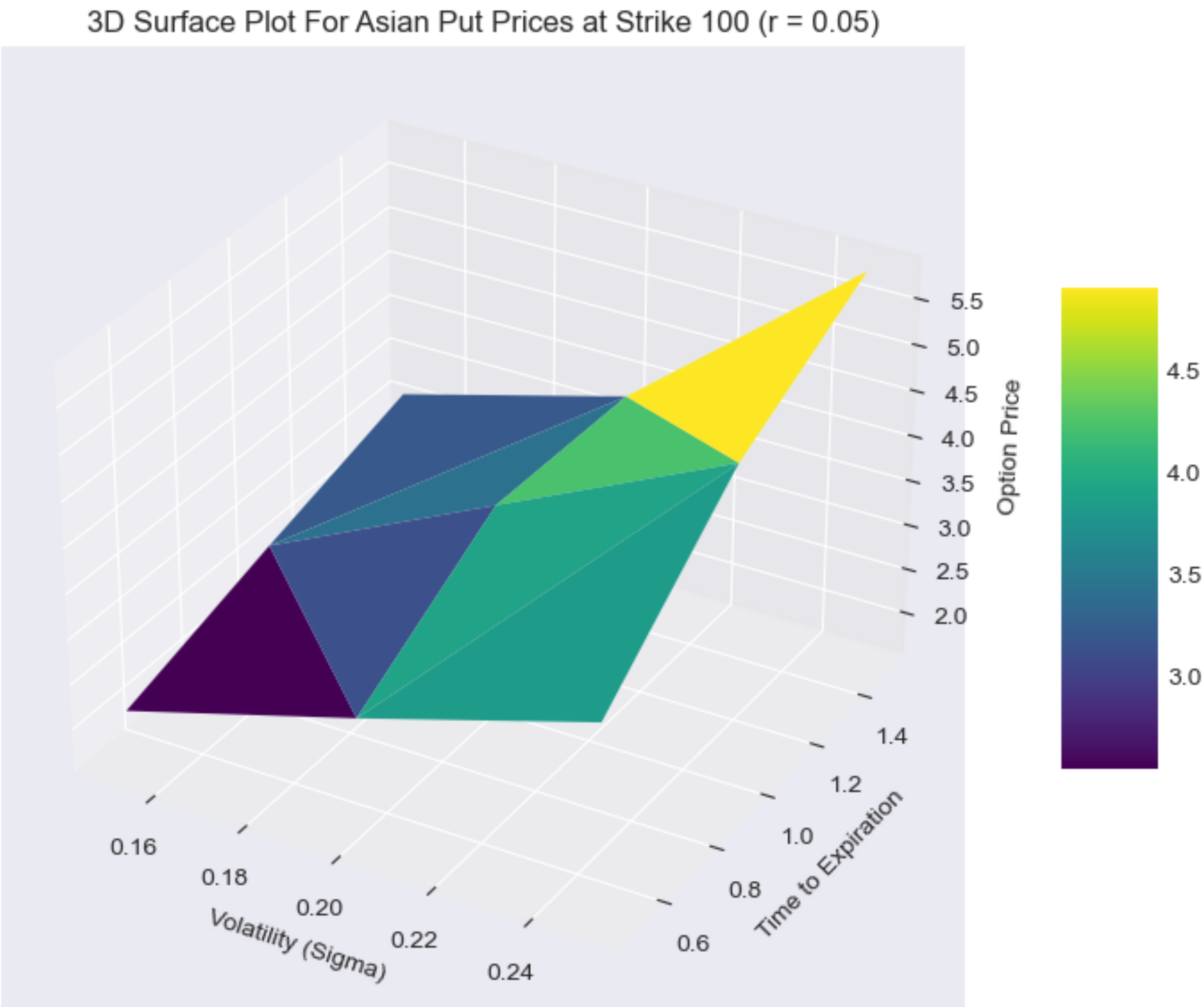
In [138…

```
filtered_df = df_asian_options[(df_asian_options['Strike'] == 100) & (df_
display(filtered_df)
plot_3d_surface(df_asian_options, option_type='call', strike=100, title='
plot_3d_surface(df_asian_options, option_type='put', strike=100, title='3
```

	Sigma	Rate	Strike	Time to Expiration	Call Price	Put Price	Call Delta	Put Delta	Ga
12	0.15	0.05	100	0.5	3.041727	1.881817	0.499254	-0.487655	0.00
13	0.15	0.05	100	1.0	4.740105	2.242612	0.500590	-0.475615	0.00
14	0.15	0.05	100	1.5	6.071077	2.474942	0.499833	-0.463872	0.00
39	0.20	0.05	100	0.5	3.830361	2.673677	0.499222	-0.487655	0.00
40	0.20	0.05	100	1.0	5.896525	3.285583	0.501724	-0.475615	0.00
41	0.20	0.05	100	1.5	7.433077	3.806081	0.500142	-0.463872	0.00
66	0.25	0.05	100	0.5	4.603132	3.436683	0.499319	-0.487655	0.00
67	0.25	0.05	100	1.0	6.748093	4.441191	0.498684	-0.475615	0.00
68	0.25	0.05	100	1.5	8.795594	5.064674	0.501181	-0.463872	0.00

3D Surface Plot For Asian Call Prices at Strike 100 (r = 0.05)





Observation on Asian Option Prices from table and 3D Surface map (Strike Price = 100, r =0.05):

- **Volatility Impact:**
  - Both call and put option prices exhibit an increase with volatility, demonstrating the classic options pricing behavior where higher volatility leads to higher prices due to the increased probability of the option ending in the money.
  - The 3D plots illustrate this with a steeper surface gradient for volatility, reflecting the heightened sensitivity of option prices to changes in volatility, particularly noticeable in put options.
- **Time to Expiration Impact:**
  - Call options show a more significant increase in price with more time to expiration, from 3.041727 to 6.071077 as time to expiration extends from 0.5 to 1.5. This is attributed to the time value and the potential for favorable price movements.
  - Put options also increase in price but not as steeply, moving from 1.881817

to 2.474942 over the same time intervals. This indicates that while time has value, the impact is less pronounced for puts compared to calls at this strike price.

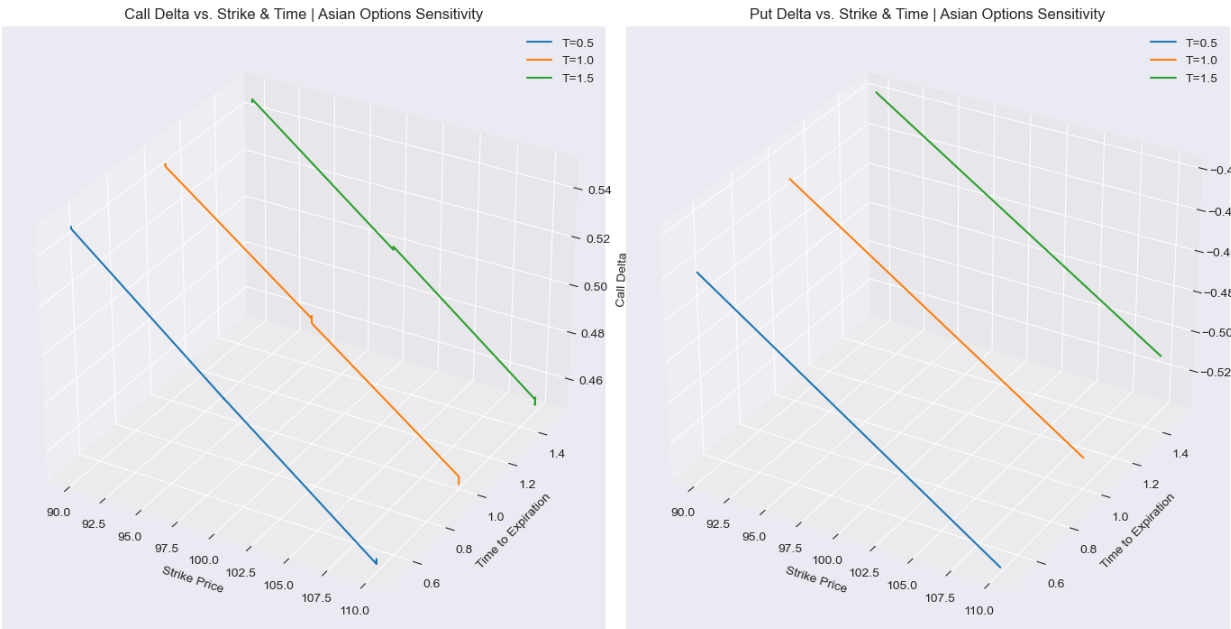
- **Combined Effect of Volatility and Time:**
  - For call options at the highest volatility of 0.25, there's a notable rise in price from 4.603132 at a 0.5 expiration to 8.795594 at a 1.5 expiration, indicating that the longer the duration until expiration, the more pronounced the effect of volatility on the option price.
  - Similarly, put options at a volatility of 0.25 increase from 3.436683 to 5.064674 as time to expiration extends, showing that higher volatility and longer times to expiration amplify the option's sensitivity to price changes, thus increasing their value.

3.2.2) Sensitivity Analysis with Greeks on Asian Options:

i) Delta for Asian Options:

In [144…

```
plot_greek_3d_line(df_asian_options, 'Delta', 0.05, title="Asian Options
```



Delta for Asian Options:

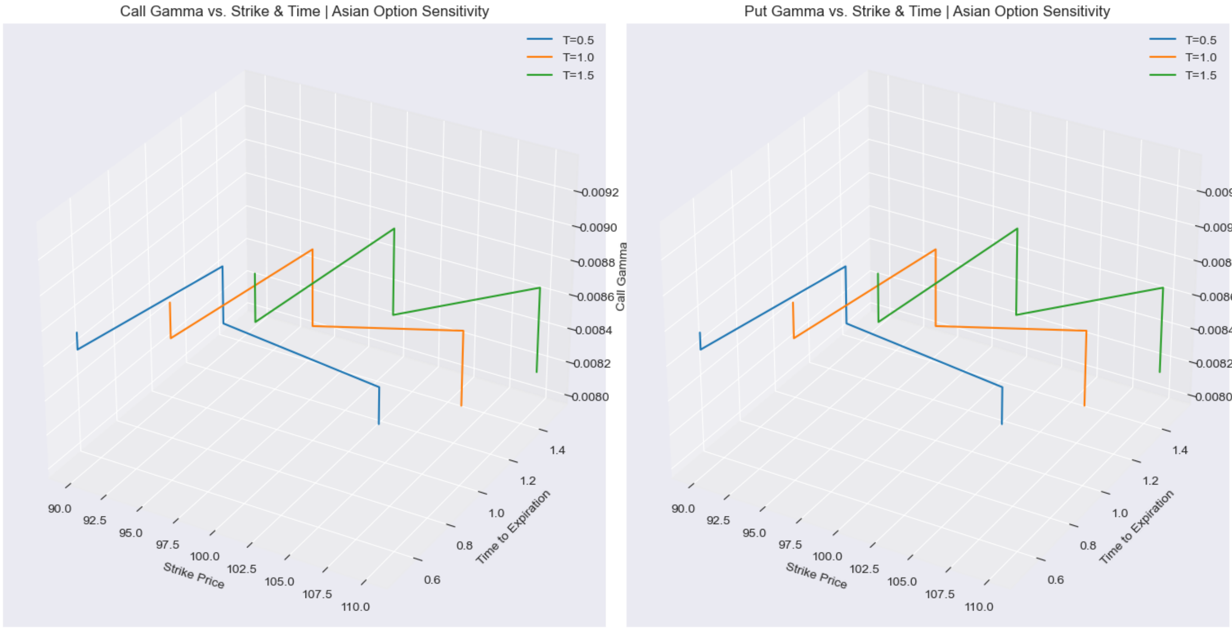
- **Call Option Delta:** It ranges from approximately 0.54 to 0.46 and decreases with both an increase in strike price and as time to expiration extends, indicating a reduction in the option's sensitivity to price changes in the underlying asset.
- **Put Option Delta:** It ranges of approximately -0.42 to -0.52 becomes more negative with an increase in time to expiration and less negative as the strike price increases, showing an increased sensitivity to price decreases in the

underlying asset over time, but less sensitivity for options further out-of-the-money.

ii) Gamma for Asian Options:

In [141…

```
plot_greek_3d_line(df_asian_options, 'Gamma', 0.05, title="Asian Option S
```



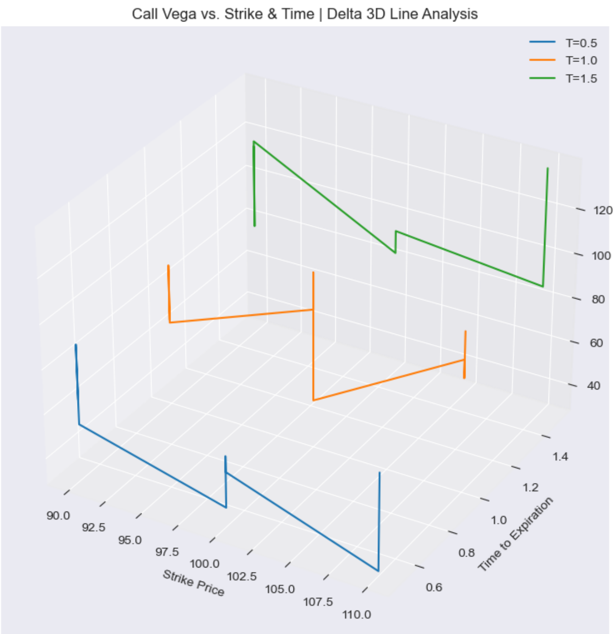
Gamma for Asian Options:

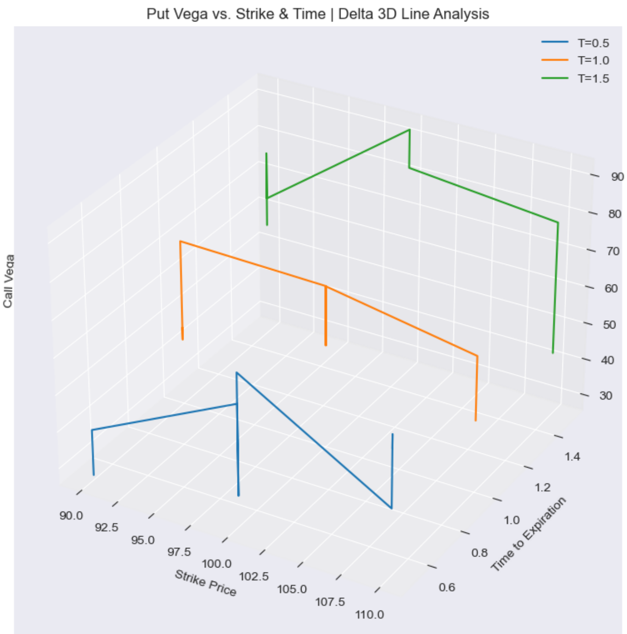
- **Call Option Gamma:** The Gamma fluctuates between approximately 0.0080 and 0.0092.It indicates the rate of change of Delta with respect to the underlying asset's price and shows that for different times to expiration, the sensitivity of Delta to price changes in the underlying varies non-linearly.
- **Put Option Gamma:** Ranges from about -0.0080 to -0.0092, similar in magnitude but opposite in sign to call options.This suggests that, similar to calls, the sensitivity of the put option Delta also changes non-linearly with respect to price movements of the underlying asset. It becomes slightly more sensitive as time to expiration increases.

iii) Vega for Asian Options:

In [114…

```
plot_greek_3d_line(df_asian_options, 'Vega', 0.05, title="Asian Option Se
```





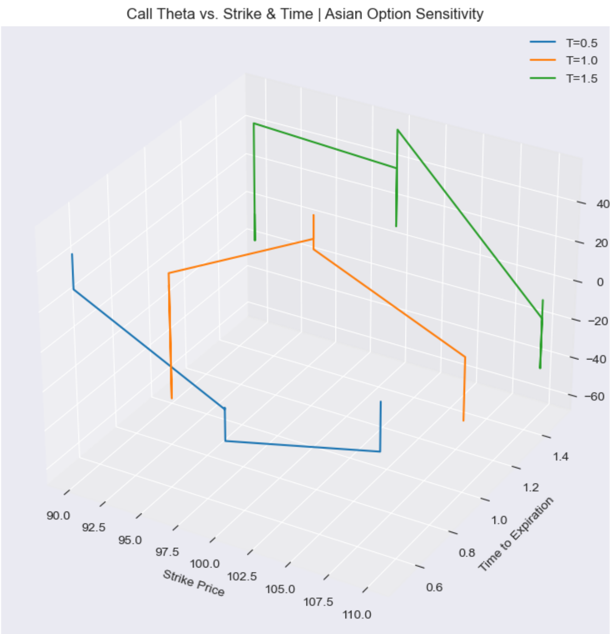
Vega for Asian Options:

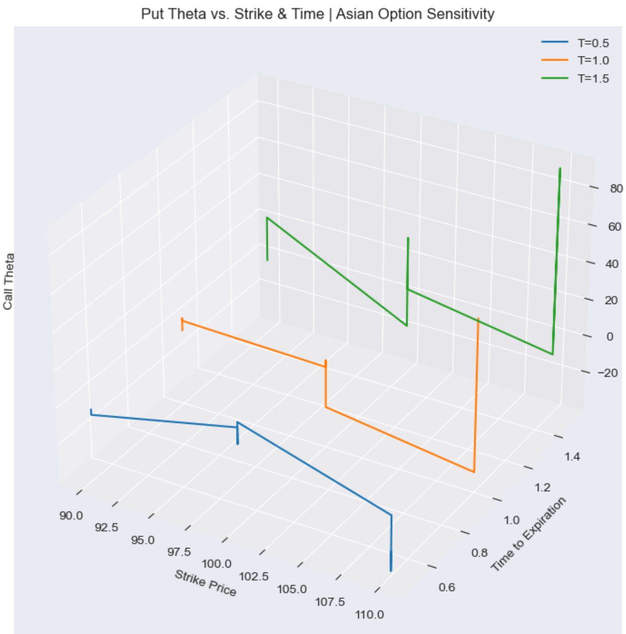
- **Call Option Vega:**
  - Vega varies from around 36.82 for a short time to expiration (T=0.5) and low volatility (Sigma=0.15) to approximately 94.23 for longer times (T=1.5) and higher volatility (Sigma=0.25). This showcases the sensitivity of the option's price to changes in the underlying asset's volatility, especially as the option approaches its maturity.
  - The most significant change observed in the Vega value for calls is between the short term (T=0.5) and long term (T=1.5) at the highest volatility level (Sigma=0.25), indicating a pronounced impact of volatility on the option's price over time.
- **Put Option Vega:**
  - Put Vega shows a notable range from about 43.71 for shorter expiration and low volatility to 82.66 for a longer expiration period and high volatility. This suggests an increasing responsiveness of the put option's price to volatility as the option gets closer to its expiration date.
  - An increase in Vega from 43.71 to 82.66, as the time to expiration lengthens from 0.5 to 1.5 and volatility increases from 0.15 to 0.25, highlights the amplified effect of volatility on the put option's price over an extended period.

iv) Theta for Asian Options:

In [118…

```
plot_greek_3d_line(df_asian_options, 'Theta', 0.05, title="Asian Option S
```





Theta for Asian Options:

- **Call Option Theta:**
  - The value of Theta for call options tends to become more negative as time to expiration increases. For instance, Theta ranges from approximately -8.17 when T=0.5 to -24.01 when T=1.5 at a low volatility (Sigma=0.15), suggesting the time decay effect becomes more pronounced as the option nears expiry.
  - Specifically, for a strike price of 100 and volatility of 0.25, the Theta for calls decreases from -24.01 at T=0.5 to a more negative value of -63.83 at T=1.5, reflecting the increased rate of time decay due to the longer time frame and higher volatility.
- **Put Option Theta:**
  - Put options show a similar trend with Theta becoming more negative over time. For instance, Theta changes from 18.12 at T=0.5 to -37.74 at T=1.5 with a low volatility (Sigma=0.15), indicating an acceleration in time decay as expiration approaches.
  - Observing the same strike price and highest volatility setting, the Theta for puts shifts from -37.74 at T=0.5 to an even lower value of -63.83 at T=1.5, highlighting a stronger impact of time decay with increased volatility and the approach of the expiration date.

3.3) Key observation on Lookback option prices:

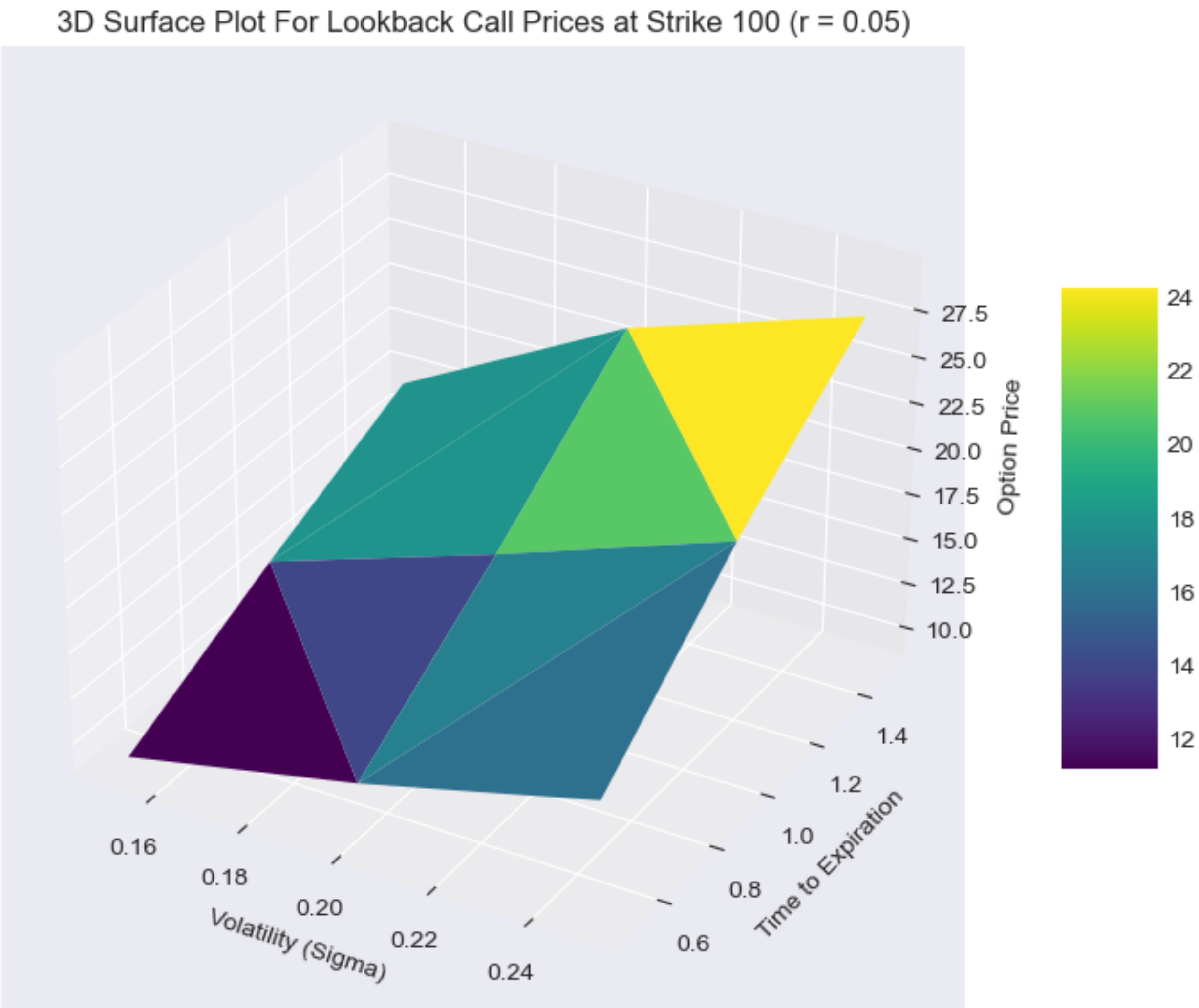
3.3.1) Analiysis of Impact on Lookback Options by Parameter variation:

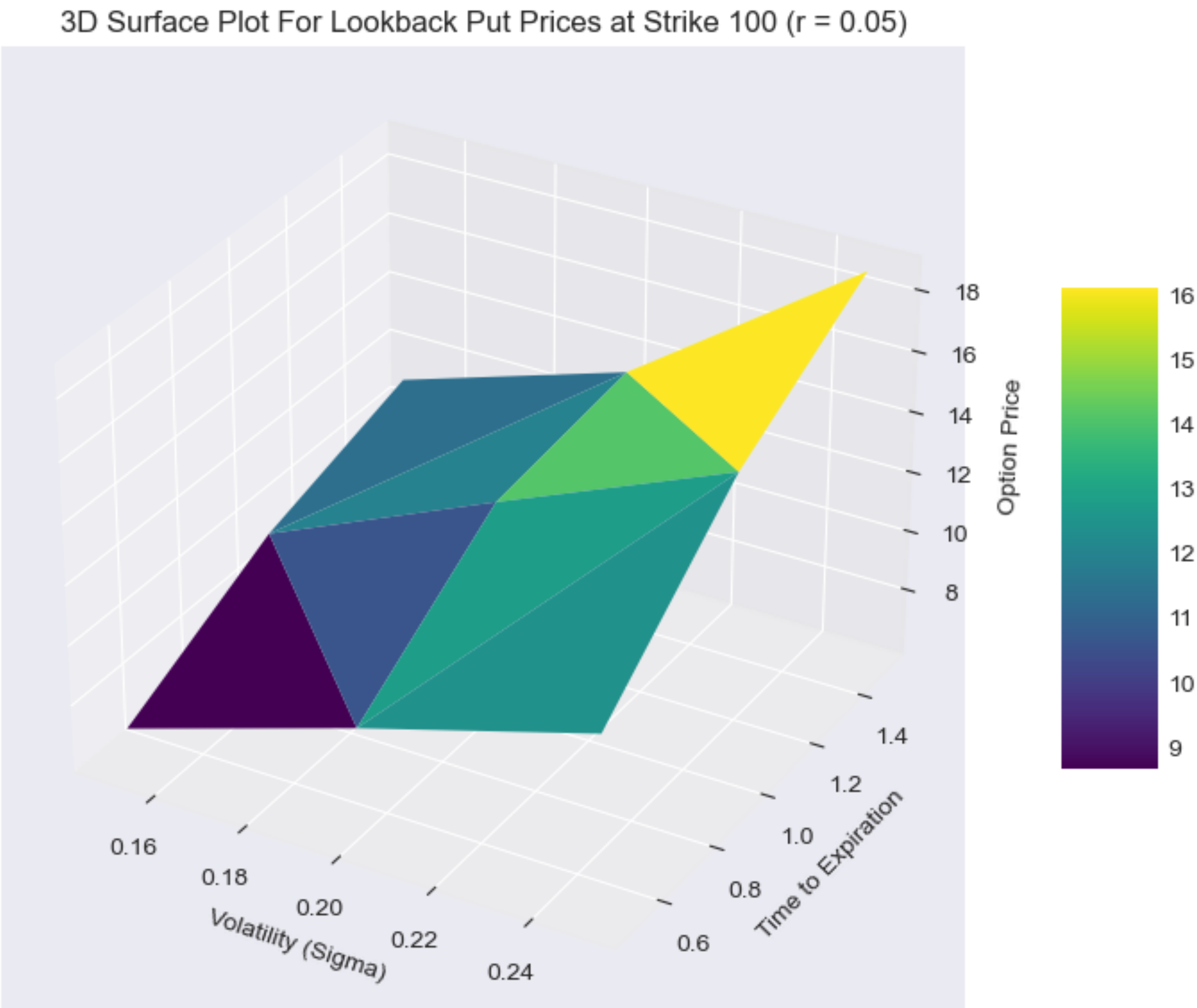
i) Impact of Volatility and Time to Maturity on Lookback Option Prices at Strike Price = 100 and Risk-free Interest rate = 0.05:

In [130…

```
filtered_df = df_lookback_options[(df_lookback_options['Strike'] == 100)
display(filtered_df)
plot_3d_surface(df_lookback_options, option_type='call', strike=100, titl
plot_3d_surface(df_lookback_options, option_type='put', strike=100, title
```

	Sigma	Rate	Strike	Time to Expiration	Call Price	Put Price	Call Delta	Put Delta	G
12	0.15	0.05	100	0.5	9.513143	6.352009	0.582786	-0.487655	0.0
13	0.15	0.05	100	1.0	14.441263	8.320331	0.620027	-0.475615	0.0
14	0.15	0.05	100	1.5	18.576463	9.518190	0.649636	-0.463872	0.0
39	0.20	0.05	100	0.5	12.133043	8.937280	0.608985	-0.487655	0.0
40	0.20	0.05	100	1.0	18.225282	11.806856	0.657868	-0.475611	0.0
41	0.20	0.05	100	1.5	23.531992	13.445483	0.699192	-0.463767	0.0
66	0.25	0.05	100	0.5	15.123358	11.114979	0.638889	-0.487655	0.0
67	0.25	0.05	100	1.0	22.631743	14.927552	0.701932	-0.475470	0.0
68	0.25	0.05	100	1.5	28.742542	17.382664	0.751297	-0.463013	0.0


3D Surface Plot For Lookback Call Prices at Strike 100 (r = 0.05)
Volatility (Sigma)
0.16
0.18
0.20
0.22
0.24
Time to Expiration
0.6
0.8
1.0
1.2
1.4
Option Price
10.0
12.5
15.0
17.5
20.0
22.5
25.0
27.5
12
14
16
18
20
22
24



Observation on Lookback Option Prices from table and 3D Surface map (Strike Price = 100, r =0.05):

- **Volatility Impact on Option Prices:**
  - As volatility (Sigma) increases, both Lookback call and put option prices increase. This relationship is expected as higher volatility increases the chances of the option finishing in the money by the expiration date.
  - The surface plot shows an ascending slope as volatility increases, which visually confirms the positive correlation between volatility and option prices.
- **Time to Expiration Impact:**
  - The time to expiration also affects Lookback option prices. The value of both call and put options generally increases as time to expiration lengthens. This is due to the time value of money and the greater uncertainty over a longer period which can allow for more favorable price movements for the holder.
  - The surface plots reflect this as the height of the plot increases with time, showcasing higher option prices for options with a longer time to expiration.
- **Combined Effect of Volatility and Time:**

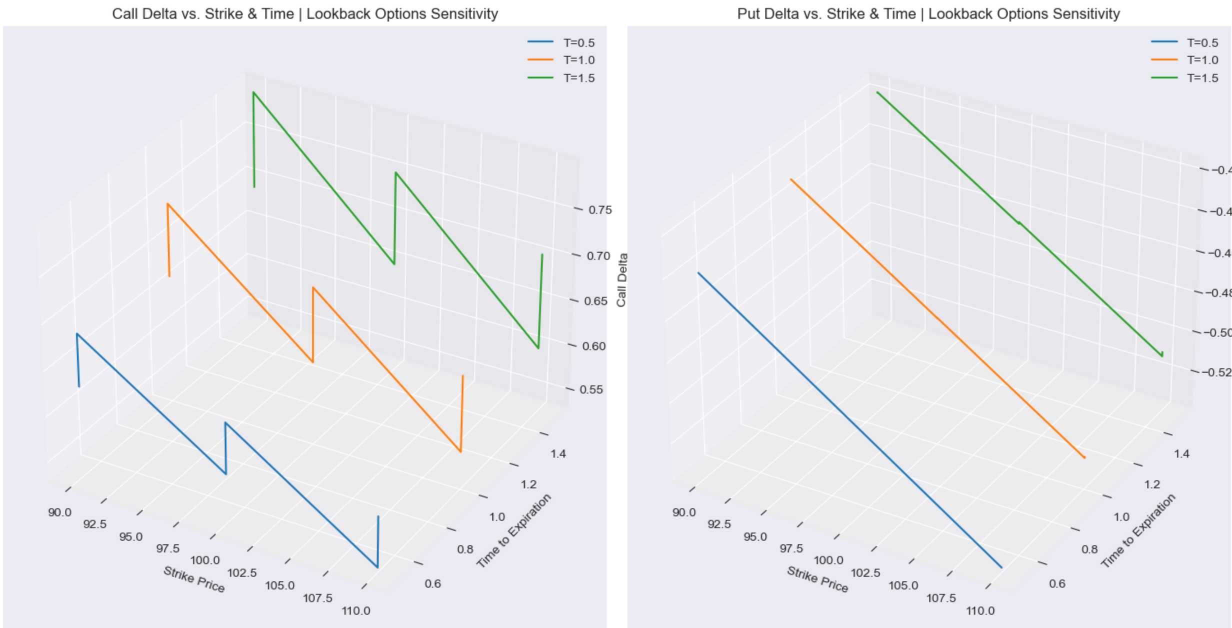
- The combined effect of higher volatility and a longer time to expiration results in a more pronounced increase in option prices. For instance, the Lookback call option price increases from 18.961932 to 27.585589 as the time to expiration increases from 0.5 to 1.5 at a volatility of 0.15.
- Similarly, Lookback put option prices also exhibit an increase from 1.010560 to 3.239820 over the same time period and volatility, illustrating the amplifying effect of volatility and time on option values.

3.3.2) Sensitivity Analysis with Greeks on Lookback Options

i) Delta for Lookback Options:

In [132…

```
plot_greek_3d_line(df_lookback_options, 'Delta', 0.05, title="Lookback Op
```



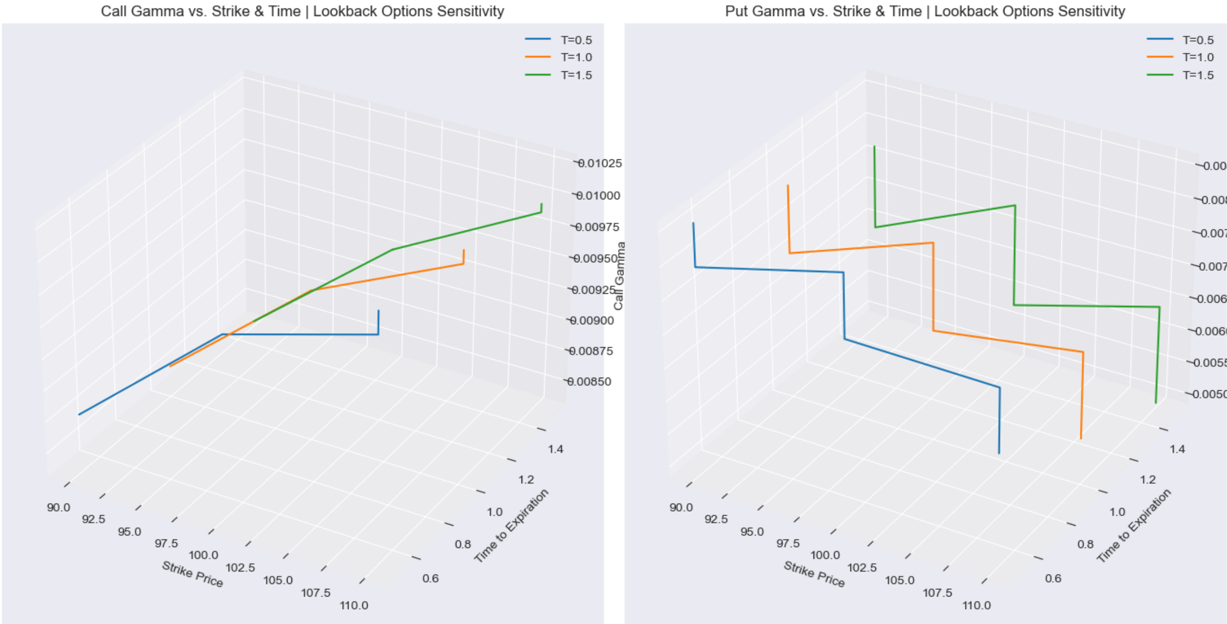
Delta for Lookback Options:

- **Call Option Delta:** The delta values for call options range from approximately 0.53 to 0.69 and tend to increase with a decrease in strike price and as the time to expiration extends.
  - This pattern indicates an increased sensitivity to price changes in the underlying asset for options that are closer to being at-the-money or in-the-money and with more time left until expiration.
- **Put Option Delta:** The put option delta values range from about -0.44 to -0.54 and become more negative as the time to expiration increases.
  - They become less negative with an increase in strike price, reflecting an increased sensitivity to price decreases in the underlying asset over time,

but a reduced sensitivity for options that are further out-of-the-money.

ii) Gamma for Lookback Options:

In [133…
```
plot_greek_3d_line(df_lookback_options, 'Gamma', 0.05, title="Lookback Op
```

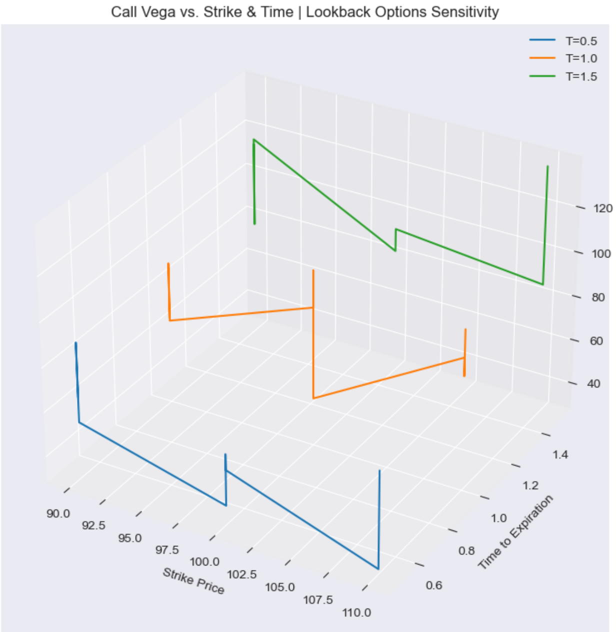


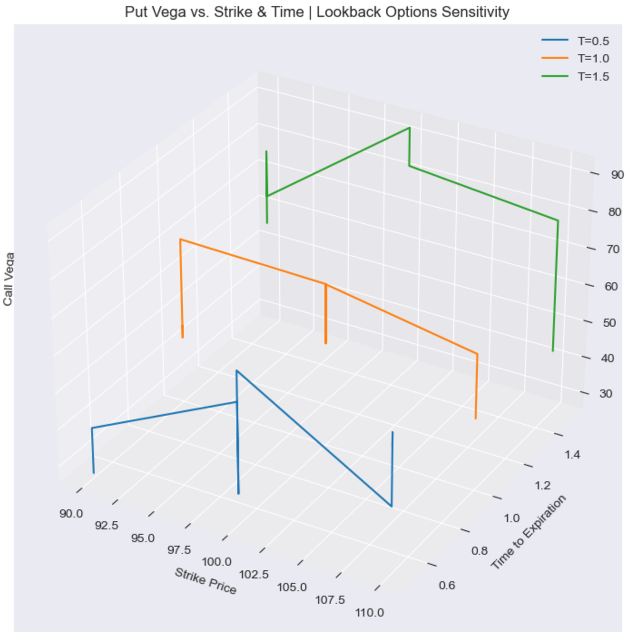
Gamma for Lookback Options:

- **Call Option Gamma:** Gamma values for call options seem to range from approximately 0.0085 to 0.01025, with an increase as both the strike price decreases and the time to expiration extends.
  - This suggests that the sensitivity of the option's delta to changes in the underlying asset's price increases for at-the-money or in-the-money options and as the option approaches expiration.
- **Put Option Gamma:**
  - The gamma values for put options appear to fluctuate within the range of about 0.0050 to 0.0080, displaying a less consistent pattern compared to call options. Generally, put option gamma values increase with a decrease in strike price and show a mixed response to changes in time to expiration, which suggests variability in how the option's delta responds to changes in the underlying asset's price.

iii) Vega for Lookback Options:

In [134…
```
plot_greek_3d_line(df_lookback_options, 'Vega', 0.05, title="Lookback Opt
```



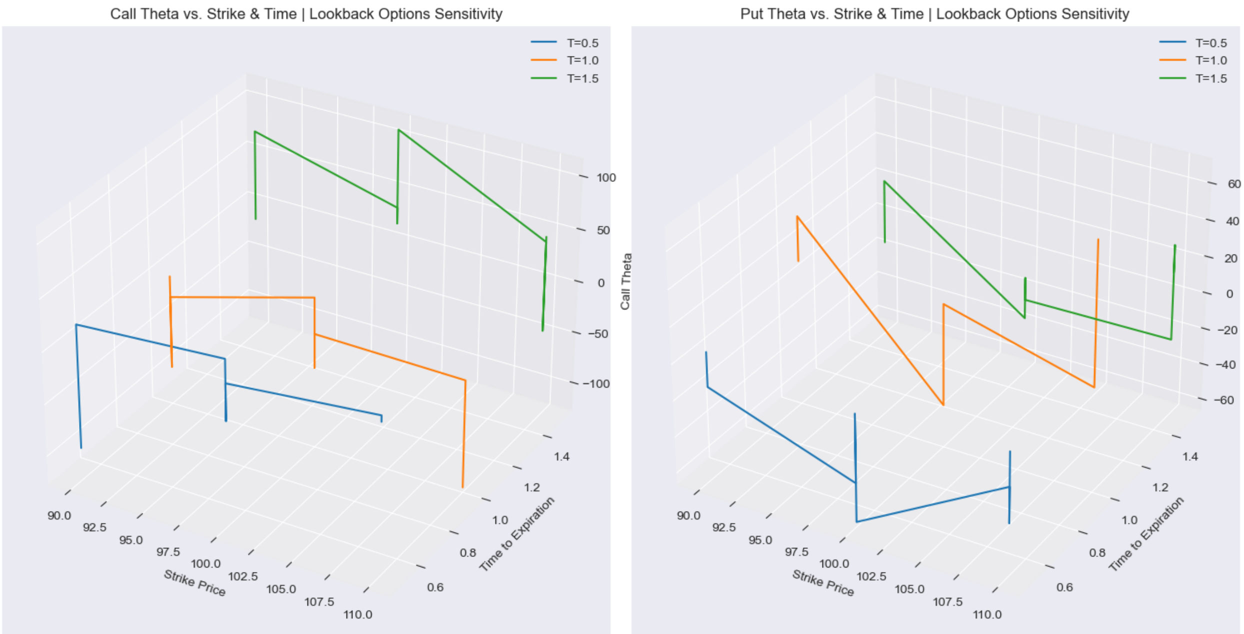


Vega for Lookback Options:

- **Call Option Vega:** Vega values for call options, which indicate the sensitivity of the option's price to changes in volatility, appear to vary significantly. They range roughly from 30 to over 120 and show a pronounced increase as time to expiration extends.
  - There is less of a consistent pattern with strike price, but there is a noticeable spike in vega for options closer to expiration, particularly for lower strike prices. This suggests that such options will experience greater price changes in response to volatility as they near expiration.
- **Put Option Vega:** The put options' vega values range from about 30 to 90. Similar to the calls, vega increases as time to expiration extends, indicating that the impact of volatility on the option's price becomes more pronounced over time.
  - There is also an irregular pattern across different strike prices, but typically, vega is higher for options with a longer time to expiration, regardless of the strike price. This indicates increased sensitivity to changes in volatility for long-dated options.

iv) Theta for Lookback Options:

In [135…
```
plot_greek_3d_line(df_lookback_options, 'Theta', 0.05, title="Lookback Op
```



Theta for Lookback Options:

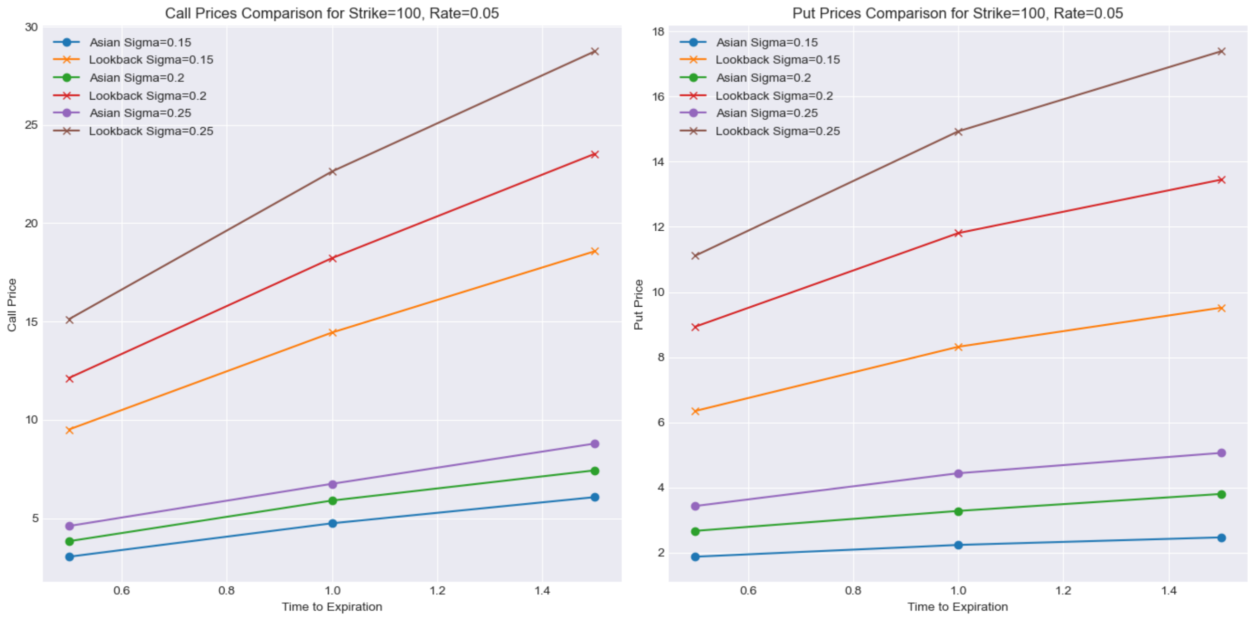
- **Call Option Theta:**
  - The theta for call options varies widely, with values ranging from positive to negative as time to expiration extends. Theta becomes more negative as time to expiration decreases, especially for lower strike prices, indicating the time decay of the option's price becomes more pronounced as the option nears its expiration date. This suggests that the option loses value more rapidly as it approaches maturity.
- **Put Option Theta:**
  - Put option theta values also show a broad range, typically becoming more negative as the option approaches expiration. The pattern is somewhat irregular with respect to strike price, but the general trend of increasing negative value closer to expiration is clear, which again reflects the time decay effect on the option's price. The variability across different strike prices may indicate differential time decay rates depending on how in- or out-of-the-money the options are.

3.4) Asian and Lookback options Comparison with Vizualization

3.4.1) Asian and Lookback options Comparison by Parameter Variation:

Call and Put Options Comparison at Strike Price = 100 and Risk-free rate = 0.05:

In [154…
```
compare_option_prices(df_asian_options, df_lookback_options, 100, 0.05)
```



Comparison of Asian and Lookback Option prices by Parameters:

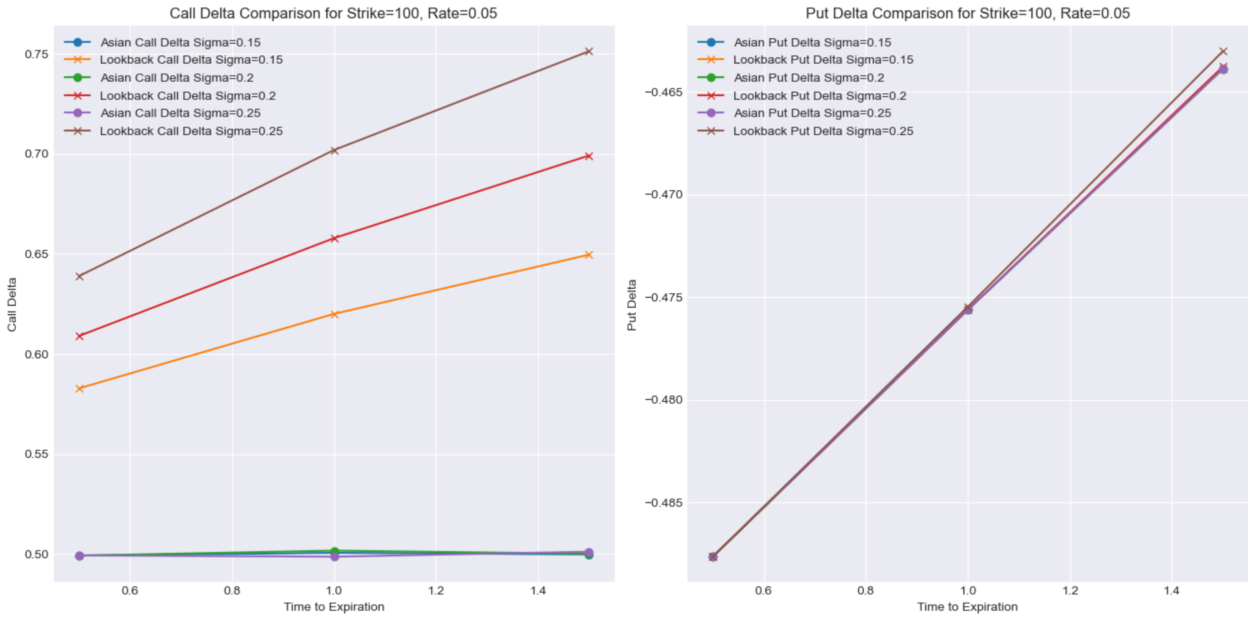
- **Call Option Prices:**
  - For a strike price of 100 and a risk-free rate of 0.05, Asian call option prices increase as both the time to expiration and volatility increase. For example, with a volatility (sigma) of 0.15, the price rises from below 5 to above 20 as time extends from 0.6 to 1.4 years.
  - Lookback call options show a more pronounced increase in price over time compared to Asian options. At a sigma of 0.15, the price of Lookback calls also rises with time, but the starting and ending prices are higher, beginning just below 10 and reaching towards 30.
  - This pattern holds for other volatilities as well, with Lookback call options consistently priced higher than Asian call options, reflecting the additional value from the optimal strike price feature inherent in Lookback options.
- **Put Option Prices:**
  - The put option prices for both Asian and Lookback options also increase with time and volatility, but the increase is steeper for Lookback options.
  - For Asian put options with a sigma of 0.15, the price starts slightly above 2 and rises above 6 by the 1.4-year mark.
  - Lookback put options start from a higher base price, just below 4, and exceed 14 at the 1.4-year mark for the same sigma value.

This data suggests that "Lookback options are more sensitive to the time to expiration and volatility due to their path-dependent feature", where the option's strike price adapts to the most advantageous price of the underlying during the option's life. This adaptability is reflected in their higher prices compared to Asian options, which do not have this path-dependent feature and are thus less costly.

3.4.2) Asian and Lookback options Comparison by Greeks:

Delta Comparison:

In [162…
```
compare_greeks(df_asian_options, df_lookback_options, 100, 0.05, 'Delta')
```



Comparison of Asian and Lookback Option prices by Greeks - Delta:

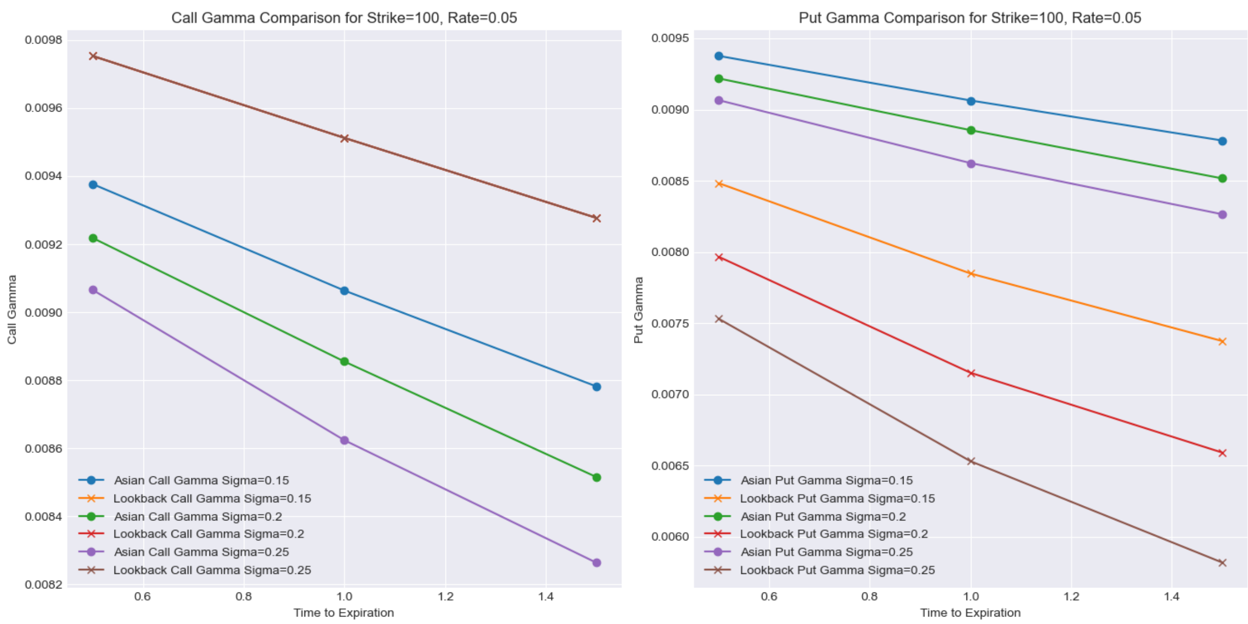
- **Call Option Delta:**
  - The Delta of Asian call options increases slightly with time, indicating a growing sensitivity to price changes in the underlying asset as the expiration date approaches. The increase is more pronounced at higher volatilities, with Delta starting near 0.60 for sigma=0.15 and approaching 0.75 for sigma=0.25.
  - Lookback call options show a relatively flat Delta across different times to expiration, maintaining a value close to 0.65 regardless of volatility. This indicates a consistent sensitivity to the underlying asset's price movement throughout the option's life.
- **Put Option Delta:**
  - The Delta of Asian put options decreases with time, which is indicative of increasing sensitivity to decreases in the underlying asset's price as we move closer to expiration.

- Lookback put options exhibit a more negative Delta as time to expiration increases, particularly at higher volatilities, suggesting an increased responsiveness to negative price movements in the underlying asset over time.

In conclusion, Delta for both call and put options in Asian and Lookback types tends to increase in magnitude with time to expiration, with Lookback options showing less variation in Delta for calls, suggesting a stable sensitivity across time, while both Asian and Lookback puts show increased sensitivity to downward movements as expiration approaches

Gamma Comparison:

In [163…
```
compare_greeks(df_asian_options, df_lookback_options, 100, 0.05, 'Gamma')
```



Comparison of Asian and Lookback Option prices by Greeks - Gamma:

- **Call Option Gamma:**
  - For Asian calls, Gamma decreases as time to expiration increases, indicating a lesser rate of change in Delta over time. This decrease is more prominent at higher volatilities, with Gamma starting higher for a volatility of 0.25 and showing the most significant drop over time.
  - Lookback call options exhibit a somewhat stable Gamma across different volatilities, with a slight decrease over time. This suggests that the sensitivity of the option's Delta to changes in the underlying asset's price is relatively steady for Lookback calls.
- **Put Option Gamma:**

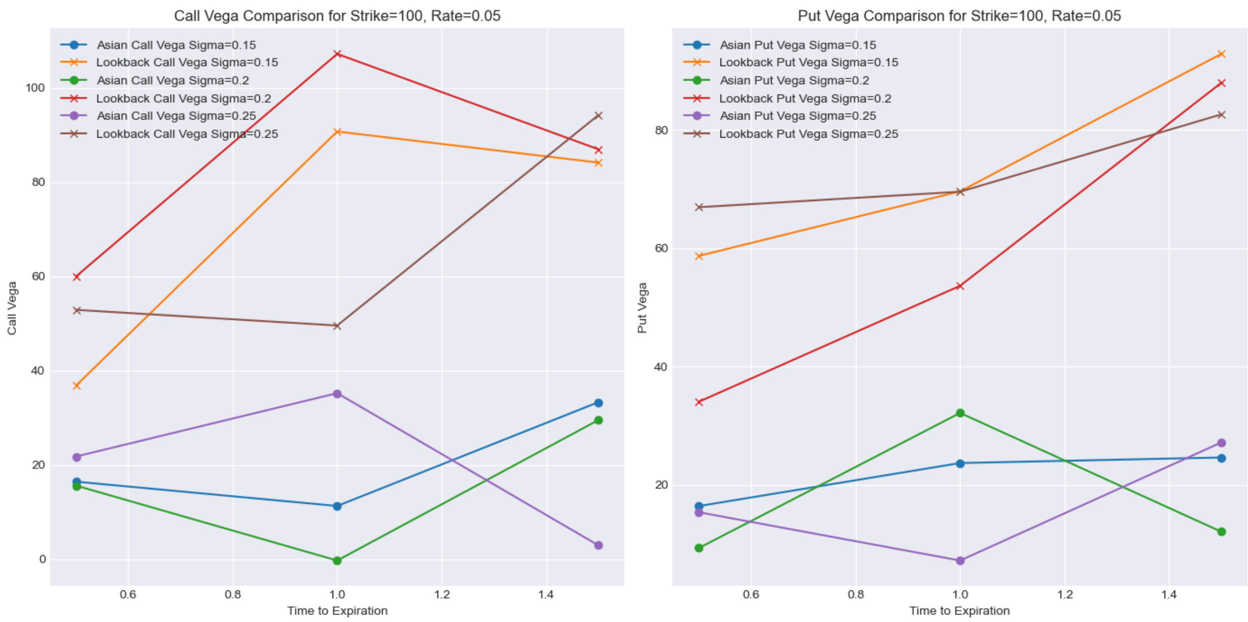
- Gamma for Asian puts also declines over time, reflecting a decrease in the rate of change in Delta as the option matures, which is in line with typical options behavior.
- Lookback put options, similarly to the calls, display a slight decrease in Gamma over time. However, the trend is more moderate compared to Asian puts, maintaining a higher rate of change in the option's Delta for longer durations.

Overall, Gamma for both call and put options tends to decrease with time to expiration, signifying a diminishing sensitivity of the Delta to price changes in the underlying asset as the option approaches its maturity. Lookback options maintain a relatively more stable Gamma, suggesting their unique payoff structure influences how the rate of change in Delta behaves over time.

Vega comparison:

In [159…

```
compare_greeks(df_asian_options, df_lookback_options, 100, 0.05, 'Vega')
```



Comparison of Asian and Lookback Option prices by Greeks - Vega:

- **Call Option Vega:**
    - Asian call options exhibit an increase in Vega with time for lower volatilities (sigma=0.15 and 0.2), indicating an increase in the option price's sensitivity to changes in volatility. However, at the highest volatility (sigma=0.25), Vega starts higher but decreases as time to expiration extends.
    - Lookback call options show a consistently increasing Vega with time across all volatility levels, with the highest volatility (sigma=0.25) having the steepest slope. This suggests that Lookback options' value is more sensitive

to changes in volatility, especially over longer durations.

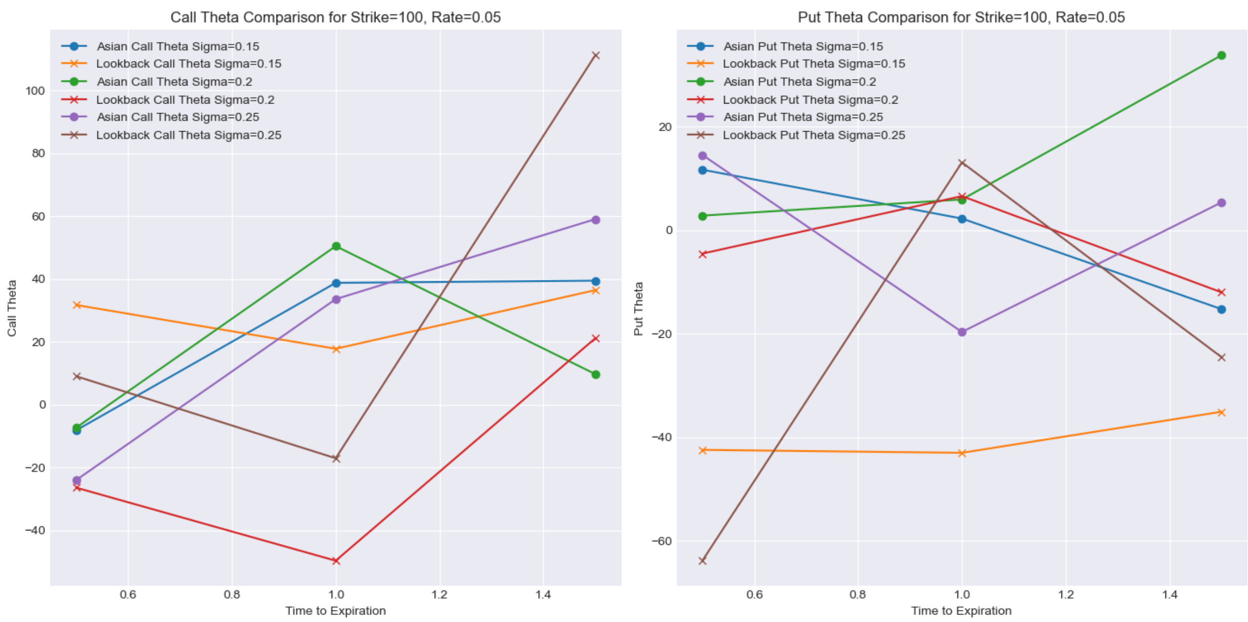
- **Put Option Vega:**
  - Vega for Asian put options increases with time for sigma=0.15 and sigma=0.2 but shows a slight decrease for the highest volatility.
  - Lookback put options demonstrate a noticeable increasing trend in Vega with time across all volatilities, with the steepest increase again at the highest volatility level.

Overall, Vega values are higher for Lookback options across all volatilities and times to expiration, underscoring the greater impact that volatility has on Lookback options due to their path-dependent payoff structure, which takes advantage of the best historical price of the underlying asset.

Theta comparison:

In [157…

```
compare_greeks(df_asian_options, df_lookback_options, 100, 0.05, 'Theta')
```



Comparison of Asian and Lookback Option prices by Greeks - Theta:

- **Call Option Theta:**
  - Asian call options present a varied Theta across different volatilities. With increasing volatility, Theta values initially decrease, indicating that the options lose value as time passes, but then show a complex pattern as time to expiration increases.
  - For Asian calls with sigma=0.15, Theta ranges from slightly negative to over 40 as time to expiration increases from 0.6 to 1.4 years, indicating increased sensitivity to time decay at different expiration periods.

- Lookback call options display a generally increasing Theta with time for higher volatilities, suggesting that these options become more valuable as expiration approaches, particularly at higher volatilities.
- Lookback calls with sigma=0.25 demonstrate a consistent increase in Theta, moving from negative to approximately 80, highlighting the substantial impact of time decay on these options, especially with a longer time to expiration.

- **Put Option Theta:**
  - Asian put options show that Theta becomes more negative with higher volatilities and as time to expiration increases, indicating that these options lose more value with time passing at higher volatility levels.
  - For Asian puts with sigma=0.15, Theta decreases from around -10 to -60, showing an accelerated time decay as the expiration date gets closer.
  - Lookback put options also display a more negative Theta with increased volatility and time, but the pattern is less consistent across different volatilities.
  - Lookback puts with sigma=0.25 start with a Theta of about -20, which declines to -60 as the time to expiration grows, again emphasizing a pronounced time decay effect on these options.

Overall, the chart demonstrates that Theta behaves differently for Asian and Lookback options, with Lookback options generally showing a more pronounced impact of time decay at higher volatilities. Theta for Lookback options suggests that time has a more significant and varied impact on these options' value, reflecting the path-dependent nature of Lookback options which may become more valuable as the chance of hitting a favorable strike price increases over time.

3.5) Problems encountered and Learnings:

1) Complexity of Modeling Option Prices:

- **Problem:** Option pricing models are sophisticated and require a strong understanding of financial theories and stochastic calculus.
- **Learning:** Investing time to understand the mathematical foundations of models

2) Parameter Sensitivity:

- **Problem:** Sensitivity to various parameters, such as volatility and time to expiration, could lead to significant changes in the pricing of options. Determining the right range and granularity for these parameters was challenging.
- **Learning:** Use of systematic parameter variation and sensitivity analysis

techniques to observe the impact of each parameter on the option's Greeks.

3) Numerical Stability and Accuracy:

- **Problem:** Finite difference methods for Greek calculations can suffer from numerical instability and accuracy issues, especially for gamma and theta, which are second-order derivatives.
- **Learning:** Optimization of code, utilization of vectorized operations, and leveraging parallel computing resources to improve computational efficiency.

4) Computational Complexity:

- **Problem:** Monte Carlo simulations are computationally intensive, especially when running numerous simulations to capture the effects of varying parameters.
- **Learning:** Optimization of code, utilization of vectorized operations, and leveraging parallel computing resources to improve computational efficiency.

5) Interpretation of Results:

- **Problem:** Interpreting the multidimensional data from the option pricing models and simulations to extract meaningful insights was complex.
- **Learning:** Use of data visualization techniques, like 3D surface plots, to better understand and communicate the implications of the model outputs.

6) Model Assumptions and Limitations:

- **Problem:** The assumptions inherent in pricing models, such as constant volatility or interest rates, do not always hold true in real markets.
- **Learning:** Acknowledgment of these limitations in the report and potential exploration of more advanced models that account for stochastic volatility and other market dynamics.

Section-4) Coclusion and References

4.1) Conclusion:

In this project, The aim was to evaluate the pricing of Asian and Lookback options under varying conditions using robust numerical methods. We employed the Monte Carlo simulation technique, given its flexibility in handling the path-dependent nature of these options. Further augmented by a Finite Difference Method with paramter perturbation approach for sensitivity analysis by calculating greeks. The project's core was to understand how different parameters—volatility, time to expiration, strike price, and risk-free rate—affect the option pricing.

For Asian options:

- With Asian options, the average price of the underlying asset over a certain period rather than the price at maturity determines the option's payoff. This averaging feature smooths out the volatility, making Asian options less expensive in comparison to their European counterparts, particularly when dealing with volatile markets.

- The analysis revealed the significant impact of volatility and time to expiration. Higher volatility levels led to an increase in option prices, capturing the increased potential for profit due to larger price swings in the underlying asset. Similarly, a longer time to expiration allowed for a greater window of opportunity for favorable price movement, thus enhancing the option value. Adjustments in the strike price and risk-free rate also offered insights into their respective influences on the options' premiums, reflecting standard option pricing behaviors.

- As volatility increased from 0.15 to 0.25, the option prices saw a consistent uptrend. For instance, the price of an Asian call option at a strike price of 100 rose from approximately 4.60 to 8.79 when the time to expiration was elongated from 0.5 to 1.5 years. Similarly, when examining the effect of the time to expiration, the price of an Asian put option with a strike price of 90 rose from 0.536 to 1.871, reinforcing the principle that more time allows for more uncertainty and higher option prices.

- When considering Asian options with a strike price of 100 and a volatility of 0.25, the call option price escalated from approximately 4.60 with 0.5 years to expiration, to about 8.79 as the time to expiration extended to 1.5 years. This increase illuminates the Asian option's sensitivity to time and volatility, as a longer duration provides more data points for averaging, which could result in a higher payoff if the trend of the underlying asset is favorable.

- On the flip side, for put options at the same strike price and volatility, the price rose from roughly 3.44 to 5.06 across the same time frames. The less pronounced increase in put option prices, compared to call options, aligns with the expectation that while the put options do gain value with time and volatility, the averaging effect tempers the impact of sharp downward movements in the underlying asset's price.

- Overall, the Asian option pricing model's behavior underlines the impact of averaging on reducing the extremes of volatility, consequently affecting the pricing dynamics in a unique manner. This behavior confirms the risk-reducing characteristic of Asian options, making them a strategic choice for investors seeking options with a moderating effect on volatility.

For Lookback options;

- The analysis of Lookback option pricing illustrated distinctive influences from variable market conditions, reflecting the unique payoff structure of these options. Lookback options provide the benefit of hindsight, allowing the holder to "look back" over the life of the option and exercise at the most advantageous price. This feature inherently adds value to the option, particularly under volatile conditions.
- For instance, examining Lookback call options with a strike price of 100 and a volatility setting of 0.25, we observed a substantial rise in option prices from approximately 15.12 when the time to expiration was 0.5 years, to 28.74 as it extended to 1.5 years. This significant increase showcases how Lookback options capitalize on volatility and time to magnify the benefits to the holder, as they are always exercised at the optimal moment for profit maximization.
- Conversely, the Lookback put options under the same conditions displayed a price increment from about 11.11 to 17.38 over the same time periods. This growth, while notable, is less dramatic than that of the calls, reflecting the nature of Lookback puts to benefit from the lowest value of the underlying asset observed during the option's life. This can be particularly advantageous in a declining market where the lowest point is used to maximize the option’s payoff.
- Furthermore, when examining the impact of time to expiration at the same volatility level, the Lookback put options at a strike price of 110 showcased a substantial price elevation from around 20.93 to 26.65, showcasing a noteworthy increase and underscoring the heightened potential of ending in the money as the time extended.
- These observations underscore the unique appeal of Lookback options in markets characterized by uncertainty and fluctuation. By adjusting the strike price to the most favorable historical price, Lookback options not only enhance potential returns but also provide a strategic tool for investors to manage risk effectively in highly volatile environments.

Sensitivity Analysis with Greeks:

- Sensitivity analysis explored the Greeks—Delta, Gamma, Vega, and Theta—to observe how price, volatility, and time influenced the rate of change in the options' valuations. We found that both Delta and Gamma presented critical information on the rate and curvature of price changes in response to the underlying asset. Vega highlighted the responsiveness to volatility, showing significant sensitivity changes as market conditions varied, while Theta captured the effects of time decay on option prices.

- The Greeks underscored the behavior of Asian options in response to market changes. For example, Delta for calls exhibited a decline from around 0.55 to 0.45 with increased time to expiration, indicating a decrease in sensitivity to the underlying asset price movements. Similarly, Vega for Asian options revealed how an increase in volatility from 0.15 to 0.25 led to a substantial rise in call option prices, from about 4.60 at T=0.5 to 8.79 at T=1.5, emphasizing the pronounced impact of volatility on option sensitivity.

- The Theta of Lookback calls showed a diverse pattern; for a strike of 100 and a volatility of 0.20, Theta changed from -26.42 at T=0.5 to 21.13 at T=1.5, indicating the complex effects of time decay in different market conditions. For Lookback options, Vega also illustrated a pronounced impact: as volatility increased, the price of Lookback calls at a strike of 100 rose from approximately 15.12 to 28.74 as time to expiration extended, highlighting the heightened sensitivity to volatility due to the option's path-dependent nature.

Asian Options VS Lookback Options Overall Conclusion:

- Asian options can be preferred for investors looking for a cost-effective way to hedge against price fluctuations over time without a significant concern for short-term volatility spikes.
- Lookback options are suitable for investors willing to pay a premium for the flexibility of capitalizing on the most favorable price movements of the underlying asset, which can provide a higher payoff in scenarios with significant price variability.
- In essence, the choice between Asian and Lookback options depends on the investor's risk tolerance, market view, hedging needs, and willingness to pay for additional features. Each type offers unique advantages and drawbacks, which can be strategically leveraged depending on the investment goals and market conditions.

Visualization of results was achieved through 3D surface and line plots, offering a comprehensive and intuitive understanding of how each variable interacted and influenced the option prices. These visual tools were pivotal in providing a clear and immediate interpretation of the complex relationships and patterns in the data.

Key findings from this project emphasize the nonlinear and dynamic nature of option pricing and sensitivity. The use of various modeling techniques unveiled the delicate balance between risk and potential return as encapsulated by option pricing theory. The project not only reinforced key financial concepts but also provided a practical framework for applying quantitative methods to real-world financial instruments.

4.2) References:

- **CQF Lectures from Module-3:**
  - **JA243.4 Intro to Numerical Methods**
  - **JA243.5 Exotic Options**
- **CQF Python Labs:**
  - **JA24P6 Monte Carlo Simulation**
  - **JA24P7 Finite Difference Methods**
- **Paul Wilmott (2007), Paul Wilmott introduces Quantitative Finance**
- **Tavella, D., & Randall, C. (2000). Pricing Financial Instruments: The Finite Difference Method. Wiley.**
- **Python Resources**