Statistical Tests and Distributions: P-Test, T-Test, Z-Test, and F-Distribution

1. P-Test

The **P-Test** is not a distinct test but relates to the calculation and interpretation of the **p-value**, a key concept in hypothesis testing.

Key Concepts

- **P-Value Definition**:
  - A measure of the probability that the observed data (or something more extreme) occurred under the null hypothesis.
  - Expressed as:

$$P = P(\text{Observed outcome or more extreme} \mid \text{Null Hypothesis true})$$

- **Threshold for Significance**:
  - The p-value is compared against a significance level (α), often 0.05 or 0.01.
    - P-value $< \alpha$: Reject the null hypothesis.
    - P-value $> \alpha$: Fail to reject the null hypothesis.

Usage

- P-values are calculated in conjunction with specific tests (e.g., T-tests, Z-tests).
- A smaller p-value indicates stronger evidence against the null hypothesis.

2. T-Test

A **T-Test** is used to determine if there is a significant difference between the means of two groups, particularly when sample sizes are small.

Types of T-Tests

1. **One-Sample T-Test**:
   - Compares the mean of a single sample to a known value or population mean.
   - Example: Testing if the average test score of a class differs from 70.
2. **Two-Sample T-Test (Independent T-Test)**:

- Compares the means of two independent groups.
- **Assumptions**:
  - Samples are independent.
  - Variances between groups are equal (homoscedasticity).
- Example: Comparing average heights of men and women.

3. **Paired T-Test**:
   - Used when measurements are taken on the same subjects under two conditions (e.g., before and after treatment).
   - Example: Measuring weight loss before and after a diet.

Key Formula

$$t = \frac{\acute{X}_1 - \acute{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where:

- $\acute{X}_1, \acute{X}_2$: Sample means.
- s_p^2: Pooled variance.
- n_1, n_2: Sample sizes.

Assumptions

- Data is approximately normally distributed.
- Samples are independent (except in paired tests).

Applications

- Comparing performance metrics.
- Testing the effect of an intervention.

3. Z-Test

A **Z-Test** evaluates whether two population means are different, based on samples from the populations. It is generally used for large sample sizes (n>30).

Types of Z-Tests

1. **One-Sample Z-Test**:

- Compares a sample mean to a known population mean when population variance is known.

2. **Two-Sample Z-Test**:
   - Compares means of two independent samples.
3. **Proportion Z-Test**:
   - Compares proportions between two groups.
   - Example: Testing if the proportion of smokers is higher in city A than city B.

Key Formula

$$z = \frac{\acute{X} - \mu}{\sigma / \sqrt{n}}$$

Where:

- $\acute{X}$: Sample mean.
- μ: Population mean.
- σ: Population standard deviation.
- n: Sample size.

Assumptions

- The data follows a normal distribution or sample size is large (Central Limit Theorem applies).
- Population variance (σ^2) is known.

Applications

- Comparing observed data to a hypothesized population value.
- Quality control and product testing.

4. F-Distribution

The **F-Distribution** is a probability distribution used primarily in variance analysis and comparing multiple group means.

Key Concepts

- **Definition**:
  - The ratio of two independent chi-squared variables divided by their degrees of freedom.

- Formula:

$$F = \frac{\left(S_1^2 / df_1\right)}{\left(S_2^2 / df_2\right)}$$

  Where:

  - S_1^2, S_2^2: Variances of the two groups.
  - df_1, df_2: Degrees of freedom of the two groups.

- **Properties**:
  - Non-negative values ($F \geq 0$).
  - Skewed distribution, becoming less skewed as sample size increases.

Applications

1. **Analysis of Variance (ANOVA)**:
   - Tests if there are significant differences among group means.
   - Example: Comparing test scores across 3 different teaching methods.
2. **Regression Analysis**:
   - Evaluates the significance of regression models by comparing explained and unexplained variances.
3. **Equality of Variances (F-Test)**:
   - Compares the variances of two datasets.

Assumptions

- Observations are independent.
- Groups are normally distributed.
- Variances between groups are equal.

Relation to Other Tests

- Used to calculate p-values in ANOVA and other statistical analyses.

Summary

- **P-Test** provides the p-value, helping decide the significance of results.
- **T-Test** and **Z-Test** compare means, differing by sample size and variance knowledge.

- **F-Distribution** is foundational for variance analysis and multi-group comparisons.