# 1 Equations and Formulas

## 1.1 Catalan Numbers, Convolution and Super

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$C_0 = 1, \quad C_1 = 1, \quad C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$$

The number of ways to completely parenthesize $n+1$ factors.
The number of triangulations of a convex polygon with $n+2$ sides.
The number of ways to connect the $2n$ points on a circle to form $n$ disjoint non-intersecting chords.
The number of permutations of $\{1, \ldots, n\}$ that avoid the pattern 123.

$$C_n^{(k)} = \frac{k+1}{n+k+1}\binom{2n+k}{n}$$

$$S(m,n) = \frac{(2m)!(2n)!}{m!n!(m+n)!}$$

$$C_n^{(k)} = \frac{(2n+k-1)(2n+k)}{n(n+k+1)} C_{n-1}^{(k)}$$

## 1.2 Stirling Numbers (First Kind)

Stirling numbers of the first kind count permutations by cycle count.

$$S(n,k) = (n-1) \cdot S(n-1,k) + S(n-1,k-1)$$

$$\sum_{k=0}^{n} S(n,k) = n!$$

$$x^{\overline{n}} = x(x+1)\ldots(x+n-1) = \sum_{k=0}^{n} S(n,k)x^k$$

## 1.3 Stirling Numbers (Second Kind)

Stirling number of the second kind counts ways to partition a set of $n$ objects into $k$ non-empty subsets.

$$S(n,k) = kS(n-1,k) + S(n-1,k-1)$$

$$S(n,2) = 2^{n-1} - 1$$

$S(n,k) \cdot k!$ = number of ways to color $n$ nodes using $k$ colors (each at lea

## 1.4 Combinatorial Identities

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$$

$$\sum_{i=0}^{k}\binom{n+i}{i} = \sum_{i=0}^{k}\binom{n+i}{n} = \binom{n+k+1}{k}$$

## 1.5 Pick's Theorem

$$A = i + \frac{b}{2} - 1$$

## 1.6 Derangements

$$d(i) = (i-1) \times (d(i-1) + d(i-2))$$

## 1.7 GCD and LCM

$$\gcd(a+mb, b) = \gcd(a,b)$$

$$\gcd(a_1 a_2, b) = \gcd(a_1, b) \cdot \gcd(a_2, b)$$

$$\gcd(a, \operatorname{lcm}(b,c)) = \operatorname{lcm}(\gcd(a,b), \gcd(a,c))$$

$$\operatorname{lcm}(a, \gcd(b,c)) = \gcd(\operatorname{lcm}(a,b), \operatorname{lcm}(a,c))$$

$$\gcd(n^a - 1, n^b - 1) = n^{\gcd(a,b)} - 1$$

$$\sum_{k|a, k|b} \phi(k) = \gcd(a,b)$$

$$\sum_{k=1}^{n} x^{\gcd(k,n)} = \sum_{d|n} x^d \phi\left(\frac{n}{d}\right)$$