1. 2D BARATROPIC EQNS ON β-PLANE

2. ENERGY BALANCE CHECK

Computing $\Gamma(U)$:

$$\Gamma(U) = U\partial_x + (\partial_y^2 U - \beta)\partial_x \Delta^{-1} + \alpha + \nu(-\Delta)^p \,.$$

Taking FT in x and y and computing for each mode k_x

$$\Gamma(\mathbf{U})_{k_x} = \mathcal{F}^{-1}[ik_x\mathbf{U}] + \mathcal{F}^{-1}[ik_x(\beta\mathbb{1} + \mathbf{U}'') * (\mathbf{k}_y^2 + k_x^2\mathbb{1})^{-1}] + \alpha\mathbb{1} + \mathcal{F}^{-1}[\nu(\mathbf{k}_y^2 + k_x^2\mathbb{1})^p] \,,$$

where $\mathbf{U}$ and $\mathbf{k}_y^2$ are diagonal matrices of size $N_y \times N_y$ with $U(y,t)$ and k_y^2 on the diagonals respectively. Then the full Γ:

$$\Gamma(U) = \sum_{k_x} \Gamma(\mathbf{U})_{k_x}.$$

We can then compute C_U by solving the lyapunov eqn:

$$\Gamma(U)C_U + C_U\Gamma(U)^\dagger = 2\sigma\sigma^\dagger$$