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<u>Chain Rule:</u>

① Given,

$f(z) = \log_e(1+z)$ where $z = x^T x$, $x \in R^d$

$\therefore$ if $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$ ~~then, $x^T =$~~

Then,

$x^T = [x_1, x_2 \dots x_d]$

$\therefore x^T x = [x_1^2 + x_2^2 + \dots\dots + x_d^2]$

Applying chain rule,

$$\frac{d}{dx} f = \frac{d}{dx} f \cdot \frac{d}{dx} z$$

$$= \frac{1}{1+z} (\log(1+z)) \cdot \frac{d}{dx} (x^T \cdot x)$$

$$= \frac{1}{1+z} \frac{d}{dz}(z) \cdot \frac{d}{dx} (x_1^2 + x_2^2 + \cdots + x_d^2)$$

$$= \frac{1}{1+z} (2x_1 + 2x_2 + \cdots 2x_d)$$

$$= \frac{1}{1+z} \cdot 2(x_1 + x_2 + \cdots + x_d$$

$$= \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

(Ans.)

② $f(z) = e^{-z/2}$, where $z = g(z)$, $g(y) = y^T s^{-1} y$

$h(x) = x - u$, $y = h(x)$

⇒ Using chain rule,

$$\frac{d}{dx} f = \frac{df}{dx} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

here, $\frac{df}{dz} = \frac{d}{dz}\left(e^{-z/2}\right) = -\frac{e^{-\frac{z}{2}}}{2}$

$$\frac{dz}{dy} = \frac{d}{dy}\left(y^T s^{-1} y\right)$$

$$= \lim_{h \to 0} \frac{y(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) s^{-1} (y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T s^{-1} + h s^{-1})(y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T s^{-1} y + y^T s^{-1} h + h s^{-1} y + h^2 s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{h(y^T s^{-1} + s^{-1} y + h s^{-1})}{h}$$

$$= \lim_{h \to 0} (y^T s^{-1} + s^{-} y + h s^{-1})$$

$$= y^T s^{-1} + s^{-1} y$$

$$\frac{dy}{dx} = \frac{d(x - w)}{dx} = 1$$

$$\therefore \frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{e^{-\frac{z}{2}}}{2} \left(y^T s^{-1} + s^{-1} y \right) \cdot 1$$

$$= -\frac{e^{-\frac{z}{2}}}{2} \cdot \frac{1}{s} \left(y^T + y \right)$$

(Ans).