Using Game Theory to Study Market Power in Simple Networks

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July 1998
Federal Energy Regulatory Commission

Abstract

"Gaming" refers to taking unanticipated advantage of market rules, and game theory provides a rigorous way to analyze and predict such chicanery. This paper provides an example of game pseudo congestion pricing and then describes the basic game-theory tools needed to analyze both this and the exercise of market power. Cournot competition between generators in one, two and three-node networks is then analyzed. This allows the illustration of (1) a single predictive equilibrium, (2) the ambiguity of multiple equilibria, and (3) a mixed-strategy equilibrium. While illustrative of basic concepts, these results also correct flaws in the influential analysis of congested electricity systems published by Shmuel Oren.

1. Gaming

When new economic mechanisms are being considered for use in the electricity market, there is much talk of "gaming" and whether the mechanism will "be gamed." This popular notion, while not precisely defined, is meaningful and that meaning derives partly from game theory. While any economic mechanism can be modeled as a game, and all evoke particular strategies from the players, certain mechanisms are considered to by particularly susceptible to gaming. These are mechanisms that evoke strategies that are unanticipated and that subvert the intentions of the mechanism's designer. One popular pseudo-economic approach to congestion pricing provides a particularly striking example of this concept.

This approach comprises three steps. First, pretend there is no congestion and clear the market under this fiction. Second, redispatch using incremental (inc) and decremental (dec) bids. Third, tax someone to pay for the redispatch costs. Although this approach can be used in a variety of contexts from bilateral to poolco, the pure nodal-pricing context is the simplest to analyze and it does not distort the analysis. An analysis of this mechanism reveals the resulting dispatch to be efficient and the inc-dec market to clear at the correct price. But this does not indicate all is well. Generators with costs above both the initial fictitious market clearing price and the inc-dec market clearing price will be paid not to generate. This will raise the short-run cost to loads and will encourage inefficient entry of new generation.

To illustrate these points it is sufficient to analyze the model illustrated in Figure 1. It consists of a one-line network with generation at A and load at B. Assume there are 15 generators at A, each with a capacity of 1 MW and a constant marginal cost of \$N/MWh, where N, $1 \le N \le 15$, is the index of the generator. Assume the line has a maximum capacity of 10 MW and that demand at bus B is fixed at 20 MW and supply at bus B costs \$20/MWh.

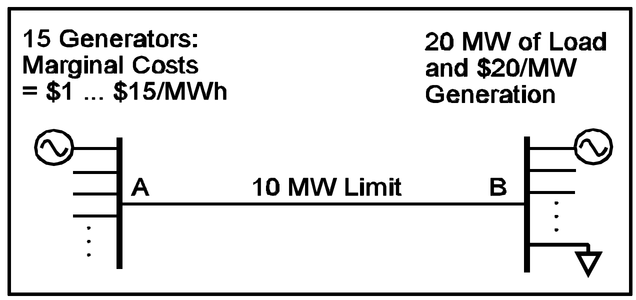

Figure 1. Pseudo-Economic Congestion Pricing

As a base case we consider the optimal dispatch of this network under a standard nodal-pricing regime. Obviously the minimum-cost dispatch runs the cheapest ten generators at A for a generation cost of \$55/h and buys the remaining power from B at a cost of \$200/h, for a total cost of \$255/h. Under nodal pricing, and absent any market power, the ISO would purchase the 10MW from A at marginal cost which is \$10/MWh and resell it at B for \$20/MWh, thus netting the congestion charge of \$100/h.

Under pseudo congestion pricing, things work as follows. In the initial (fictitious) un-constrained market the market-clearing price is \$20/MWh and all of the generators at A are dispatched for a cost of $\sum_{N=1}^{15} N = \$120/\text{h}$. Five MW are dispatched from B for a cost of \$100/h, so the total fictitious cost of generation is \$220. This is \$35/h less than the actual least possible cost of supply which tells us that the cost of congestion (or of the line constraint) is \$35/h. Note that this is much less than the congestion charges collected by the ISO.

In the second phase of pseudo congestion pricing, the ISO recognizes the constraints and is forced to decrement five of the generators at A. For now we will assume that they have bid their marginal costs in the initial auction and that these carry over as decremental bids for redispatch. The meaning of a decremental bid in a nodal-price setting is that when the ISO accepts a decremental bid the generator must reduce its output (to zero in this example) and pay the ISO its bid price. What it gets in return is a release from its original obligation to generate without giving up the payment for that generation. Thus, when the $15 generator is decremented, it is paid $20/h (the market-clearing price) but must pay $15/h (its bid price). So in net it is paid $5/h not to generate.

So how will the generators at A behave? To get a definite answer we need to know what bids are allowed, so assume for simplicity that bids must be in tenths of dollars. In this case a solution is that all generators at A up to the $10 generator bid truthfully while the remainder bid $10.10/h. These generators know that they will be decremented by the ISO and they want to minimize the price they pay when they are decremented. On the other hand if they bid any lower they might not be decremented and so would have to produce. In every case this would cost them more than $10.10/h. (Once the definition of a Nash equilibrium has been introduced, these strategies can be checked to see that they describe such an equilibrium and are thus a solution to the game in a formal sense.)

The result is that the five expensive generators are decremented and each of these earns $9.90/h (20.0-10.1) for not generating. When the ISO decrements the five generators at A it increments five a B, each at a cost of $20/MWh. The ISO's net cost is $49.50/h (5×20 – 5×10.1). This is cheaper than the congestion charges paid under nodal pricing ($100/h), plus we end up with the optimal dispatch. What could be better?

But something seems amiss. Generators that should obviously not be generating are being paid not to generate. In fact if the marginal cost of the $15 generator at A were to increase to $150/MWh it would still bid $10.10 and the ISO would still pay it $9.90/h not to generate. This is "gaming." In fact it would make sense for an entrepreneur to build a cheap-to-build but expensive-to-run generator just to get paid not to run. This is also gaming.

Someone must pay for this game, and so someone must be hurt by it. The difference between the $100/h congestion charge under nodal pricing and the $49.50/h redispatch cost under pseudo-economic pricing is that the ISO collects the $100/h, but pays out the $49.50/h. Thus the $100/h would be used to reduce the cost of power to the load, while the $49.50 would increase the cost of power to the load. Unlike the $35/h congestion cost (from out-of-merit dispatch) the $49.50/h redispatch cost is not a social cost, it is simply a transfer payment from loads to generators. But in the long run these pseudo-economic charges will cause a social loss because they will encourage generation to be built where it is not needed.

An interesting extension of this model includes demand-side bidding. Although there is insufficient room for that analysis, it would demonstrate that the gaming problems become even more severe.

2. Game Theory

So far we have discussed gaming without any reference to game theory. But often in these situations economists do use Von Neuman's concept of a game, Borel's concept of a "mixed strategy," and Nash's theorem stating that every finite game has an equilibrium if mixed strategies are considered. But perhaps the best approach to game theory for an engineer is to view it as the economist's way of analyzing feedback. For instance when two firms (say generation companies) interact in the market place each reacts to the others strategy. If one firm produces Q_1 MW and the other Q_2, then the market determines a price which is an input to each firm's strategic production decision. If either firm changes its output, this affects the input price of the other firm. Thus we have a complete feedback loop. If there is a stable operating point for such an economic system it is called a Nash equilibrium.

Often economists describe such a system in terms of the players "reaction" functions. These summarize each player's behavior in terms of the other players' outputs. For instance in the above system, player 1's reaction function would be $R_1(Q_2)$ while player 2's would be $R_2(Q_1)$. This description of the game emphasizes the feedback. Any pair of strategies, $(Q_1{}^*, Q_2{}^*)$ such that

$$Q_1^* = R_1(Q_2^*) \text{ and } Q_2^* = R_2(Q_1^*)$$

is a Nash equilibrium.

Games are also commonly presented in what is called strategic (or normal) form. In this form the same game would be represented as two payoff functions $\pi_1(Q_1, Q_2)$ and $\pi_2(Q_1, Q_2)$, which give the players' payoffs (e.g. profit) in terms of the strategic choices of all players. Economics assumes that players will try to maximize their payoffs. (If this does not appear to be the case it is usually because some hard-to-measure cost or benefit has been omitted from the definition of payoff.) If $\pi_1(Q_1, Q_2)$ is maximized by a unique Q_1, given Q_2, and similarly for $\pi_2(.,.)$, then these payoff functions define reaction functions. But when there are multiple strategies that produce a profit maximum, the payoff functions provide a more general approach. This turns out to be particularly important when we generalize the concept of a strategy to include mixed strategies, for then there will often be infinite sets of strategies with the same payoff, and reaction functions become wholly inadequate.

If a game has only one stable (Nash) equilibrium, then game theory predicts that equilibrium to be the game's "solution." A solution is what game theory predicts will happen when experienced players actually play the game. This is a little different than an engineering approach which might start with initial conditions and system dynamics before making such a prediction. The argument in economics is that it is too difficult to predict the dynamics of learning, but that intelligent players will eventually find their way to the equilibrium, so that is predictable.[1]

If a game has many Nash equilibria then the situation is much more difficult. In general, for such games, game theory makes no assertion about a game's solution and thus no prediction of its outcome. However there are special cases in which an outcome is predicted. For instance if there is one equilibrium in which all players are better off than in any other, that equilibrium is often predicted as the outcome. But there are many games in which the players would disagree strenuous over which is the preferred equilibrium and for these games, game theory makes no prediction.[2]

Another possibility is that the game has no stable equilibrium in "pure strategies." Pure strategies are the type that we have considered so far, such as "produce exactly *Q*." In the absence of such equilibria, economics and engineering take somewhat different views. The economic approach is to define the concept of a "mixed strategy." This is a composite strategy in which a single pure strategy is picked randomly (but using a specific probability function) from the set of available pure strategies. For example, if Q=1 and Q=2 are two pure strategies, then the strategy of choosing one or the other with a 50/50 chance of each is a mixed strategy. If x(Q) is the probability function (mixed strategy) for player 1's pure strategies and y(Q) is a mixed strategy for player 2, then we can define the payoff function for player i, as

$$payoff_i = \iint x(q_1)\cdot p_i(q_1,q_2)\cdot y(q_2)\, dq_1 dq_2$$

In a slight abuse of notation we will no refer to these new payoff function simply as $\pi_i(x, y)$.

We can now redefine an equilibrium (typically called a Nash equilibrium) as a pair(x^*, y^*) such that

$$x^* \text{ maximizes } p_1(x^*, y^*) \text{ and}$$
$$y^* \text{ maximizes } p_2(x^*, y^*)\ .$$

These definitions are easily generalized to more players, and with such a generalization Nash's theorem assures us that any finite game has at least one equilibrium. A finite game is one with a finite number of players each of which has a finite number of pure strategies.

One may wonder if the concept of a mixed strategy is not too subtle to describe the behavior of untutored marketers. But even ten-year-olds quickly learn to play according to such strategies, and they do this with no help from game theorists. In the common children's game paper-scissors-stone, paper beats stone which beats scissors which beats paper. The two players move simultaneously. Children quickly learn there is no worthwhile pure strategy and start randomizing. Since leaning towards one pure strategy or another increases the loss rate on that strategy, players soon converge on something quite near the Nash equilibrium strategy, a mixed strategy with equal chances of playing each pure strategy.

3. Market-Power Games

The first section explored an example of "gaming," that is behavior that is dishonest and unanticipated but rewarding for the individual player. A related form of behavior, much studied by economists, is the exercise of market power. While this behavior is typically analyzed with game theory, the strategies involved, raising price and withholding output, are so common and well understood as not to be particularly deceptive or unanticipated. When the artist exercises market power by offering a limited edition of 100 lithographs, no one believes that more could not have been produced, while in our first example the offer to sell at $10.10/MWh is meant to be taken as a genuine offer, which it is not. Probably for this reason, the exercise of market power is not so often referred to as gaming.

The first example will demonstrate standard Cournot competition in which there is only one Nash equilibrium. The second will demonstrate the ambiguity of multiple equilibria, and the third will demonstrate a case in which there is a mixed strategy equilibrium.

[1] Fudenberg and Levine's new book, *The Theory of Learning in Games (1998),* is an exception.

[2] "First, a major conceptual problem occurs when there are multiple equilibria, for in the absence of an explanation of how players come to expect the same equilibrium, their play need not correspond to any equilibrium at all." –Fudenberg (1998) page 1.

4. A Simple Cournot-Nash Game

Cournot published his theory of oligopolistic competition in the 1830's, and it has only recently been interpreted as a strategic game and the prediction of a Nash equilibrium. This reinterpretation has not changed the basic model. Although Cournot's model is sometimes presented as one of producers choosing output levels, that description misses the point. The crucial point is that each producer, when selecting its strategy, *assumes that other producers have fixed their output levels*. In fact if all producers set prices and all believe that the others set quantities, the market will still arrive at the Cournot equilibrium. This second interpretation is often used to simplify calculation.

For example, consider two producers, Gen-1 with a constant marginal cost of \$20/MWh and Gen-2 with a constant marginal cost of \$40/MWh. Assume they have no capacity constraints and that demand is $Q_D = 2\times(100-P)$.[3] Their two profit functions are:

$$p_1 = P \cdot Q_1 - 20 \cdot Q_1$$
$$p_2 = P \cdot Q_2 - 40 \cdot Q_2$$

Price is just given by $P = 100 - (Q_1 + Q_2)/2$, so by substitution we obtain both profit functions in terms of only the strategic variables Q_1 and Q_2. To maximize profit we differentiate each function and set the derivative to zero, and this is where the Cournot assumption enters. According to the Cournot model, when we differentiate π_1 with respect to Q_1 we hold Q_2 constant.[4] Thus, in equilibrium, Q_1^* maximizes producer 1's payoff given producer 2's optimal strategy, Q_2^*. The reverse is also true. But these two conditions are just the conditions for a Nash equilibrium. In this simple system there is a unique Nash equilibrium and it is given by

$$Q_1^* = 66\tfrac{2}{3}, \quad \text{and} \quad Q_2^* = 26\tfrac{2}{3}.$$

This implies a price of \$53.3/MWh, which is well above marginal cost. Notice that excess capacity is required by this model, but plays no direct role in the calculation.

[3] I have chosen a rather elastic demand curve because what matters to a Cournot firm is not the aggregate elasticity of demand but its own elasticity of demand. Because this increases with the number of firms, and I have choose an excessively small number of firms (2) for simplicity I have compensated by using an elastic demand.

[4] This is sometimes described as producer 1's "conjectural variation" in Q_2 being zero.

5. Cournot Competition with a Congested Line

In order to illustrate a frequent shortcoming of game theory, I will now introduce congestion in its starkest form by assuming that the two generators are located at one end of a congested line whose capacity is only 80MW and that all demand is located at the other end. This makes the previous solution infeasible.

The line constraint requires some type of central control and for simplicity I will assume this is standard nodal-pricing ISO. Now the strategic variables Q_1 and Q_2, must be reinterpreted as bids in the ISO's auction. This still leaves some complex economic questions unresolved. Generators are required to bid supply functions which involve both price and quantity. To build a Cournot model in this context we need to make an assumption about the price part of the bid, which is both reasonable and in keeping with the Cournot model.[5] The most obvious assumption is that generators bid their marginal cost up to a certain quantity limit.[6] This allows them to act as Cournot competitors while refusing to generate at a loss. I will make this assumption for the remainder of the paper.

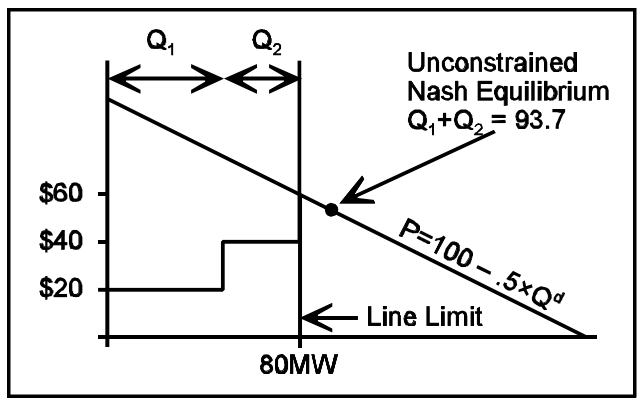

Figure 2. Constrained Nash Equilibrium

The ISO evaluates the generators' bids by constructing a supply curve and intersecting this with the demand curve. Recall that demand at the generation bus is limited by the line constraint. Figure 2 shows the equilibrium case in which the two generator bids are just sufficient to use 100 percent of the line's capacity. Since this is feasible, the line is uncongested and price is determined by the unconstrained demand function

[5] If generators bid only quantities in a standard nodal-pricing auction, there is no way for the ISO to determine nodal prices without imputing a bid price. While it might seem that zero is an obvious choice for such an imputed price, that is unreasonably low in most cases and too high in a few peculiar circumstances.

[6] This is also consistent with Oren's (1997) assumption that generators bid prices and that "prices at each supply node are bounded below by the corresponding marginal costs."

to be \$60/MWh. Note that if either generator increased its bid by the smallest amount the line would be congested and the price would be determined to be \$40/MWh by the intersection of the constrained part of the demand curve and the supply curve.

While it is often difficult to find a Nash equilibrium, it is usually not hard to check whether or not a proposed strategy pair is an equilibrium. To check the proposed equilibrium shown in Figure 2, first consider Gen-1 and assume a large line capacity. It is easy to show that Gen-1 wishes to increase production ($d\pi_1/dQ_1 > 0$) unless $Q_1 > 2\cdot(P - C)$, where C is marginal cost. In other words Gen-1 will wish to increase production unless Q_1 is already greater than 80, which it is not. But we have shown already that with the line constrained any increase in Gen-1's bid will cause a drastic decrease in price, so Gen-1 is at a profit maximum given Gen-2's output. The situation of Gen-2 is identical except that Gen-2 wants to increase output only so long as $Q_2 < 40$. Thus, provided $Q_2 < 40$, which it is, the bid strategy pair depicted in Figure 2 represents a Nash equilibrium.

Unfortunately, the above calculation indicates that there is an infinity of Nash equilibria. All of these equilibria are characterized by $40 \le Q_1 \le 80$ and $Q_2 = 80 - Q_1$. These equilibria have one thing in common which is that the price is \$60/MWh. So one might hope that even though game theory makes no prediction of actual strategies it could at least tell us that one of the Nash equilibria will prevail and that the resulting price will therefore be \$60/MWh. Unfortunately, game theory does not even make this prediction. In generally game theory makes no prediction when there are multiple Nash equilibria.[7] In special cases, there are particular arguments that quite convincingly argue for a particular outcome among a group of Nash equilibria. But in this game there is no convincing argument that any particular equilibrium will prevail and no convincing argument that the outcome will be one of the Nash equilibria.

What makes this game so ambiguous is that the players rank the equilibria in exactly opposite orders. Gen-1 most prefers the $Q_1 = 80$ equilibrium while Gen-2 most prefer's the $Q_1 = 40$ equilibrium. It is possible that they will “split the difference,” but many other considerations could enter the bargaining process: output capacity, financial strength and more. Now it would not be surprising if, after some time, the two suppliers did manage to agree on a particular equilibrium, but the point to remember is that there is no credible game-theory argument that predicts this. This same multiplicity of equilibria can occur with any number of suppliers and as the number of competing suppliers and the amount of excess capacity increases, the chance that they will all agree on one particular equilibrium, becomes increasingly remote.

Part of Argentina’s transmission grid mimics this game almost perfectly. A number of generators with a total output capacity of 5255 MW are located in Comahue which has a maximum demand of only 388 MW. Thus local demand is small enough that its elasticity probably plays no role in disciplining prices. Comahue is connected to Buenos Aires by a transmission path rated at about 4000 MW. This constraint often causes low prices in Comahue when the price in Buenos Aires is high. This indicates that the generators have not been able to agree on one of the Nash equilibria and instead are fighting over which one should be adopted.

6. Competition in a Three-Line Network

This section transplants the same two suppliers and the load into the standard three-line network. This produces a game with no pure-strategy Nash equilibrium instead of an infinity of them. The two suppliers are located at different buses connected by a transmission line with a 5MW capacity. These two buses are each connected by a high capacity transmission line to the load bus. To see that there is no pure-strategy equilibrium we first consider the one proposed by Oren (1997). He asserts that this game has a pure Nash equilibrium in which the line is not congested but is loaded to full capacity. In the symmetrical network of our example that implies that $Q_1 – Q_2 = 15$, because Gen-1 will have the greater output. As Oren points out, lack of congestion also means that all nodal prices are equal. If generators ignore congestion, both generators will adjust their bids to satisfy the standard Nash-Cournot conditions and this will cause congestion. But because the congestion would force down the price at Gen-1, Gen-1 will “unilaterally (and profitably) force the equilibrium price to be at the kink ... by slightly ‘backing off’.” This leaves Gen-2, the load, and the above constraint equation to determine the price. This is Oren’s proposed equilibrium. As demonstrated in the appendix, Gen-2 maximizes its profit by choosing $Q_2 = 35$, which induces $Q1 = 50$, and a price of \$57.50.

The trouble with this proposal is that Gen-2 has the ability to congest the line by reducing its bid ever so slightly and thereby raising the price at its bus to \$60 from \$57.50. This is profitable and consequently undermines the proposed equilibrium because, by definition of a Nash equilibrium, no player can gain by a unilateral change of strategy.

[7] “... in the absence of an explanation of how players come to expect the same equilibrium, their play need not correspond to any equilibrium at all." –Fudenberg (1998) page 1.

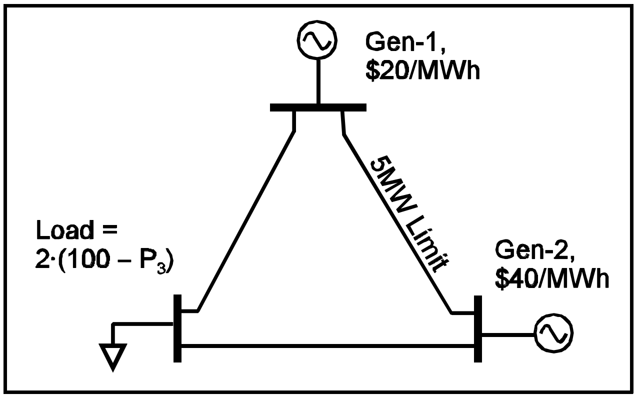

Figure 3. Three-Bus Model with Congestion

Although Oren recognizes that "Competitors at DEARGEN [Gen-2] would prefer a higher price on the vertical segment of their demand function ...", he argues that "such prices would not support a Nash equilibrium" for the reason cited above. This is also correct but the conclusion drawn in the next sentence misses the whole point of Nash's theorem. "It follows that the only possible congested Cournot-Nash equilibrium with generation at both supply nodes is at the kinks of the demand function shown in Figure 4..." The logic of this statement is that since a congested line (and a high-price for Gen-2) is not an equilibrium then the equilibrium must leave the line uncongested. But Oren has considered only pure-strategy equilibria and forgotten that there is no guarantee that one exists.

What Oren has shown is that if Q_1–Q_2 = 15, Gen-2 will back off and congest the line making Q_1–Q_2 > 15. This in turn will cause Gen-1 to back off to the point where Q_1–Q_2 = 15. Thus neither situation is an equilibrium. But Nash assures us that in this case there must be a mixed-strategy equilibrium. We will now find that equilibrium which is the correct solution to this game.

The simplest algorithm for finding mixed-strategy equilibria is called "fictitious play." The procedure is similar to the naive dynamics just described. First a pair of pure strategies is picked as the initial guess, in this case ($Q_1^1; Q_2^1$). Then each player computes an optimal strategy based on the other's initial strategy. We now have four strategies, ($Q_1^1, Q_1^2; Q_2^1, Q_2^2$). Next each player computes a third strategy based on the other's first two strategies. To do this player 2 assumes that player 1 will select Q_1^1 or Q_1^2, each with a 50 percent probability, and then computes the single optimal pure strategy based on this assumption. Each computes the fourth strategy by assuming the opponents previous three strategies will be played with a probability of 1/3. At any point, the set of past optimal strategies can be viewed as a sample from the probability function that describes the equilibrium strategy. This process is continued until an estimate of the probability with the desired accuracy is obtained. It has been demonstrated that if this process converges it converges to a Nash equilibrium. Although it often does converge, it does not converge for the present game. Consequently we must turn to a more sophisticated and complex algorithm.

A Nash equilibrium satisfies a set of inter-related maximization conditions. In the case of a two-player game where player A chooses a mixed strategy x and player 1 chooses a mixed strategy y, x maximizes $\Pi_1(x, y)$ while y maximizes $\Pi_2(x, y)$. Because of this inter-relationship we cannot use standard optimization techniques such as quadratic programming even when $\pi_1(.,.)$ and $\pi_2(.,.)$ are bilinear in x and y. Instead we must solve a complementarity problem.

Our first step is to recast our continuous game as a discrete-choice (finite) game by replacing the continuous decision variables with discrete variables. In other words we will restrict output Q to vary over the set of integers from 0 to 60 instead of over the set of real numbers. This gives each player 61 discrete moves (pure strategies) and for each pair of moves we can compute the profit to each player. These profit levels are the payoffs and they are recorded in the payoff matrixes **A** and **B**. Now x and y are defined to be 61 dimensional vectors of probabilities that represent the two generators mixed strategies. Thus $\pi_1(x, y) = x'\boldsymbol{A}y$, and $\pi_2(x, y) = x'\boldsymbol{B}y$. The next step is to derive the first-order conditions for the Nash optimization problem. These conditions have the standard form of a linear complementarity problem (LCP), and Lemke and Howson (1964) have provided an algorithm that is guaranteed to find one solution of this problem.

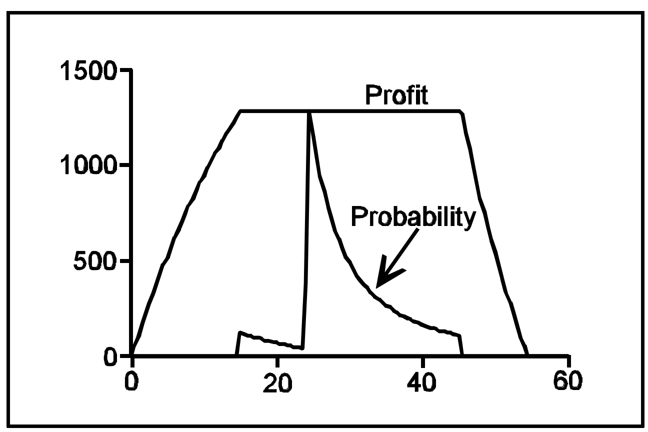

Figure 4. Mixed Strategy of Cheap Generator

By applying the Lemke algorithm to our problems we can find one Nash equilibrium, but we cannot prove that this is the only equilibrium for our game. Nevertheless I believe it is and because it is quite informative it is displayed in the Figures 4 and 5. The appendix gives the MatLab code used to construct the payoff matrixes since this is the only economic part of the problem. This code is equivalent to the following approach. First draw the demand function and the supply function from the submitted bids. Then find Q_1, Q_2, and P as determined by their intersection. Next determine if the line constraint has been violated. If so reduce the output of the generator causing the forward flow. Set the price for this generator to its marginal cost and the compute the price at the demand node from the total output and the demand function. Then compute the price at the unconstrained generations bus to satisfy $(P_1 + P_2)/2 = P_D$. If there is no congestion, accept the original Q_1, Q_2 and set all bus prices to P.

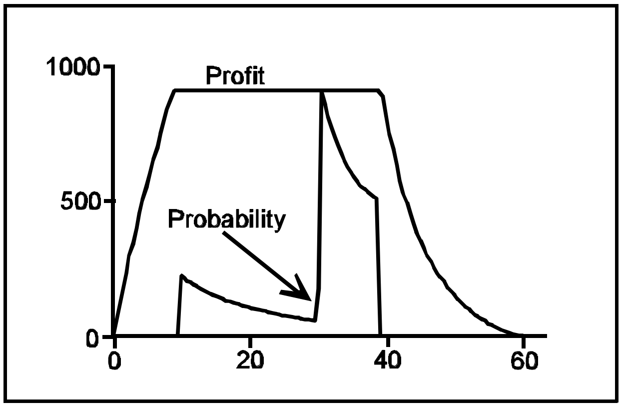

Figure 5. Mixed Strategy of Gen-2 ($40/MWh)

In Figures 4 and 5 the probability density functions have been scaled to have the same height as the profit function and that scale is not shown. The scale on the vertical axis indicates expected profit and the horizontal axis represents bid quantity. The probability functions give the mixed strategies that constitute the Nash equilibrium. Note that every pure strategy with a positive probability has the same maximum expected profit. The profit function is computed from the payoff matrixes after the Lemke algorithm finds the equilibrium strategies x, and y. For Gen-1, profit as a function of Q_1 is given by $\boldsymbol{A}y$, while for Gen-2 profit is given $x'\boldsymbol{B}$. The fact that profit is maximized for all non-zero probabilities is confirmation of the success of the algorithm because players will not randomize among strategies that are not all best.

Considering that the mixed-strategy equilibrium is motivated by Gen-2 reducing its bid minutely in order to raise its price from $57.50 to $60, the differences between the proposed pure-strategy equilibrium and the true Nash equilibrium are surprisingly large. Most notably, far more market power is exercised under the mixed strategy. Average output is reduced from 85 to 59 which causes average price to increase from $57.50 to $71. As a consequence, consumer surplus is reduced from $1806/h to $870/h. Since Oren's "equilibrium" is based on Gen-1 taking advantage of the line limit but Gen-2 failing to take such advantage it is no surprise that the true equilibrium shows more profit for Gen-2 and less for Gen-1.

A qualitative difference occurs in congestion. Oren's pure strategies are constructed to produce no congestion, while the explanation of the mixed solution is Gen-2's motivation to cause it. Surprisingly the two generators produce almost exactly the same output on average: Gen-1 produces 29.5 on average while Gen-2 produces 29.1. But the randomness in output choice is such that the line is congested 11% of the time in the expected direction and 3% of the time in the reverse direction.

Expected Outcomes	With Nash Strategies	With Pure Strategies
E(Total Supply)	59 MW	85 MW
E(Consumer Price)	$71 /MWh	$57.5 /MWh
E(Consumer Surplus)	$870 /h	$1806 /h
E(Gen-1 Profit)	$1283 /h	$1875 /h
E(Gen-2 Profit)	$908 /h	$612 /h
E(Congestion)	14.3 %	0 %

Table 1. Nash Outcomes vs. Incorrect Pure-Strategy Predictions

7. Conclusions

When market design does not imitate the principles of a competitive market, for instance when physical constraints are ignored, participants are taxed, and payments are made for not producing, we should expect "gaming." This gaming should be analyzed by computing the payoffs to participants for all available strategies (including those that are illegal or provoke penalties) and then computing the Nash equilibria.

When there is only one Nash equilibrium or when a good case can be made that the participants (players) will agree on a particular Nash equilibrium, then, and only then, game theory predicts this equilibrium will be the game's outcome. Although such predictions are not foolproof, they are often quite accurate.

Even when a market is well designed and is not susceptible to deceptive forms of "gaming," game theory is still useful particularly for the analysis of market power. One result of such analysis is that networks can produce situations in which generators randomize their output. These "mixed strategies" can produce a surprising amount of market power. Often they can be computed using the simple "fictitious play" algorithm, but sometimes they must be found by solving a linear complementarity problem.

8. Appendix: Details of the Three-Bus Model

Oren's proposed solution to the three-line problem is based on three equilibrium conditions, one for each market participant. For the load we have the demand equation. For the cheap generator we have maximum-flow constraint, and for the expensive generator we have profit maximization under the Cournot assumption. These conditions are represented as follows.

$$
\begin{aligned}
&(1)\ \text{Demand: } P = 100 - (Q_1 + Q_2)/2 \\
&(2)\ \text{Gen}-1\text{: } Q_1 - Q_2 = 15 \\
&(3)\ \text{Gen}-2\text{: } Q_2 \text{ maxes } [\mathrm{P}(Q_1, Q_2) - 40] \cdot Q_2 \\
&\qquad\qquad\text{given } Q_1 \\
&\qquad (1) \Rightarrow dP/dQ_2 = -0.5 \\
&(4)\quad (3) \Rightarrow -0.5Q_2 + (P - 40) = 0
\end{aligned}
$$

Equations (1), (2) and (4) above can now be solved for the values given in Section 6.

In order to fully specify the three-line model and allow checking of the mixed-strategy equilibrium, this appendix presents the method of calculating profits for both generators. The function is written in MatLab, but this language is sufficiently generic that it should be generally accessible. The market participants are defined by marginal costs of 20 and 40 for generators 1 and 2 respectively, and a demand function of Q = (100 – P)/0.5. Because the flow limit on the line from generator 1 to generator 2 is 5, there can be different prices at the three buses. These are P1, P2 and P, where P is the price at the load bus.

```
function [pay1, pay2]= pay(Q1bid, Q2bid)
LLim = 5;
c1   = 20;       c2 = 40;
D0   = 100;   slope = 1/2;
Qtot = Q1bid+Q2bid;

Q1star = (D0 - c1)/slope;
Q2star = (D0 - c2)/slope;
if      Q1star < Q1bid
   P  = c1;
   Q1 = Q1star; Q2 = 0;
elseif  Q2star < Q1bid
   P  = D0-slope*Q1bid;
   Q1 = Q1bid; Q2 = 0;
elseif  Q2star < Qtot
   P  = c2;
   Q1 = Q1bid; Q2 = Q2star-Q1bid;
else
   P=D0-slope*Qtot;
   Q1 = Q1bid; Q2 = Q2bid;
end

if (Q1-Q2)/3 > LLim   % If Congestion
   Q1=Q2+3*LLim;
   P1=c1;
   P=D0-slope*(Q1+Q2);
   P2=2*P-P1;
elseif (Q2-Q1)/3 > LLim
   Q2=Q1+3*LLim;
   P2=c2;
   P=D0-slope*(Q1+Q2);
   P1=2*P-P2;
else
   P1=P; P2=P;
end

pay1 = (P1-c1)*Q1;
pay2 = (P2-c2)*Q2;
```

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